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INVESTIGATION OF ROUNDOFF NOISE IN IIR DIGITAL FILTERS USING MATLAB

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INVESTIGATION OF ROUNDOFF NOISE IN IIR DIGITAL FILTERS USING MATLAB

A Thesis
Presented to
the Graduate School of
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In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Computer Engineering

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by
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Accepted by:
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CHAPTER ONE

INTRODUCTION

As technology increases, more and more devices are becoming digital. Digital filters are common in the everyday devices used such as radios, cell phones, and mp3 players. Digital filters are also used for other signal processing applications such as satellites. The typical purpose of a digital filter is to detect unwanted signals and noise and remove them by filtering.

Digital filters with an infinite-duration unit sample response have characteristics that make them useful in many applications. The infinite impulse response (IIR) filter is also called a recursive filter because of the feedback necessary during implementation. IIR filters are sometimes called an autoregressive moving-average filter (ARMA). Unlike the FIR filter, the IIR filter has finite poles as well as zeros. Having finite poles and zeros gives a filter more flexibility and power [1].

1.1 The Problem

A filter can be expressed as a cascade of second-order sections (SOS). Each of the sections has numerator and denominator coefficients associated with the subsystems. The optimum ordering and pairing of the numerator and denominator coefficients for second-order sections of digital filters is a task that has yet to be completely resolved. The reason is that there are so many different possibilities for ordering and pairing the sections. A recursive algorithm based on minimizing the external normalization coefficients is used in [2] to obtain the optimum ordering of second-order sections. However, this study did not investigate the different pairings of the poles and zeros. A heuristic approach was done in [3]. For N sections, there are (N!) ways to pair the poles and zeros. There are also (N!) ways to order the resulting second-order sections, for a total of $(N!)^2$ different filter configurations for implementation [4]. In this thesis, for a 6th order filter with three second-order sections, there are $(3!)^2$ or 36 different ordering and pairings.

1.2 The Work

The focus of this thesis is on the effects of quantization error or roundoff noise in the different IIR digital lowpass filters. The filter types included in this thesis are Butterworth, Chebyshev, and Elliptic filters. Each filter type was analyzed for both direct-forms I and II structures and normalized frequency cutoffs of $.2\pi$, $.5\pi$, and $.8\pi$. These three normalized frequency cutoffs represent low, mid, and high cutoffs.

Chapter 2 explains in detail the background information on digital IIR filters and roundoff noise. Some of the subsections of this chapter include an explanation of cascade, direct form I (DF1) and direct form II (DF2) structures.

Chapter 3 describes the method used to create the data for the different filter configurations. This chapter includes the table of ordering and pairing for the filters being tested. This chapter also includes more information on the scaling used and the error sources for each direct form implementation.

Chapter 4 explains the results of the different filter configurations. Analysis was done by filter type, direct form implementation, and normalized cutoff frequency. Several tables are in this chapter for clarity.

Chapter 5 summarizes the conclusions of the work done. It also presents a possible new “rule of thumb”.

Chapter 6 describes what additional work can be done in the future with the data collected or what methods can be added to extend the work of the thesis.

CHAPTER TWO

BACKGROUND

2.1 IIR Digital Filters

The analysis of roundoff noise for IIR filters proceeds in the same way as for FIR filters. The analysis for IIR filters is more complicated because roundoff noise computed internally must be propagated through a transfer function from the point of the quantization to the filter output. This is usually not necessary for FIR filters where all of the quantizations are typically injected at the output. Internal scaling is another complication in the case of IIR filters. Internal scaling is sometimes necessary to prevent overflow [5].

IIR filters realized in the parallel or cascade connection of first- and second- order subfilters are almost always superior in terms of roundoff noise to a high-order direct form realization. Choosing optimal or near optimal realizations in a roundoff noise sense is possible. These realizations generally require more computation and therefore suboptimal realizations with slightly higher roundoff noise are often preferred [5].

2.2 Filter Structures

2.2.1 Cascade

A cascade of second-order sections is one structure for realizing digital filters. The second-order sections can either be in direct form I or direct form II. The reason for choosing a cascade of second-order sections is that a second-order section is required to realize complex conjugate poles or zeros, with the real filter coefficients [6].

In a cascade realization, H(z) is factored into a product of transfer functions[7].

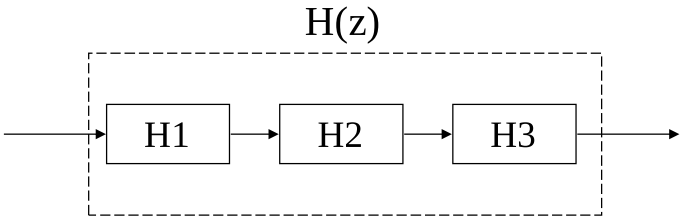

Figure 2.2.1 Cascade Implementation

A sixth order filter can be factored as H(z) = H1 * H2 * H3, where H1, H2, and H3 are second-order sections shown in Figure 2.2.1.

As stated in the Section 1.1, the difficulty with the cascade structure is that we must decide which poles to pair with which zeros. The shape of the output noise power spectrum and the total output noise variance depends on the way the zeros and poles are paired to form the second-order sections and on the order of the second-order sections in the cascade-form realization. On the other hand, the order of the different sections in cascade is an even more complicated problem [6]. In this thesis, the only focus is second-order sections.

In [8], Jackson found that good results are almost always obtained by applying simple rules of the following form:

1. The pole that is closest to the unit circle should be paired with the zero that is closest to it in the z-plane.

2. Rule 1 should be repeatedly applied until all the poles and zeros have been paired.
3. The resulting second-order sections should be ordered according to either increasing closeness to the unit circle or decreasing closeness to the unit circle.

The first two rules are about the pairing of the poles and zeros and the third rule is about the ordering. By implementing the different filter configurations, analysis of the proposed rules by Jackson can be evaluated for further validity and possible creation of a new rule. The filter configurations that are selected by Jackson's rules will be referred to as the "rule of thumb" filters throughout this thesis.

Scaling is another difficulty with the cascade structure. Scaling multipliers between the individual sections in the cascade are necessary to prevent the filter variables from becoming too small or too large [6].

2.2.2 Direct Form I (DF1)

The general difference equation for the direct form I structure is

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \tag{2.1}$$

where a_k are the denominator coefficients and b_k are the numerator coefficients of H(z). In Equation 2.1, N is the number of poles, M is the number of zeros, x[n] is the input and y[n] is the output.

The corresponding system function is

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} \tag{2.2}$$

The variance of the output noise can be shown [4] to be

$$\sigma_f^2 = \sigma_e^2 \frac{1}{2\pi} \int_{-\pi}^{\pi} | H(e^{j\omega}) |^2 d\omega \tag{2.3}$$

Using Parseval's theorem we can also express the variance of the output noise as

$$\sigma_f^2 = \sum_{n=-\infty}^{\infty} | h[n] |^2 \tag{2.4}$$

Assume that all signal values and coefficients are represented by (B +1)-bit fixed point binary numbers. In implementing Eq. (2.1) with a (B+1)-bit adder, the length of the (2B +1)-bit products would be reduced to (B+1) bits. All numbers are treated as proper fractions and the least significant B bits are discarded either by rounding or truncation. Roundoff noise or quantization error is caused by rounding or truncation of bits.

The simplicity of the structure and its direct relation to the z transform are some attributes of DF1. The direct-form I implementation has the zeros before the poles. In this thesis the zeros are denoted as N and the poles as D, for the numerator and denominator coefficients respectively. Figure 2.2.2 shows a cascade of three second-order sections implemented using DF1. In cases when the poles H (z) are reasonably close to each other or to the unit circle, serious problems occur and the use of the DF1 structure is generally avoided in most practical situations [6]. Figure 2.2.3 is the linear-noise model for DF1 which shows quantization errors for both the numerator and denominator coefficients for one second-order section.

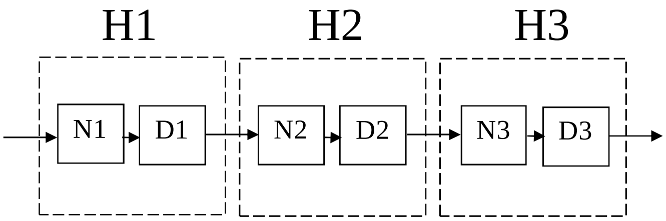

Figure 2.2.2 Direct-Form I with Coefficients

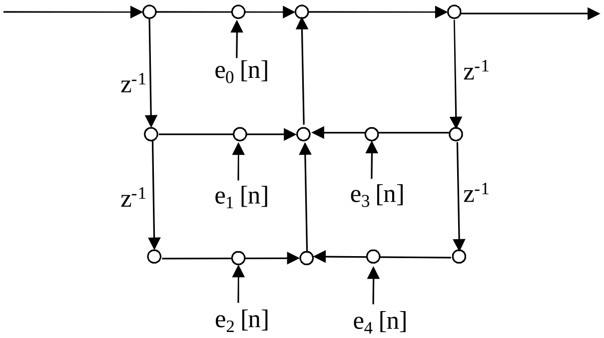

Figure 2.2.3 Linear-Noise Model for Direct Form I

There are a few assumptions that are normally made about the quantization noise sources.

1.) Each quantization noise source e[n] is a wide-sense-stationary white-noise process.
2.) Each quantization noise source has a uniform distribution of amplitudes over one quantization interval.
3.) Each quantization noise source is uncorrelated with the input to the corresponding quantizer, all other quantization noise sources, and the input to the system.

The simple linear-noise model facilitates analysis of the noise in the output due to internally generated roundoff noise.

For (B+1)-bit quantization, for rounding

$$-\frac{1}{2}2^{-B} < e[n] \le \frac{1}{2}2^{-B} \tag{2.5}$$

and for two's complement truncation

$$-2^{-B} < e[n] \le 0 \tag{2.6}$$

The individual noise sources can be combined into larger noise sources as shown in Figure 2.2.4. For DF1, there are three "larger" sources of error that occur between the poles and zeros. The transfer function at each error source depends on what part of the total system those sources actually propagate through.

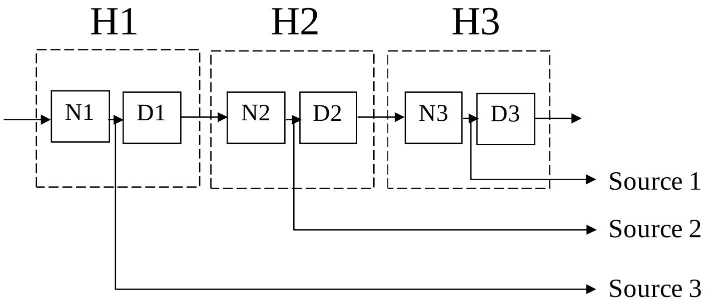

Figure 2.2.4 Direct-Form I with Combined Error Sources

The transfer function seen by the three sources of error for direct form I include:

1. The denominator of H3
2. The denominator of H2 multiplied by H3
3. The denominator of H1 multiplied by H2 * H3

2.2.3 Direct Form II (DF2)

Direct form II, this is a canonical form, has the minimum number of multiplier, adder, and delay elements [6]. This does not mean that it is the best realization. Immunity to round off and quantization errors are very important considerations for DF2 [7]. Equations 2.1 –2.4 apply to direct form II implementation. In a direct form II implementation, the input signal propagates through the poles first and then the zeros as shown in Figure 2.2.5. Just as for DF1, the transfer function seen by roundoff noise depends on what part of the total system those noise sources actually propagate through.

The same assumptions are made about e[n]. In Figure 2.2.6, sources can be combined into a single larger noise source.

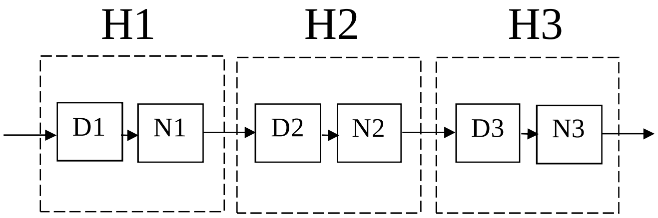

Figure 2.2.5 Direct-Form II with Coefficients

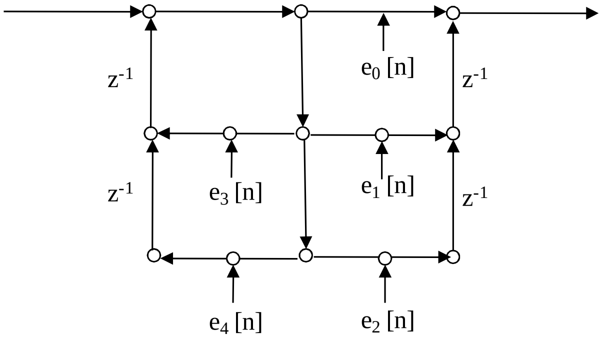

Figure 2.2.6 Linear-Noise Model for Direct Form II

There are four “larger” noise sources of error for direct-form II implementation of a sixth order filter, as shown in Figure 2.2.7.

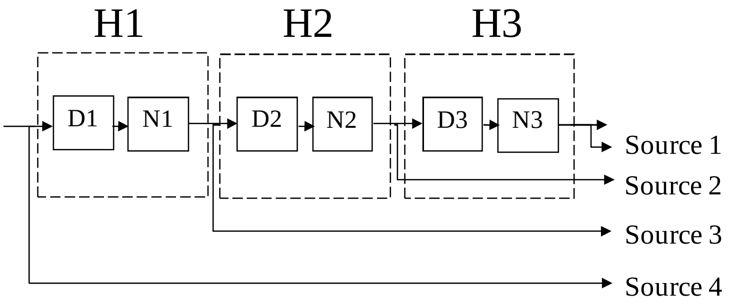

Figure 2.2.7 Direct-Form II with Sources

The transfer function seen by the four sources of error for direct form II are:

1. One at the output
2. H3
3. H2 * H3
4. H1 * H2 * H3

CHAPTER THREE

METHOD OF SOLUTIONS

3.1 The Plan

The plan was developed to narrow the scope to lowpass 6th order IIR digital filters. In order to ensure accuracy for the results, we evaluated different methods. The first method used was based on the sum of squares method of h(n). The second method used was the partial fraction expansion method discussed in further detail in Appendix A.5 of [4]. The second method could only readily be used for a two pole system but with further analysis a proper method could be devised for higher order filters. After testing several second-order filters using the two methods described and comparing the results, we were comfortable that the MATLAB commands implementation of the sum of the squares method provided the intended results.

3.2 Development of Systems

A sixth order filter will have thirty-six different configurations of second-order sections. These thirty-six possibilities were created and saved in a text file in order to import into MATLAB. The text file consists of the three possible numerators and three possible denominators, paired and ordered to create the 36 systems.

Table 3.2.1 The 36 Systems: Ordering and Pairings

	H1		H2		H3	
1	N1	D1	N2	D2	N3	D3
2	N1	D1	N2	D3	N3	D2
3	N1	D1	N3	D2	N2	D3
4	N1	D1	N3	D3	N2	D2
5	N1	D2	N2	D1	N3	D3
6	N1	D2	N2	D3	N3	D1
7	N1	D2	N3	D1	N2	D3
8	N1	D2	N3	D3	N2	D1
9	N1	D3	N2	D1	N3	D2
10	N1	D3	N2	D2	N3	D1
11	N1	D3	N3	D1	N2	D2
12	N1	D3	N3	D2	N2	D1
13	N2	D1	N1	D2	N3	D3
14	N2	D1	N1	D3	N3	D2
15	N2	D1	N3	D2	N1	D3
16	N2	D1	N3	D3	N1	D2
17	N2	D2	N1	D1	N3	D3
18	N2	D2	N1	D3	N3	D1
19	N2	D2	N3	D1	N1	D3
20	N2	D2	N3	D3	N1	D1
21	N2	D3	N1	D1	N3	D2
22	N2	D3	N1	D2	N3	D1
23	N2	D3	N3	D1	N1	D2
24	N2	D3	N3	D2	N1	D1
25	N3	D1	N1	D2	N2	D3
26	N3	D1	N1	D3	N2	D2
27	N3	D1	N2	D2	N1	D3
28	N3	D1	N2	D3	N1	D2
29	N3	D2	N1	D1	N2	D3
30	N3	D2	N1	D3	N2	D1
31	N3	D2	N2	D1	N1	D3
32	N3	D2	N2	D3	N1	D1
33	N3	D3	N1	D1	N2	D2
34	N3	D3	N1	D2	N2	D1
35	N3	D3	N2	D1	N1	D2
36	N3	D3	N2	D2	N1	D1

3.3 Filter Design

Using the built-in MATLAB functions: butter, cheby1, and ellip; sixth-order filters were created with the low, mid, and high cutoff frequencies. The value of the low cutoff frequency is .2π. The value of the mid cutoff frequency is .5π. The value of the high cutoff frequency is .8π. The filters were also implemented for both DF1 and DF2. Therefore, eighteen different filters were evaluated as shown in Table 3.3.1. All eighteen of the different filters are shown in the appendix. Each filter is created in analog and converted to a digital filter which is the best way is to design a filter.

Table 3.3.1 The Eighteen Filter Implementations

Filter Type	Cutoff Frequency	Direct Form
Butterworth	0.2	DF1
Butterworth	0.5	DF1
Butterworth	0.8	DF1
Butterworth	0.2	DF2
Butterworth	0.5	DF2
Butterworth	0.8	DF2
Chebyshev	0.2	DF1
Chebyshev	0.5	DF1
Chebyshev	0.8	DF1
Chebyshev	0.2	DF2
Chebyshev	0.5	DF2
Chebyshev	0.8	DF2
Elliptic	0.2	DF1
Elliptic	0.5	DF1
Elliptic	0.8	DF1
Elliptic	0.2	DF2
Elliptic	0.5	DF2
Elliptic	0.8	DF2

3.4 Scaling

The scaling for each of the filter configurations was done externally instead of internally. After the filter is designed and separated into second-order sections, a scale value created using MATLAB function scale is applied to the transfer function.

3.5 Error Sources

For each of the sources, the MATLAB function tfdata was used to extract the numerator and denominator coefficient of the transfer function as seen by each of the sources of error. The output sum the squares of h (n) is calculated to determine the error due to that particular source. This method is repeated for the remainder of the error sources.

After the errors due to all sources have been calculated and their values stored, the total error for the system must be calculated. For DF1, the error at each source shown in Figure 2.2.4 has a multiplier of 5 since 5 sources of error are seen at the point. For DF2, the error sources shown in Figure 2.2.7 have multipliers of 3, 5, 5, and 2 respectively. The final error sources are exported to an excel worksheet for further analysis.

CHAPTER FOUR

RESULTS

Many tables were created for analysis. The tables used as results have eight columns. The first column contains the name of the filter type, the second column contains the value of the normalized cutoff frequency, and the third column tells whether the filter is direct form I or direct form II. The third column contains the actual system with the orderings and pairings that make up the second-order sections of the filter and follow the “rule of thumb” discussed in [8]. The fifth column contains the value of the error and the sixth column has the ranking when the systems were ranked according to lowest roundoff noise. The seventh column has the range of the error which is value of the system that had the lowest roundoff and the highest roundoff. The eighth column has a ratio of error to the total range of the error. These values range between 0 and 1. If the “rule of thumb” filter has the lowest roundoff noise for the range of error of a particular configuration, then the value of the ratio to error is 0. If the “rule of thumb” filter has the highest roundoff noise for the range of error of a particular configuration, then the value of the ratio to error is 1. Therefore, the lower the value of the eighth column, the ratio of error, the better the performance of the “rule of thumb” filter.

For the Butterworth and Chebyshev filter types, at each normalized cutoff frequency there were six systems that actually followed the “rule of thumb”. In the case of the elliptic filters, there were only two “rule of thumb” options for each of the normalized cutoff frequencies.

4.1 Analysis by Filter Type

As stated in Chapter 3, each filter type was evaluated at a low, mid, and high normalized cutoff frequency for both direct-form I and direct-form II implementation. Table 4.1 has the results for the Butterworth filters, Table 4.2 has the results for the Chebyshev filter, and Table 4.3 has the results for the elliptic filter.

The “rule of thumb” filters for the Butterworth and Chebyshev filters are {N3 D3 N2 D2 N1 D1}, {N3 D3 N1 D2 N2 D1}, {N2 D3 N3 D2 N1 D1}, {N1 D3 N3 D2 N2 D1}, {N2 D3 N1 D2 N3 D1}, and {N1 D3 N2 D2 N3 D1}. The “rule of thumb” filters for the elliptic filters are {N3 D3 N2 D2 N1 D1} and {N3 D3 N1 D2 N2 D1}.

Table 4.1.1 “Rule of Thumb” Filters for Butterworth Filter Type

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Butterworth	0.5	DF2	N3	D3	N1	D2	N2	D1	7.58932	4	7.48552	14.6203	0.015
Butterworth	0.5	DF2	N3	D3	N2	D2	N1	D1	8.46584	10	7.48552	14.6203	0.137
Butterworth	0.5	DF1	N3	D3	N2	D2	N1	D1	8.62954	1	8.62954	18.9401	0
Butterworth	0.5	DF2	N1	D3	N3	D2	N2	D1	8.94879	14	7.48552	14.6203	0.205
Butterworth	0.5	DF1	N3	D3	N1	D2	N2	D1	9.24465	4	8.62954	18.9401	0.06
Butterworth	0.5	DF1	N2	D3	N3	D2	N1	D1	10.0915	8	8.62954	18.9401	0.142
Butterworth	0.5	DF2	N2	D3	N3	D2	N1	D1	11.2986	22	7.48552	14.6203	0.534
Butterworth	0.5	DF1	N1	D3	N3	D2	N2	D1	11.7519	13	8.62954	18.9401	0.303
Butterworth	0.2	DF2	N3	D3	N2	D2	N1	D1	11.861	1	11.861	64.1942	0
Butterworth	0.5	DF2	N1	D3	N2	D2	N3	D1	11.9654	27	7.48552	14.6203	0.628
Butterworth	0.2	DF2	N3	D3	N1	D2	N2	D1	12.2323	4	11.861	64.1942	0.007
Butterworth	0.5	DF1	N2	D3	N1	D2	N3	D1	12.8049	19	8.62954	18.9401	0.405
Butterworth	0.5	DF2	N2	D3	N1	D2	N3	D1	13.4387	34	7.48552	14.6203	0.834
Butterworth	0.5	DF1	N1	D3	N2	D2	N3	D1	13.8502	25	8.62954	18.9401	0.506
Butterworth	0.2	DF2	N2	D3	N3	D2	N1	D1	22.6436	13	11.861	64.1942	0.206
Butterworth	0.8	DF2	N1	D3	N3	D2	N2	D1	23.0594	11	21.9911	44.0897	0.048
Butterworth	0.8	DF2	N3	D3	N1	D2	N2	D1	23.0594	12	21.9911	44.0897	0.048
Butterworth	0.8	DF1	N3	D3	N2	D2	N1	D1	23.5543	1	23.5543	58.5448	0
Butterworth	0.8	DF1	N3	D3	N1	D2	N2	D1	24.3249	2	23.5543	58.5448	0.022
Butterworth	0.8	DF1	N2	D3	N3	D2	N1	D1	25.8422	4	23.5543	58.5448	0.065
Butterworth	0.2	DF2	N1	D3	N3	D2	N2	D1	27.5681	17	11.861	64.1942	0.3
Butterworth	0.8	DF1	N1	D3	N3	D2	N2	D1	28.1477	8	23.5543	58.5448	0.131
Butterworth	0.8	DF1	N2	D3	N1	D2	N3	D1	28.7402	9	23.5543	58.5448	0.148
Butterworth	0.8	DF2	N1	D3	N2	D2	N3	D1	29.3464	23	21.9911	44.0897	0.333
Butterworth	0.8	DF2	N3	D3	N2	D2	N1	D1	29.3464	24	21.9911	44.0897	0.333
Butterworth	0.8	DF1	N1	D3	N2	D2	N3	D1	30.2749	13	23.5543	58.5448	0.192
Butterworth	0.2	DF1	N3	D3	N2	D2	N1	D1	34.9164	1	34.9164	78.87	0
Butterworth	0.2	DF2	N2	D3	N1	D2	N3	D1	36.195	23	11.861	64.1942	0.465
Butterworth	0.2	DF1	N3	D3	N1	D2	N2	D1	37.038	3	34.9164	78.87	0.048
Butterworth	0.2	DF1	N2	D3	N3	D2	N1	D1	39.9554	6	34.9164	78.87	0.115
Butterworth	0.2	DF2	N1	D3	N2	D2	N3	D1	40.7482	28	11.861	64.1942	0.552
Butterworth	0.8	DF2	N2	D3	N1	D2	N3	D1	44.0897	35	21.9911	44.0897	1
Butterworth	0.8	DF2	N2	D3	N3	D2	N1	D1	44.0897	36	21.9911	44.0897	1
Butterworth	0.2	DF1	N1	D3	N3	D2	N2	D1	45.4572	11	34.9164	78.87	0.24
Butterworth	0.2	DF1	N2	D3	N1	D2	N3	D1	47.9332	14	34.9164	78.87	0.296
Butterworth	0.2	DF1	N1	D3	N2	D2	N3	D1	51.3134	18	34.9164	78.87	0.373

For the Butterworth DF1 with a normalized cutoff frequency of .2π, the range of error is between 34.91 and 78.87 as shown in Table 6.2.1. The “rule of thumb” filter {N3 D3 N2 D2 N1 D1} includes the lowest error for these specifications as shown in Figure 6.2.1, but when compared to the round-off error for all the Butterworth filters, falls short of being the best as shown in Table 4.1.1. With a round-off error of 34.9, the “rule of thumb” filter falls in the bottom 1/3 based on the value of the error for filter type Butterworth. Four of the six other “rule of thumb” filters are in the top 25% and all are in the top 50% for the specifications in Table 6.2.1. Based on the value of error alone, are all ranked in the bottom 1/3.

For the Butterworth DF2 with a normalized cutoff frequency of .2π, the range of error is between 11.86 and 64.19 as shown in Table 6.2.2. The “rule of thumb” filter {N3 D3 N2 D2 N1 D1} includes the lowest roundoff error for these specifications. Four of the six “rule of thumb” filters are in the top 50% for these specifications as shown in Figure 6.2.2. When compared to all configurations for filter type Butterworth, {N2 D3 N1 D2 N3 D1} and {N1 D3 N2 D2 N3 D1} are ranked in the bottom 1/4 as shown in Table 4.1.1.

For the Butterworth DF1 with a normalized cutoff frequency of .5π, the range of error is between 8.62 and 18.94 as shown in Table 6.3.1. This is a small range of error by comparison to the other ranges for the Butterworth filters. The “rule of thumb” filter {N3 D3 N2 D2 N1 D1} includes the lowest roundoff error for these specifications as shown in Figure 6.3.1and when compared to the roundoff error for all the Butterworth filters is ranked 3rd. Four of the six “rule of thumb” filters for this configuration are ranked in the top 1/4 for lowest Butterworth roundoff error as shown in Table 4.1.1.

The range of error for the Butterworth DF2 with a normalized cutoff frequency of .5π is between 7.48 and 14.62 as shown in Table 6.3.2. This is also a small range of error. The “rule of thumb” filters does not include the filter with the lowest roundoff for these specifications as shown in Figure 6.3.2. When compared to the Butterworth roundoff errors, {N3 D3 N1 D2 N2 D1} has the lowest roundoff for Butterworth filters with a value of 7.589 and is followed by another filter with these specifications{N3 D3 N2 D2 N1 D1} with a roundoff value of 8.465. Four of the six “rule of thumb” filters for this configuration rank in the top one-fourth for lowest Butterworth roundoff error.

The range of error for the Butterworth DF1 with a normalized cutoff frequency of .8π is between 23.55 and 58.54 as shown in Table 6.4.1. Five of the six “rule of thumb” filters are in the top one-fourth for these specifications as shown in Figure 6.4.1. The “rule of thumb” filter {N3 D3 N2 D2 N1 D1} has the lowest roundoff for these specifications but ranks near the bottom one-half for Butterworth roundoff noise with an error of 23.55.

The range of error for the Butterworth DF2 with a normalized cutoff frequency of .8π is between 21.99 and 44.08 as shown in Table 6.4.2. The “rule of thumb” filters do not account for the first ten lowest error configurations. Based on these specifications, there is a tie for 11th, 23rd and 35th places. The “rule of thumb” filters {N3 D3 N2 D2 N1 D1} and {N2 D3 N1 D2 N3 D1}, which tie for 35th , accounts for the worst roundoff errors for these specifications as shown in Figure 6.4.2 and is ranked in the bottom 5 when compared with Butterworth roundoff errors as shown in Table 4.1.1.

Table 4.1.2 “Rule of Thumb” Filters for Chebyshev Filter Type

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Chebyshev	0.8	DF2	N3	D3	N1	D2	N2	D1	6.19178	6	5.12951	48.762	0.024
Chebyshev	0.8	DF2	N3	D3	N2	D2	N1	D1	8.84719	14	5.12951	48.762	0.085
Chebyshev	0.8	DF1	N3	D3	N2	D2	N1	D1	9.91066	1	9.91066	462.581	0
Chebyshev	0.8	DF2	N1	D3	N3	D2	N2	D1	10.2768	16	5.12951	48.762	0.118
Chebyshev	0.8	DF1	N2	D3	N3	D2	N1	D1	11.2527	3	9.91066	462.581	0.003
Chebyshev	0.8	DF1	N3	D3	N1	D2	N2	D1	17.5399	5	9.91066	462.581	0.017
Chebyshev	0.8	DF2	N2	D3	N3	D2	N1	D1	18.9403	22	5.12951	48.762	0.317
Chebyshev	0.8	DF1	N2	D3	N1	D2	N3	D1	23.1278	7	9.91066	462.581	0.029
Chebyshev	0.5	DF1	N3	D3	N2	D2	N1	D1	27.6678	6	15.3816	220.585	0.06
Chebyshev	0.5	DF1	N2	D3	N3	D2	N1	D1	29.2889	8	15.3816	220.585	0.068
Chebyshev	0.8	DF2	N1	D3	N2	D2	N3	D1	42.754	33	5.12951	48.762	0.862
Chebyshev	0.8	DF1	N1	D3	N3	D2	N2	D1	42.9634	13	9.91066	462.581	0.073
Chebyshev	0.8	DF1	N1	D3	N2	D2	N3	D1	47.2091	15	9.91066	462.581	0.082
Chebyshev	0.8	DF2	N2	D3	N1	D2	N3	D1	48.762	36	5.12951	48.762	1
Chebyshev	0.5	DF1	N3	D3	N1	D2	N2	D1	59.0361	16	15.3816	220.585	0.213
Chebyshev	0.5	DF2	N3	D3	N1	D2	N2	D1	63.3173	12	37.6362	194.96	0.163
Chebyshev	0.5	DF2	N3	D3	N2	D2	N1	D1	68.2359	14	37.6362	194.96	0.195
Chebyshev	0.5	DF1	N2	D3	N1	D2	N3	D1	113.186	24	15.3816	220.585	0.477
Chebyshev	0.5	DF2	N1	D3	N3	D2	N2	D1	115.398	28	37.6362	194.96	0.494
Chebyshev	0.5	DF1	N1	D3	N3	D2	N2	D1	123.871	26	15.3816	220.585	0.529
Chebyshev	0.5	DF2	N2	D3	N3	D2	N1	D1	138.608	34	37.6362	194.96	0.642
Chebyshev	0.5	DF1	N1	D3	N2	D2	N3	D1	176.4	34	15.3816	220.585	0.785
Chebyshev	0.5	DF2	N1	D3	N2	D2	N3	D1	176.67	35	37.6362	194.96	0.884
Chebyshev	0.5	DF2	N2	D3	N1	D2	N3	D1	194.96	36	37.6362	194.96	1
Chebyshev	0.2	DF2	N3	D3	N2	D2	N1	D1	423.074	11	379.524	2800.67	0.018
Chebyshev	0.2	DF2	N3	D3	N1	D2	N2	D1	450.219	12	379.524	2800.67	0.029
Chebyshev	0.2	DF1	N3	D3	N2	D2	N1	D1	461.947	13	124.75	3339.4	0.105
Chebyshev	0.2	DF1	N2	D3	N3	D2	N1	D1	479.807	15	124.75	3339.4	0.11
Chebyshev	0.2	DF1	N3	D3	N1	D2	N2	D1	993.139	19	124.75	3339.4	0.27
Chebyshev	0.2	DF2	N2	D3	N3	D2	N1	D1	1011.25	23	379.524	2800.67	0.261
Chebyshev	0.2	DF2	N2	D3	N1	D2	N3	D1	1520.17	32	379.524	2800.67	0.471
Chebyshev	0.2	DF1	N1	D3	N3	D2	N2	D1	1934.85	29	124.75	3339.4	0.563
Chebyshev	0.2	DF2	N1	D3	N3	D2	N2	D1	2318.89	34	379.524	2800.67	0.801
Chebyshev	0.2	DF1	N2	D3	N1	D2	N3	D1	2415.55	34	124.75	3339.4	0.713
Chebyshev	0.2	DF2	N1	D3	N2	D2	N3	D1	2800.67	36	379.524	2800.67	1
Chebyshev	0.2	DF1	N1	D3	N2	D2	N3	D1	3339.4	36	124.75	3339.4	1

The range of error for the Chebyshev DF1 with a normalized cutoff frequency of .2π is between 124.75 and 3339.4 as shown in Table 6.2.3. The “rule of thumb” filters exclude the top one-third of the orderings and pairings as shown in Figure 6.2.3. The “rule of thumb” filter {N1 D3 N2 D2 N3 D1} includes the worst filter for both these specifications and for the Chebyshev filter type. For the Chebyshev filters, the “rule of thumb” filters for this configuration ranks near the bottom 1/4 with roundoff errors between 461.94 and 3339.4 as shown in Table 4.1.2.

The range of error for the Chebyshev DF2 with a normalized cutoff frequency of .2π is between 379.524 and 2800.67 as shown in Table 6.2.4. The “rule of thumb” filters excludes the first 25% of low round-off errors. The “rule of thumb” filter {N1 D3 N2 D2 N3 D1} includes the worst error for these specifications as shown in Figure 6.2.4. The “rule of thumb” filters for this configuration also ranks near the bottom 1/4 for all Chebyshev filter as shown in Table 4.1.2.

The range of error for the Chebyshev DF1 with a normalized cutoff frequency of .5 is between 15.38 and 220.585 as shown in Table 6.3.3. The “rule of thumb” filters {N3 D3 N2 D2 N1 D1} and {N2 D3 N3 D2 N1 D1} are in the top 1/3 of lowest errors based on these specification and are near the top one-fourth for filter type Chebyshev. Three of the six filters fall in the bottom one-half of rankings for these specifications as shown in Figure 6.3.3. The “rule of thumb” filters that are in the bottom for these specifications are still not near the highest possible error of 220.585. Even the smallest matches are close to being the best pairing and ordering but when compared to all the Chebyshev filters, the "rule of thumb" filters for these specifications rank in the middle as shown in Table 4.1.2.

The range of error for the Chebyshev DF2 with a normalized cutoff frequency of .5π is between 37.63 and 194.96 as shown in Table 6.3.4. The "rule of thumb" includes systems that are up to 25.68 higher than that lowest round-off error. The "rule of thumb" includes four filters that in the bottom one-fourth for these specifications as shown in Figure 6.3.4. One of the "rule of thumb" filters, {N2 D3 N1 D2 N3 D1}, has the worst round-off. The advantage of using these specifications is the lower numeric round-off errors in comparison to some of the other Chebyshev designs.

The range of error for the Chebyshev DF1 with a normalized cutoff frequency of .8π is between 9.9 and 462.58 as shown in Table 6.4.3. The "rule of thumb" filters include four systems in the top ¼ of round-off error for these specifications. {N3 D3 N2 D2 N1 D1} is a true "rule of thumb" with a ranking of 1 for these specifications as shown in Figure 6.4.3 and an overall ranking of 3rd for the Chebyshev filters as shown in Table 4.1.2. The "rule of thumb" filters include errors that are up to 10% of the maximum error and are therefore not close to the highest round-off error of 462.58.

The range of error for the Chebyshev DF2 with a normalized cutoff frequency of .8π is between 5.129 and 48.762 as shown in Table 6.4.4. Three of the six "rule of thumb" filters are in the bottom one-half of error rankings for these specifications as shown in Figure 6.4.4. One of the "rule of thumb" filters, {N2 D3 N1 D2 N3 D1}, has the worst round-off error for these specifications. On the other hand, two of the six "rule of thumb” filters for this configuration actually are the highest ranked for the Chebyshev filters. They are {N3 D3 N1 D2 N2 D1} and {N3 D3 N2 D2 N1 D1} with a total roundoff error of 6.19 and 8.84 respectively.

Table 4.1.3 “Rule of Thumb” Filters for Elliptic Filter Type

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.8	DF2	N3	D3	N1	D2	N2	D1	3.03475	4	3.02382	15.1254	9E-04
Elliptic	0.2	DF2	N3	D3	N1	D2	N2	D1	3.03559	6	3.02349	15.5397	1E-03
Elliptic	0.5	DF2	N3	D3	N1	D2	N2	D1	3.04809	4	3.02603	46.3081	5E-04
Elliptic	0.8	DF2	N3	D3	N2	D2	N1	D1	3.06059	8	3.02382	15.1254	0.003
Elliptic	0.2	DF2	N3	D3	N2	D2	N1	D1	3.12436	14	3.02349	15.5397	0.008
Elliptic	0.5	DF2	N3	D3	N2	D2	N1	D1	3.14372	9	3.02603	46.3081	0.003
Elliptic	0.5	DF1	N3	D3	N2	D2	N1	D1	46.6754	9	23.7997	2375.16	0.01
Elliptic	0.5	DF1	N3	D3	N1	D2	N2	D1	46.8374	11	23.7997	2375.16	0.01
Elliptic	0.8	DF1	N3	D3	N1	D2	N2	D1	107.848	8	38.2288	9762.22	0.007
Elliptic	0.8	DF1	N3	D3	N2	D2	N1	D1	126.764	11	38.2288	9762.22	0.009
Elliptic	0.2	DF1	N3	D3	N2	D2	N1	D1	284.409	9	156.404	11963.5	0.011
Elliptic	0.2	DF1	N3	D3	N1	D2	N2	D1	304.462	11	156.404	11963.5	0.013

The range of error for the Elliptic DF1 with a normalized cutoff frequency of .2π is between 156.4 and 11963.5 as shown in Table 6.2.5. None of the “rule of thumb” filters include the lowest round off error. The “rule of thumb” filters {N3 D3 N2 D2 N1 D1} and {N3 D3 N1 D2 N2 D1} are actually ranked 9th and 11th as shown in Figure 6.2.5. The error values for these filters are 128 and 148 more, respectively, than the lowest error. Even though these configurations have higher error than the minimum they are still far from being the worst filter implementations for these specifications as shown in Table 6.2.5. They are ranked the worst in comparison to the roundoff errors for the elliptic filter type as shown in Table 4.1.3.

The range of error for the Elliptic DF2 with a normalized cutoff frequency of .2π is between 3.02 and 15.53 as shown in Table 6.2.6. The range of error for this configuration is very small. The “rule of thumb” filters do not include the lowest error but because the range of error is so small the difference between configurations are sometimes negligible. The “rule of thumb” filters {N3 D3 N1 D2 N2 D1} and {N3 D3

N2 D2 N1 D1} are ranked 6th and 14th filter for these specifications as shown in Figure 6.2.6 but the difference between the lowest error and the “rule of thumb” filters are .01 and .1, respectively. {N3 D3 N1 D2 N2 D1} is ranked 2nd for lowest roundoff error for the elliptic filter type.

The range of error for the elliptic DF1 with a normalized cutoff frequency of .5π is between 23.79 and 2375.16 as shown in Table 6.3.5. The “rule of thumb” does not include the lowest error but are ranked 9th and 11th best errors as shown in Figure 6.3.5. The errors are still not close to the highest error. Their error values are actually not more than 23 more than the lowest error.

The range of error for the elliptic DF2 filters with a normalized cutoff frequency of .5π is between 3.02 and 46.30 as shown in Table 6.3.6. The range of error for this configuration is very small as well. The “rule of thumb” filters are ranked 4th and 9th as shown in Figure 6.3.6 but the difference between the lowest error and the “rule of thumb” is .02 and .12, respectively. {N3 D3 N1 D2 N2 D1} is ranked 3rd for the lowest roundoff error for the elliptic filter type as shown in Table 4.1.3.

The range of error for the elliptic DF1 with a normalized cutoff frequency of .8π is between 38.22 and 9762.22 as shown in Table 6.4.5. The “rule of thumb” does not include the lowest error but the 8th and 11th best errors as shown in Figure 6.4.5. The errors are not close to the highest error for this configuration.

The range of error for the Elliptic DF2 filters with a normalized cutoff frequency of .8π is between 3.02 and 15.12 as shown in Table 6.4.6. The range of error for this configuration is very small as well. The “rule of thumb” filters are ranked 4th and 8th as shown in Figure 6.4.6 but the difference between the lowest error and the “rule of thumb” is .01 and .04, respectively. {N3 D3 N1 D2 N2 D1} is ranked 1st for the overall lowest roundoff error for the elliptic filter type, as shown in Table 4.1.3.

4.2 Analysis by Direct Form

In order to get a closer look at the results from the direct-form point of view, analysis was done by separate analysis of direct-form I and direct-form II for each filter type. The sections are divided by direct-form.

Section 4.2.1 shows the analysis for direct-form I. Table 4.2.1presents the Butterworth results, Table 4.2.2, presents the Chebyshev results, and Table 4.2.3 presents the elliptic results.

Section 4.2.2 shares the analysis for direct form II. Table 4.2.4 presents the Butterworth results, Table 4.2.5 presents the Chebyshev results, and Table 4.2.6 presents the elliptic results.

4.2.1 Direct-Form I

Table 4.2.1 “Rule of Thumb” Filters for Butterworth Direct-Form I

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Butterworth	0.5	DF1	N3	D3	N2	D2	N1	D1	8.62954	1	8.62954	18.9401	0
Butterworth	0.5	DF1	N3	D3	N1	D2	N2	D1	9.24465	4	8.62954	18.9401	0.06
Butterworth	0.5	DF1	N2	D3	N3	D2	N1	D1	10.0915	8	8.62954	18.9401	0.142
Butterworth	0.5	DF1	N1	D3	N3	D2	N2	D1	11.7519	13	8.62954	18.9401	0.303
Butterworth	0.5	DF1	N2	D3	N1	D2	N3	D1	12.8049	19	8.62954	18.9401	0.405
Butterworth	0.5	DF1	N1	D3	N2	D2	N3	D1	13.8502	25	8.62954	18.9401	0.506
Butterworth	0.8	DF1	N3	D3	N2	D2	N1	D1	23.5543	1	23.5543	58.5448	0
Butterworth	0.8	DF1	N3	D3	N1	D2	N2	D1	24.3249	2	23.5543	58.5448	0.022
Butterworth	0.8	DF1	N2	D3	N3	D2	N1	D1	25.8422	4	23.5543	58.5448	0.065
Butterworth	0.8	DF1	N1	D3	N3	D2	N2	D1	28.1477	8	23.5543	58.5448	0.131
Butterworth	0.8	DF1	N2	D3	N1	D2	N3	D1	28.7402	9	23.5543	58.5448	0.148
Butterworth	0.8	DF1	N1	D3	N2	D2	N3	D1	30.2749	13	23.5543	58.5448	0.192
Butterworth	0.2	DF1	N3	D3	N2	D2	N1	D1	34.9164	1	34.9164	78.87	0
Butterworth	0.2	DF1	N3	D3	N1	D2	N2	D1	37.038	3	34.9164	78.87	0.048
Butterworth	0.2	DF1	N2	D3	N3	D2	N1	D1	39.9554	6	34.9164	78.87	0.115
Butterworth	0.2	DF1	N1	D3	N3	D2	N2	D1	45.4572	11	34.9164	78.87	0.24
Butterworth	0.2	DF1	N2	D3	N1	D2	N3	D1	47.9332	14	34.9164	78.87	0.296
Butterworth	0.2	DF1	N1	D3	N2	D2	N3	D1	51.3134	18	34.9164	78.87	0.373

The distinction between the different cutoffs for DF1 is clear. The most desirable cutoff frequency for Butterworth DF1 is the mid range cutoff frequency. All of the mid-range cutoff frequencies are ranked in the top tier. The second best cutoff for Butterworth DF1 is the high frequency cutoff. The low frequency cutoff is the most undesirable for Butterworth DF1. For all of the frequency cutoffs the “rule of thumb” filter {N3 D3 N2 D2 N1 D1} which ranked first for their respective cutoffs was in direct-form I.

Table 4.2.2 “Rule of Thumb” Filters for Chebyshev Direct-Form I

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Chebyshev	0.8	DF1	N3	D3	N2	D2	N1	D1	9.91066	1	9.91066	462.581	0
Chebyshev	0.8	DF1	N2	D3	N3	D2	N1	D1	11.2527	3	9.91066	462.581	0.003
Chebyshev	0.8	DF1	N3	D3	N1	D2	N2	D1	17.5399	5	9.91066	462.581	0.017
Chebyshev	0.8	DF1	N2	D3	N1	D2	N3	D1	23.1278	7	9.91066	462.581	0.029
Chebyshev	0.5	DF1	N3	D3	N2	D2	N1	D1	27.6678	6	15.3816	220.585	0.06
Chebyshev	0.5	DF1	N2	D3	N3	D2	N1	D1	29.2889	8	15.3816	220.585	0.068
Chebyshev	0.8	DF1	N1	D3	N3	D2	N2	D1	42.9634	13	9.91066	462.581	0.073
Chebyshev	0.8	DF1	N1	D3	N2	D2	N3	D1	47.2091	15	9.91066	462.581	0.082
Chebyshev	0.5	DF1	N3	D3	N1	D2	N2	D1	59.0361	16	15.3816	220.585	0.213
Chebyshev	0.5	DF1	N2	D3	N1	D2	N3	D1	113.186	24	15.3816	220.585	0.477
Chebyshev	0.5	DF1	N1	D3	N3	D2	N2	D1	123.871	26	15.3816	220.585	0.529
Chebyshev	0.5	DF1	N1	D3	N2	D2	N3	D1	176.4	34	15.3816	220.585	0.785
Chebyshev	0.2	DF1	N3	D3	N2	D2	N1	D1	461.947	13	124.75	3339.4	0.105
Chebyshev	0.2	DF1	N2	D3	N3	D2	N1	D1	479.807	15	124.75	3339.4	0.11
Chebyshev	0.2	DF1	N3	D3	N1	D2	N2	D1	993.139	19	124.75	3339.4	0.27
Chebyshev	0.2	DF1	N1	D3	N3	D2	N2	D1	1934.85	29	124.75	3339.4	0.563
Chebyshev	0.2	DF1	N2	D3	N1	D2	N3	D1	2415.55	34	124.75	3339.4	0.713
Chebyshev	0.2	DF1	N1	D3	N2	D2	N3	D1	3339.4	36	124.75	3339.4	1

The distinct separation of cutoffs for Chebyshev DF1 is not as clear as it is for the Butterworth. The most desirable cutoff for the Chebyshev DF1 would be the high cutoff. The second best would be the mid-range frequency cutoff. Two of the six mid-range “rule of thumb” filters {N3 D3 N2 D2 N1 D1} and {N2 D3 N3 D2 N1 D1} have lower roundoff error than the high cutoff frequency but outside of those two designs. The high cutoff frequency ranks about the rest for DF1. The low cutoff frequency is ranked at the bottom for all of its “rule of thumb” filters.

Table 4.2.3 “Rule of Thumb” Filters for Elliptic Direct-Form I

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.5	DF1	N3	D3	N2	D2	N1	D1	46.6754	9	23.7997	2375.16	0.01
Elliptic	0.5	DF1	N3	D3	N1	D2	N2	D1	46.8374	11	23.7997	2375.16	0.01
Elliptic	0.8	DF1	N3	D3	N1	D2	N2	D1	107.848	8	38.2288	9762.22	0.007
Elliptic	0.8	DF1	N3	D3	N2	D2	N1	D1	126.764	11	38.2288	9762.22	0.009
Elliptic	0.2	DF1	N3	D3	N2	D2	N1	D1	284.409	9	156.404	11963.5	0.011
Elliptic	0.2	DF1	N3	D3	N1	D2	N2	D1	304.462	11	156.404	11963.5	0.013

The most desirable cutoff for the Elliptic DF1 is the mid range frequency cutoff. The second best is the high cutoff. Like the Butterworth and Chebyshev, the low cutoff is the most undesirable. When comparing the roundoff between the best elliptic {N3 D3 N2 D2 N1 D1} at the mid range and {N3 D3 N1 D2 N2 D1} at the low cutoff, the low roundoff is almost seven times the roundoff error for the mid-range cutoff. The “rule of thumb” filters at all ranges are still closer to optimum than some of the other filter configurations because they are the roundoff error is still minor compared to the maximum in their respective ranges.

4.2.2 Direct-Form II

Table 4.2.4 “Rule of Thumb” Filters for Butterworth Direct-Form II

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Butterworth	0.5	DF2	N3	D3	N1	D2	N2	D1	7.58932	4	7.48552	14.6203	0.015
Butterworth	0.5	DF2	N3	D3	N2	D2	N1	D1	8.46584	10	7.48552	14.6203	0.137
Butterworth	0.5	DF2	N1	D3	N3	D2	N2	D1	8.94879	14	7.48552	14.6203	0.205
Butterworth	0.5	DF2	N2	D3	N3	D2	N1	D1	11.2986	22	7.48552	14.6203	0.534
Butterworth	0.2	DF2	N3	D3	N2	D2	N1	D1	11.861	1	11.861	64.1942	0
Butterworth	0.5	DF2	N1	D3	N2	D2	N3	D1	11.9654	27	7.48552	14.6203	0.628
Butterworth	0.2	DF2	N3	D3	N1	D2	N2	D1	12.2323	4	11.861	64.1942	0.007
Butterworth	0.5	DF2	N2	D3	N1	D2	N3	D1	13.4387	34	7.48552	14.6203	0.834
Butterworth	0.2	DF2	N2	D3	N3	D2	N1	D1	22.6436	13	11.861	64.1942	0.206
Butterworth	0.8	DF2	N1	D3	N3	D2	N2	D1	23.0594	11	21.9911	44.0897	0.048
Butterworth	0.8	DF2	N3	D3	N1	D2	N2	D1	23.0594	12	21.9911	44.0897	0.048
Butterworth	0.2	DF2	N1	D3	N3	D2	N2	D1	27.5681	17	11.861	64.1942	0.3
Butterworth	0.8	DF2	N1	D3	N2	D2	N3	D1	29.3464	23	21.9911	44.0897	0.333
Butterworth	0.8	DF2	N3	D3	N2	D2	N1	D1	29.3464	24	21.9911	44.0897	0.333
Butterworth	0.2	DF2	N2	D3	N1	D2	N3	D1	36.195	23	11.861	64.1942	0.465
Butterworth	0.2	DF2	N1	D3	N2	D2	N3	D1	40.7482	28	11.861	64.1942	0.552
Butterworth	0.8	DF2	N2	D3	N1	D2	N3	D1	44.0897	35	21.9911	44.0897	1
Butterworth	0.8	DF2	N2	D3	N3	D2	N1	D1	44.0897	36	21.9911	44.0897	1

For Butterworth DF2, the mid-range frequency cutoff has the lowest roundoff error. Four of the six “rule of thumb” filters are tanked in the top for Butterworth DF2. One of the worst mid-range Butterworth DF2 “rule of thumb” filters, {N3 D2 N1 D2 N1 D1}, is still ranked above all the high range frequency cutoffs and four of the six low range cutoffs. The most undesirable cutoff for Butterworth DFII is the high cutoff. All of its “rule of thumb” filters fall in the lower ½ of roundoff errors. The worst two “rule of thumb” filters {N2 D3 N1 D2 N3 D1} and {N2 D3 N3 D2 N1 D1}, for Butterworth DF2 are high frequency cutoffs. The low frequency cutoff has a true “rule of thumb” filter, {N3 D3 N2 D2 N1 D1}, but this filter still only ranks 5th when compared with roundoff error for all Butterworth DF2 filters.

Table 4.2.5 “Rule of Thumb” Filters for Chebyshev Direct-Form II

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Chebyshev	0.8	DF2	N3	D3	N1	D2	N2	D1	6.19178	6	5.12951	48.762	0.024
Chebyshev	0.8	DF2	N3	D3	N2	D2	N1	D1	8.84719	14	5.12951	48.762	0.085
Chebyshev	0.8	DF2	N1	D3	N3	D2	N2	D1	10.2768	16	5.12951	48.762	0.118
Chebyshev	0.8	DF2	N2	D3	N3	D2	N1	D1	18.9403	22	5.12951	48.762	0.317
Chebyshev	0.8	DF2	N1	D3	N2	D2	N3	D1	42.754	33	5.12951	48.762	0.862
Chebyshev	0.8	DF2	N2	D3	N1	D2	N3	D1	48.762	36	5.12951	48.762	1
Chebyshev	0.5	DF2	N3	D3	N1	D2	N2	D1	63.3173	12	37.6362	194.96	0.163
Chebyshev	0.5	DF2	N3	D3	N2	D2	N1	D1	68.2359	14	37.6362	194.96	0.195
Chebyshev	0.5	DF2	N1	D3	N3	D2	N2	D1	115.398	28	37.6362	194.96	0.494
Chebyshev	0.5	DF2	N2	D3	N3	D2	N1	D1	138.608	34	37.6362	194.96	0.642
Chebyshev	0.5	DF2	N1	D3	N2	D2	N3	D1	176.67	35	37.6362	194.96	0.884
Chebyshev	0.5	DF2	N2	D3	N1	D2	N3	D1	194.96	36	37.6362	194.96	1
Chebyshev	0.2	DF2	N3	D3	N2	D2	N1	D1	423.074	11	379.524	2800.67	0.018
Chebyshev	0.2	DF2	N3	D3	N1	D2	N2	D1	450.219	12	379.524	2800.67	0.029
Chebyshev	0.2	DF2	N2	D3	N3	D2	N1	D1	1011.25	23	379.524	2800.67	0.261
Chebyshev	0.2	DF2	N2	D3	N1	D2	N3	D1	1520.17	32	379.524	2800.67	0.471
Chebyshev	0.2	DF2	N1	D3	N3	D2	N2	D1	2318.89	34	379.524	2800.67	0.801
Chebyshev	0.2	DF2	N1	D3	N2	D2	N3	D1	2800.67	36	379.524	2800.67	1

The distinction is clear for Chebyshev DF2. The high cutoff is the most desirable with roundoff between 6.19 and 48.76. Even though there are many other filters that are better than some of the “rule of thumb” filters, these high cutoff “rule of thumb” filters still ranks above all “rule of thumb” filters for the low and mid-range cutoffs. The second best cutoff for DF2 is the mid range frequency. Two of the “rule of thumb” filters {N1 D3 N2 D2 N3 D1} and {N2 D3 N1 D2 N3 D1} are the worst for Chebyshev DF2 mid range but they still rank above all low frequency cutoffs. Low frequency cutoffs are the most undesirable for Chebyshev DF2. The roundoff error for these filters can become quite large.

Table 4.2.6 “Rule of Thumb” Filters for Elliptic Direct-Form II

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.8	DF2	N3	D3	N1	D2	N2	D1	3.03475	4	3.02382	15.1254	9E-04
Elliptic	0.2	DF2	N3	D3	N1	D2	N2	D1	3.03559	6	3.02349	15.5397	1E-03
Elliptic	0.5	DF2	N3	D3	N1	D2	N2	D1	3.04809	4	3.02603	46.3081	5E-04
Elliptic	0.8	DF2	N3	D3	N2	D2	N1	D1	3.06059	8	3.02382	15.1254	0.003
Elliptic	0.2	DF2	N3	D3	N2	D2	N1	D1	3.12436	14	3.02349	15.5397	0.008
Elliptic	0.5	DF2	N3	D3	N2	D2	N1	D1	3.14372	9	3.02603	46.3081	0.003

A key result for elliptic DF2 filters is for all cutoff frequencies the first ordering and pairing should be {N3 D3}. The best elliptic “rule of thumb” filters follow {N3 D3 N1 D2 N2 D1} and the worst follow {N3 D3 N2 D2 N1 D1}. For the top three filters with the lowest roundoff error, they represent each of the normalized frequency categories. The high cutoff had the lowest roundoff error, but the second best roundoff error was from the low frequency cutoff. The difference between the high and low cutoff is only .001. The third best roundoff error was from the mid range, and its roundoff error is only .01 from both the high and low cutoff frequencies.

4.3 Analysis by Cutoff Frequency

The value of the normalized cutoff frequency was also a key parameter in evaluating the filters. Tables were also created to get a closer look at the effect each cutoff had depending on the filter type. Three sections were created for further analysis. Section 4.3.1 shares the analysis for the low normalized cutoff which had a value of .2π. Section 4.3.2 shows the analysis for the mid normalized cutoff frequency which had a value of .5π. Section 4.3.3 provides the analysis for the high cutoff frequency which had a value of .8π.

4.3.1 Low Normalized Cutoff

Table 4.3.1 “Rule of Thumb” Filters for Low Normalized Cutoff Frequency

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.2	DF2	N3	D3	N1	D2	N2	D1	3.03559	6	3.02349	15.5397	1E-03
Elliptic	0.2	DF2	N3	D3	N2	D2	N1	D1	3.12436	14	3.02349	15.5397	0.008
Butterworth	0.2	DF2	N3	D3	N2	D2	N1	D1	11.861	1	11.861	64.1942	0
Butterworth	0.2	DF2	N3	D3	N1	D2	N2	D1	12.2323	4	11.861	64.1942	0.007
Butterworth	0.2	DF2	N2	D3	N3	D2	N1	D1	22.6436	13	11.861	64.1942	0.206
Butterworth	0.2	DF2	N1	D3	N3	D2	N2	D1	27.5681	17	11.861	64.1942	0.3
Butterworth	0.2	DF1	N3	D3	N2	D2	N1	D1	34.9164	1	34.9164	78.87	0
Butterworth	0.2	DF2	N2	D3	N1	D2	N3	D1	36.195	23	11.861	64.1942	0.465
Butterworth	0.2	DF1	N3	D3	N1	D2	N2	D1	37.038	3	34.9164	78.87	0.048
Butterworth	0.2	DF1	N2	D3	N3	D2	N1	D1	39.9554	6	34.9164	78.87	0.115
Butterworth	0.2	DF2	N1	D3	N2	D2	N3	D1	40.7482	28	11.861	64.1942	0.552
Butterworth	0.2	DF1	N1	D3	N3	D2	N2	D1	45.4572	11	34.9164	78.87	0.24
Butterworth	0.2	DF1	N2	D3	N1	D2	N3	D1	47.9332	14	34.9164	78.87	0.296
Butterworth	0.2	DF1	N1	D3	N2	D2	N3	D1	51.3134	18	34.9164	78.87	0.373
Elliptic	0.2	DF1	N3	D3	N2	D2	N1	D1	284.409	9	156.404	11963.5	0.011
Elliptic	0.2	DF1	N3	D3	N1	D2	N2	D1	304.462	11	156.404	11963.5	0.013
Chebyshev	0.2	DF2	N3	D3	N2	D2	N1	D1	423.074	11	379.524	2800.67	0.018
Chebyshev	0.2	DF2	N3	D3	N1	D2	N2	D1	450.219	12	379.524	2800.67	0.029
Chebyshev	0.2	DF1	N3	D3	N2	D2	N1	D1	461.947	13	124.75	3339.4	0.105
Chebyshev	0.2	DF1	N2	D3	N3	D2	N1	D1	479.807	15	124.75	3339.4	0.11
Chebyshev	0.2	DF1	N3	D3	N1	D2	N2	D1	993.139	19	124.75	3339.4	0.27
Chebyshev	0.2	DF2	N2	D3	N3	D2	N1	D1	1011.25	23	379.524	2800.67	0.261
Chebyshev	0.2	DF2	N2	D3	N1	D2	N3	D1	1520.17	32	379.524	2800.67	0.471
Chebyshev	0.2	DF1	N1	D3	N3	D2	N2	D1	1934.85	29	124.75	3339.4	0.563
Chebyshev	0.2	DF2	N1	D3	N3	D2	N2	D1	2318.89	34	379.524	2800.67	0.801
Chebyshev	0.2	DF1	N2	D3	N1	D2	N3	D1	2415.55	34	124.75	3339.4	0.713
Chebyshev	0.2	DF2	N1	D3	N2	D2	N3	D1	2800.67	36	379.524	2800.67	1
Chebyshev	0.2	DF1	N1	D3	N2	D2	N3	D1	3339.4	36	124.75	3339.4	1

Low cutoff Chebyshev filters for both DF1 and DF2 are the worst “rule of thumb” filters. An elliptic DF2 is the best for a low cutoff. Both of the “rule of thumb” filters for elliptic DF2 had the smallest roundoff error. The “rule of thumb” for a low cutoff elliptic DF2 does not include many ordering and pairings that have lower roundoff than the “rule of thumb” filters. For a low cutoff, the Butterworth DF2 gives the lowest roundoff error above the elliptic DF2 filters. All the Butterworth filter types are better than the elliptic DF1 filters though.

4.3.2 Mid Normalized Cutoff

Table 4.3.2 "Rule of Thumb" Filters for Mid Normalized Cutoff Frequency

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.5	DF2	N3	D3	N1	D2	N2	D1	3.04809	4	3.02603	46.3081	5E-04
Elliptic	0.5	DF2	N3	D3	N2	D2	N1	D1	3.14372	9	3.02603	46.3081	0.003
Butterworth	0.5	DF2	N3	D3	N1	D2	N2	D1	7.58932	4	7.48552	14.6203	0.015
Butterworth	0.5	DF2	N3	D3	N2	D2	N1	D1	8.46584	10	7.48552	14.6203	0.137
Butterworth	0.5	DF1	N3	D3	N2	D2	N1	D1	8.62954	1	8.62954	18.9401	0
Butterworth	0.5	DF2	N1	D3	N3	D2	N2	D1	8.94879	14	7.48552	14.6203	0.205
Butterworth	0.5	DF1	N3	D3	N1	D2	N2	D1	9.24465	4	8.62954	18.9401	0.06
Butterworth	0.5	DF1	N2	D3	N3	D2	N1	D1	10.0915	8	8.62954	18.9401	0.142
Butterworth	0.5	DF2	N2	D3	N3	D2	N1	D1	11.2986	22	7.48552	14.6203	0.534
Butterworth	0.5	DF1	N1	D3	N3	D2	N2	D1	11.7519	13	8.62954	18.9401	0.303
Butterworth	0.5	DF2	N1	D3	N2	D2	N3	D1	11.9654	27	7.48552	14.6203	0.628
Butterworth	0.5	DF1	N2	D3	N1	D2	N3	D1	12.8049	19	8.62954	18.9401	0.405
Butterworth	0.5	DF2	N2	D3	N1	D2	N3	D1	13.4387	34	7.48552	14.6203	0.834
Butterworth	0.5	DF1	N1	D3	N2	D2	N3	D1	13.8502	25	8.62954	18.9401	0.506
Chebyshev	0.5	DF1	N3	D3	N2	D2	N1	D1	27.6678	6	15.3816	220.585	0.06
Chebyshev	0.5	DF1	N2	D3	N3	D2	N1	D1	29.2889	8	15.3816	220.585	0.068
Elliptic	0.5	DF1	N3	D3	N2	D2	N1	D1	46.6754	9	23.7997	2375.16	0.01
Elliptic	0.5	DF1	N3	D3	N1	D2	N2	D1	46.8374	11	23.7997	2375.16	0.01
Chebyshev	0.5	DF1	N3	D3	N1	D2	N2	D1	59.0361	16	15.3816	220.585	0.213
Chebyshev	0.5	DF2	N3	D3	N1	D2	N2	D1	63.3173	12	37.6362	194.96	0.163
Chebyshev	0.5	DF2	N3	D3	N2	D2	N1	D1	68.2359	14	37.6362	194.96	0.195
Chebyshev	0.5	DF1	N2	D3	N1	D2	N3	D1	113.186	24	15.3816	220.585	0.477
Chebyshev	0.5	DF2	N1	D3	N3	D2	N2	D1	115.398	28	37.6362	194.96	0.494
Chebyshev	0.5	DF1	N1	D3	N3	D2	N2	D1	123.871	26	15.3816	220.585	0.529
Chebyshev	0.5	DF2	N2	D3	N3	D2	N1	D1	138.608	34	37.6362	194.96	0.642
Chebyshev	0.5	DF1	N1	D3	N2	D2	N3	D1	176.4	34	15.3816	220.585	0.785
Chebyshev	0.5	DF2	N1	D3	N2	D2	N3	D1	176.67	35	37.6362	194.96	0.884
Chebyshev	0.5	DF2	N2	D3	N1	D2	N3	D1	194.96	36	37.6362	194.96	1

At the mid cutoff, elliptic DF2 "rule of thumb" filters are ranked the highest. The Butterworth "rule of thumb" filters are all ranked together above all Chebyshev filters and the elliptic DF1 according to their roundoff error. Unlike at the low cutoff, two Chebyshev "rule of thumb" filters, {N3 D3 N2 D2 N1 D1} and {N3 D3 N1 D2 N2 D1}, keep Chebyshev filters from being the overall worst filter type with regard to roundoff errors.

4.3.3 High Normalized Cutoff

Table 4.3.3 “Rule of Thumb” Filters for High Normalized Cutoff Frequency

Filter Type	Normalized Cutoff	Direct Form	Rule of Thumb Filter						Error	Actual Rank (of 36)	Range of Error		Ratio of Error
Elliptic	0.8	DF2	N3	D3	N1	D2	N2	D1	3.03475	4	3.02382	15.1254	0.0009
Elliptic	0.8	DF2	N3	D3	N2	D2	N1	D1	3.06059	8	3.02382	15.1254	0.003
Chebyshev	0.8	DF2	N3	D3	N1	D2	N2	D1	6.19178	6	5.12951	48.762	0.0243
Chebyshev	0.8	DF2	N3	D3	N2	D2	N1	D1	8.84719	14	5.12951	48.762	0.0852
Chebyshev	0.8	DF1	N3	D3	N2	D2	N1	D1	9.91066	1	9.91066	462.581	0
Chebyshev	0.8	DF2	N1	D3	N3	D2	N2	D1	10.2768	16	5.12951	48.762	0.118
Chebyshev	0.8	DF1	N2	D3	N3	D2	N1	D1	11.2527	3	9.91066	462.581	0.003
Chebyshev	0.8	DF1	N3	D3	N1	D2	N2	D1	17.5399	5	9.91066	462.581	0.0169
Chebyshev	0.8	DF2	N2	D3	N3	D2	N1	D1	18.9403	22	5.12951	48.762	0.3165
Butterworth	0.8	DF2	N1	D3	N3	D2	N2	D1	23.0594	11	21.9911	44.0897	0.0483
Butterworth	0.8	DF2	N3	D3	N1	D2	N2	D1	23.0594	12	21.9911	44.0897	0.0483
Chebyshev	0.8	DF1	N2	D3	N1	D2	N3	D1	23.1278	7	9.91066	462.581	0.0292
Butterworth	0.8	DF1	N3	D3	N2	D2	N1	D1	23.5543	1	23.5543	58.5448	0
Butterworth	0.8	DF1	N3	D3	N1	D2	N2	D1	24.3249	2	23.5543	58.5448	0.022
Butterworth	0.8	DF1	N2	D3	N3	D2	N1	D1	25.8422	4	23.5543	58.5448	0.0654
Butterworth	0.8	DF1	N1	D3	N3	D2	N2	D1	28.1477	8	23.5543	58.5448	0.1313
Butterworth	0.8	DF1	N2	D3	N1	D2	N3	D1	28.7402	9	23.5543	58.5448	0.1482
Butterworth	0.8	DF2	N1	D3	N2	D2	N3	D1	29.3464	23	21.9911	44.0897	0.3328
Butterworth	0.8	DF2	N3	D3	N2	D2	N1	D1	29.3464	24	21.9911	44.0897	0.3328
Butterworth	0.8	DF1	N1	D3	N2	D2	N3	D1	30.2749	13	23.5543	58.5448	0.1921
Chebyshev	0.8	DF2	N1	D3	N2	D2	N3	D1	42.754	33	5.12951	48.762	0.8623
Chebyshev	0.8	DF1	N1	D3	N3	D2	N2	D1	42.9634	13	9.91066	462.581	0.073
Butterworth	0.8	DF2	N2	D3	N1	D2	N3	D1	44.0897	35	21.9911	44.0897	1
Butterworth	0.8	DF2	N2	D3	N3	D2	N1	D1	44.0897	36	21.9911	44.0897	1
Chebyshev	0.8	DF1	N1	D3	N2	D2	N3	D1	47.2091	15	9.91066	462.581	0.0824
Chebyshev	0.8	DF2	N2	D3	N1	D2	N3	D1	48.762	36	5.12951	48.762	1
Elliptic	0.8	DF1	N3	D3	N1	D2	N2	D1	107.848	8	38.2288	9762.22	0.0072
Elliptic	0.8	DF1	N3	D3	N2	D2	N1	D1	126.764	11	38.2288	9762.22	0.0091

At the high cutoff, elliptic DF1 “rule of thumb” filters have the highest roundoff error and are ranked the worst, but elliptic DF2filters are ranked the best just as they are for the low and mid cutoffs. For the high cutoff frequencies, the Chebyshev filters perform better than the Butterworth filters for the majority of the “rule of thumb” cases.

CHAPTER FIVE

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

For all Butterworth and Chebyshev filters, the zeros are at -1 in the z-plane. Ideally, this knowledge makes the rule of thumb more about the ordering and less about the pairing. From the data, this conclusion seems less straight forward. When all of the filter configurations are sorted by roundoff error, a few of the poles seems to consistently group together.

At the low frequency cutoff of .2π, for Butterworth DF1 and DF2 filters, the “rule of thumb” does include the filter configuration with the lowest roundoff noise. The Chebyshev DF1 and DF2 filters for the same cutoff do not include the filter configuration with the lowest cutoff. This is also true for the elliptic DF1 and DF2 filter configurations. For Chebyshev DF1, the more optimal arrangement of the poles is {D1 D3 D2} and {D2 D1 D3} for Chebyshev DF2. For the elliptic DF2 configuration the difference between the “rule of thumb” filters is negligible. For elliptic DF1 filters the optimal ordering and pairing is {N2 D3 N3 D2 N1 D1}.

At the mid frequency cutoff of .5π, the “rule of thumb” was true only for the Butterworth DF1. The more optimal arrangement of the poles for Butterworth DF2 is {D2 D3 D1}. For Chebyshev DF1 the optimal arrangement is {D3 D1 D2} and {D2 D1 D3}. For the elliptic DF2 configuration the difference between the “rule of thumb” is negligible. For elliptic DF1filters the optimal ordering and pairing is {N1 D2 N2 D3 N3 D1}.

At the high cutoff frequency of .8π, “the rule of thumb” was true only for Butterworth and Chebyshev DF1.For elliptic DF2 at the same cutoff; the “rule of thumb” filters selected do not have the lowest error but are negligibly higher. A better arrangement for the Butterworth DF2 configuration would have been ordering the poles with the poles farthest from the unit circle first, then the closest to the unit circle, and then the last pole for an arrangement of {D1 D3 D2}. For Chebyshev DF2 the {D3 D1 D3} arrangement would be more optimal. For the elliptic DF2 configuration the difference between the “rule of thumb” is negligible. For elliptic DF1 the optimal ordering and pairing is {N1 D2 N2 D1 N3 D1}.

5.2 Future Work

Using the partial fraction expansion method discussed in Appendix A.5 of [4], the effects of roundoff noise on these same filters can be analyzed. Further analysis can also be done on the filter configurations discussed for highpass and bandpass filters. Other cutoff frequencies could also be included in filter analysis.

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APPENDIX

6.1 MATLAB Functions

6.1.1 butter

Purpose:

butter designs lowpass, bandpass, highpass, and bandstop digital and analog Butterworth filters. Butterworth filters are characterized by a magnitude response that is maximally flat in the passband and monotonic overall.

Syntax:

[z,p,k] = butter(n,Wn)

[b,a] = butter(n,Wn)

Description:

[z,p,k] = butter(n,Wn) designs an order n lowpass digital Butterworth filter with normalized cutoff frequency Wn. It returns the zeros and poles in length n column vectors z and p, and the gain in the scalar k.

[b,a] = butter(n,Wn) designs an order n lowpass digital Butterworth filter with normalized cutoff frequency Wn. It returns the filter coefficients in length n+1 row vectors b and a, with coefficients in descending powers of *z*.

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}$$

For digital filter design, butter uses bilinear to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at Wn.

6.1.2 cheby1

Purpose:

cheby1 designs lowpass, bandpass, highpass, and bandstop digital and analog Chebyshev Type I filters. Chebyshev Type I filters are equiripple in the passband and monotonic in the stopband.

Syntax:

[z,p,k] = cheby1(n,R,Wp)

[b,a] = cheby1(n,R,Wp)

Description:

[z,p,k] = cheby1(n,R,Wp) designs an order n Chebyshev lowpass digital Chebyshev filter with normalized passband edge frequency Wp and R dB of peak-to-peak ripple in the passband. It returns the zeros and poles in length n column vectors z and p and the gain in the scalar k.

[b,a] = cheby1(n,R,Wp) designs an order n Chebyshev lowpass digital Chebyshev filter with normalized passband edge frequency Wp and R dB of peak-to-peak ripple in the

passband. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of *z*.

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}$$

6.1.3 ellip

Purpose:

ellip designs lowpass, bandpass, highpass, and bandstop digital and analog elliptic filters. Elliptic filters offer steeper roll off characteristics than Butterworth or Chebyshev filters, but are equiripple in both the pass- and stopbands.

Syntax:

[z,p,k] = ellip(n,Rp,Rs,Wp)

[b,a] = ellip(n,Rp,Rs,Wp)

Description:

[z,p,k] = ellip(n,Rp,Rs,Wp) designs an order n lowpass digital elliptic filter with normalized passband edge frequency Wp, Rp dB of ripple in the passband, and a stopband Rs dB down from the peak value in the passband. It returns the zeros and poles in length n column vectors z and p and the gain in the scalar k.

The normalized passband edge frequency is the edge of the passband, at which the magnitude response of the filter is -Rp dB. For ellip, the normalized cutoff frequency Wp is a number between 0 and 1, where 1 corresponds to half the sampling frequency (Nyquist frequency). Smaller values of passband ripple Rp and larger values of stopband attenuation Rs both lead to wider transition widths (shallower roll off characteristics).

If Wp is a two-element vector, Wp = [w1 w2], ellip returns an order 2*n bandpass filter with passband w1 < ω < w2.

[b,a] = ellip(n,Rp,Rs,Wp) designs an order n lowpass digital elliptic filter with normalized passband edge frequency Wp, Rp dB of ripple in the passband, and a stopband Rs dB down from the peak value in the passband. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of z.

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}$$

6.1.4 zp2sos

Purpose:

zp2sos converts a discrete-time zero-pole-gain representation of a given digital filter to an equivalent second-order section representation.

Syntax:

[sos,g] = zp2sos(z,p,k)

Description:

Use [sos,g] = zp2sos(z,p,k) to obtain a matrix sos in second-order section form with gain g equivalent to the discrete-time zero-pole-gain filter represented by input arguments z, p, and k. Vectors z and p contain the zeros and poles of the filter's transfer function *H*(*z*), not necessarily in any particular order.

$$H(z) = k\frac{(z-z_1)(z-z_2)\cdots(z-z_n)}{(z-p_1)(z-p_2)\cdots(z-p_m)}$$

where *n* and *m* are the lengths of z and p, respectively, and k is a scalar gain. The zeros and poles must be real or complex conjugate pairs. sos is an *L*-by-6 matrix

$$sos = \begin{bmatrix} b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L} \end{bmatrix}$$

whose rows contain the numerator and denominator coefficients b_{ik} and a_{ik} of the second-order sections of *H*(*z*).

$$H(z) = g\prod_{k=1}^{L} H_k(z) = g\prod_{k=1}^{L}\frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1+a_{1k}z^{-1}+a_{2k}z^{-2}}$$

The number *L* of rows of the matrix sos is the closest integer greater than or equal to the maximum of *n*/2 and *m*/2.

6.1.5 tfdata

Purpose:

Numerator and denominator of transfer function from idmodel object

Syntax:

[num,den] = tfdata(m,'v')

Description:

m is a model given as any idmodel object with ny output channels and nu input channels.

num is a cell array of dimension ny-by-nu. num{ky,ku} (note the curly brackets) contains the numerator of the transfer function from input ku to output ky. This numerator is a row vector whose interpretation is described below.

Similarly, den is an ny-by-nu cell array of the denominators. If m is a SISO model, adding an extra input argument 'v' (for vector) will return num and den as vectors rather than cell arrays.

6.1.6 scale

Purpose:

Scale sections of SOS (second-order section) filter

Syntax:

scale(hd)

Description:

scale(hd) scales the second-order section filter hd using peak magnitude response scaling (L-infinity, Linf), to reduce the possibility of overflows when your filter hd operates in fixed-point arithmetic mode.

6.2 Low Cutoff Filter Design for All Data: Showing “Rule of Thumbs”

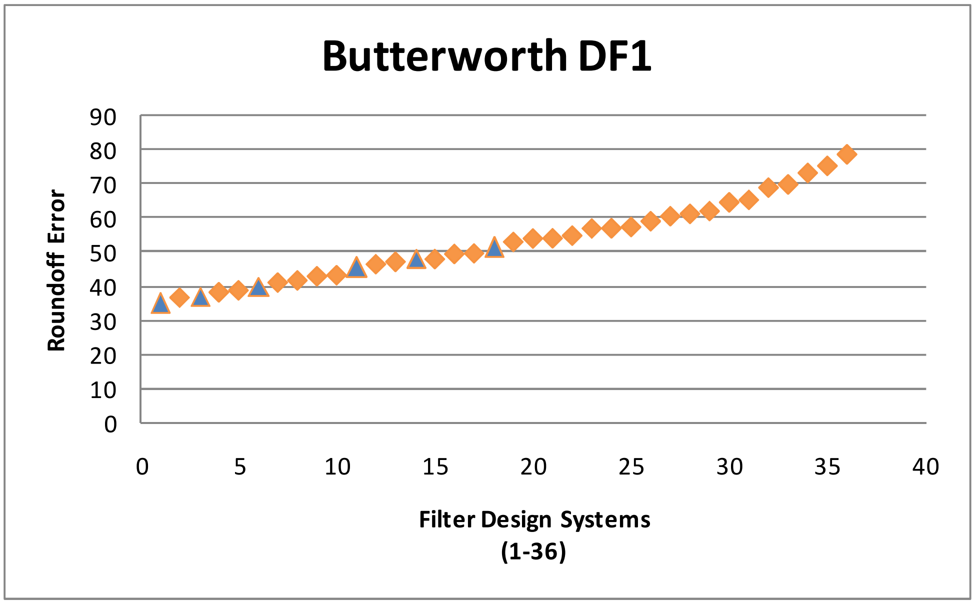


Figure 6.2.1 Roundoff Error for Butterworth DF1 with .2π Cutoff Frequency

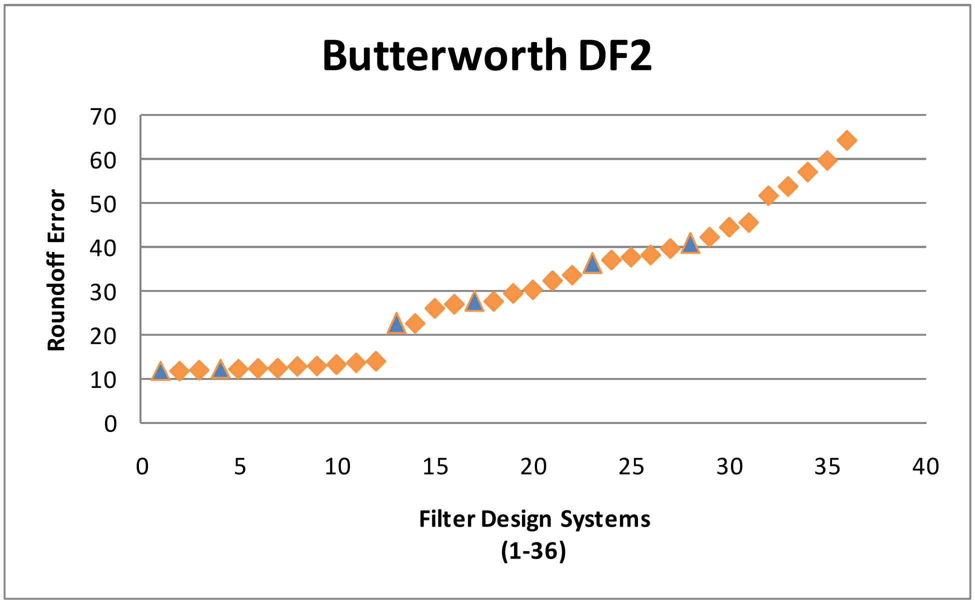


Figure 6.2.2 Roundoff Error for Butterworth DF2 with .2π Cutoff frequency

Table 6.2.1 Roundoff Error for Butterworth DF1 with .2π Cutoff Frequency

Butterworth DF1												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	34.9164
N3	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	36.6985
N3	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	37.038
N3	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	38.2543
N3	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	38.8201
N2	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	39.9554
N3	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	41.1318
N2	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	41.7375
N3	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	42.9904
N2	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	43.2933
N1	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	45.4572
N3	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	46.537
N1	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	47.2392
N2	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	47.9332
N2	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	48.0294
N1	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	49.551
N2	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	49.7153
N1	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	51.3134
N1	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	53.0955
N2	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	54.1137
N3	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	54.1605
N1	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	54.9561
N3	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	57.038
N3	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	57.1146
N1	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	57.4939
N2	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	59.1995
N3	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	60.6611
N2	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	61.3656
N2	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	62.1536
N1	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	64.7458
N1	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	65.4572
N1	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	69.0803
N2	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	70.02
N1	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	73.4001
N2	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	75.4898
N1	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	78.87

Table 6.2.2 Roundoff Error for Butterworth DF2 with .2π Cutoff Frequency

Butterworth DF2												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	11.861
N3	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	11.9637
N3	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	12.184
N3	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	12.2323
N3	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	12.3743
N3	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	12.5553
N3	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	12.5898
N3	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	13.0004
N3	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	13.122
N3	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	13.4251
N3	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	13.8528
N3	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	14.1559
N2	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	22.6436
N2	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	22.7463
N2	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	26.1441
N2	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	27.0821
N1	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	27.5681
N1	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	27.7101
N2	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	29.5309
N2	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	30.3662
N1	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	32.4103
N1	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	33.7078
N2	D3	0.85785	1.77209	N1	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	36.195
N1	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	37.0953
N2	D3	0.85785	1.77209	N1	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	37.7343
N1	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	38.2508
N2	D2	0.6425	1.59868	N1	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	39.6954
N1	D3	0.85785	1.77209	N2	D2	0.6425	1.59868	N3	D1	0.52508	1.51914	40.7482
N1	D3	0.85785	1.77209	N2	D1	0.52508	1.51914	N3	D2	0.6425	1.59868	42.2875
N2	D1	0.52508	1.51914	N1	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	44.5189
N1	D2	0.6425	1.59868	N2	D3	0.85785	1.77209	N3	D1	0.52508	1.51914	45.5903
N1	D1	0.52508	1.51914	N2	D3	0.85785	1.77209	N3	D2	0.6425	1.59868	51.6727
N2	D2	0.6425	1.59868	N1	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	53.7563
N2	D1	0.52508	1.51914	N1	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	57.0405
N1	D2	0.6425	1.59868	N2	D1	0.52508	1.51914	N3	D3	0.85785	1.77209	59.6513
N1	D1	0.52508	1.51914	N2	D2	0.6425	1.59868	N3	D3	0.85785	1.77209	64.1942

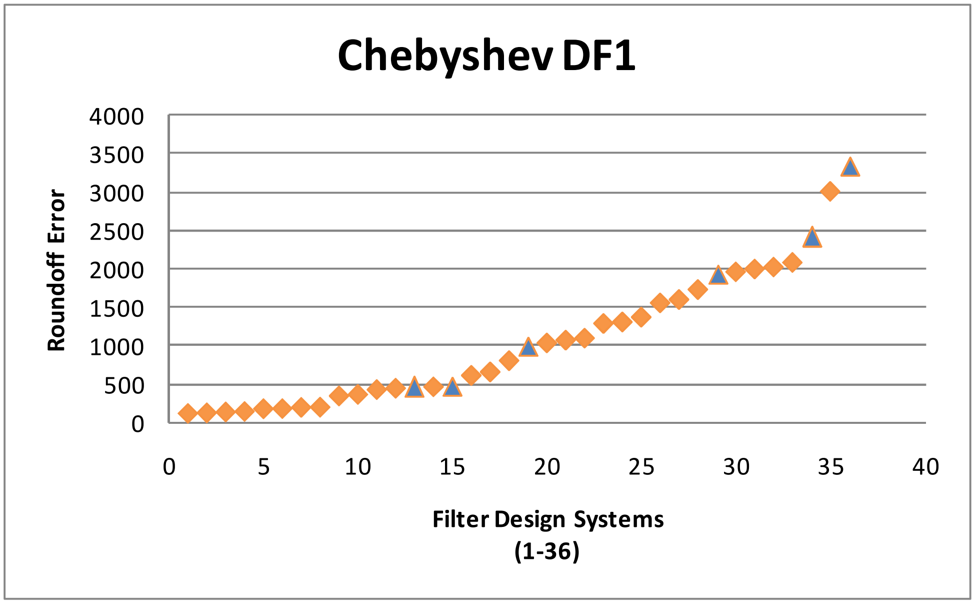


Figure 6.2.3 Roundoff Error for Chebyshev DF1 with .2π Cutoff Frequency

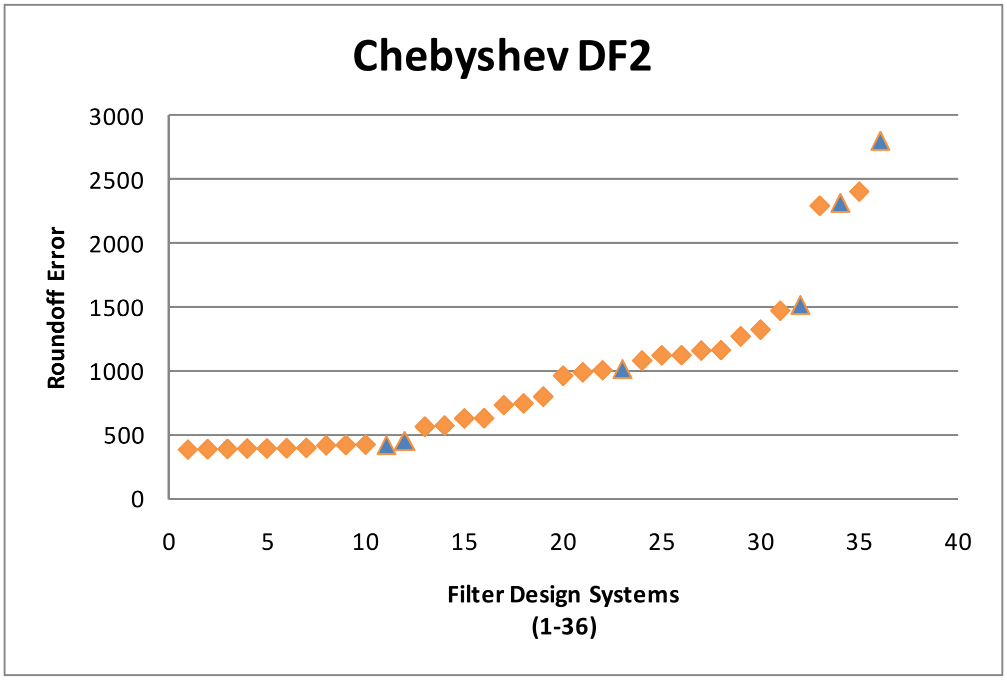


Figure 6.2.4 Roundoff Error for Chebyshev DF2 with .2π Cutoff Frequency

Table 6.2.3 Roundoff Error for Chebyshev DF1 with .2π Cutoff Frequency

Chebyshev DF1												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D1	0.912	1.90512	N2	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	124.75
N3	D3	0.97768	1.88508	N2	D1	0.912	1.90512	N1	D2	0.93761	1.88733	132.393
N2	D1	0.912	1.90512	N3	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	142.609
N2	D3	0.97768	1.88508	N3	D1	0.912	1.90512	N1	D2	0.93761	1.88733	150.253
N3	D2	0.93761	1.88733	N2	D1	0.912	1.90512	N1	D3	0.97768	1.88508	185.441
N3	D1	0.912	1.90512	N2	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	187.581
N2	D2	0.93761	1.88733	N3	D1	0.912	1.90512	N1	D3	0.97768	1.88508	203.301
N2	D1	0.912	1.90512	N3	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	205.441
N3	D2	0.93761	1.88733	N1	D1	0.912	1.90512	N2	D3	0.97768	1.88508	352.117
N3	D1	0.912	1.90512	N1	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	371.17
N3	D1	0.912	1.90512	N1	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	434.002
N3	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	N1	D1	0.912	1.90512	452.164
N3	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	N1	D1	0.912	1.90512	461.947
N2	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	N1	D1	0.912	1.90512	470.023
N2	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	N1	D1	0.912	1.90512	479.807
N3	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	N2	D1	0.912	1.90512	618.839
N3	D3	0.97768	1.88508	N1	D1	0.912	1.90512	N2	D2	0.93761	1.88733	663.585
N2	D2	0.93761	1.88733	N1	D1	0.912	1.90512	N3	D3	0.97768	1.88508	810.692
N3	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	N2	D1	0.912	1.90512	993.139
N2	D1	0.912	1.90512	N1	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	1040.6
N2	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	N3	D1	0.912	1.90512	1077.41
N2	D1	0.912	1.90512	N1	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	1103.44
N1	D2	0.93761	1.88733	N3	D1	0.912	1.90512	N2	D3	0.97768	1.88508	1293.83
N1	D1	0.912	1.90512	N3	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	1312.88
N1	D1	0.912	1.90512	N3	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	1375.71
N1	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	N2	D1	0.912	1.90512	1560.55
N1	D3	0.97768	1.88508	N3	D1	0.912	1.90512	N2	D2	0.93761	1.88733	1605.29
N1	D2	0.93761	1.88733	N2	D1	0.912	1.90512	N3	D3	0.97768	1.88508	1734.54
N1	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	N2	D1	0.912	1.90512	1934.85
N1	D1	0.912	1.90512	N2	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	1964.45
N1	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	N3	D1	0.912	1.90512	2001.26
N1	D1	0.912	1.90512	N2	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	2027.28
N2	D3	0.97768	1.88508	N1	D1	0.912	1.90512	N3	D2	0.93761	1.88733	2086
N2	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	N3	D1	0.912	1.90512	2415.55
N1	D3	0.97768	1.88508	N2	D1	0.912	1.90512	N3	D2	0.93761	1.88733	3009.85
N1	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	N3	D1	0.912	1.90512	3339.4

Table 6.2.4 Roundoff Error for Chebyshev DF2 with .2π Cutoff Frequency

Chebyshev DF2												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D2	0.93761	1.88733	N2	D1	0.912	1.90512	N1	D3	0.97768	1.88508	379.524
N3	D1	0.912	1.90512	N2	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	383.357
N3	D1	0.912	1.90512	N2	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	384.722
N3	D2	0.93761	1.88733	N1	D1	0.912	1.90512	N2	D3	0.97768	1.88508	388.549
N3	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	N1	D1	0.912	1.90512	388.55
N3	D1	0.912	1.90512	N1	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	389.641
N3	D1	0.912	1.90512	N1	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	393.747
N3	D3	0.97768	1.88508	N2	D1	0.912	1.90512	N1	D2	0.93761	1.88733	412.683
N3	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	N2	D1	0.912	1.90512	415.695
N3	D3	0.97768	1.88508	N1	D1	0.912	1.90512	N2	D2	0.93761	1.88733	418.967
N3	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	N1	D1	0.912	1.90512	423.074
N3	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	N2	D1	0.912	1.90512	450.219
N2	D2	0.93761	1.88733	N3	D1	0.912	1.90512	N1	D3	0.97768	1.88508	558.688
N2	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	N1	D1	0.912	1.90512	567.714
N2	D1	0.912	1.90512	N3	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	624.106
N2	D1	0.912	1.90512	N3	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	625.471
N2	D2	0.93761	1.88733	N1	D1	0.912	1.90512	N3	D3	0.97768	1.88508	727.885
N2	D1	0.912	1.90512	N1	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	741.918
N2	D1	0.912	1.90512	N1	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	794.668
N1	D2	0.93761	1.88733	N3	D1	0.912	1.90512	N2	D3	0.97768	1.88508	957.769
N1	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	N2	D1	0.912	1.90512	984.915
N2	D3	0.97768	1.88508	N3	D1	0.912	1.90512	N1	D2	0.93761	1.88733	1000.86
N2	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	N1	D1	0.912	1.90512	1011.25
N2	D2	0.93761	1.88733	N1	D3	0.97768	1.88508	N3	D1	0.912	1.90512	1076.64
N1	D2	0.93761	1.88733	N2	D1	0.912	1.90512	N3	D3	0.97768	1.88508	1117.94
N2	D3	0.97768	1.88508	N1	D1	0.912	1.90512	N3	D2	0.93761	1.88733	1118.67
N1	D1	0.912	1.90512	N3	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	1154.52
N1	D1	0.912	1.90512	N3	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	1158.63
N1	D1	0.912	1.90512	N2	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	1266.05
N1	D1	0.912	1.90512	N2	D2	0.93761	1.88733	N3	D3	0.97768	1.88508	1318.8
N1	D2	0.93761	1.88733	N2	D3	0.97768	1.88508	N3	D1	0.912	1.90512	1466.69
N2	D3	0.97768	1.88508	N1	D2	0.93761	1.88733	N3	D1	0.912	1.90512	1520.17
N1	D3	0.97768	1.88508	N3	D1	0.912	1.90512	N2	D2	0.93761	1.88733	2287.64
N1	D3	0.97768	1.88508	N3	D2	0.93761	1.88733	N2	D1	0.912	1.90512	2318.89
N1	D3	0.97768	1.88508	N2	D1	0.912	1.90512	N3	D2	0.93761	1.88733	2399.17
N1	D3	0.97768	1.88508	N2	D2	0.93761	1.88733	N3	D1	0.912	1.90512	2800.67

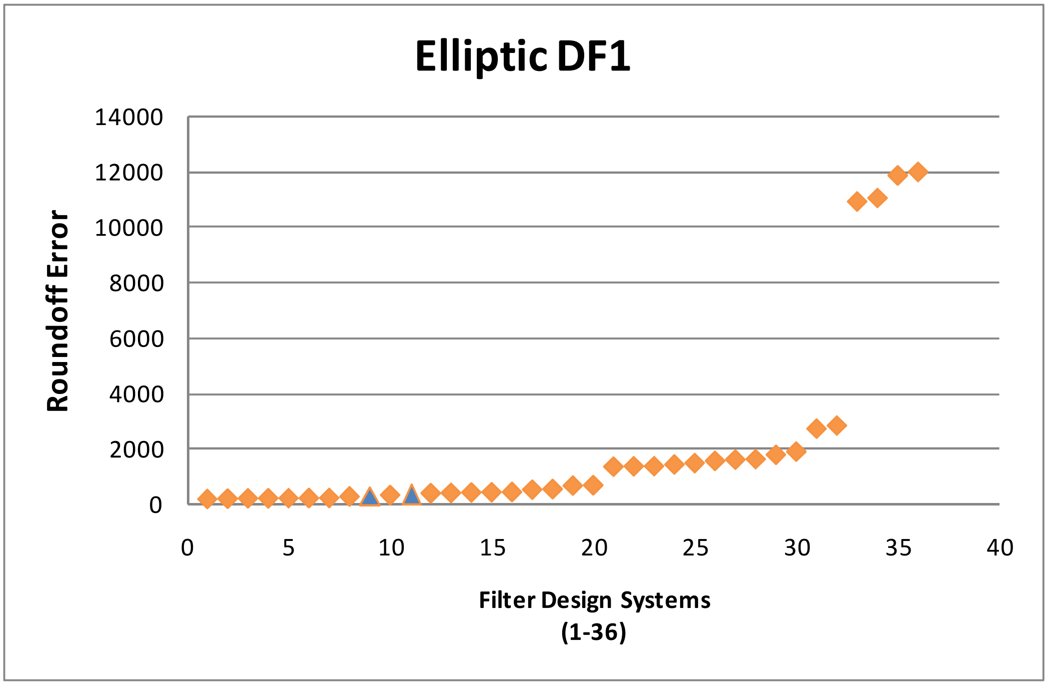


Figure 6.2.5 Roundoff Error for Elliptic DF1 with .2π Cutoff Frequency

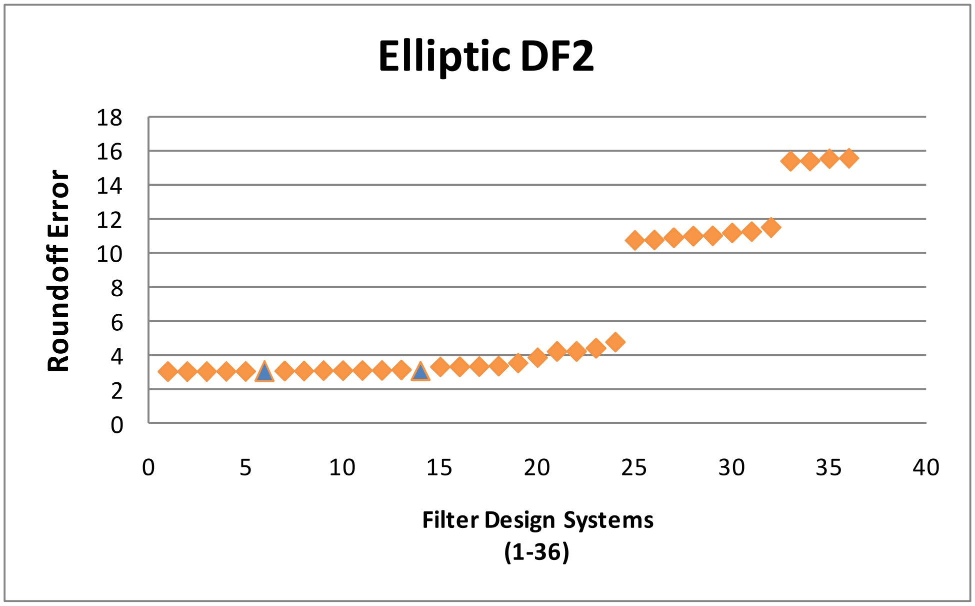


Figure 6.2.6 Roundoff Error for Elliptic DF2 with .2π Cutoff Frequency

Table 6.2.5 Roundoff Error for Elliptic DF1 with .2π Cutoff Frequency

EllipticDF1												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N2	D3	0.99908	0.02261	N3	D2	0.98599	0.02194	N1	D1	0.85759	0.5667	156.404
N2	D2	0.98599	0.03925	N1	D3	0.99908	0.38475	N3	D1	0.85759	0.24573	171.503
N2	D3	0.99908	0.02261	N1	D2	0.98599	0.39642	N3	D1	0.85759	0.24573	181.866
N1	D2	0.98599	0.39642	N2	D3	0.99908	0.02261	N3	D1	0.85759	0.24573	187.251
N1	D2	0.98599	0.39642	N3	D3	0.99908	0.00284	N2	D1	0.85759	0.26088	189.497
N1	D3	0.99908	0.38475	N3	D2	0.98599	0.02194	N2	D1	0.85759	0.26088	192.205
N1	D3	0.99908	0.38475	N2	D2	0.98599	0.03925	N3	D1	0.85759	0.24573	197.614
N2	D2	0.98599	0.03925	N3	D3	0.99908	0.00284	N1	D1	0.85759	0.5667	249.308
N3	D3	0.99908	0.00284	N2	D2	0.98599	0.03925	N1	D1	0.85759	0.5667	284.409
N3	D2	0.98599	0.02194	N1	D3	0.99908	0.38475	N2	D1	0.85759	0.26088	301.754
N3	D3	0.99908	0.00284	N1	D2	0.98599	0.39642	N2	D1	0.85759	0.26088	304.462
N2	D3	0.99908	0.02261	N3	D1	0.85759	0.24573	N1	D2	0.98599	0.39642	370.026
N3	D2	0.98599	0.02194	N2	D3	0.99908	0.02261	N1	D1	0.85759	0.5667	377.313
N2	D3	0.99908	0.02261	N1	D1	0.85759	0.5667	N3	D2	0.98599	0.02194	395.487
N1	D3	0.99908	0.38475	N3	D1	0.85759	0.24573	N2	D2	0.98599	0.03925	405.826
N1	D3	0.99908	0.38475	N2	D1	0.85759	0.26088	N3	D2	0.98599	0.02194	411.235
N3	D3	0.99908	0.00284	N2	D1	0.85759	0.26088	N1	D2	0.98599	0.39642	498.031
N3	D3	0.99908	0.00284	N1	D1	0.85759	0.5667	N2	D2	0.98599	0.03925	518.083
N2	D1	0.85759	0.26088	N1	D3	0.99908	0.38475	N3	D2	0.98599	0.02194	641.695
N1	D1	0.85759	0.5667	N2	D3	0.99908	0.02261	N3	D2	0.98599	0.02194	657.443
N2	D2	0.98599	0.03925	N1	D1	0.85759	0.5667	N3	D3	0.99908	0.00284	1326.25
N1	D2	0.98599	0.39642	N2	D1	0.85759	0.26088	N3	D3	0.99908	0.00284	1342
N1	D2	0.98599	0.39642	N3	D1	0.85759	0.24573	N2	D3	0.99908	0.02261	1344.24
N2	D2	0.98599	0.03925	N3	D1	0.85759	0.24573	N1	D3	0.99908	0.38475	1404.05
N3	D2	0.98599	0.02194	N1	D1	0.85759	0.5667	N2	D3	0.99908	0.02261	1456.5
N3	D2	0.98599	0.02194	N2	D1	0.85759	0.26088	N1	D3	0.99908	0.38475	1532.06
N2	D1	0.85759	0.26088	N1	D2	0.98599	0.39642	N3	D3	0.99908	0.00284	1582.82
N1	D1	0.85759	0.5667	N2	D2	0.98599	0.03925	N3	D3	0.99908	0.00284	1598.57
N1	D1	0.85759	0.5667	N3	D3	0.99908	0.00284	N2	D2	0.98599	0.03925	1756.98
N3	D1	0.85759	0.24573	N1	D3	0.99908	0.38475	N2	D2	0.98599	0.03925	1869.24
N1	D1	0.85759	0.5667	N3	D2	0.98599	0.02194	N2	D3	0.99908	0.02261	2698.11
N3	D1	0.85759	0.24573	N1	D2	0.98599	0.39642	N2	D3	0.99908	0.02261	2810.37
N2	D1	0.85759	0.26088	N3	D3	0.99908	0.00284	N1	D2	0.98599	0.39642	10894.4
N3	D1	0.85759	0.24573	N2	D3	0.99908	0.02261	N1	D2	0.98599	0.39642	11022.4
N2	D1	0.85759	0.26088	N3	D2	0.98599	0.02194	N1	D3	0.99908	0.38475	11835.5
N3	D1	0.85759	0.24573	N2	D2	0.98599	0.03925	N1	D3	0.99908	0.38475	11963.5

Table 6.2.6 Roundoff Error for Elliptic DF2 with .2π Cutoff Frequency

EllipticDF2												
H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.99908	0.00284	N1	D1	0.85759	0.5667	N2	D2	0.98599	0.03925	3.02349
N3	D2	0.98599	0.02194	N1	D1	0.85759	0.5667	N2	D3	0.99908	0.02261	3.02461
N3	D1	0.85759	0.24573	N1	D3	0.99908	0.38475	N2	D2	0.98599	0.03925	3.03115
N3	D1	0.85759	0.24573	N1	D2	0.98599	0.39642	N2	D3	0.99908	0.02261	3.03233
N3	D2	0.98599	0.02194	N1	D3	0.99908	0.38475	N2	D1	0.85759	0.26088	3.03553
N3	D3	0.99908	0.00284	N1	D2	0.98599	0.39642	N2	D1	0.85759	0.26088	3.03559
N1	D1	0.85759	0.5667	N3	D3	0.99908	0.00284	N2	D2	0.98599	0.03925	3.05798
N1	D1	0.85759	0.5667	N3	D2	0.98599	0.02194	N2	D3	0.99908	0.02261	3.05917
N1	D2	0.98599	0.39642	N3	D1	0.85759	0.24573	N2	D3	0.99908	0.02261	3.07735
N1	D3	0.99908	0.38475	N3	D1	0.85759	0.24573	N2	D2	0.98599	0.03925	3.08109
N1	D2	0.98599	0.39642	N3	D3	0.99908	0.00284	N2	D1	0.85759	0.26088	3.08826
N1	D3	0.99908	0.38475	N3	D2	0.98599	0.02194	N2	D1	0.85759	0.26088	3.09319
N3	D2	0.98599	0.02194	N2	D3	0.99908	0.02261	N1	D1	0.85759	0.5667	3.12429
N3	D3	0.99908	0.00284	N2	D2	0.98599	0.03925	N1	D1	0.85759	0.5667	3.12436
N3	D3	0.99908	0.00284	N2	D1	0.85759	0.26088	N1	D2	0.98599	0.39642	3.3005
N3	D1	0.85759	0.24573	N2	D3	0.99908	0.02261	N1	D2	0.98599	0.39642	3.30815
N2	D2	0.98599	0.03925	N3	D3	0.99908	0.00284	N1	D1	0.85759	0.5667	3.31753
N2	D3	0.99908	0.02261	N3	D2	0.98599	0.02194	N1	D1	0.85759	0.5667	3.35013
N2	D3	0.99908	0.02261	N3	D1	0.85759	0.24573	N1	D2	0.98599	0.39642	3.52627
N2	D1	0.85759	0.26088	N3	D3	0.99908	0.00284	N1	D2	0.98599	0.39642	3.84588
N3	D2	0.98599	0.02194	N2	D1	0.85759	0.26088	N1	D3	0.99908	0.38475	4.20503
N3	D1	0.85759	0.24573	N2	D2	0.98599	0.03925	N1	D3	0.99908	0.38475	4.21275
N2	D2	0.98599	0.03925	N3	D1	0.85759	0.24573	N1	D3	0.99908	0.38475	4.39827
N2	D1	0.85759	0.26088	N3	D2	0.98599	0.02194	N1	D3	0.99908	0.38475	4.75049
N1	D1	0.85759	0.5667	N2	D2	0.98599	0.03925	N3	D3	0.99908	0.00284	10.7216
N1	D2	0.98599	0.39642	N2	D1	0.85759	0.26088	N3	D3	0.99908	0.00284	10.7397
N2	D2	0.98599	0.03925	N1	D1	0.85759	0.5667	N3	D3	0.99908	0.00284	10.8802
N1	D1	0.85759	0.5667	N2	D3	0.99908	0.02261	N3	D2	0.98599	0.02194	10.9707
N1	D3	0.99908	0.38475	N2	D1	0.85759	0.26088	N3	D2	0.98599	0.02194	10.9938
N2	D3	0.99908	0.02261	N1	D1	0.85759	0.5667	N3	D2	0.98599	0.02194	11.162
N2	D1	0.85759	0.26088	N1	D2	0.98599	0.39642	N3	D3	0.99908	0.00284	11.2325
N2	D1	0.85759	0.26088	N1	D3	0.99908	0.38475	N3	D2	0.98599	0.02194	11.4816
N1	D2	0.98599	0.39642	N2	D3	0.99908	0.02261	N3	D1	0.85759	0.24573	15.3666
N1	D3	0.99908	0.38475	N2	D2	0.98599	0.03925	N3	D1	0.85759	0.24573	15.3715
N2	D2	0.98599	0.03925	N1	D3	0.99908	0.38475	N3	D1	0.85759	0.24573	15.5071
N2	D3	0.99908	0.02261	N1	D2	0.98599	0.39642	N3	D1	0.85759	0.24573	15.5397

6.3 Mid Cutoff Filter Design for All Data: Showing “Rule of Thumbs”

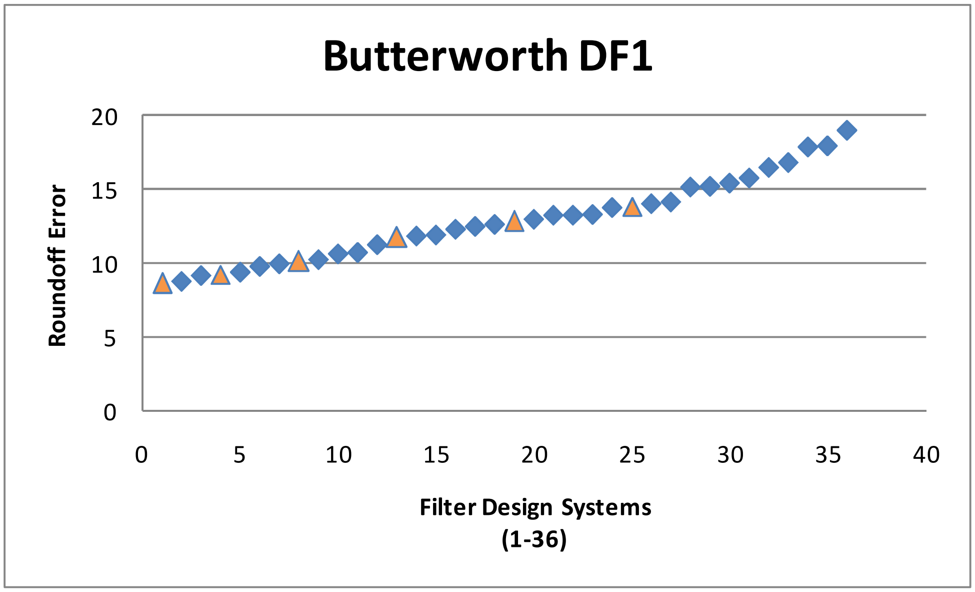


Figure 6.3.1 Roundoff Error for Butterworth DF1 with .5π Cutoff Frequency

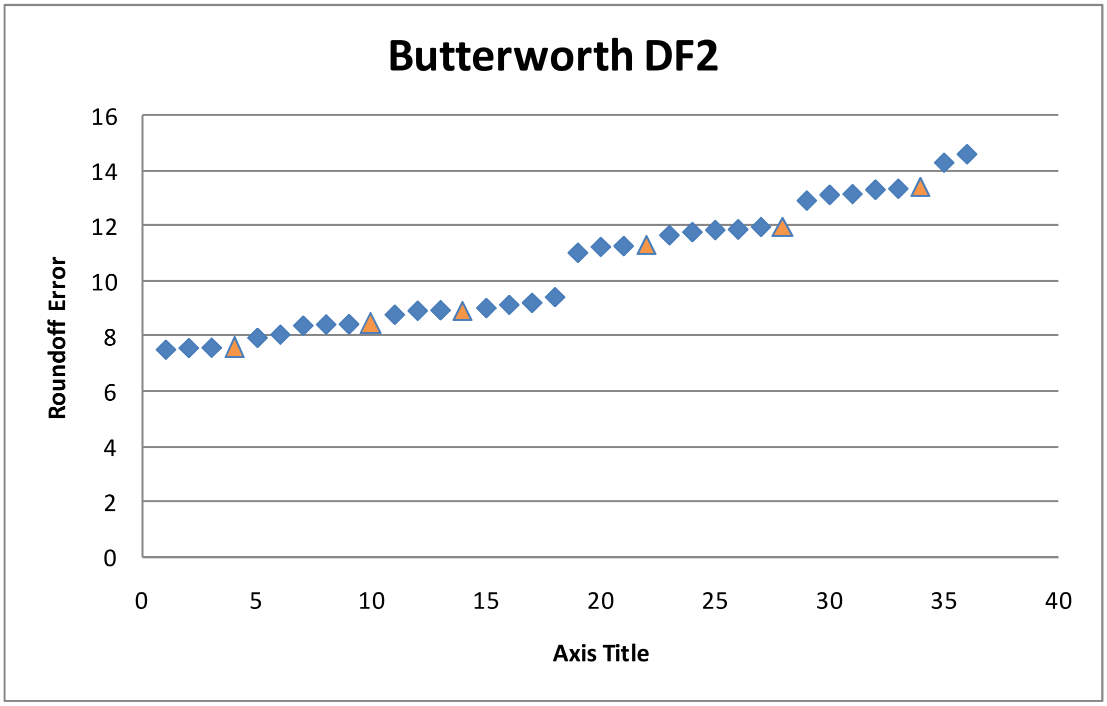
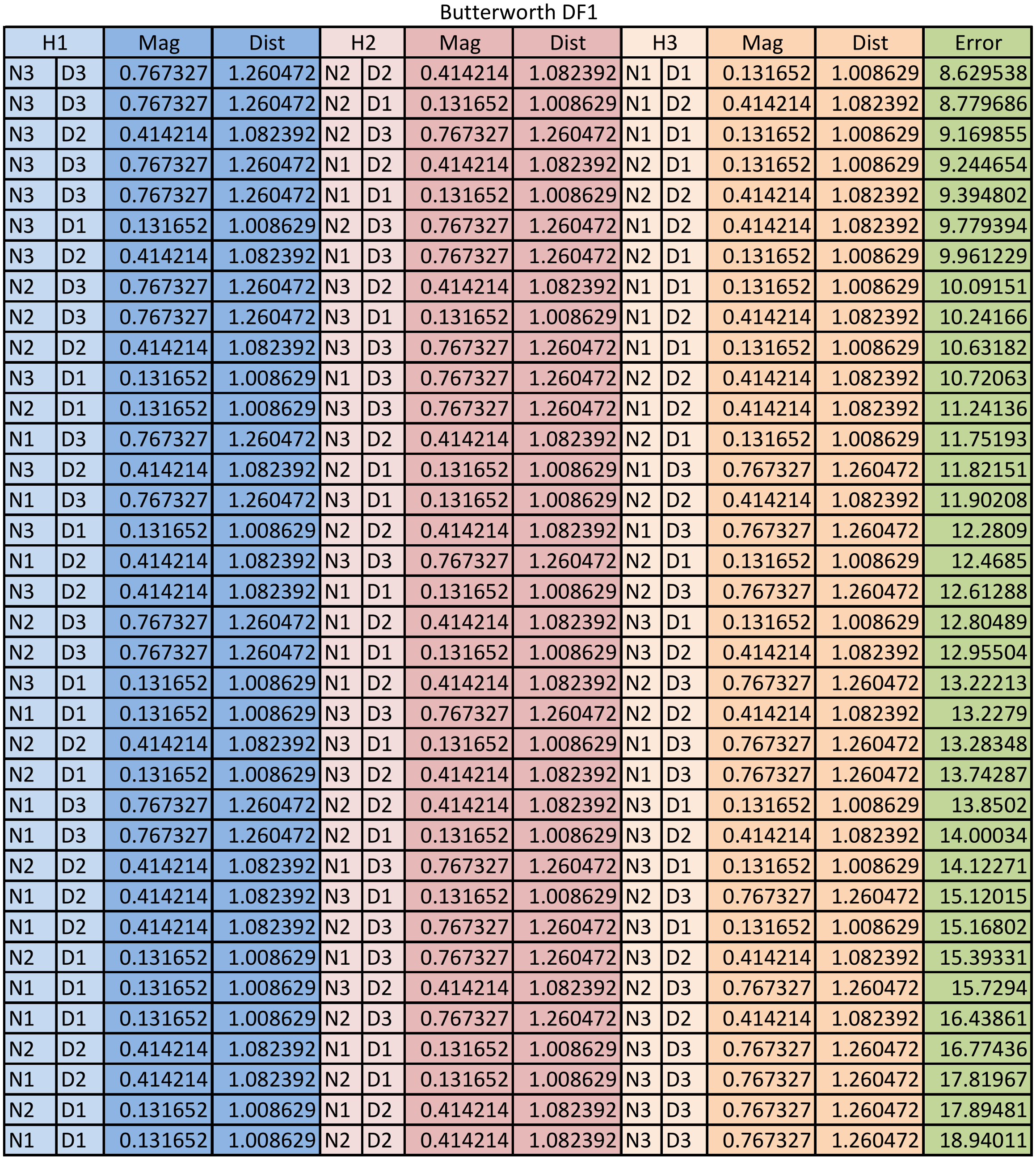


Figure 6.3.2 Roundoff Error for Butterworth DF2 with .5π Cutoff Frequency

Table 6.3.1 Roundoff Error for Butterworth DF1 with .5π Cutoff Frequency



Butterworth DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	8.629538
N3	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	8.779686
N3	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	9.169855
N3	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	9.244654
N3	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	9.394802
N3	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	9.779394
N3	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	9.961229
N2	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	10.09151
N2	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	10.24166
N2	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	10.63182
N3	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	10.72063
N2	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	11.24136
N1	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	11.75193
N3	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	11.82151
N1	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	11.90208
N3	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	12.2809
N1	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	12.4685
N3	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	12.61288
N2	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	12.80489
N2	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	12.95504
N3	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	13.22213
N1	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	13.2279
N2	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	13.28348
N2	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	13.74287
N1	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	13.8502
N1	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	14.00034
N2	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	14.12271
N1	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	15.12015
N1	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	15.16802
N2	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	15.39331
N1	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	15.7294
N1	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	16.43861
N2	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	16.77436
N1	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	17.81967
N2	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	17.89481
N1	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	18.94011

Table 6.3.2 Roundoff Error for Butterworth DF2 with .5π Cutoff Frequency

Butterworth DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	7.485516
N3	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	7.546715
N3	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	7.558919
N3	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	7.589315
N3	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	7.919872
N3	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	8.035875
N3	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	8.362043
N3	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	8.402869
N3	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	8.415072
N3	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	8.465843
N1	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	8.762994
N1	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	8.906188
N1	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	8.928031
N1	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	8.948788
N3	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	9.004137
N3	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	9.12014
N1	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	9.19735
N1	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	9.404988
N2	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	11.02395
N2	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	11.23563
N2	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	11.26792
N2	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	11.2986
N2	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	11.66604
N1	D2	0.414214	1.082392	N2	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	11.77964
N1	D3	0.767327	1.260472	N2	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	11.85271
N1	D1	0.131652	1.008629	N2	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	11.87455
N1	D3	0.767327	1.260472	N2	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	11.96543
N2	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	11.97299
N1	D2	0.414214	1.082392	N2	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	12.92894
N1	D1	0.131652	1.008629	N2	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	13.13658
N2	D2	0.414214	1.082392	N1	D3	0.767327	1.260472	N3	D1	0.131652	1.008629	13.16406
N2	D3	0.767327	1.260472	N1	D1	0.131652	1.008629	N3	D2	0.414214	1.082392	13.326
N2	D1	0.131652	1.008629	N1	D3	0.767327	1.260472	N3	D2	0.414214	1.082392	13.35829
N2	D3	0.767327	1.260472	N1	D2	0.414214	1.082392	N3	D1	0.131652	1.008629	13.43872
N2	D2	0.414214	1.082392	N1	D1	0.131652	1.008629	N3	D3	0.767327	1.260472	14.31337
N2	D1	0.131652	1.008629	N1	D2	0.414214	1.082392	N3	D3	0.767327	1.260472	14.62031

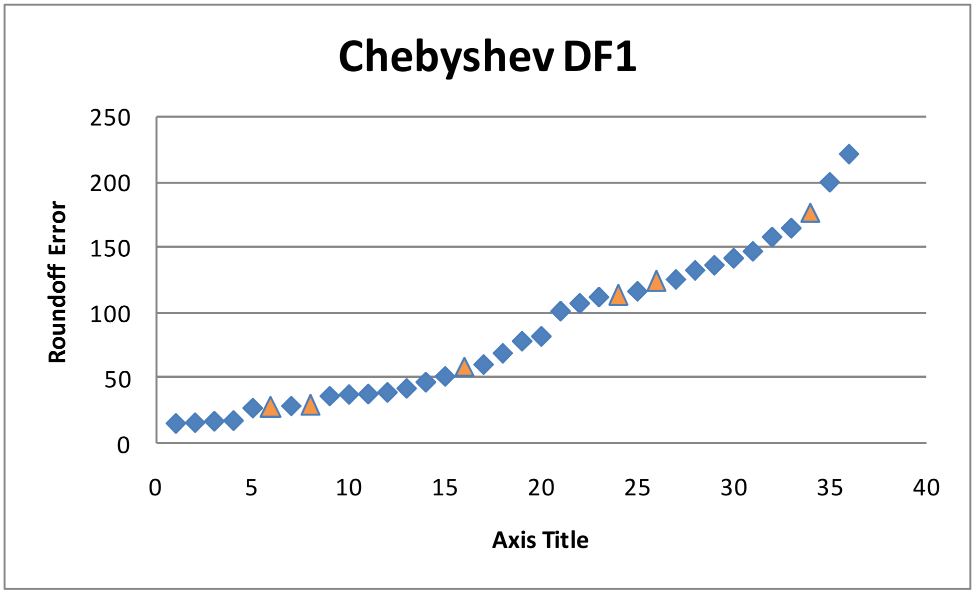


Figure 6.3.3 Roundoff Error for Chebyshev DF1 with .5π Cutoff Frequency

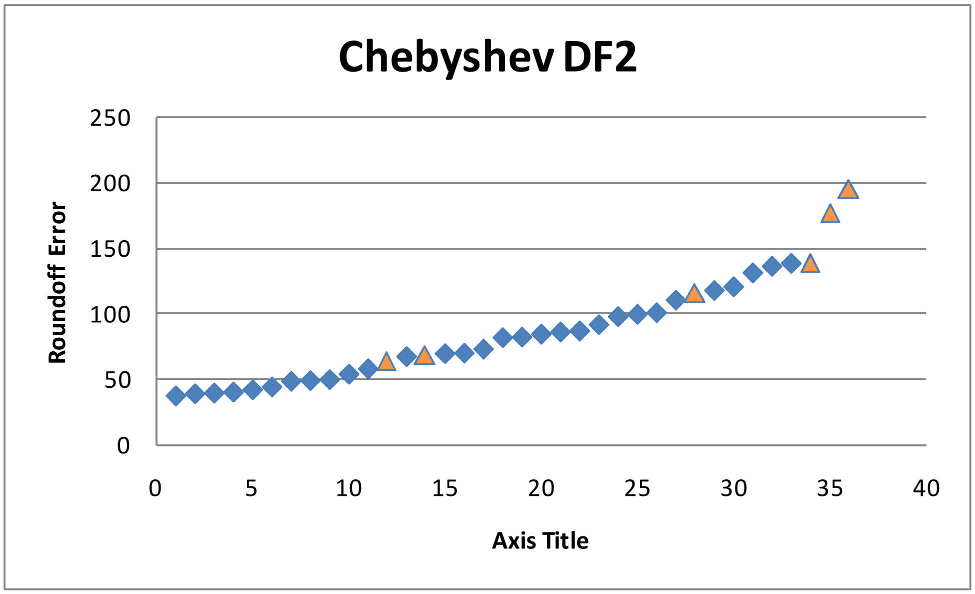


Figure 6.3.4 Roundoff Error for Chebyshev DF2 with .5π Cutoff Frequency

Table 6.3.3 Roundoff Error for Chebyshev DF1 with .5π Cutoff Frequency

Chebyshev DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	15.38156
N3	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	15.91776
N2	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	17.00267
N2	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	17.53887
N3	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	26.98797
N3	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	27.66779
N2	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	28.60908
N2	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	29.2889
N3	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	36.15941
N3	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	37.37543
N2	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	37.78052
N2	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	38.99654
N3	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	42.01277
N3	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	46.74988
N3	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	51.18421
N3	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	59.03611
N3	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	60.17665
N2	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	68.79423
N2	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	77.96567
N3	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	81.63432
N2	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	100.9
N1	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	106.8478
N1	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	111.5849
N2	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	113.1862
N1	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	116.0192
N1	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	123.8711
N1	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	125.0117
N1	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	132.0081
N2	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	135.9132
N1	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	141.1796
N1	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	146.4693
N2	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	157.3709
N1	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	164.1139
N1	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	176.4001
N1	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	199.1271
N1	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	220.5848

Table 6.3.4 Roundoff Error for Chebyshev DF2 with .5π Cutoff Frequency

Chebyshev DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	37.63624
N3	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	39.14949
N3	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	39.74893
N3	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	40.50396
N3	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	42.27326
N3	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	44.38595
N3	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	48.63359
N3	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	49.19619
N3	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	49.98807
N3	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	54.11476
N1	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	58.11754
N3	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	63.31731
N2	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	67.42309
N3	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	68.23588
N1	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	69.67749
N1	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	70.00737
N1	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	73.13114
N2	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	81.78892
N2	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	82.19887
N1	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	84.43561
N2	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	86.08086
N1	D1	0.764651	1.706131	N2	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	86.88023
N2	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	N3	D3	0.961623	1.403279	91.62848
N2	D1	0.764651	1.706131	N1	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	97.71726
N1	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	99.44921
N1	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	N2	D2	0.870981	1.520261	100.7147
N2	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	N3	D3	0.961623	1.403279	110.2862
N1	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	N2	D1	0.764651	1.706131	115.3985
N1	D3	0.961623	1.403279	N2	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	117.5876
N2	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	N1	D2	0.870981	1.520261	120.3597
N1	D2	0.870981	1.520261	N2	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	130.9489
N2	D3	0.961623	1.403279	N1	D1	0.764651	1.706131	N3	D2	0.870981	1.520261	135.8781
N2	D2	0.870981	1.520261	N1	D3	0.961623	1.403279	N3	D1	0.764651	1.706131	138.1418
N2	D3	0.961623	1.403279	N3	D2	0.870981	1.520261	N1	D1	0.764651	1.706131	138.6075
N1	D3	0.961623	1.403279	N2	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	176.6699
N2	D3	0.961623	1.403279	N1	D2	0.870981	1.520261	N3	D1	0.764651	1.706131	194.9604

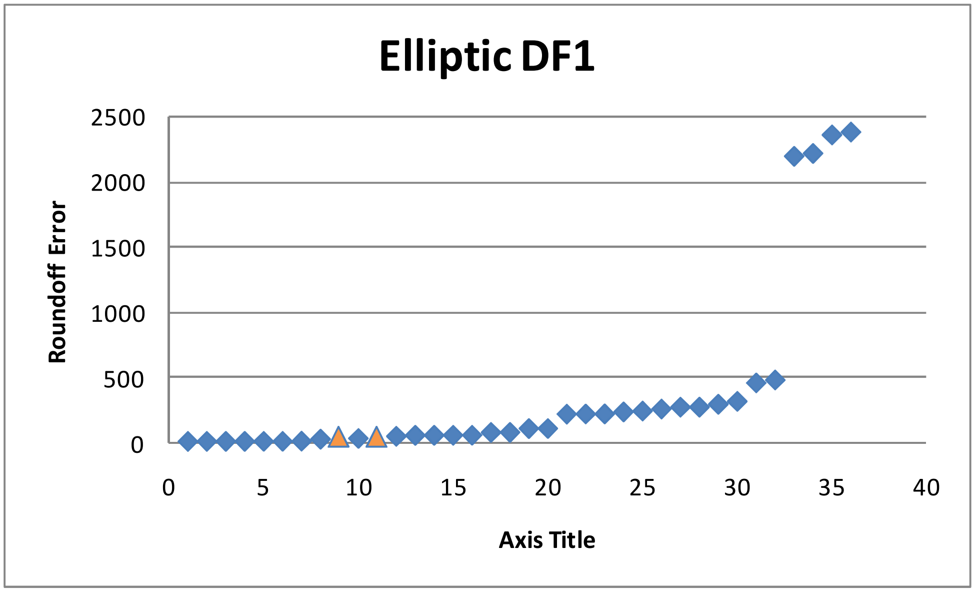


Figure 6.3.5 Roundoff Error for Elliptic DF1 with .5π Cutoff Frequency

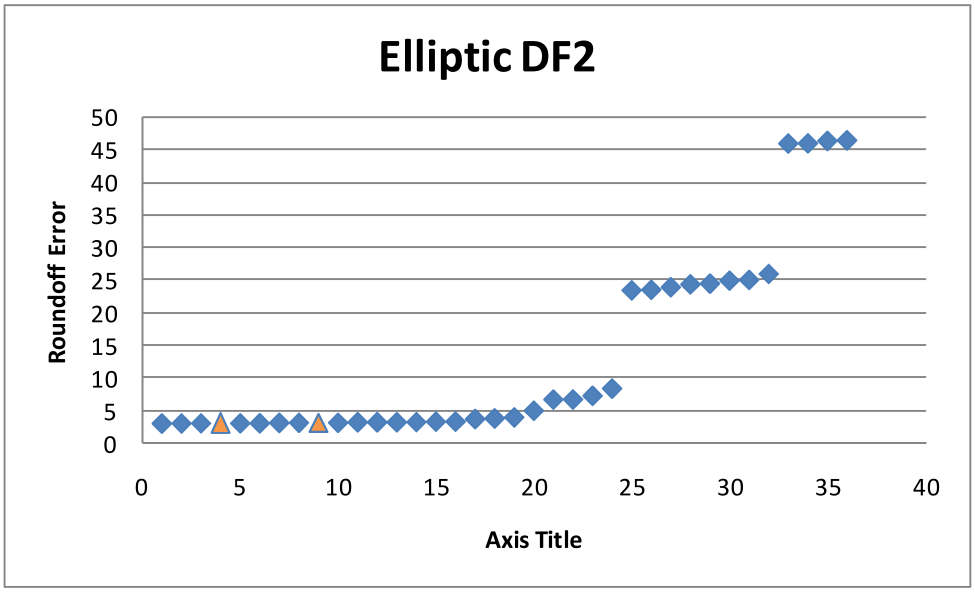


Figure 6.3.6 Roundoff Error for Elliptic DF2 with .5π Cutoff Frequency

Table 6.3.5 Roundoff Error for Elliptic DF1 with .5π Cutoff Frequency

Elliptic DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N1	D2	0.975804	0.526116	N2	D3	0.998435	0.037881	N3	D1	0.706301	0.440442	23.79971
N1	D2	0.975804	0.526116	N3	D3	0.998435	0.004821	N2	D1	0.706301	0.461234	24.33635
N2	D2	0.975804	0.066084	N1	D3	0.998435	0.508097	N3	D1	0.706301	0.440442	24.38849
N1	D3	0.998435	0.508097	N3	D2	0.975804	0.037457	N2	D1	0.706301	0.461234	24.44094
N1	D3	0.998435	0.508097	N2	D2	0.975804	0.066084	N3	D1	0.706301	0.440442	24.85103
N2	D3	0.998435	0.037881	N3	D2	0.975804	0.037457	N1	D1	0.706301	0.789749	24.86775
N2	D3	0.998435	0.037881	N1	D2	0.975804	0.526116	N3	D1	0.706301	0.440442	25.43982
N2	D2	0.975804	0.066084	N3	D3	0.998435	0.004821	N1	D1	0.706301	0.789749	40.89385
N3	D3	0.998435	0.004821	N2	D2	0.975804	0.066084	N1	D1	0.706301	0.789749	46.67542
N3	D2	0.975804	0.037457	N1	D3	0.998435	0.508097	N2	D1	0.706301	0.461234	46.7328
N3	D3	0.998435	0.004821	N1	D2	0.975804	0.526116	N2	D1	0.706301	0.461234	46.8374
N3	D2	0.975804	0.037457	N2	D3	0.998435	0.037881	N1	D1	0.706301	0.789749	62.70153
N1	D3	0.998435	0.508097	N3	D1	0.706301	0.440442	N2	D2	0.975804	0.066084	70.02375
N1	D3	0.998435	0.508097	N2	D1	0.706301	0.461234	N3	D2	0.975804	0.037457	70.43385
N2	D3	0.998435	0.037881	N3	D1	0.706301	0.440442	N1	D2	0.975804	0.526116	70.45056
N2	D3	0.998435	0.037881	N1	D1	0.706301	0.789749	N3	D2	0.975804	0.037457	71.02263
N3	D3	0.998435	0.004821	N2	D1	0.706301	0.461234	N1	D2	0.975804	0.526116	92.25823
N3	D3	0.998435	0.004821	N1	D1	0.706301	0.789749	N2	D2	0.975804	0.066084	92.42021
N1	D1	0.706301	0.789749	N2	D3	0.998435	0.037881	N3	D2	0.975804	0.037457	121.6606
N2	D1	0.706301	0.461234	N1	D3	0.998435	0.508097	N3	D2	0.975804	0.037457	122.2494
N1	D2	0.975804	0.526116	N2	D1	0.706301	0.461234	N3	D3	0.998435	0.004821	231.4855
N1	D2	0.975804	0.526116	N3	D1	0.706301	0.440442	N2	D3	0.998435	0.037881	232.0222
N2	D2	0.975804	0.066084	N1	D1	0.706301	0.789749	N3	D3	0.998435	0.004821	232.0743
N2	D2	0.975804	0.066084	N3	D1	0.706301	0.440442	N1	D3	0.998435	0.508097	248.5797
N3	D2	0.975804	0.037457	N1	D1	0.706301	0.789749	N2	D3	0.998435	0.037881	254.4186
N3	D2	0.975804	0.037457	N2	D1	0.706301	0.461234	N1	D3	0.998435	0.508097	270.3873
N1	D1	0.706301	0.789749	N2	D2	0.975804	0.066084	N3	D3	0.998435	0.004821	283.7636
N2	D1	0.706301	0.461234	N1	D2	0.975804	0.526116	N3	D3	0.998435	0.004821	284.3524
N1	D1	0.706301	0.789749	N3	D3	0.998435	0.004821	N2	D2	0.975804	0.066084	305.97
N3	D1	0.706301	0.440442	N1	D3	0.998435	0.508097	N2	D2	0.975804	0.066084	328.3664
N1	D1	0.706301	0.789749	N3	D2	0.975804	0.037457	N2	D3	0.998435	0.037881	468.073
N3	D1	0.706301	0.440442	N1	D2	0.975804	0.526116	N2	D3	0.998435	0.037881	490.4694
N2	D1	0.706301	0.461234	N3	D3	0.998435	0.004821	N1	D2	0.975804	0.526116	2191.247
N3	D1	0.706301	0.440442	N2	D3	0.998435	0.037881	N1	D2	0.975804	0.526116	2213.054
N2	D1	0.706301	0.461234	N3	D2	0.975804	0.037457	N1	D3	0.998435	0.508097	2353.35
N3	D1	0.706301	0.440442	N2	D2	0.975804	0.066084	N1	D3	0.998435	0.508097	2375.157

Table 6.3.6 Roundoff Error for Elliptic DF2 with .5π Cutoff Frequency

Elliptic DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.998435	0.004821	N1	D1	0.706301	0.789749	N2	D2	0.975804	0.066084	3.026033
N3	D2	0.975804	0.037457	N1	D1	0.706301	0.789749	N2	D3	0.998435	0.037881	3.039955
N3	D1	0.706301	0.440442	N1	D3	0.998435	0.508097	N2	D2	0.975804	0.066084	3.04691
N3	D3	0.998435	0.004821	N1	D2	0.975804	0.526116	N2	D1	0.706301	0.461234	3.048089
N3	D2	0.975804	0.037457	N1	D3	0.998435	0.508097	N2	D1	0.706301	0.461234	3.048127
N3	D1	0.706301	0.440442	N1	D2	0.975804	0.526116	N2	D3	0.998435	0.037881	3.060794
N1	D1	0.706301	0.789749	N3	D3	0.998435	0.004821	N2	D2	0.975804	0.066084	3.126691
N1	D1	0.706301	0.789749	N3	D2	0.975804	0.037457	N2	D3	0.998435	0.037881	3.140575
N3	D3	0.998435	0.004821	N2	D2	0.975804	0.066084	N1	D1	0.706301	0.789749	3.14372
N3	D2	0.975804	0.037457	N2	D3	0.998435	0.037881	N1	D1	0.706301	0.789749	3.143758
N1	D2	0.975804	0.526116	N3	D1	0.706301	0.440442	N2	D3	0.998435	0.037881	3.225821
N1	D3	0.998435	0.508097	N3	D1	0.706301	0.440442	N2	D2	0.975804	0.066084	3.232498
N1	D2	0.975804	0.526116	N3	D3	0.998435	0.004821	N2	D1	0.706301	0.461234	3.233993
N1	D3	0.998435	0.508097	N3	D2	0.975804	0.037457	N2	D1	0.706301	0.461234	3.254554
N3	D3	0.998435	0.004821	N2	D1	0.706301	0.461234	N1	D2	0.975804	0.526116	3.309772
N3	D1	0.706301	0.440442	N2	D3	0.998435	0.037881	N1	D2	0.975804	0.526116	3.330649
N2	D2	0.975804	0.066084	N3	D3	0.998435	0.004821	N1	D1	0.706301	0.789749	3.722987
N2	D3	0.998435	0.037881	N3	D2	0.975804	0.037457	N1	D1	0.706301	0.789749	3.805025
N2	D3	0.998435	0.037881	N3	D1	0.706301	0.440442	N1	D2	0.975804	0.526116	3.971078
N2	D1	0.706301	0.461234	N3	D3	0.998435	0.004821	N1	D2	0.975804	0.526116	4.995467
N3	D2	0.975804	0.037457	N2	D1	0.706301	0.461234	N1	D3	0.998435	0.508097	6.688891
N3	D1	0.706301	0.440442	N2	D2	0.975804	0.066084	N1	D3	0.998435	0.508097	6.70973
N2	D2	0.975804	0.066084	N3	D1	0.706301	0.440442	N1	D3	0.998435	0.508097	7.268121
N2	D1	0.706301	0.461234	N3	D2	0.975804	0.037457	N1	D3	0.998435	0.508097	8.374548
N1	D1	0.706301	0.789749	N2	D2	0.975804	0.066084	N3	D3	0.998435	0.004821	23.38786
N1	D2	0.975804	0.526116	N2	D1	0.706301	0.461234	N3	D3	0.998435	0.004821	23.4731
N2	D2	0.975804	0.066084	N1	D1	0.706301	0.789749	N3	D3	0.998435	0.004821	23.86647
N1	D1	0.706301	0.789749	N2	D3	0.998435	0.037881	N3	D2	0.975804	0.037457	24.31067
N1	D3	0.998435	0.508097	N2	D1	0.706301	0.461234	N3	D2	0.975804	0.037457	24.41648
N2	D3	0.998435	0.037881	N1	D1	0.706301	0.789749	N3	D2	0.975804	0.037457	24.87132
N2	D1	0.706301	0.461234	N1	D2	0.975804	0.526116	N3	D3	0.998435	0.004821	24.9729
N2	D1	0.706301	0.461234	N1	D3	0.998435	0.508097	N3	D2	0.975804	0.037457	25.89571
N1	D2	0.975804	0.526116	N2	D3	0.998435	0.037881	N3	D1	0.706301	0.440442	45.83273
N1	D3	0.998435	0.508097	N2	D2	0.975804	0.066084	N3	D1	0.706301	0.440442	45.85329
N2	D2	0.975804	0.066084	N1	D3	0.998435	0.508097	N3	D1	0.706301	0.440442	46.2261
N2	D3	0.998435	0.037881	N1	D2	0.975804	0.526116	N3	D1	0.706301	0.440442	46.30814

6.4 High Cutoff Filter Design for All Data: Showing “Rule of Thumbs”

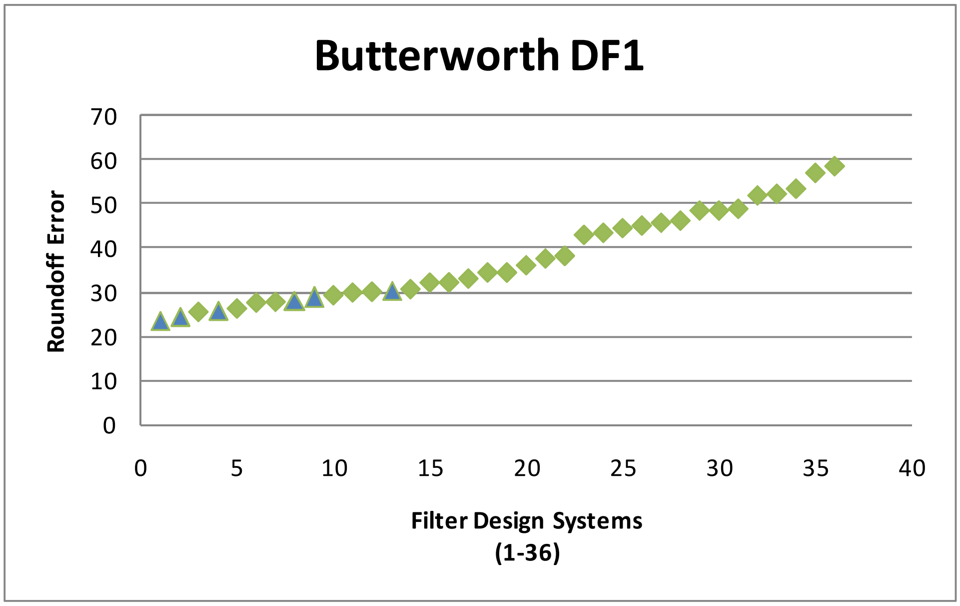


Figure 6.4.1 Roundoff Error for Butterworth DF1 with .8π Cutoff Frequency

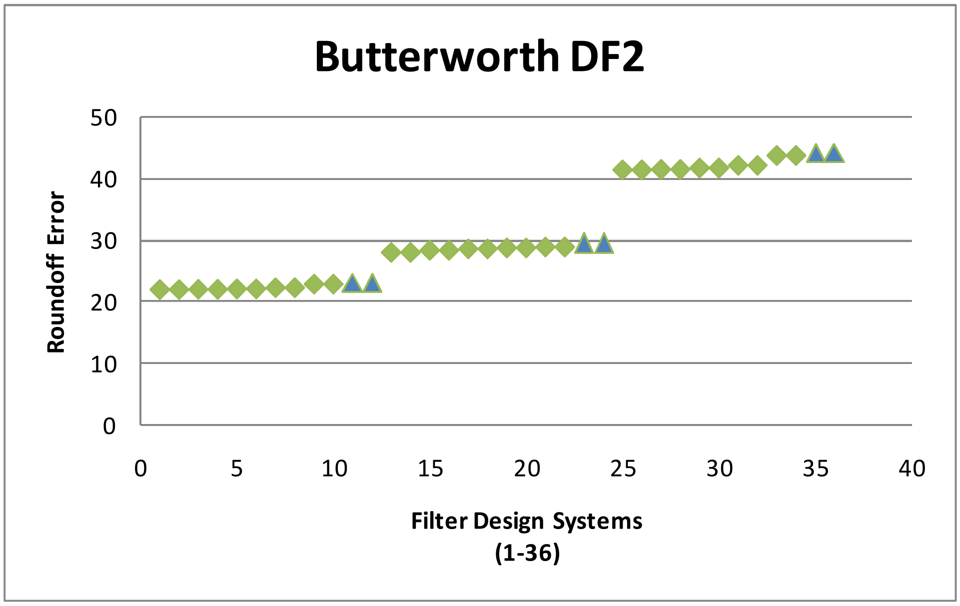


Figure 6.4.2 Roundoff Error for Butterworth DF2 with .8π Cutoff Frequency

Table 6.4.1 Roundoff Error for Butterworth DF1 with .8π Cutoff frequency

Butterworth DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	23.5543
N3	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	24.3249
N3	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	25.3363
N2	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	25.8422
N3	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	26.107
N3	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	27.4598
N2	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	27.6243
N1	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	28.1477
N2	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	28.7402
N3	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	29.115
N2	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	29.7478
N1	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	29.9298
N1	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	30.2749
N2	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	30.5222
N3	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	32.0135
N1	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	32.057
N1	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	32.9378
N3	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	34.2964
N2	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	34.3015
N2	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	35.9718
N1	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	37.5066
N1	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	38.1191
N2	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	42.8858
N3	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	43.3661
N1	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	44.4206
N3	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	45.0213
N2	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	45.6541
N3	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	46.1377
N3	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	48.4205
N2	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	48.4256
N1	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	48.844
N2	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	51.8781
N1	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	52.2433
N1	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	53.4129
N2	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	57.01
N1	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	58.5448

Table 6.4.2 Roundoff Error for Butterworth DF2 with .8π Cutoff Frequency

Butterworth DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N1	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	21.9911
N3	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	21.9911
N1	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	22.0206
N3	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	22.0206
N1	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	22.1125
N3	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	22.1125
N1	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	22.2727
N3	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	22.2727
N1	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	22.8696
N3	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	22.8696
N1	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	23.0594
N3	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	23.0594
N1	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	27.9671
N3	D1	0.52508	0.4936	N2	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	27.9671
N1	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	28.3075
N3	D2	0.6425	0.51944	N2	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	28.3075
N1	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	28.5501
N3	D2	0.6425	0.51944	N2	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	28.5501
N1	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	28.7103
N3	D1	0.52508	0.4936	N2	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	28.7103
N1	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	28.8457
N3	D3	0.85785	0.57579	N2	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	28.8457
N1	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	29.3464
N3	D3	0.85785	0.57579	N2	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	29.3464
N2	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	41.2709
N2	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	41.2709
N2	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	41.3488
N2	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	41.3488
N2	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	N3	D3	0.85785	0.57579	41.5913
N2	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	N1	D3	0.85785	0.57579	41.5913
N2	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	N3	D3	0.85785	0.57579	42.0141
N2	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	N1	D3	0.85785	0.57579	42.0141
N2	D3	0.85785	0.57579	N1	D1	0.52508	0.4936	N3	D2	0.6425	0.51944	43.589
N2	D3	0.85785	0.57579	N3	D1	0.52508	0.4936	N1	D2	0.6425	0.51944	43.589
N2	D3	0.85785	0.57579	N1	D2	0.6425	0.51944	N3	D1	0.52508	0.4936	44.0897
N2	D3	0.85785	0.57579	N3	D2	0.6425	0.51944	N1	D1	0.52508	0.4936	44.0897

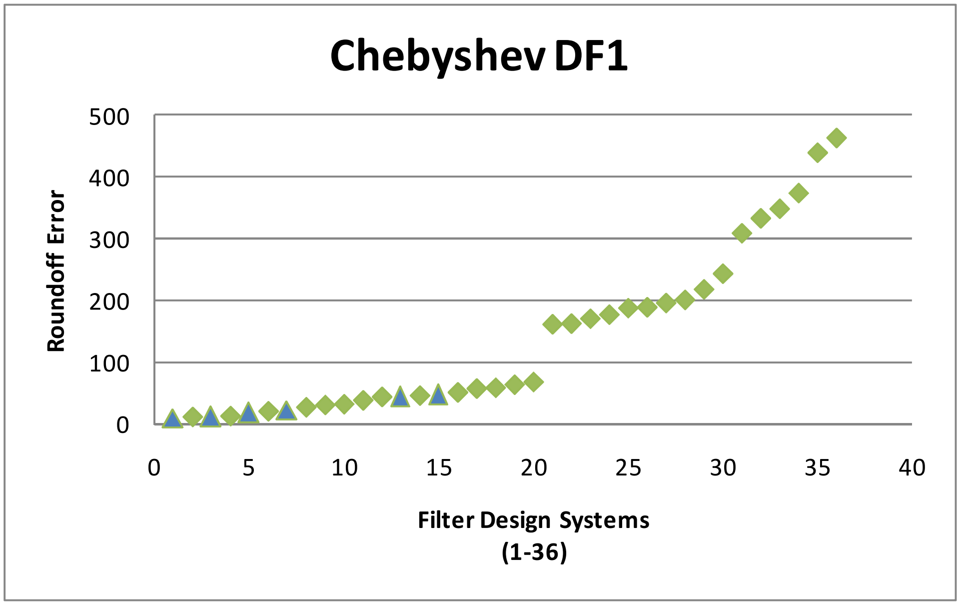


Figure 6.4.3 Roundoff Error for Chebyshev DF1 with .8π Cutoff Frequency

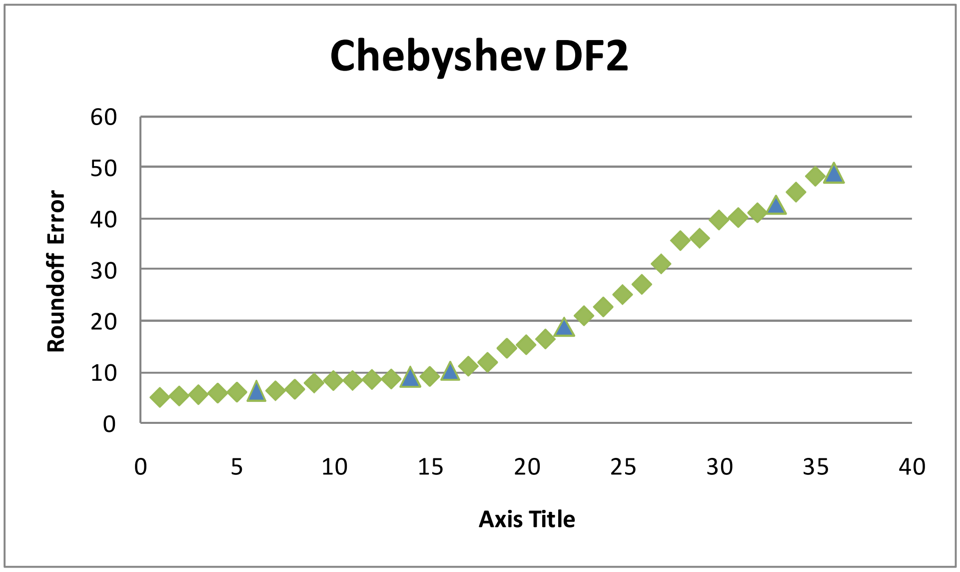


Figure 6.4.4 Roundoff Error for Chebyshev DF1 with .8π Cutoff Frequency

Table 6.4.3 Roundoff Error for Chebyshev DF1 with .8π Cutoff Frequency

Chebyshev DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.97684	0.6238	N2	D2	0.89708	0.77935	N1	D1	0.59507	1.21282	9.91066
N3	D2	0.89708	0.77935	N2	D3	0.97684	0.6238	N1	D1	0.59507	1.21282	10.1804
N2	D3	0.97684	0.6238	N3	D2	0.89708	0.77935	N1	D1	0.59507	1.21282	11.2527
N2	D2	0.89708	0.77935	N3	D3	0.97684	0.6238	N1	D1	0.59507	1.21282	11.5224
N3	D3	0.97684	0.6238	N1	D2	0.89708	0.77935	N2	D1	0.59507	1.21282	17.5399
N3	D2	0.89708	0.77935	N1	D3	0.97684	0.6238	N2	D1	0.59507	1.21282	19.3666
N2	D3	0.97684	0.6238	N1	D2	0.89708	0.77935	N3	D1	0.59507	1.21282	23.1278
N2	D2	0.89708	0.77935	N1	D3	0.97684	0.6238	N3	D1	0.59507	1.21282	25.8208
N3	D3	0.97684	0.6238	N2	D1	0.59507	1.21282	N1	D2	0.89708	0.77935	29.4977
N2	D3	0.97684	0.6238	N3	D1	0.59507	1.21282	N1	D2	0.89708	0.77935	30.8397
N3	D3	0.97684	0.6238	N1	D1	0.59507	1.21282	N2	D2	0.89708	0.77935	37.1269
N2	D3	0.97684	0.6238	N1	D1	0.59507	1.21282	N3	D2	0.89708	0.77935	42.7147
N1	D3	0.97684	0.6238	N3	D2	0.89708	0.77935	N2	D1	0.59507	1.21282	42.9634
N1	D2	0.89708	0.77935	N3	D3	0.97684	0.6238	N2	D1	0.59507	1.21282	44.79
N1	D3	0.97684	0.6238	N2	D2	0.89708	0.77935	N3	D1	0.59507	1.21282	47.2091
N1	D2	0.89708	0.77935	N2	D3	0.97684	0.6238	N3	D1	0.59507	1.21282	49.9022
N3	D1	0.59507	1.21282	N2	D3	0.97684	0.6238	N1	D2	0.89708	0.77935	56.0647
N2	D1	0.59507	1.21282	N3	D3	0.97684	0.6238	N1	D2	0.89708	0.77935	57.4068
N1	D3	0.97684	0.6238	N3	D1	0.59507	1.21282	N2	D2	0.89708	0.77935	62.5503
N1	D3	0.97684	0.6238	N2	D1	0.59507	1.21282	N3	D2	0.89708	0.77935	66.7961
N3	D2	0.89708	0.77935	N2	D1	0.59507	1.21282	N1	D3	0.97684	0.6238	160.266
N2	D2	0.89708	0.77935	N3	D1	0.59507	1.21282	N1	D3	0.97684	0.6238	161.608
N3	D2	0.89708	0.77935	N1	D1	0.59507	1.21282	N2	D3	0.97684	0.6238	169.452
N2	D2	0.89708	0.77935	N1	D1	0.59507	1.21282	N3	D3	0.97684	0.6238	175.907
N3	D1	0.59507	1.21282	N2	D2	0.89708	0.77935	N1	D3	0.97684	0.6238	186.564
N2	D1	0.59507	1.21282	N3	D2	0.89708	0.77935	N1	D3	0.97684	0.6238	187.906
N1	D2	0.89708	0.77935	N3	D1	0.59507	1.21282	N2	D3	0.97684	0.6238	194.876
N1	D2	0.89708	0.77935	N2	D1	0.59507	1.21282	N3	D3	0.97684	0.6238	199.988
N3	D1	0.59507	1.21282	N1	D3	0.97684	0.6238	N2	D2	0.89708	0.77935	217.062
N1	D1	0.59507	1.21282	N3	D3	0.97684	0.6238	N2	D2	0.89708	0.77935	242.486
N2	D1	0.59507	1.21282	N1	D3	0.97684	0.6238	N3	D2	0.89708	0.77935	308.001
N1	D1	0.59507	1.21282	N2	D3	0.97684	0.6238	N3	D2	0.89708	0.77935	332.082
N3	D1	0.59507	1.21282	N1	D2	0.89708	0.77935	N2	D3	0.97684	0.6238	347.561
N1	D1	0.59507	1.21282	N3	D2	0.89708	0.77935	N2	D3	0.97684	0.6238	372.985
N2	D1	0.59507	1.21282	N1	D2	0.89708	0.77935	N3	D3	0.97684	0.6238	438.5
N1	D1	0.59507	1.21282	N2	D2	0.89708	0.77935	N3	D3	0.97684	0.6238	462.581

Table 6.4.4 Roundoff Error for Chebyshev DF1 with .8π Cutoff Frequency

Chebyshev DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.97684	0.623795	N1	D1	0.59507	1.212815	N2	D2	0.897083	0.779348	5.129509
N3	D1	0.59507	1.212815	N1	D3	0.97684	0.623795	N2	D2	0.897083	0.779348	5.405144
N3	D2	0.897083	0.779348	N1	D1	0.59507	1.212815	N2	D3	0.97684	0.623795	5.676485
N3	D2	0.897083	0.779348	N1	D3	0.97684	0.623795	N2	D1	0.59507	1.212815	5.977897
N3	D1	0.59507	1.212815	N1	D2	0.897083	0.779348	N2	D3	0.97684	0.623795	6.166005
N3	D3	0.97684	0.623795	N1	D2	0.897083	0.779348	N2	D1	0.59507	1.212815	6.191782
N3	D3	0.97684	0.623795	N2	D1	0.59507	1.212815	N1	D2	0.897083	0.779348	6.437183
N3	D1	0.59507	1.212815	N2	D3	0.97684	0.623795	N1	D2	0.897083	0.779348	6.712818
N3	D2	0.897083	0.779348	N2	D1	0.59507	1.212815	N1	D3	0.97684	0.623795	7.949485
N1	D2	0.897083	0.779348	N3	D1	0.59507	1.212815	N2	D3	0.97684	0.623795	8.418236
N3	D1	0.59507	1.212815	N2	D2	0.897083	0.779348	N1	D3	0.97684	0.623795	8.439005
N3	D2	0.897083	0.779348	N2	D3	0.97684	0.623795	N1	D1	0.59507	1.212815	8.633305
N1	D2	0.897083	0.779348	N3	D3	0.97684	0.623795	N2	D1	0.59507	1.212815	8.719647
N3	D3	0.97684	0.623795	N2	D2	0.897083	0.779348	N1	D1	0.59507	1.212815	8.847191
N1	D3	0.97684	0.623795	N3	D1	0.59507	1.212815	N2	D2	0.897083	0.779348	9.214549
N1	D3	0.97684	0.623795	N3	D2	0.897083	0.779348	N2	D1	0.59507	1.212815	10.27682
N1	D1	0.59507	1.212815	N3	D3	0.97684	0.623795	N2	D2	0.897083	0.779348	11.22128
N1	D1	0.59507	1.212815	N3	D2	0.897083	0.779348	N2	D3	0.97684	0.623795	11.98214
N2	D2	0.897083	0.779348	N3	D1	0.59507	1.212815	N1	D3	0.97684	0.623795	14.72364
N2	D2	0.897083	0.779348	N3	D3	0.97684	0.623795	N1	D1	0.59507	1.212815	15.40746
N2	D3	0.97684	0.623795	N3	D1	0.59507	1.212815	N1	D2	0.897083	0.779348	16.53026
N2	D3	0.97684	0.623795	N3	D2	0.897083	0.779348	N1	D1	0.59507	1.212815	18.94026
N2	D1	0.59507	1.212815	N3	D3	0.97684	0.623795	N1	D2	0.897083	0.779348	21.08298
N2	D1	0.59507	1.212815	N3	D2	0.897083	0.779348	N1	D3	0.97684	0.623795	22.80917
N1	D3	0.97684	0.623795	N2	D1	0.59507	1.212815	N3	D2	0.897083	0.779348	25.20815
N1	D1	0.59507	1.212815	N2	D3	0.97684	0.623795	N3	D2	0.897083	0.779348	27.21488
N2	D3	0.97684	0.623795	N1	D1	0.59507	1.212815	N3	D2	0.897083	0.779348	31.21618
N2	D1	0.59507	1.212815	N1	D3	0.97684	0.623795	N3	D2	0.897083	0.779348	35.76891
N1	D2	0.897083	0.779348	N2	D1	0.59507	1.212815	N3	D3	0.97684	0.623795	36.21832
N1	D1	0.59507	1.212815	N2	D2	0.897083	0.779348	N3	D3	0.97684	0.623795	39.78223
N2	D2	0.897083	0.779348	N1	D1	0.59507	1.212815	N3	D3	0.97684	0.623795	40.25072
N1	D2	0.897083	0.779348	N2	D3	0.97684	0.623795	N3	D1	0.59507	1.212815	41.19681
N1	D3	0.97684	0.623795	N2	D2	0.897083	0.779348	N3	D1	0.59507	1.212815	42.75399
N2	D2	0.897083	0.779348	N1	D3	0.97684	0.623795	N3	D1	0.59507	1.212815	45.22922
N2	D1	0.59507	1.212815	N1	D2	0.897083	0.779348	N3	D3	0.97684	0.623795	48.33626
N2	D3	0.97684	0.623795	N1	D2	0.897083	0.779348	N3	D1	0.59507	1.212815	48.76202

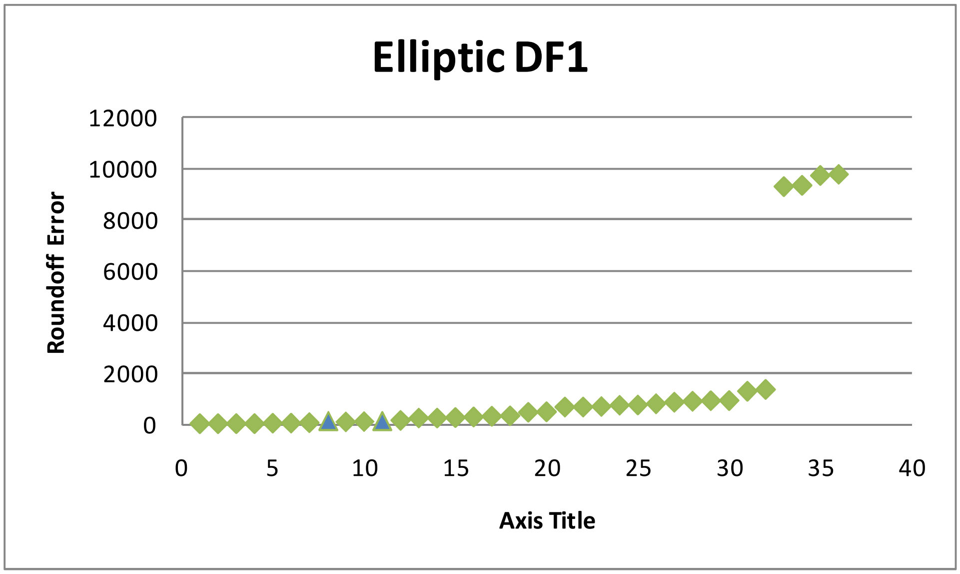


Figure 6.4.5 Roundoff Error for Elliptic DF1 with .8π Cutoff Frequency

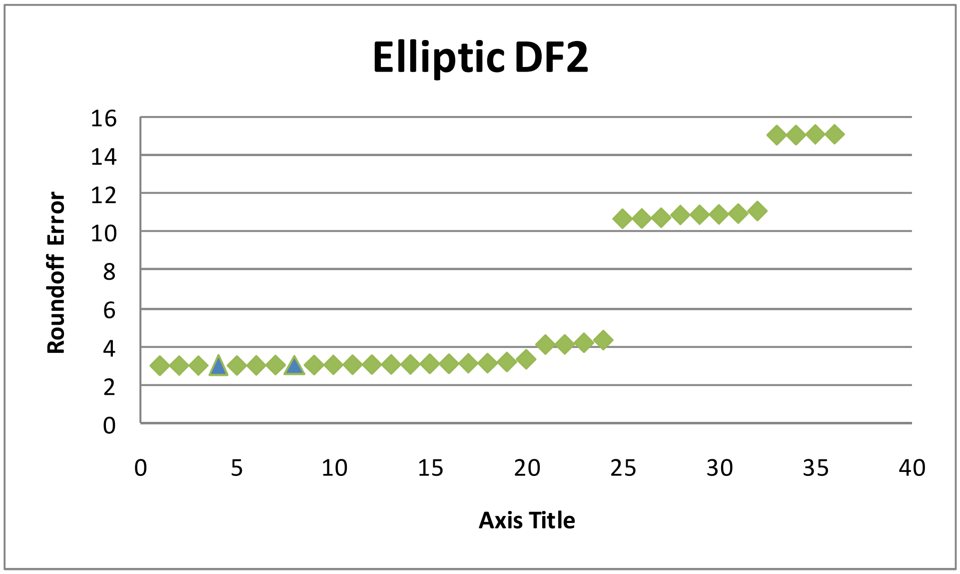


Figure 6.4.6 Roundoff Error for Elliptic DF2 with .8π Cutoff Frequency

Table 6.4.5 Roundoff Error for Elliptic DF1 with .8π Cutoff Frequency

Elliptic DF1

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N1	D2	0.985415	0.26556	N2	D3	0.99908	0.021951	N3	D1	0.752841	0.313465	38.22876
N1	D3	0.99908	0.252787	N2	D2	0.985415	0.038851	N3	D1	0.752841	0.313465	38.86514
N1	D3	0.99908	0.252787	N3	D2	0.985415	0.022314	N2	D1	0.752841	0.323941	39.1768
N1	D2	0.985415	0.26556	N3	D3	0.99908	0.002831	N2	D1	0.752841	0.323941	39.21828
N2	D2	0.985415	0.038851	N1	D3	0.99908	0.252787	N3	D1	0.752841	0.313465	58.61657
N2	D3	0.99908	0.021951	N1	D2	0.985415	0.26556	N3	D1	0.752841	0.313465	59.25295
N2	D3	0.99908	0.021951	N3	D2	0.985415	0.022314	N1	D1	0.752841	0.47623	78.48055
N3	D3	0.99908	0.002831	N1	D2	0.985415	0.26556	N2	D1	0.752841	0.323941	107.8482
N3	D2	0.985415	0.022314	N1	D3	0.99908	0.252787	N2	D1	0.752841	0.323941	107.8897
N2	D2	0.985415	0.038851	N3	D3	0.99908	0.002831	N1	D1	0.752841	0.47623	119.5501
N3	D3	0.99908	0.002831	N2	D2	0.985415	0.038851	N1	D1	0.752841	0.47623	126.7642
N3	D2	0.985415	0.022314	N2	D3	0.99908	0.021951	N1	D1	0.752841	0.47623	167.8337
N1	D3	0.99908	0.252787	N2	D1	0.752841	0.323941	N3	D2	0.985415	0.022314	257.4869
N1	D3	0.99908	0.252787	N3	D1	0.752841	0.313465	N2	D2	0.985415	0.038851	257.7985
N2	D3	0.99908	0.021951	N1	D1	0.752841	0.47623	N3	D2	0.985415	0.022314	277.8747
N2	D3	0.99908	0.021951	N3	D1	0.752841	0.313465	N1	D2	0.985415	0.26556	297.1023
N3	D3	0.99908	0.002831	N1	D1	0.752841	0.47623	N2	D2	0.985415	0.038851	326.47
N3	D3	0.99908	0.002831	N2	D1	0.752841	0.323941	N1	D2	0.985415	0.26556	345.3859
N1	D1	0.752841	0.47623	N2	D3	0.99908	0.021951	N3	D2	0.985415	0.022314	482.9523
N2	D1	0.752841	0.323941	N1	D3	0.99908	0.252787	N3	D2	0.985415	0.022314	503.3401
N1	D2	0.985415	0.26556	N2	D1	0.752841	0.323941	N3	D3	0.99908	0.002831	686.9951
N1	D2	0.985415	0.26556	N3	D1	0.752841	0.313465	N2	D3	0.99908	0.021951	687.9847
N2	D2	0.985415	0.038851	N1	D1	0.752841	0.47623	N3	D3	0.99908	0.002831	707.383
N3	D2	0.985415	0.022314	N1	D1	0.752841	0.47623	N2	D3	0.99908	0.021951	756.6561
N2	D2	0.985415	0.038851	N3	D1	0.752841	0.313465	N1	D3	0.99908	0.252787	768.3165
N3	D2	0.985415	0.022314	N2	D1	0.752841	0.323941	N1	D3	0.99908	0.252787	816.6001
N1	D1	0.752841	0.47623	N3	D3	0.99908	0.002831	N2	D2	0.985415	0.038851	873.2845
N1	D1	0.752841	0.47623	N2	D2	0.985415	0.038851	N3	D3	0.99908	0.002831	913.0969
N2	D1	0.752841	0.323941	N1	D2	0.985415	0.26556	N3	D3	0.99908	0.002831	933.4847
N3	D1	0.752841	0.313465	N1	D3	0.99908	0.252787	N2	D2	0.985415	0.038851	941.9559
N1	D1	0.752841	0.47623	N3	D2	0.985415	0.022314	N2	D3	0.99908	0.021951	1303.429
N3	D1	0.752841	0.313465	N1	D2	0.985415	0.26556	N2	D3	0.99908	0.021951	1372.101
N2	D1	0.752841	0.323941	N3	D3	0.99908	0.002831	N1	D2	0.985415	0.26556	9283.791
N3	D1	0.752841	0.313465	N2	D3	0.99908	0.021951	N1	D2	0.985415	0.26556	9332.075
N2	D1	0.752841	0.323941	N3	D2	0.985415	0.022314	N1	D3	0.99908	0.252787	9713.936
N3	D1	0.752841	0.313465	N2	D2	0.985415	0.038851	N1	D3	0.99908	0.252787	9762.22

Table 6.4.6 Roundoff Error for Elliptic DF2 with .8π Cutoff Frequency

Elliptic DF2

H1		Mag	Dist	H2		Mag	Dist	H3		Mag	Dist	Error
N3	D3	0.99908	0.002831	N1	D1	0.752841	0.47623	N2	D2	0.985415	0.038851	3.023822
N3	D1	0.752841	0.313465	N1	D3	0.99908	0.252787	N2	D2	0.985415	0.038851	3.029807
N3	D2	0.985415	0.022314	N1	D1	0.752841	0.47623	N2	D3	0.99908	0.021951	3.030704
N3	D3	0.99908	0.002831	N1	D2	0.985415	0.26556	N2	D1	0.752841	0.323941	3.034751
N3	D2	0.985415	0.022314	N1	D3	0.99908	0.252787	N2	D1	0.752841	0.323941	3.034752
N3	D1	0.752841	0.313465	N1	D2	0.985415	0.26556	N2	D3	0.99908	0.021951	3.036688
N1	D1	0.752841	0.47623	N3	D3	0.99908	0.002831	N2	D2	0.985415	0.038851	3.060208
N3	D3	0.99908	0.002831	N2	D2	0.985415	0.038851	N1	D1	0.752841	0.47623	3.060586
N3	D2	0.985415	0.022314	N2	D3	0.99908	0.021951	N1	D1	0.752841	0.47623	3.060586
N1	D1	0.752841	0.47623	N3	D2	0.985415	0.022314	N2	D3	0.99908	0.021951	3.067089
N1	D3	0.99908	0.252787	N3	D1	0.752841	0.313465	N2	D2	0.985415	0.038851	3.078656
N1	D2	0.985415	0.26556	N3	D1	0.752841	0.313465	N2	D3	0.99908	0.021951	3.082917
N1	D2	0.985415	0.26556	N3	D3	0.99908	0.002831	N2	D1	0.752841	0.323941	3.086966
N1	D3	0.99908	0.252787	N3	D2	0.985415	0.022314	N2	D1	0.752841	0.323941	3.089585
N3	D3	0.99908	0.002831	N2	D1	0.752841	0.323941	N1	D2	0.985415	0.26556	3.121987
N3	D1	0.752841	0.313465	N2	D3	0.99908	0.021951	N1	D2	0.985415	0.26556	3.127972
N2	D2	0.985415	0.038851	N3	D3	0.99908	0.002831	N1	D1	0.752841	0.47623	3.152689
N2	D3	0.99908	0.021951	N3	D2	0.985415	0.022314	N1	D1	0.752841	0.47623	3.158423
N2	D3	0.99908	0.021951	N3	D1	0.752841	0.313465	N1	D2	0.985415	0.26556	3.219824
N2	D1	0.752841	0.323941	N3	D3	0.99908	0.002831	N1	D2	0.985415	0.26556	3.35745
N3	D2	0.985415	0.022314	N2	D1	0.752841	0.323941	N1	D3	0.99908	0.252787	4.130294
N3	D1	0.752841	0.313465	N2	D2	0.985415	0.038851	N1	D3	0.99908	0.252787	4.136279
N2	D2	0.985415	0.038851	N3	D1	0.752841	0.313465	N1	D3	0.99908	0.252787	4.222396
N2	D1	0.752841	0.323941	N3	D2	0.985415	0.022314	N1	D3	0.99908	0.252787	4.365757
N1	D1	0.752841	0.47623	N2	D2	0.985415	0.038851	N3	D3	0.99908	0.002831	10.70642
N1	D2	0.985415	0.26556	N2	D1	0.752841	0.323941	N3	D3	0.99908	0.002831	10.72225
N2	D2	0.985415	0.038851	N1	D1	0.752841	0.47623	N3	D3	0.99908	0.002831	10.76213
N2	D1	0.752841	0.323941	N1	D2	0.985415	0.26556	N3	D3	0.99908	0.002831	10.90549
N1	D1	0.752841	0.47623	N2	D3	0.99908	0.021951	N3	D2	0.985415	0.022314	10.91603
N1	D3	0.99908	0.252787	N2	D1	0.752841	0.323941	N3	D2	0.985415	0.022314	10.93447
N2	D3	0.99908	0.021951	N1	D1	0.752841	0.47623	N3	D2	0.985415	0.022314	10.97748
N2	D1	0.752841	0.323941	N1	D3	0.99908	0.252787	N3	D2	0.985415	0.022314	11.1151
N1	D2	0.985415	0.26556	N2	D3	0.99908	0.021951	N3	D1	0.752841	0.313465	15.07978
N1	D3	0.99908	0.252787	N2	D2	0.985415	0.038851	N3	D1	0.752841	0.313465	15.0824
N2	D2	0.985415	0.038851	N1	D3	0.99908	0.252787	N3	D1	0.752841	0.313465	15.11967
N2	D3	0.99908	0.021951	N1	D2	0.985415	0.26556	N3	D1	0.752841	0.313465	15.1254