8-2015

THE PARTIALLY RECHARGEABLE ELECTRIC VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND CAPACITATED CHARGING STATIONS

Navid Matin Moghaddam
Clemson University, nmatinm@clemson.edu

THE PARTIALLY RECHARGEABLE ELECTRIC VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND CAPACITATED CHARGING STATIONS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Industrial Engineering

---

by
Navid Matin Moghaddam
August 2015

Accepted by:
Dr. Scott J Mason, Committee Chair
Dr. Amin Khademi
Dr. Yongxi Huang

ABSTRACT

Electric vehicles are potentially beneficial for both the environment and an organization's bottom line. These benefits include, but are not limited to, reduced fuel costs, government tax incentives, reduced greenhouse gas emissions, and the ability to promote a company's "green" image. In order to decide whether or not to convert or purchase electric trucks and install charging facilities, decision makers need to consider many factors including onboard battery capacity, delivery or service assignments, scheduling and routes, as well as weather and traffic conditions in a well-defined modeling framework. We develop a model to solve the partially rechargeable electric vehicle routing problem with time windows and capacitated charging stations. Given destination data and vehicle properties, our model determines the optimal number of vehicles or charging stations needed to meet the network's requirements. Analyzing the model shows the relationships between vehicle range, battery recharge time, and fleet size.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is concerned with finding effective routes for a set of vehicles. These vehicles must visit a number of customers in different geographical locations. Each customer has a demand and the objective typically associated with a VRP is to satisfy this demand with minimum cost of vehicle travel from a depot.

Electric vehicles, especially battery-powered electric trucks, carry potential long-term economic and environmental benefits for reduced fuel cost, government tax incentives, reduced greenhouse gas emissions, and the ability to promote a company's "green" image. There are many benefits of electrification that companies can take advantage of today. However, limited travel range ("range anxiety") and intense capital investment have hindered progress. The typical electric truck's 50-100 mile travel range makes electric trucks particularly suitable for urban trips, given well-calibrated routing plans. The investments to convert or purchase electric trucks and install charging facilities at depots (e.g., local distribution centers or warehouses) depend on a number of major factors, including onboard battery capacity, delivery or service assignments, scheduling, and routes, as well as weather and traffic conditions. When making investment strategies, decision makers need to take into account these factors in a well-defined modeling framework.

According to a report by the Union of Concerned Scientists (2012), "freight-hauling trucks consumed 2.3 million barrels of oil per day … and emitted 348 million metric tons of carbon dioxide." There are nine million medium- and heavy-duty trucks on the road in the United States. Today electrification of trucks can significantly reduce those environmentally sensitive numbers. As electric trucks run 50-100 miles per full charge, this is ideal for urban deliveries (e.g., UPS) and services (e.g., AT&T). However, sizable investments in these trucks (at a cost of ~$100,000) need justification. One of the barriers to adoption is the lack of public charging facilities. These trucks must be charged at depots and the charging facilities need to assure the trucks' completion of trips under a variety of conditions: traffic congestions, weather conditions, routes, and scheduling.

In this thesis research, we develop a model to solve the general electric vehicle routing problem. Given destinations data and vehicles properties, the model determines the optimal number of vehicles or charging stations needed for meeting delivery requirements. Real world constraints in the model include vehicle charge limits and delivery time windows within which goods have to be delivered to each customer. Analyzing this model will give insights about the optimal design combination of charging stations and electric vehicle fleet size needed for delivery.

2. LITERATURE REVIEW

The VRP is an integer programming and optimization problem in which a number of customers have to be served by a limited number of resources. Dantzig and Ramser (1959) consider a limited number of trucks that have to serve some stations—these trucks travel between a terminal and the stations. The demand of the stations and the distance between different points is given and the objective is to find the shortest total distance traveled by the trucks. Linear programming is used to find the optimal solution. Some models focus on distance limitations. Ichimori *et al.* (1983) propose an algorithm to find the minimum range needed to travel all the customers without the need to refuel. Mehrez and Stern (1985) consider a military problem in which the fuel can be transferred between trucks.

Multi-depot VRP (MDVRP) is discussed in Crevier *et al.* (2007), the extension is called MDVRP with inter-depot routes (MDVRPI) and is motivated by the deliveries of groceries in Montreal. The model considers intermediate depots at which vehicles can be replenished with goods. Goncalves *et al.* (2011) consider a VRP with pickup and delivery (VRPPD) with a mixed fleet that consist of electric vehicles and regular vehicles. They do not incorporate the actual location of recharging stations into their model. Tarantilis *et al.* (2008) revise this model and name it VRPIRF (VRP with intermediate replenishment facilities).

A recharging version of VRP is presented by Conrad and Figliozzi (2011). They consider that vehicles can only travel a limited distance. Some of the customer nodes could be considered as charging stations and the charging time is a fixed amount of time.

They also consider time window constraints. They present problem instances solved by a modified iterative construction and improvement algorithm. An environment related objective function is considered in Jabali *et al.* (2012). In their objective function they consider fuel and environment-related cost and also travel time. Travel speed is considered as a variable and the model finds the optimal speed during periods of time in order to minimize environment-related cost. Similarly, Bektas and Laporte (2011) consider a fuel and environment-related objective function. Fuel and environment cost variable is based on vehicle speed and type.

VRP with the possibility of refueling a vehicle at a station along the route is called G-VRP, Erdogan and Miller-Hooks (2012). The G-VRP is modeled as an extension to the MDVRPI. Their objective is to minimize total distance traveled. The G-VRP seeks to find at most m tours. In constraint description it states that at most m vehicles return to the depot in a given day. So it means that there's no multi-trip considered for each vehicle. When refueling is undertaken, it is assumed that the tank is filled to the capacity. Since charging stations may be visited more than once, some dummy vertices are associated with every charging station. This technique was introduced by Bard *et al.* (1998) for their application involving stops at intermediate depots for reloading vehicles with goods for delivery. Service time parameter is considered in the model, for the charging stations the refueling time is equal to the service time. Refueling time is considered to be constant, which means that if a vehicle arrives at the depot with 100% battery or 0% battery, the same amount of time is required to charge. All of the vehicles are considered to be the same; this homogeneous vehicle assumption means that all of them have the same battery capacity. No time window and capacity constraint is considered and partial charging is not allowed. No resource limitation constraint is considered which means that unlimited number of vehicles could be at a particular charging station at the same time. A different formulation for the exact solution is proposed by Taha *et al.* (2014) in which all the constraints are linear and it also permits return paths that visit more than one charging station.

Electric Vehicle Routing Problem with Time Windows and Recharging Stations (E-VRPTW) is introduced by Schneider *et al.* (2014). It is stated that the objective function is to minimize the total traveled distance. All of the vehicles are considered to be the same; this homogeneous vehicle assumption means that all of them have the same battery capacity and the same cargo capacity. Multi-trip is considered in this model, a set of instances of depot is defined to avoid using decision variable with four dimensions. Capacity and time window constraints are considered. They don't introduce departure and delivery time in their model. Using this model no information is available about the arrival and departure time and we only know the delivery time and a sum of waiting time. They use instances from Solomon (1987) to run the model and test their solving algorithm. Partial charging is not allowed and at each charging station charge goes to maximum, but in contrast to G-VRP in this model the charging time is not constant and depends on the available charge at the arrival at the charging station. The recharging process makes the calculations complex because the charging time depends on the available charge. The problem is solved by a variable neighborhood search (VNS) approach using tabu search (TS).

A more general problem with heterogeneous vehicles is defined by Hiermann *et al.* (2014). They introduce the Electric Fleet Size and Mix Vehicle Routing Problem with Time Windows and recharging stations (E-FSMVRPTW). Vehicles have different capacity, battery size and acquisition cost. They present a MIP model to solve for small instances, which is done after some preprocessing and symmetry breaking. In order to solve for larger instances they present a metaheuristic approach based on Adaptive Large Neighborhood Search (ALNS) with embedded local search and labeling procedures. The objective is to minimize acquisition cost and the total distance traveled. As the authors use dummy nodes representing recharging stations, they cannot count number of charging stations used and do not avoid overlaps at the charging station for different vehicles. Therefore, an unlimited number of vehicles could be present at the charging station at the same time. When the vehicle arrives at a charging station it is recharged to full capacity and the charging time is dependent on the vehicle's remaining charge upon arrival, so no partial charging is considered. Compared to best results found in Schneider *et al* (2014) their approach is able to find 12 new, best-known solutions.

A location-routing problem is defined by Yang and Sun (2014) for EVs considering the existence of battery swap stations. They simultaneously determine the location of stations and vehicle routes. Battery driving range and capacity limitations are considered. Time windows and station limitations are not considered, which means that unlimited number of EVs could be present at the same time at each battery swap station. In order to find the locations of the stations, they consider a set of candidate locations for stations. Objective function is to minimize the cost which is a combination of two parts;

fixed construction for constructing each of the stations and the unit shipping cost for each route considered. It is assumed that each route starts and ends at the depot, which means that multi-trip property is not considered in the model. Battery power is reset when the vehicle leaves the station, which means that no partial charging is considered in the model. They present two exact models, the first one assumes that every vehicle may pass a station only once, the second one eliminates this assumption and they consider station revisit in the extended model. They present a four-phase heuristic named SIGNAL and a Two-phase Tabu search-modified Clarke-Wright Savings heuristic to solve the model for large instances.

Pickup and delivery for solar-recharged vehicles is modeled in Albrecht and Pudney (2013). In the problem definition they consider electric vehicles called African Solar Taxis to take people from villages to healthcare facilities. Charging stations are located at these healthcare facilities. The range limitations of these vehicles make it important to schedule the vehicles. In this article only a single-vehicle schedule is provided. Two objectives are considered: 1) maximize total trip distance completed in a day and 2) minimize the schedule span. They consider constraints on how much energy each charging station can deliver in a day. Partial charging is also considered in their scheduling. While they do this scheduling they don't consider overlaps at charging stations.

Vehicle routing in networks for electric vehicles is considered in Cassandras *et al.* (2014a). The authors consider a single-vehicle in their modeling and try to minimize the total elapsed time for vehicles to reach their destinations. They formulate a mixed integer nonlinear programming (MINLP) and prove some properties for the optimal solution and divide the problem into two less complex problems. Then they consider a multi-vehicle problem and group vehicles into subflows and present an alternative formulation. They do not impose full recharging constraints. This problem is an optimal path finding for electric vehicles considering charging stations so no customer, depot, time window or capacity constraint is considered in the formulation. In their formulation they consider the potential energy recuperation effect during the routes (it means that the energy consumption could be negative during the route). In their latest article, Cassandras *et al.* (2014b) consider inhomogeneous charging nodes. Charging rates at different charging nodes are not the same, they use Society of Automotive Engineering (SAE) classification of charging stations.

VRP with intermediate stops is considered in Schneider *et al.* (2014). They define three kinds of intermediate stops; replenishment of goods to be delivered, recharging and unloading of collected goods or disposal of waste. They define a dummy set of vertices for stop locations, which means that they are unable to count the number of stops or consider overlaps at stations. All the vehicles are considered to be homogeneous and the capacity constraints are considered. Arriving at a recharging station resets the battery to its capacity (no partial charging) and the time for recharge is dependent on the arrival remaining charge or fuel. Arriving at other types of stop locations fully replenishes or unloads the vehicle. They present a mixed-integer program as an exact solution. An adaptive variable neighborhood search algorithm (AVNS) is proposed to solve the model.

Energy-optimized vehicle routing for EVs is considered in Preis *et al.* (2013). They consider minimizing fuel consumption depending on vehicle weight and payload. Energy consumption functions are defined for empty vehicles and payload. In their mixed integer program they only consider charging stops by vehicles and not the actual locations of charging stations. Time window and capacity constraints are considered, charging time is supposed to be a fix amount of time. They propose a tabu search heuristic to find the optimal solutions for large instances. In the latest version of this book, Preis *et al.* (2014) consider actual locations of charging stations and dummy sets for them. Charging time is considered to be zero. First, they propose a two-index formulation in which the objective is to minimize total distance cost, and then they propose a revised formulation in which dummy vertices for charging stations are not considered. A set-partitioning formulation is offered. They use column generation approach to add feasible routes to all possible routes generated in master problem MP. It is shown that model formulation without use of dummy sets has a positive impact on the solving time ratio.

Simultaneous vehicle routing and charging station siting in considered in Worley *et al.* (2014) in which they formulate a model in order to locate charging stations and design vehicle routes. The charging stations are chosen among a set of candidates. The objective function is to minimize sum of total travel, recharging and charging station construction. Vehicles are considered to be heterogeneous, no time constraint is considered. The charging time is assumed to be zero. When the vehicle arrives at the charging station it would be immediately fully charged (no partial charging). After completing our review of the literature, it is clear that no previous researchers have addressed the problem studied in this thesis research. The formal description of our research problem follows in the next section.

3. PROBLEM DESCRIPTION

Consider a set of geographically dispersed customers, each of which has their own demand for a package of some size that must be delivered within a specific period of time (i.e., time window). Packages deliveries are sourced from a single depot by a fleet of electric vehicles. A limited number of charging stations located at different locations are required to recharge a vehicle when needed—the amount of time required to recharge a vehicle is directly proportional to the amount of charge to be input into the vehicle. Each electric vehicle can be characterized by its recharging property/rate, and maximum range (as measured by distance driven).

The research problem of interest focuses on determining the required routing for each electric vehicle, including required driving, waiting, and charging times, such that all customer demands are satisfied within the required time windows. Any feasible solution must not violate maximum range limitations, or customer-required delivery requirements. Our goal is to develop an optimal solution for each of the following objective functions individually:

- Given a set of available charging stations, minimize the total number of vehicles required
- Given a fleet of available vehicles, minimize the total number of charging stations required

4. MATHEMATICAL MODEL

This section contains our model formulation, pertinent notation, and an explanation of the model's constraint sets.

4.1. Notation

The notation used in the mixed-integer program model formulation is as follows:

Sets

N	set of nodes. This set includes the depot $\{0\}$, customers $\{1, \ldots, c\}$ and charging stations $\{c+1, \ldots, c+h\}$. Indexed by i, j
C	set of customers $\{1, \ldots, c\}$. Indexed by q
V	set of vehicles $\{1, \ldots, v\}$. Indexed by m, n
T	set of trips $\{1, \ldots, t\}$. Indexed by f, g, u
S	set of charging stations $\{c+1, \ldots, c+h\}$. Indexed by k

Parameters

c	number of customers
v	number of vehicles
h	number of charging stations
t	number of trips possible
$d_{i,j}$	distance between node i and node j
δ_m	maximum distance vehicle m could travel with full battery
$\tau_{i,j}$	time it takes to travel from node i to node j
α_i	starting point in time window for node i
β_i	ending point in time window for node i
μ	maximum charge time for empty battery
M_1	parameter for constraint sets
M_2	parameter for constraint sets
M_3	parameter for constraint sets
e_1	parameter for objective function coefficient
e_2	parameter for constraint sets
e_3	parameter for constraint sets

Variables

$x_{i,j,m,f}$	1 if vehicle m travels from node i to node j in its trip number f else 0.
$p_{i,m,f}$	clock time at which vehicle m delivers package to node i in its trip number f.
$a_{i,m,f}$	clock time at which vehicle m arrives at node i in its trip number f.
$l_{i,m,f}$	clock time at which vehicle m leaves node i in its trip number f.
$b_{m,f}$	battery charge of vehicle m when it leaves depot or a charging station at the beginning of its trip number f.
$e_{m,f,n,g}$	binary variable used in resource limitation constraint.
o_m	binary variable used in counting number of vehicles used in the solution.
w_k	binary variable used in counting number of charging stations used in the solution.

4.2. Model Formulation

Our model is formulated as a mixed-integer program as follows:

$$\min \quad \left(\textstyle\sum_{m \in V} o_m\right) + e_1\left(\textstyle\sum_{m \in V} a_{0mt}\right) \tag{1}$$

$$\min \quad \left(\textstyle\sum_{k \in S} w_k\right) + e_1\left(\textstyle\sum_{m \in V} a_{0mt}\right) \tag{2}$$

s.t.

$$\sum_{\substack{m \in V \\ f \in T \\ i \in N}} x_{iqmf} = 1 \qquad \forall\, q \in C \tag{3}$$

$$\sum_{i \in N} x_{iqmf} = \sum_{j \in N} x_{qjmf} \qquad \forall\, q \in C, \quad m \in V, \quad f \in T \tag{4}$$

$$\sum_{i \in N} x_{kimf} = \sum_{i \in N} x_{ikmg} \qquad \forall\, k \in \{0\} \cup S, \quad m \in V, \quad f \in \{2, \dots, t\}, \quad g = f - 1 \tag{5}$$

$$\sum_{\substack{k \in \{0\} \cup S \\ i \in N}} x_{kimf} + \sum_{\substack{k \in \{0\} \cup S \\ i \in N}} x_{ikmf} = 2 \qquad \forall\, m \in V, \quad f \in T \tag{6}$$

$$\sum_{i \in N} x_{0im1} = 1 \qquad \forall\, m \in V \tag{7}$$

$$\sum_{i \in N} x_{i0mt} = 1 \qquad \forall\, m \in V \tag{8}$$

$$\left[\sum_{i,j \in N} d_{ij} x_{ijmf}\right] \Big/ \delta_m \leq b_{mf} \qquad \forall\, m \in V, \quad f \in T \tag{9}$$

$$l_{jmf} + \tau_{ji} - M_1\big(1 - x_{jimf}\big) \leq a_{imf} \qquad \forall\, i,j \in N, \quad m \in V, \quad f \in T \tag{10}$$

$$a_{imf} \leq l_{jmf} + \tau_{ji} + M_1\big(1 - x_{jimf}\big) \qquad \forall\, i,j \in N, \quad m \in V, \quad f \in T \tag{11}$$

$$a_{imf} \leq M_2 \sum_{j \in N} x_{jimf} \qquad \forall\, i \in N, \quad m \in V, \quad f \in T \tag{12}$$

$$-(\sum_{j \in N} x_{jimf}) \leq a_{imf} \qquad \forall\, i \in N, \quad m \in V, \quad f \in T \tag{13}$$

$$l_{imf} \leq M_2 \sum_{j \in N} x_{jimf} \qquad \forall\, i \in N, \quad m \in V, \quad f \in T \tag{14}$$

$$-(\sum_{j \in N} x_{jimf}) \leq l_{imf} \qquad \forall\, i \in N, \quad m \in V, \quad f \in T \tag{15}$$

$$p_{imf} \leq l_{imf} \qquad \forall\, i \in N, \quad m \in V, \quad f \in T \tag{16}$$

$$a_{qmf} \leq p_{qmf} \qquad \forall\, q \in C, \quad m \in V, \quad f \in T \tag{17}$$

$$p_{km1} = 0 \qquad \forall\, k \in \{0\} \cup S, \quad m \in V \tag{18}$$

$$a_{kmf} = p_{kmg} \qquad \forall\, k \in \{0\} \cup S, \quad m \in V, \quad f \in \{1, \ldots, t-1\}, \quad g = f + 1 \tag{19}$$

$$\sum_{\substack{m \in V \\ f \in T}} p_{imf} \leq \beta_i \qquad \forall\, i \in N \tag{20}$$

$$\alpha_i \leq \sum_{\substack{m \in V \\ f \in T}} p_{imf} \qquad \forall\, i \in N \tag{21}$$

$$b_{m1} = 1 \qquad \forall\, m \in V \tag{22}$$

$$0 \leq b_{mf} \leq 1 \qquad \forall\, m \in V, \quad f \in T \tag{23}$$

$$b_{mg} = b_{mf} - \left[\left(\sum_{i,j \in N} d_{ij} x_{ijmf}\right) \Big/ \delta_m\right] + \left[\left(\sum_{k \in S} l_{kmg} - \sum_{k \in S} a_{kmf}\right) \Big/ \mu\right] \qquad \forall\, m \in V, f \in \{1, \dots, t-1\}, g = f+1 \qquad (24)$$

$$a_{imf} - a_{inu} \leq M_3 e_{mfng} - e_2 \qquad \forall\, i \in S, \forall\, m, n \in V, f \in \{1, \dots, t-1\}, g \in \{2, \dots, t\}, u = g-1, m \neq n \qquad (25)$$

$$l_{ing} - a_{imf} \leq M_3\left(1 - e_{mfng}\right) \qquad \forall\, i \in S, \forall\, m, n \in V, f \in \{1, \dots, t-1\}, g \in \{2, \dots, t\}, m \neq n \qquad (26)$$

$$o_m \geq e_3\, a_{0mt} \qquad \forall\, m \in V \qquad (27)$$

$$w_k \geq e_3 \sum_{\substack{m \in V \\ f \in T}} a_{kmf} \qquad \forall\, k \in S \qquad (28)$$

Our model contains two individual (candidate) objective functions. Objective function (1) minimizes total number of vehicles used in the routes to serve customers; the second part of the objective function (with a very small coefficient) is used to make the vehicles return to the depot as soon as possible. Objective function (2) minimizes the total number of charging stations used in the routes to serve customers.

In constraint set (3) there’s exactly one vehicle visits each customer q and the vehicle passes through that customer on only one of its trips. Constraint set (4) allows at most one arrival and one departure at each customer. Constraint sets (5) and (6) are constraints for beginning and ending each trip. The first set (5) ensures that when a vehicle enters a node at the end of a trip, the vehicle would exit the same node at the start of next trip. The second set (6) ensures that only the starting node and the ending node of each trip are non-customer nodes in that trip. Constraint set (7) ensures that each vehicle begins its route from the depot, while constraint set (8) ensures that each vehicle returns to the depot at the end of its route.

In constraint set (9) the charge vehicle m uses in trip f to travel to customers should be less than its charge at the beginning of this trip. Constraint set (10) prescribes that a vehicle departing from node j at time l that takes τ hours to travel from node j to node i would arrive at node i at time $l + \tau$. If there's an arc between node i and j, constraint sets (10) and (11) become activated. If vehicle m doesn't pass through node i in any of its trips, the arrival time would be zero. Otherwise, constraint sets (12) and (13) become inactive.

Constraint sets (14) and (15) serve the same purpose as constraint sets (13) and (14), but they are for departure time calculations. In constraint sets (16) and (17), a vehicle should deliver to customer q at some time between the vehicle's arrival and departure times at the customer's node. Constraint set (18) sets the start value of the model's clock to zero. Constraint set (19) manages the calculation of pertinent times so that the departure time from a node at the beginning of any trip occurs after the vehicle's arrival time to that same node at the end of the previous trip. We also consider stations for the starting and ending nodes of each trip.

Constraint sets (20) and (21) specify that the delivery of any package should be within the specified time window. Next, constraint set (22) fixes charge of each vehicle at the beginning of its first trip to one (i.e., 100%). In constraint set (23), the charge of each vehicle is restricted to be between zero and one. Constraint set (24) ensures that at the end of each trip, if the vehicle enters a charging station, it is charged proportionally according to the time spent at the station. However, no charging occurs at the depot.

Constraint sets (25) and (26) are resource limitation constraints. Constraint sets (27) and (28) calculate the values of two variables. In set (27), if a particular vehicle doesn't go to any nodes, its arrival time to the depot at the final trip would be zero -- this constraint becomes inactive. However, if it travels to any node, the vehicle's arrival time at the depot would be greater than zero and this constraint becomes active and the value for the variable would be equal to one. It follows that this is used to compute the total number of vehicles used. The same holds for set (28). If a charging station is used the value of the second variable becomes equal to one, so sum of this variable for all of the charging stations is equal to the number of stations used in the solution.

5. MODEL VALIDATION

A sample problem instance is defined to validate the model. Consider a fleet of vehicles and a demand to deliver two packages each belonging to a unique customer. Each vehicle may travel a maximum of 30 miles using a fully charged battery. The distance between customers is given in Figure 1. By construction, it is not possible for any vehicle to deliver both packages without recharging. Three charging stations are available. It's possible for any vehicle to leave the charging station with partially charged battery. Full charge of an empty battery takes 8 hours, while partial charging time is proportional to the charge gained during the recharge process. According to Table 1, Customer 1's package has to be delivered at 7 AM (exact time) and the package belonging to Customer 2 has to be delivered within the time window from 10 AM to 10 PM.

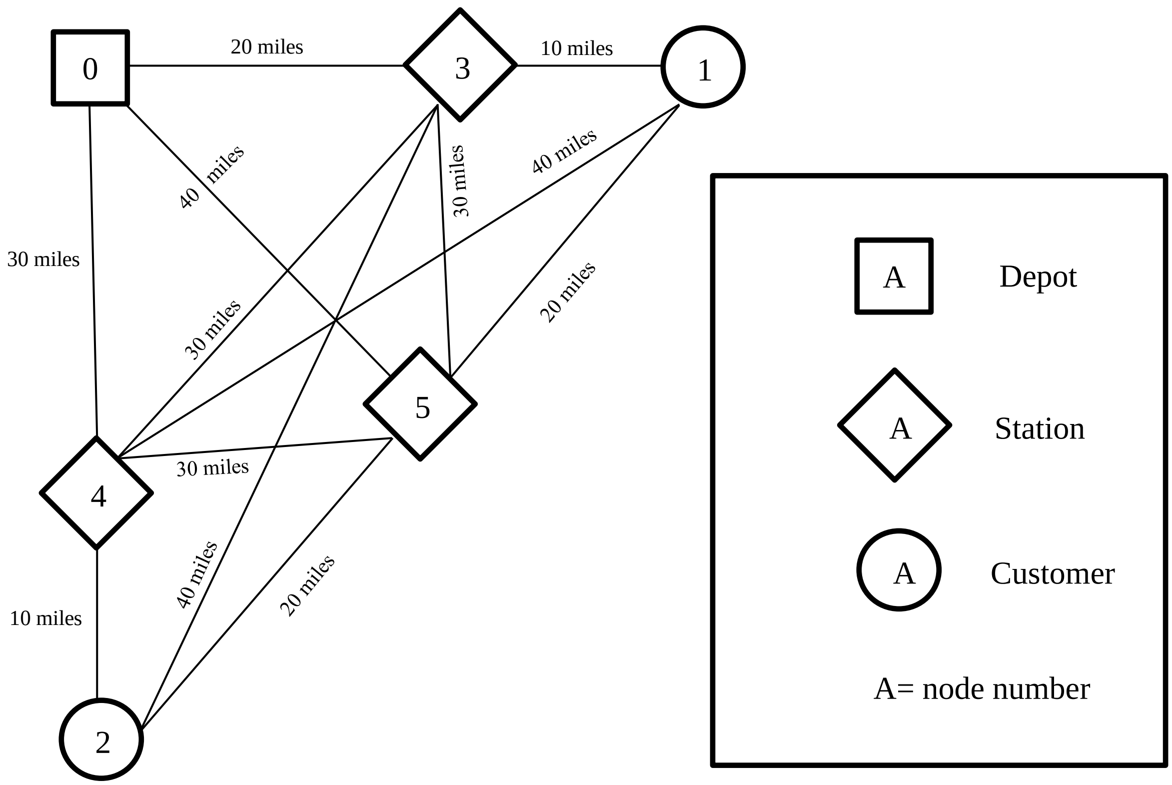

Figure 1: Data for Test Instance

Table 1: Input Data for Example Problem

Time window (Customer 1)	7 AM – 7:01 AM
Time window (Customer 2)	10 AM – 10 PM
Vehicle range	30 miles
Recharge time	8 hours

After implementing the model in AMPL, the two-customer instance is evaluated to minimize total number of vehicles required. The solution was produced using Gurobi v.5.6 solver on a Windows 7 Enterprise platform with an Intel® Core™ i7-2600 CPU processor @3.40 GHz. Optimal vehicle routes, remaining charge percentage, arrival and departure times are shown in Figure 2. The optimal objective function value is 1.03 as only one vehicle is used throughout the delivery process. The reason that the objective value is 1.03 instead of 1 is that we have defined a small objective function term which rewards the shortest return time to the depot (i.e., it eliminates long-journey solutions). The vehicle first travels to customer one's location because the package must be delivered at 7 AM, before package two.

Because of its distance limitation associated with its range, the vehicle uses a charging station after delivering the first package. The vehicle does a full charge due to the distance to the next charging station within its route being equal to the maximum vehicle range of 30 miles. The vehicle travels to customer two and uses a second charging station in this trip, again because of the maximum distance limitation. Then, the vehicle returns to the depot and again needs to use the third charging station due to its maximum distance limitation. Therefore, the model proposes a travel schedule in which the number of vehicles used is the minimum (one) and the number of charging stations needed is three.

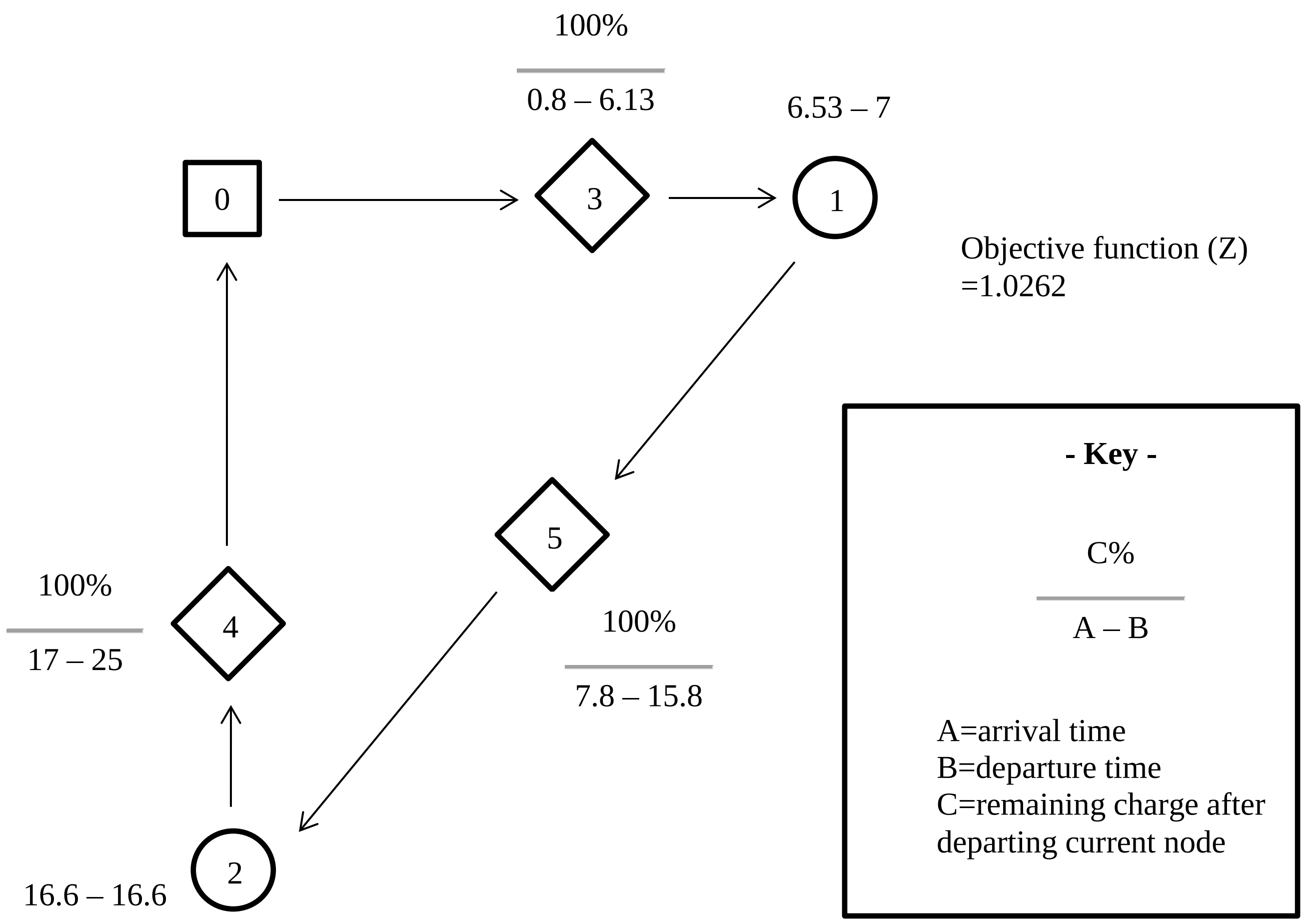

Figure 2: Results for Test Instance (objective is to minimize number of vehicles)

Now we run the model with a second objective: to minimize the number of charging stations. Now, the objective function is equal to 2.03 (i.e., the model uses two vehicles during the delivery process). Each vehicle delivers one package and each uses one charging station in their route due to the maximum distance limit (Figure 3). The vehicle delivering package one does not need a full charge when returning to the depot because its distance traveled is 10 miles less than the maximum distance limit. However, the vehicle delivering package two requires a full charge while returning to the depot. Therefore, two vehicles and minimum number of charging stations (two) are used to deliver the packages. A comparison of the two models' results is presented in Table 2. It is clear that the different objective functions lead to completely different routes and schedules.

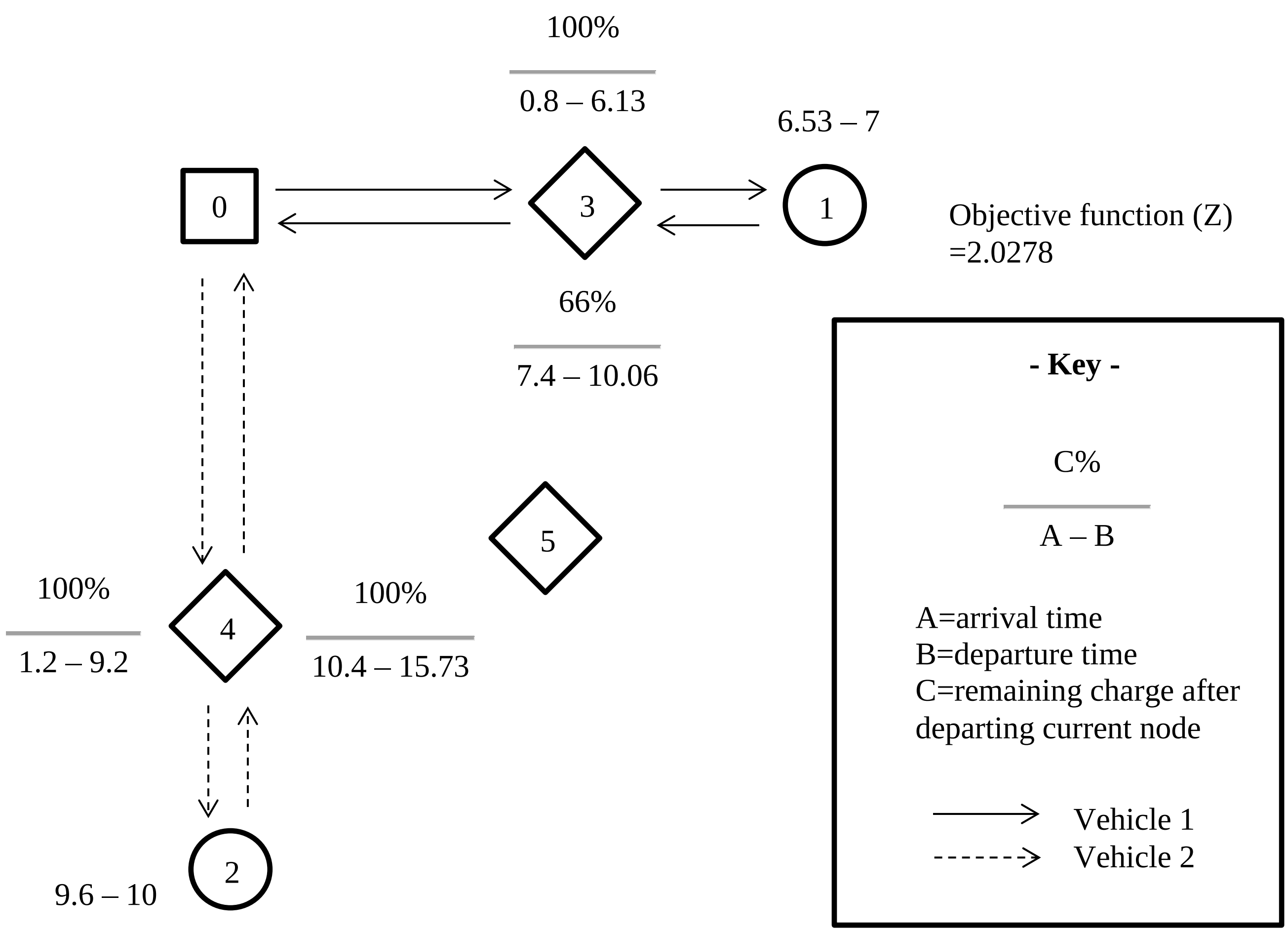

Figure 3: Results for Test Instance (objective is to minimize number of stations)

Table 2: Comparison between Results

	Objective	
	Min number of vehicles	Min number of stations
Number of vehicles used	1	2
Number of stations used	3	2

6. MODEL PARAMETER ESTIMATION

In the proposed MIP model we use a number of M (“big M”) parameters in the constraint right hand sides. The use of M as a constraint coefficient could result in rounding errors that may cause the basis matrix (B) to become singular. Also, there could exist precision errors when computing B^{-1}. Since M is sometimes used to make constraints active or inactive, even without basis matrix errors, large values of M could cause commercial branch-and-bound solvers to work inefficiently or slower than desired. Loose bounds can make it harder to prune nodes based on the objective function. In this case, more nodes will be explored and the solving process can slow down. With this in mind, we try to find smallest value for M that works for the model.

In the optimization model, parameter e_1 is used in objective function (1) and (2). Parameter e_1 is a coefficient multiplied by the summation of arrival times of all vehicles to the depot. This parameter is important to ensure that each vehicle returns to the depot as soon as possible and in order to eliminate unwanted idle time. We must make sure that this coefficient does not impact the count of vehicles or charging stations used in the model. In turn, we must ensure the value of second term in each objective function should be less than one.

In the worst case, consider customer j who has the maximum value of (any customer time window upper bound value) + (travel time from customer to the depot). In this case, $e_1 * \left(\sum_V a_{0,j,t}\right)$ would be maximal. This leads to the following estimation for parameter e_1:

$$e_1 < \frac{1}{v * Max_{i \in C}\,(\beta_i + \tau_{i,0})}. \tag{29}$$

Parameter M_1 is used in constraint sets (10) and (11). This parameter is important to make these constraints active or inactive as necessary. We need to estimate M_1 only if x_{ji} is zero (if not, the parameter would have a zero multiplier and would be eliminated). In the worst case, we estimate the value for parameter M_1 as follows:

$$Max_{i \in C}\left(\beta_i + \tau_{i,0}\right) \leq M_1\,. \tag{30}$$

Parameter M_2 is used in constraint sets (12) and (14). This parameter is important in order to allow these constraints to become inactive as necessary. We must estimate a value for this parameter only if $\sum_{nodes} x_{ji}$ is not equal to zero, so it could be any positive integer (if not, parameter would have zero multiplier and would be eliminated). In worst case, this term would be one; therefore, our estimate for M_2 would be as follows:

$$M_2 \geq\ a_{imf}\,. \tag{31}$$

Further, as arrival time (a_{imf}) is estimated in the same way as for our M_1 estimation, we conclude that M_2 is equal to M_1.

Parameter M_3 is used in constraint sets (25) and (26) in order to make these constraints active or inactive. If e_{mfng} is equal to one, in the worst case, parameter M_3 would be greater than a_{imf}. If e_{mfng} is equal to zero then parameter M_3 would be greater than l_{ing}. Arrival time (and similarly, departure time) is estimated in the same way we did for our parameter M_1 estimation. In general, M_3 may be less than M_1 but for simplicity, we conclude that M_3 is equal to M_1.

Parameter e_2 is used in constraint set (25). This constraint does not allow simultaneous charging for distinct vehicles at the same station. In addition, we need this parameter to avoid the possibility of similar arrival times (without this parameter if two vehicles arrive at the same time, this constraint set does not work). In fact we block e_2 time units before and after each arrival. A worst-case estimate for this parameter is as follows:

$$e_2 < \frac{\min(\tau_{i,j})}{\frac{max\,(\delta_m)}{vehicle\ speed}} * \mu\,. \tag{32}$$

Parameter e_3 is used in constraint sets (27) and (28). We need to estimate this parameter only if a_{0mt} is not equal to zero (otherwise, the parameter would have a zero multiplier and would be eliminated). In the worst case, our estimate for this parameter is as follows:

$$e_3 < \frac{1}{Max_{i\in C}\,(\beta_i + \tau_{i,0})}\ . \tag{33}$$

7. EXPERIMENTAL STUDY

7.1. Experimental Plan

We analyze a set of 180 test instances, each with one depot, five customers, and one charging station to evaluate the proposed model's performance. All instances are created based on the benchmark instances of Solomon (1987). The location of the depot and customers is fixed in each instance, but various factor levels are considered: two levels for vehicle range, three levels for recharge time, two levels for charging station location, and two levels for time windows (Table 3). Instance set $R1$ has a short scheduling horizon (time windows $t1 - t8$) while instance set $R2$ has a long scheduling horizon (time windows $t9 - t15$). Figure 4 depicts a representation of depot, customers, and candidate charging station locations.

Table 3: Experimental Design Parameters

Parameter	**Levels**
Station Locations	(61.7, 72.4) (42.1, 64.9)
Time Windows	See Table 4
Vehicle Range (δ)	100 150
Recharge Time (μ)	8 2.5 1

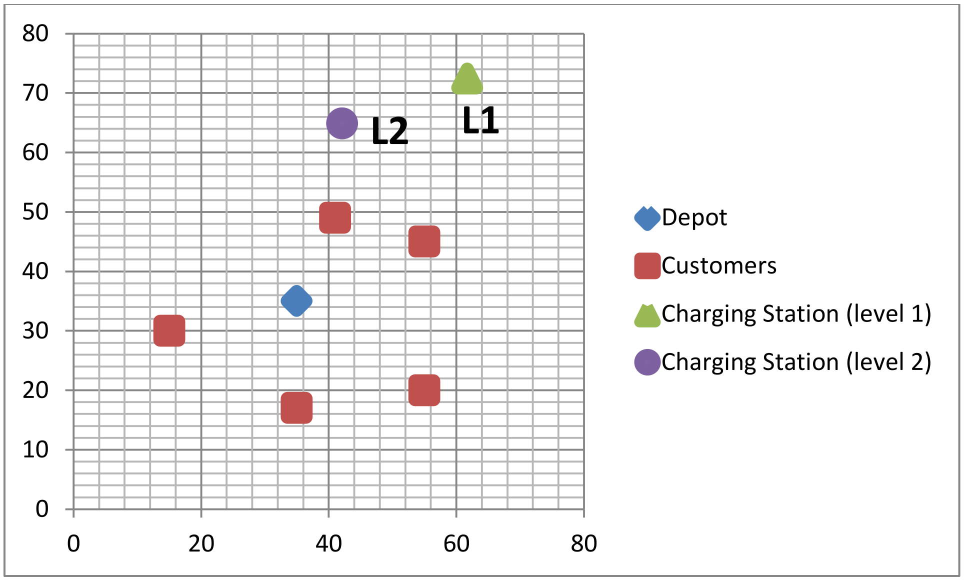

Figure 4: Instance Representation

Table 4: Experimental Time Window Data

		Customer1	Customer2	Customer3	Customer4	Customer5
t1	start	161	50	116	149	34
	end	171	60	126	159	44
t2	start	0	0	0	149	0
	end	204	202	197	159	199
t3	start	151	40	106	139	24
	end	181	70	136	169	54
t4	start	0	0	0	139	0
	end	204	202	197	169	199
t5	start	133	22	98	123	20
	end	198	87	143	184	93
t6	start	130	20	106	71	20
	end	201	89	135	195	107
t7	start	15	18	54	138	20
	end	204	202	187	169	199
t8	start	73	18	76	73	20
	end	204	147	165	195	167
t9	start	707	143	527	678	34
	end	848	282	584	801	209
t10	start	0	0	0	149	0
	end	974	972	967	801	969
t11	start	658	93	436	620	20
	end	898	333	676	860	260
t12	start	0	0	0	620	0
	end	974	972	967	860	969
t13	start	636	74	498	470	20
	end	919	351	613	965	370
t14	start	190	18	289	679	20
	end	974	792	822	800	906
t15	start	451	18	378	477	20
	end	974	534	733	965	610

7.2. Experimental Results and Analysis

Solutions are found by implementing the optimization model in AMPL and solving it with Gurobi after imposing an optimization time limit, in order to minimize number of vehicles used in the solution. The solution is produced using Gurobi solver on a Windows 7 Enterprise platform with Intel® Core™ i7-2600 CPU @3.40 GHz. After running our test instances, we see that in most cases, the objective function reaches a stable value after 5-10 minutes of optimization time that does not change for several hours. With this in mind, the optimization time limit for each run is set to 15 minutes. A summary of the experimental results is shown in Tables 5 and 6 for the two time window levels considered ($R1$ and $R2$). The relative optimality gap is shown in Tables 7 and 8 for the two time window levels considered ($R1$ and $R2$).

For all instances, the near-optimal solution is to use either one or two vehicles to deliver packages. Table 9 shows the percentage of test instances in which the resulting near-optimal objective function recommends using one vehicle. This confirms that we would need fewer vehicles if either vehicle range is increased, recharge time is decreased, or scheduling horizon is increased.

Table 5: Experimental Results for Time Windows Set R1

		μ=8 δ=100	μ=8 δ=150	μ=2.5 δ=100	μ=2.5 δ=150	μ=1 δ=100	μ=1 δ=150
Charging Station Location = L1	**t1**	2.000059	2.000058	2.000059	2.000059	2.00006	2.000058
	t2	2.00004	1.00003	2.000036	1.00003	2.000041	1.000029
	t3	2.000044	2.000032	2.000044	2.000039	2.000044	1.000031
	t4	2.000038	1.000028	2.000038	1.000027	2.000028	1.000026
	t5	2.000037	2.000036	2.000041	2.000033	2.000039	1.000029
	t6	2.000024	1.000054	2.000023	1.000024	2.000023	1.000024
	t7	2.000029	1.000028	2.000039	1.000027	2.000029	1.000027
	t8	2.000022	1.000023	2.000023	1.000022	2.000023	1.000023
Charging Station Location = L2	**t1**	2.00006	2.000056	2.000059	2.000059	2.000059	2.000058
	t2	2.00004	1.000029	2.00004	1.000029	2.000036	1.00003
	t3	2.000044	2.00004	2.000045	2.000036	2.000044	2.000039
	t4	2.000033	1.000027	2.000037	1.000027	1.000035	1.000027
	t5	2.000042	2.000033	2.000042	2.000035	2.000042	1.00003
	t6	2.000023	1.000025	2.000033	1.000025	2.000028	1.000024
	t7	2.000029	1.000028	2.000029	2.000036	2.000039	1.000025
	t8	2.000023	1.000022	2.000029	1.000022	2.000027	1.000015

Table 6: Experimental Results for Time Windows Set R2

		μ=8 δ=100	μ=8 δ=150	μ=2.5 δ=100	μ=2.5 δ=150	μ=1 δ=100	μ=1 δ=150
Charging Station Location = L1	**t9**	2.000148	1.000122	2.000135	1.000121	2.000132	1.000121
	t10	1.000085	1.000026	1.00005	1.00003	1.000043	1.00003
	t11	2.000114	1.000112	2.00012	1.00011	2.000114	1.000112
	t12	1.00011	1.000108	1.000111	1.000107	1.00011	1.000108
	t13	2.000108	1.000109	2.000111	1.000109	2.000114	1.000109
	t14	1.00012	1.000117	1.00012	1.000118	1.00012	1.000117
	t15	2.000093	1.000087	2.000093	1.000086	2.00009	1.000087
Charging Station Location = L2	**t9**	2.000132	1.00012	1.000121	1.000122	1.000121	1.000121
	t10	1.000076	1.000029	1.000044	1.00003	1.000035	1.000028
	t11	1.000112	1.000112	1.000112	1.000113	1.000111	1.000112
	t12	1.000108	1.000108	1.000107	1.000108	1.000108	1.000108
	t13	1.000109	1.000108	1.000109	1.000108	1.000108	1.000109
	t14	1.000118	1.000117	1.000118	1.000118	1.000118	1.000118
	t15	1.000088	1.000087	1.000086	1.000086	1.000087	1.000085

Table 7: Relative Optimality Gap for Time Windows Set R1

		μ=8 δ=100	μ=8 δ=150	μ=2.5 δ=100	μ=2.5 δ=150	μ=1 δ=100	μ=1 δ=150
Charging Station Location = L1	**t1**	0.500970	0.002791	0.002791	0.500783	0.500970	0.002791
	t2	0.501004	0.002973	0.501000	0.002973	0.501004	0.002973
	t3	0.000048	0.501018	0.000086	0.501017	0.000096	0.000174
	t4	0.500963	0.002808	0.500962	0.002808	0.500961	0.002808
	t5	0.000100	0.001513	0.000090	0.001584	0.000077	0.500939
	t6	0.500818	0.002463	0.000874	0.002463	0.001185	0.002791
	t7	0.500970	0.002791	0.500970	0.002791	0.500970	0.002791
	t8	0.500783	0.002383	0.500783	0.002383	0.500783	0.002372
Charging Station Location = L2	**t1**	0.500963	0.001945	0.002808	0.001945	0.002808	0.001977
	t2	0.501002	0.002969	0.501004	0.002973	0.501002	0.002960
	t3	0.000069	0.000732	0.000075	0.000322	0.000097	0.000647
	t4	0.500963	0.002808	0.500963	0.002806	0.500962	0.002808
	t5	0.002305	0.001977	0.001191	0.471952	0.001945	0.001963
	t6	0.500820	0.000322	0.500822	0.002464	0.001870	0.002452
	t7	0.500970	0.002790	0.500970	0.002791	0.500970	0.002789
	t8	0.500783	0.002383	0.500783	0.002370	0.500783	0.002790

Table 8: Relative Optimality Gap for Time Windows Set R2

		μ=8 δ=100	μ=8 δ=150	μ=2.5 δ=100	μ=2.5 δ=150	μ=1 δ=100	μ=1 δ=150
Charging Station Location = L1	**t9**	0.000985	0.012099	0.503452	0.010787	0.007017	0.010985
	t10	0.501004	0.002972	0.005359	0.002973	0.004245	0.002973
	t11	0.503044	0.011155	0.503045	0.011156	0.503045	0.011156
	t12	0.010975	0.010717	0.010975	0.010713	0.010975	0.010716
	t13	0.502915	0.010787	0.005968	0.010787	0.006066	0.010787
	t14	0.503114	0.011678	0.011925	0.011679	0.011937	0.011674
	t15	0.502455	0.008585	0.502333	0.008585	0.502334	0.008583
Charging Station Location = L2	**t9**	0.402777	0.012100	0.010786	0.012098	0.011679	0.010717
	t10	0.007534	0.002974	0.004484	0.002972	0.003663	0.011679
	t11	0.011295	0.011152	0.011145	0.011149	0.011155	0.011155
	t12	0.010716	0.010717	0.010716	0.010716	0.010717	0.011295
	t13	0.010985	0.010786	0.010786	0.010786	0.010784	0.011679
	t14	0.011679	0.011679	0.011679	0.011678	0.011677	0.010975
	t15	0.008731	0.008586	0.008586	0.008586	0.008586	0.005968

Table 9: Percentage of Instances Able to Use Only One Electric Vehicle

	Time Window Set R1		**Time Window Set R2**	
	δ=100	**δ=150**	**δ=100**	**δ=150**
μ=8	0 %	62 %	64 %	100 %
μ=2.5	0 %	56 %	71 %	100 %
μ=1	6%	81 %	71 %	100 %

CONCLUSIONS AND FUTURE WORK

A mixed-integer programing (MIP) formulation is developed to solve the partially rechargeable electric vehicle routing problem with time windows and capacitated charging stations. The analysis of this model gives insights about the optimal design combination of charging stations and electric vehicle fleet sizing needed for parcel delivery. This analysis confirms that fewer vehicles are needed if systems designers are able to increase vehicle range, decrease recharge time, or increase their scheduling horizon.

Due to the problem's NP-hard complexity (via reduction to the classical VRP), our solution is not optimal. In fact, for larger test instances with more than five customers, we may not reach even a good solution in an appropriate amount of time. In future work, this research can be extended to develop heuristic or metaheuristic algorithms to achieve better results for larger instances in a timely fashion.

REFERENCES

[1] G. B. Dantzig and J. H. Ramser (1959) The truck dispatching problem. *Management Science*, 6:80-91

[2] B. Crevier, J.-F. Cordeau, and G. Laporte. (2007) The multi-depot vehicle routing problem with inter-depot routes. *European Journal of Operational Research*, 176(2):756-773

[3] F. Goncalves, S. R. Cardoso, Relvas S., and A. P. F. D Barbosa-P_ovoa. Optimization of a distribution network using electric vehicles: A VRP problem. *Technical report*, CEG-IST, UTL, Lisboa, 2011.

[4] S. Erdogan and E. Miller-Hooks. A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):100-114, 2012.

[5] Bard, J., Huang, L., Dror, M., Jaillet, P., 1998. A branch and cut algorithm for the VRP with satellite facilities. *IIE Transactions* 30, 821–834.

[6] M. M. Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, 35(2):254-265, 1987.

[7] Schneider M, Stenger A, Goeke D (2014) The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science*

[8] Tarantilis CD, Zachariadis EE, Kiranoudis CT (2008) A hybrid-guided local search for the vehicle routing problem with intermediate replenishment facilities. *INFORMS J. Comput*.20(1):154–168.

[9] Bektas T, Laporte G (2011) The pollution-routing problem. *Transportation Res. Part B* 45(8):1232-1250

[10] Mehrez A, Stern HI (1985) Optimal refueling strategies for a mixed vehicle fleet. *Naval Res. Logist. Quart*. 32(2):315–328.

[11] Ichimori T, Ishii H, Nishida T (1983) Two routing problems with the limitation of fuel. *Discrete Appl. Math*. 6(1):85–89.

[12] Jabali O, Van Woensel T, de Kok AG (2012) Analysis of travel times and CO2 emissions in time dependent vehicle routing. *Production Oper. Management* 21(6):1060–1074

[13] Conrad RG, Figliozzi MA (2011) The recharging vehicle routing problem. *Proc. 2011 Indust. Engrg. Res. Conf., Reno, CA.*

[14] Taha M, Fors N, Shoukry A (2014) An Exact solution for a class of green vehicle routing problem. *Ind Eng and Operations Management. Conf., Bali, Indonesia*.

[15] Hiermann G, Puchinger J, Hartl R (2014) The electric fleet size and mix vehicle routing problem with time windows and recharging stations. *Submitted to Transportation Science*

[16] Yang J, Sun H (2014) Battery swap station location-routing problem with capacitated electric vehicles. *Computers & operations research (accepted manuscript)*

[17] Albrecht A, Pudney P (2013) Pickup and delivery with a solar-recharged vehicle. *National conference of the Australian society for operations research*

[18] Cassandras C, Wang T, Pourazarm S (2014a) Energy-aware vehicle routing in networks with charging nodes. *Working paper*

[19] Cassandras C, Wang T, Pourazarm S (2014b) Optimal routing of energy-aware vehicles in networks with inhomogeneous charging nodes. *Working paper*

[20] Worley O, Klabjan D, Sweda T (2014) Simultaneous vehicle routing and charging station siting for commercial electric vehicles. *Working paper*
[21] Schneider M, Stenger A, Hof J (2014) An Adaptive VNS algorithm for Vehicle Routing Problems with Intermediate Stops. *Technical report*
[22] Preis H, Frank S, Nachtigall K (2013) Energy-optimized routing of electric vehicles in urban delivery systems. *Operations Research Proceedings, pp 583-588*
[23] Preis H, Frank S, Nachtigall K (2014) On the modeling of recharging stops in context of vehicle routing problems. *Operations Research Proceedings, pp 129-135*