# MULTIVALUED SUBSETS UNDER INFORMATION THEORY 

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# MULTIVALUED SUBSETS UNDER INFORMATION THEORY 

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Industrial Engineering

by<br>Indraneel Chandrasen Dabhade<br>August 2011

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#### Abstract

In the fields of finance, engineering and varied sciences, Data Mining/ Machine Learning has held an eminent position in predictive analysis. Complex algorithms and adaptive decision models have contributed towards streamlining directed research as well as improve on the accuracies in forecasting. Researchers in the fields of mathematics and computer science have made significant contributions towards the development of this field. Classification based modeling, which holds a significant position amongst the different rule-based algorithms, is one of the most widely used decision making tools. The decision tree has a place of profound significance in classification-based modeling. A number of heuristics have been developed over the years to prune the decision making process. Some key benchmarks in the evolution of the decision tree could to attributed to the researchers like Quinlan (ID3 and C4.5), Fayyad (GID3/3*, continuous value discretization), etc. The most common heuristic applied for these trees is the entropy discussed under information theory by Shannon. The current application with entropy covered under the term 'Information Gain' is directed towards individual assessment of the attribute-value sets. The proposed study takes a look at the effects of combining the attribute-value sets, aimed at improving the information gain. Couple of key applications have been tested and presented with statistical conclusions. The first being the application towards the feature selection process, a key step in the data mining process, while the second application is targeted towards the discretization of data. A search-based heuristic tool is applied towards identifying the subsets sharing a better gain value than the ones presented in the GID approach.


## DEDICATION

I dedicate this work to Amma, Baba and Sir.

## ACKNOWLEDGEMENT

I thank my committee for this opportunity.

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## Chapter 1

## Introduction

Data Mining is defined as a process of making meaningful conclusions from complex databases. Fayyad (1996) refers to the process as making patterns, associations, anomalies and statistically significant structures from the databases depending on the type of rule applied for class identification.

The data mining process is composed of three primary steps:
Data pre-processing
Pattern recognition
Interpreting results
The data pre-processing stage provides meaning to raw data by removing noise and identifying attributes for the cases in the population. Pattern recognition identifies rules for including the classes in the database. This phase of data mining identifies the type of the data-mining tool, to be used for identifying the classes for the population set. Finally, the extracted patterns are interpreted as knowledge, sometimes referred to as visual validation.

Data mining has been seen as an important tool for various sectors varying from industrial application, marketing, and medical to achieving advances in technology like image recognition, accident investigations and biometric applications. With the increasing popularity of the World Wide Web, advances in data mining have seen an increase in popularity among web developers.

The use of this tool is broadly divided into 2 main categories namely Web Usage Mining (WUM) and Web Structure Mining (WSM) (Srivastava et al. 2000), (Gong \& Miguel 2005). WUM identifies prominent searches made by the user over the Internet to recognize popular/emerging trends and individual user needs. With the advent of the social networking sites over the past decade, companies have been able to target specific users based on their preferences, which resulted in increasing use of the Internet as a marketing tool. Web marketers' use advanced data mining algorithms to classify the user search history on the web and offer products/services as per the user search patterns. WHOWEDA ( $W$ are $H O$ use of $W E \mathrm{~b} D A$ ta) (Madria 1999) is one of the prominent projects in the field of web structure mining. The project explored the use of the basic data mining architecture of links and nodes for creating a hyperlink structure of the web as an information source. Data mining, with its close proximity with the areas of machine learning and artificial intelligence, finds extensive use in the field of robotics. Data mining in soft computing makes use of tools such as fuzzy sets, neural networks and genetic algorithms to highly complex mixed mode/media datasets (Mitra 2002). The dynamic natures of the decision-making process and combinatorial massive search spaces have led to the refinement and development of complex algorithms. These algorithms will be dealt in the later sections. Jang (1995) focuses on evolving emotions and decisions making behavior within machines and has tried to combine the human behavior via fuzzy sets with learning structures of neural networks to create hybrid data mining algorithms, namely the 'Neuro-Fuzzy' systems.

### 1.1. Data Mining Background

Given below are the summaries of some of the most commonly used data mining methods, which include clustering, classification, regressions and association rules.

Clustering: Clustering, or grouping, creates sets of data that are identical in specific characteristics. This methodology is also sometimes referred to as k-means clustering (MacQueen 1967), where ' $k$ ' refers to the number of clusters that are created with each cluster centric about a mean. The two main type of clustering techniques are partitionbased and hierarchical. The partition-based technique could create either completely exclusive groups or overlapping set of groups. The partition method deals with creating rules based on the similarity of the attribute-value sets chosen, to group similar points together. The hierarchical method is associated with creating a tree of clusters based on close proximity of the near-by objects/points/other clusters. Commonly referred to as 'dendrogram' (Forina \& Raggio 2002), based on the way the tree is built, either "updown" (Jain \& Dubes 1988) or "down-up" (Kaufmann \& Rouseeuw 1990), referred to as agglomerative or divisive. Apart from these two main categories, some lesser-known techniques include grid and constraint based, scalable and different algorithms, dealing with other categories of data. The literature for these techniques can be found in Han \& Kambler (2001).

Classification: Under this methodology of data mining, the examples/items are placed in differing groups based on the inherent characteristics indicated by the attribute-value sets taken by the individual items. The algorithms behind this methodology create rules as per the individual datasets. This characteristic along with rule creation for individual datasets classifies the methodology under supervised machine learning techniques. A number of
algorithms have been developed for classification based data mining techniques. Some of them include k-Nearest Neighbor, Bayesian and Neural-Net based classifiers. For the kNearest Neighbor (KNN), the new data simply assumes the class of the nearest item/group (Wasito \& Mirkin 2006). This is different from what class boundaries identified by the decision tree learner based classification identifies, in which constrained boundaries identify the classes. KNN is one such popular memory based classification system. Bayesian-based classifiers use probability as a tool to identify the classes for the dataset. These classifiers show exclusiveness in terms of identifying classes for the datasets (Zhang 2004). They use the maximum likelihood function as a tool to identify the rule. In Neural Net-based methods, the model, the activation function and the learning algorithm help in the pattern recognition and hence prove to be a useful utility in the process of data mining (Fausett 1994). More information on its applicability and usage can be found in Bigus (1996).

Regression: Regression in data mining works on the principle of predicting the class of the example based on the rule generated by the regression function. Based on the property of error reduction for pattern development, a number of rule-based algorithms have been developed both for data mining using linear as well as non-linear regression. CART is a well-known linear regression-based algorithm, whereas the Support Vector Machine (SVM) is a good example for the non-linear regression based algorithm. Association Rules: This technique uses decision support as a measure to weigh the relationship between attributes and establish rules to predict the classes. Some prominent work in this field has been conducted by Agarwal \& Srikant (1994), some of which include studying the purchasing patterns in supermarkets and creating a 'Point of Sale'
(POS) systems.
The focus of this research is on the refinement of a classification-based decision tree. The following section deals with introducing the features of a decision tree.

### 1.2 The Decision Tree

The decision tree is defined as a decision tool constructed after learning the data patterns and associations within a data set. Hence most of the algorithms defined for decision tree generation are referred to as learning algorithms. Since the optimality for the decision making process is dependent on how accurate the tree has been constructed, the data set in use are divided into two parts. The first part deals with the training data, which are used to learn the algorithm, or in the technical sense, to define the rules for the rest of the data set. The second part of the data set is used to evaluate the quality of the rules. Apart from the mentioned approach of data handling, another approach, the k-fold cross validation, is also quite popular among researchers. But for the purpose of this study, the dataset would be subjected to the Test-Train Split approach. The datasets available in the real world vary ranging from categorical, ordinal, and nominal classified under continuous and discrete. The following section deals with handling the datasets for the decision tree based data mining technique.

### 1.3 Discretizing the Data Sets.

Researchers have attempted to use continuous attributes directly into the datamining algorithm. The Genetic Network Program (GNP) (Taboada et al. 2007) is one such algorithm that directly handles continuous attribute-value sets for data mining. Most of the other algorithms first convert the continuous attributed data to discrete data intervals before creating rules. These algorithms apply discretizing methods for
converting continuous valued attributes into discrete attributes. The process of discretization, in its attempt to reduce the dimensionality of the dataset, is subject to loss of information. Popular algorithms like Ant Miner use an external function like C4.5 (Quinlan 1992) based discretizer (C4.5-Disc) at the pre-processing stage for discretizing the continuous data sets. An entropy based discretization approach is then applied for the original ant miner algorithm. The 'c-Ant Miner' (Otero 2008), which had the entropy measure as a function to discretize the continuous attributed data set, proved to be better than the C4.5-Disc algorithm in only two of the eight datasets that were used for the experiment. Fayyad \& Irani (1993) introduces the 'Multi-Interval Discretization of Continuous Valued Attributes', another discretization method, which uses the Minimum Description Length Principle (MDLP) to achieve a supervised discretization scheme. The 'Class Attribute Interdependence Maximization' (CAIM) (Kurgan \& Cios 2004) proved to be a better discretization tool than the entropy maximization algorithm for discretizing continuous attributes. The details on the discretization process and the contribution of this research towards this field will be discussed in greater detail in the section 'Discretizers'.

### 1.4 Survey of Classification-based Algorithms

A number of classification-based algorithms have been developed. Some of the most prominent ones are described below. However, it is important to first review the concept of entropy, which forms a key measure to estimate the information gain.

Shannon's entropy is defined as the uncertainty about the source of a message. As per Shannon (1951), it would take $\log n$ queries to fully encode a message. This gives rise to extremities; if every message to be encoded is different, results in maximum number of
queries required to encode the next unknown message. Similar is the case for the class prediction; if every item from the dataset coming in for classification is identified to be different, resulting in maximum amount of information to predict the class for the next case. For the other extremity, if the same type of message keeps on repeating, no additional queries will be required to encode the next incoming message, hence no additional information would be required to predict the class for the new item. There are certain advantages that have been observed with the use of entropy as a heuristic for decision-making process. Since it uses a log function, it provides a weight to the heuristic to make the right decision. Also, entropy will distinguish probabilities. For the purposes to measure the information in terms of bits, this study uses log to base value 2 .

$$
\text { Entropy }=-\sum_{i=1}^{n} p_{i} \log _{2} p_{i}
$$

Conclusions on the usage of the entropy as the decision heuristic take different meanings based on the objective of the algorithm. Though the goal of the study is maximizing the information gain or conversely, minimizing entropy, there are applications that use the convex optimality of entropy maximization (Guiasu \& Shenitzer 1968).

A description on a few major classification algorithms is as follows.

### 1.4.1 ID 3 Algorithm

The Iterative Dichotomiser 3 (ID3) algorithm uses information gain as a measure to make decisions on training the rule and then predicting the classes. Information gain is defined as the difference between the entropy needed to collect the information about a class $H(T)$ and the entropy needed to conclude about a class given an attribute value $\mathrm{H}(\mathrm{X}, \mathrm{T})$. Gain $=H(T)-H(X, T)$

Where,
$H(T)=$ Entropy for probability distribution of the classes
$H(X, T)=$ Entropy for probability distribution of the classes knowing the dataset partition on the basis of attribute value X .

The algorithm recursively checks for the gains using different attribute-value sets and keeps track of the attributes providing the highest gain. This attribute ultimately forms the node on the decision tree. More details on the working of the algorithm can be found in Quinlan (1986). The algorithm faces problems regarding large values carried by attributes since the gain tends to favor attributes with larger values. Appendix C. 1 provides the program used for calculating the information gain under the ID3 algorithm.

### 1.4.2 CART Algorithm

The classification and regression tree algorithm CART, (Brieman 1984), considered 3 different splitting criteria namely the GINI criterion, Twoing criterion and the Ordered Twoing criterion. All the three, dealt with change in impurity levels of sending the items from the dataset either to the left of the node or the right. There were a few problems that were associated with using ID3 as well as the CART algorithms that led to the formation of the refined GID algorithm (Fayyad 1988).

### 1.4.3 ASSISSTANT Algorithm

This algorithm follows a similar classification criterion as the ID 3, but provides an improvement on the noise handling capacity (Kononenko 1984). The algorithm tests each leaf node for further branching. The termination criterion is the test of reduction in classification accuracy for any further branching.

### 1.4.4 The AQ Algorithm

This classification based data-mining algorithm follows the simple (if-then) rule creation technique (Michalski \& Larson 1983). The main heuristic checks for the purity i.e. the maximum number of examples covered for the class. The one problem with the use of this algorithm is that it is less easy to modify, based on its dependency on specific training examples.

### 1.4.5 The CN2 Induction Algorithm

The algorithm possesses properties of both the ID3 and the AQ algorithm (Clark \& Niblett 1989). This algorithm uses entropy as criterion for creating an ordered set of rules. This particular feature created a problem on its scope on general applicability. Using 'Laplace Error Estimate' as an alternative evaluation function, an unordered list of rules could be derived using the algorithm (Clark \& Boswell 1991). One common problem observed for the CN2 algorithm was with the specificity of the rule selection algorithm.

### 1.4.6 The Ant-Miner Algorithm

The Ant-Miner algorithm developed on the basis of ant colony optimization, proved to be a better suite against the CN2 algorithm considering the reduction and the simplification in the number of rules (Parpinelli 2002).

### 1.4.7 GID3 Algorithm

The GID3 algorithm developed by Fayyad (1988) considers binary partitions of the attribute-value pairs. The attribute is divided into two discrete subsets, one that contains the test-attribute value pair $\left(A=a_{i}\right)$ and the other containing the rest of the values $\left(A \neq a_{i}\right)$.

The values for the information gain are collected for all such pairs and the highest value is selected amongst them. Based on the user-provided threshold limit value, the algorithm creates a measure to filter out the attribute-value set displaying gain values greater than the threshold limit. These values are then collected together in what is known as a Phantom Attribute (PA). Hence this temporary attribute contains the attribute-value sets that would significantly contribute towards creating purer class sets. This procedure proves better than branching at each individual attribute-value pairs. The 'Threshold Limit' is a user-defined value. The program for calculating the information gain under the GID rule is included in the Appendix C.2.

### 1.4.8 GID3* Algorithm

A later refinement of the algorithm, introduced as GID3*, provides a tear measure to automatically select the threshold level based on how effectively the subset of the attribute-value pair manages to discretely segregate and separate the distinct classes. A comparative performance measurement on the functioning of the two algorithms shows an improvement in the following features of the decision tree:
a. Increase in the number of examples in individual final leaves.
b. Decrease in the average number of leaves.
c. Decrease in the error rate.
d. Increase in the number of decisions per example.
e. Reduction in the number of nodes.

This refined algorithm too works on the same principle of binary partitioning on individual values of the attribute. The rest of the values are simply grouped together corresponding to forming a different set.

### 1.5 Critique of Current Research

As mentioned above, the ID3 algorithm uses entropy as a criterion to select the appropriate attribute-value pair for branching at the node. As a part of the algorithm, branches are being created at individual attribute-value sets. A comparison between information gains for the pairs is done among pair values. One disadvantage of this methodology is that the algorithm suffers from lack of relevance from the created branches on the attribute pair values. Also this algorithm suffers from the missing values/incomplete dataset. But at the same time the use of information gain, a function of information entropy, proves to be of good use to get an estimate of the contribution of the attribute-value pair in the purity of the class (Fayyad 1991).

The key factors that affect these features of the decision tree are as follows:
a. The test conducted on the node.
b. Number of branches per node.
c. Distribution of the examples across the leaves.
d. Number of examples carried per branch of the tree.

## Chapter 2

## Research Introduction

### 2.1 Area of Research

The approach of this research is the application of binary partitions to multivalue sets. Consider a data set for which the attributes in question are $A$ and $B$ possessing values ( $\mathrm{a} 1, \mathrm{a} 2$, a 3 , a 4 , a 5 ) and ( $\mathrm{b} 1, \mathrm{~b} 2, \mathrm{~b} 3, \mathrm{~b} 4, \mathrm{~b} 5$ ) respectively. After applying the partition at the attribute node, the attribute A gets divided into two branches/partitions picking up values (a1, a2) and (a3, a4, a5). Correspondingly the attribute B branches out to form 2 sets namely ( $\mathrm{b} 1, \mathrm{~b} 2$ ) and ( $\mathrm{b} 3, \mathrm{~b} 4, \mathrm{~b} 5$ ). The problem, as can be seen considering the number of partition sets to be considered for the attributes, grows exponentially as the number of values for the attribute increases. Hence for the above case, the attribute A consisting of 5 values, there are $2^{5}$ possible partition sets to be evaluated. To check for the right partition, the algorithm needs to execute the loop for 32 different partition combinations. Overall, if the attribute possesses ' $r$ ' values, there are $2^{r}$ possible partition combinations to be evaluated for each attribute. This further has an implication on the choice of the attribute to create a node. This choice is based on the impurity measure associated with the attribute. Most of the algorithms, especially the ones mentioned above, use information gain as the purity measure. This measure is a function of the attribute-value subsets. This is an adaptation of a problem suggested by Fayyad (1991).

Hence the need is to develop a heuristic that would potentially perform fewer searches. The task remains to define the binary decision vector. The efficiency of the heuristic would be based on the purity of the class.

A heuristic-based search approach has been adopted to identify attribute-value
subsets, which provide a higher information gain value than the existing GID3 algorithm. Though these values do not exceed the ID3 information gain values, an application towards ranking the features has been implemented. Since the application deals with a greedy search of attribute value pairs showing higher information gain, a semi-supervised approach has been implemented to discretize attributes. This application has been tested against the existing unsupervised algorithms of Equal Width (EW) and Equal Frequency (EF) along with a supervised discretizer CAIM. The developed heuristic in this application has also been tested for change in interval size values varying from 4 to 20 . For the final section, a reverse build order categorized under 'Un-supervising the Supervised' has been implemented which focuses on building the semi-supervised algorithm on top of the unsupervised algorithm. The current study proposes the following models/algorithms, details of which are presented in the sections to follow.

- Multivalued Subset (MVS)
- Multivalued Discrete Frequency/Width (MDF/MDW)
- Frequency Multivalued Frequency/Width (FMF/FMW)


### 2.2 Measures

The proposed algorithms are subject to percentage classification errors under different classifiers. The details and distinguishing factors of these classifiers can be found in the section 'Classifiers'. A separate section has been dedicated to identifying the behavior of these classifiers subject to varying constraints as introduced earlier.

### 2.3 Datasets

This study is aimed at using continuous datasets. The continuous data will be discretized prior to its usage. Since the algorithm deals with studying the effect of
considering more than one attribute-value pair in the testing subset, datasets with comparatively fewer to none numbers of binary attributes are chosen. The proposed algorithms are a function of a search heuristic hence, for a few instances as indicated in their corresponding section; the datasets need to be trimmed prior to subjecting them to classifiers.

For the purpose of the study, 5 different categories of datasets have been used. All datasets were obtained from the Machine Learning repository online (http://archive.ics.uci.edu/ml/). The reason for doing so was to establish a common platform to compare new models with the existing ones.

One of the motives of the study was to analyze the effect of discretizing the data. More information on this can be found in the section 'Discretization'. The choice of the datasets was made based on the following features.

- Varying number of instances across the different datasets.
- Varying number of unique values per dataset.
- Integers and fractional values to be considered.
- No binary-valued attributes.
- The attribute-value sets were continuous and numeric in nature.

The table [Table 1] briefly summarizes the features of the datasets.
The column 4 in Table 1 indicates the range of the unique values found for individual datasets. A high value is important to this study since it provides a higher search space for the multivalued subsets. The other key characteristics for dataset consideration such as outliers, spread and clustering were too taken into consideration while making the selection.

Table 1. Dataset Characteristics

| Dataset | Instances | Attributes | Unique <br> Values | Data Type | Missing <br> Values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Iris | 150 | 4 | $22-43$ | Fractional | No |
| Glass | 214 | 9 | $32-178$ | Fractional | No |
| Images | 4435 | 36 | $49-104$ | Integer | Missing <br> Class '6' |
| PenDigits | 7494 | 16 | $96-101$ | Integer | No |
| Vehicles | 846 | 18 | $13-424$ | Integer | No |

### 2.4 Classifiers

The classifiers play an important part especially when comparing practices (algorithms) tested on standardized datasets. For the purposes of the current study, four distinct classifiers have been chosen. This section introduces these classifiers in a moderate depth and showcases certain distinguishing characteristics between them.

### 2.4.1 AdaBoost (Freund \& Schapire 1996)

Commonly referred to as the AdaBoost (Adaptive Boosting), the key learning algorithm converges to a single rule by 'adaptively boosting' rules created on the smaller sample sizes. In other words, the algorithm converges to a stronger classifier from a number of weaker classifiers.

## AdaBoost

1. Divide the training set into $n$ distributions $\left(D_{1} \ldots D_{n}\right)$.
2. Converge to a local optimal rule on these individual distributions.
3. Adapt the distribution over the training set

$$
D_{t+1}=\frac{D_{t} e^{-\alpha_{t} y y_{i}\left(x_{t}\right)}}{N_{t}}
$$

Where,
$\alpha_{t}=$ Choice of the weight for individual training distributions contributing towards the final rule.
$h_{t}=$ Hypothesis/rule for the distribution $t$.
$h_{t}\left(x_{t}\right)=$ Performance of the local rule on the subset of the sample $(x \in X)$
4. Identify the convergent value summing the local results.

$$
H(x)=\sum_{t=1}^{T} \alpha_{t} h_{t}(x)
$$

Key Features:

- Designed for binary classification $y \in(1,-1)$
- Overfitting\# is a function of the noise carried in the dataset. Low noise seldom leading to overfitting whereas high noise often leads to considerable overfitting (Ratsch et al. 2001) attributed towards the boosting nature of the algorithm.


### 2.4.2 Iterative Dichotomize 3 -ID3 (Quinlan 1986)

As explained in the previous chapter, the key criterion for the functioning of the ID3 decision tree is the information gain. As per the rankings identified under feature selection, the decision tree based hierarchical rule structure provides a discrete measure to identify misclassifications on the testing set.

ID3 Classifier

1. Calculate the information gain for individual attributes.
2. Rank the attributes with increasing values of the information gain.
3. Subdivide each set of examples on the following attribute-value sets in the ranking scheme based on the subset of the examples in the current attribute scheme.

Key Features:

- Main criterion is 'Information Gain'.
- Prone to 'overfitting \#',


### 2.4.3 Regression

A regression function is used to split the clustered data.

1. A weighted regression function is identified based on the least mean squared error.

$$
f(x, a)=a_{0}+a_{1} x_{1}+\ldots+a_{d} x_{d}
$$

2. For every new query, provide a 'decision_class'.

$$
\begin{aligned}
& f(x, a)>0.5 \Rightarrow \text { decision_class }=1 \\
& f(x, a)<=0.5 \Rightarrow \text { decision_class }=0
\end{aligned}
$$

### 2.4.4 Naïve Bayesian Classifier

This classifier provides a decision class on the basis of rule generated with the string of literals consisting conditional probability, function of the attribute-value and the class. Consider the following example.

Jimmy wants to make a decision, whether he could go out in the field and play. The following literal string affects his decision.
$P($ play $=1 \mid$ condition $=$ hot $) \times P($ play $=0 \mid$ homework $=$ not_completed $) \times P($ play $=1 \mid$ Jake $=$ available $)$

### 2.4.5 Summary of the Classifiers

The table provided below compares the characteristics of these classifiers.
Table 2. Classifier Characteristics

| Classifier | Time to <br> compute | Nature | Rule Generation |
| :---: | :---: | :---: | :---: |
| AdaBoost | Low | Stochastic | Function of Sample <br> Size and weighted <br> predictions |
| ID3 | Low | Deterministic | Robust Rule |
| Regression | High | Deterministic | Robust Rule |
| Naïve Bayesian | Low | Probabilistic | Robust Rule |

### 2.5 Testing Conditions

Since most of the programs involved are conformant to the use of random number generators, it would make good sense that all the random number generators for the different programs follow the same stream and with identical starting positions. As mentioned in the later section 'Adaptive Simulated Annealing', the 'Mersenne Twister' is the pseudorandom number generator implemented for running the programs. The programs were run on the Palmetto high-performance computing (HPC) environment at Clemson University with a wall time of 50 hours per run. While most of the datasets were able to adhere to the fixed wall time, it had been observed that datasets with larger set of instances needed multiple runs. As mentioned in the section 'Discretization', the algorithms representing the four different set of classifiers required the data fed to be of the alphanumeric type; an additional set of macros in Excel were utilized to achieve this.

The evaluation of the algorithms was based on the measure of classification errors identified for the datasets. The datasets were divided into two parts in the ratio 70:30, the former representing the training data while the later the testing data.

### 2.6 Adaptive Simulated Annealing (ASA)

This section will provide the information on the metaheuristic tool used for this study. The key goal of the study is to identify a subset from the attribute-value sets providing a higher value of the information gain. An adaptive version of the Simulated Annealing was applied, for reasons pertaining to the time taken to reach an optimal value. The algorithm shown below has been modified for a maximization function.

## Algorithm A. 1 : The Adaptive Simulated Annealing (ASA)

generate initial solution
initialize $F_{l}, F_{h}=$ initial solution
begin
initialize $T_{o}, T_{\text {end }}$
while $T_{o}>T_{\text {end }}\{$
begin
initialize $L_{b}, I, L_{t}$
while $L_{t}<\left(L_{b}+I\right)$
begin
generate solution (Solcurr )
if solution $<F_{l}$ then change $F_{l}$
if solution $>=F_{h}$ then change $F_{h}$
evaluate $\Delta=$ Sol $_{\text {curr }}-L \_$Solcurr
if $\Delta>0$ then $L_{-}$Solcurr $=$Solcurr
if $\Delta<0$ then if $e^{\Delta / T_{o}}>\operatorname{Rand}(1)$ then $L_{-}$Sol $_{\text {curr }}=$ Sol $_{\text {curr }}$
then $E_{\text {best }}=$ Solcurr
end

$$
L_{t}=L_{b}+\left(L_{b} .(1-e)^{\frac{-\left(F_{h}-F_{i}\right)}{F_{h}}}\right)
$$

lower $T_{o}$
end
end

The term $L_{t}=L_{b}+\left(L_{b} .(1-e)^{\frac{-\left(F h-F_{i}\right)}{F_{h}}}\right)$ represents the adaptive equilibrium condition. With the inclusion of the information gain criterion, the final version of the algorithm is interpreted below. One of the keys tasks for evaluating the objective function was to define the Class Quanta Identity (CQI). The CQI represents the distribution of the attribute-value sets against the classes. There are few more factors with regards to the generation of the solution under the multivalued subset scheme, which are covered in Algorithm A.2. The matrix is built as binary values (rows) against the classes (columns). Appendix C. 1 provides the program used for performing the 'Adaptive Simulated Annealing'. Shown below [Algorithm A.2], is the multisubset variant of the Adaptive Simulated Annealing.

Algorithm A.2: Multisubset variant using the Adaptive Simulated Annealing

```
generate initial solution
initialize \(F_{l}, F_{h}, E_{\text {best }}, E_{\text {config }}\)
begin
    initialize \(T_{o}, T_{\text {end }}\)
    while \(T_{o}>T_{\text {end }}\) \{
    begin
            initialize \(L_{b}, I, L_{t}\)
            while \(L_{t}<\left(L_{b}+I\right)\)
            begin
                Binary-Rand ( \(n_{x} 1\) )
                form CQI for the binary subsets
                if solution \(<F_{l}\) then change \(F_{l}\)
                if solution \(>=F_{h}\) then change \(F_{h}\)
                evaluate \(\Delta=\) Sol curr \(^{-L}\) _Solcurr
                if \(\Delta>0\) then \(L_{-}\)Sol \(_{\text {curr }}=\) Solcurr
                    if \(\Delta<0\) then if \(e^{\Delta / T_{o}}>\operatorname{Rand}(1)\) then \(L_{-}\)Sol \(_{\text {curr }}=\) Sol \(_{\text {curr }}\)
                        then \(E_{\text {best }}=\) Solcurr
```

            end
    $$
L_{t}=L_{b}+\left(L_{b} .(1-e)^{\frac{-\left(F_{h}-F_{l}\right)}{F_{h}}}\right)
$$

lower $T_{o}$
end
end

### 2.6.1 Random Number Generation.

One group of random binary multipliers was generated using the same stream with changing seed values. Each stream was generated of the 'Mersenne Twister' pseudorandom number generation. The seed values provided to the random number generation played an important role to show adequate variance in the generation of the solution. Another key observation to be noted with the implementation of this method was in the area of utilization of the equilibrium condition. Though this condition was meant to provide self-adjustability in the speed of implementing the algorithm, provided little to no significant contribution towards improving a solution at the expense of time to traverse with the decreasing temperature values.

## Chapter 3

## Multivalue subset based Feature Selection

What makes this problem interesting? Is it NP hard ?
When thinking from the perspective of building a decision tree, one of the most familiar heuristic as mentioned in the previous sections is that of information gain. The previously defined algorithms (ID3, GID3 and GID3*) have used this heuristic to determine their own individual decision trees. Though the approach of this research doesn't define a new decision tree, it does ask the question, what if more than a single attribute value pairs were to be combined? How would that influence the information gain? By doing so, the search space of combining the attribute-value sets against the objective function of improving the information gain makes the problem NP Hard.

The research initially started with the intention of having built a decision tree that would hold leaves being tested for more than a single attribute at a time. The earlier authors who thought about doing this did arrive at a conclusion of it being an NP-hard problem. It is much more like inferring on a multicolored leaf. Though the intention is to have a tree with leaves of different color, the hardness of the problem arrives with deciding on the pureness of the tree based on a multicolored leaf. Does this mean the end of search for multivalued sets? No. A number of researchers in the field of Computer Science, Finance, and Engineering etc. focus on the aspect of directly using classifiers to identify the rules. The basic problem with this approach is from perspective that the number of attributes being considered is finite. The increase in the number of attributes emphasizes the need to rank the attributes as per a decision heuristic. Why 'selecting'?

Would one want to consider an attribute impurely dividing the classes in the decision tree or rather for that matter, any sort of classifier? What is the problem in doing so? As had been discussed earlier, the ultimate aim for any new algorithm for generating the decision tree would be to keep the number of leaves to a minimum. The reason, lesser the number of leaves, the purer the rules and hence better the classification. Fayyad (1991) provides the proof for the same. Hence, the approach at this point is to identify an alternative approach towards the pre-processing stage for the classifiers. The stage commonly known as the 'Feature Selection' has played a vital role in a number of fields ranging from Biology (Sundaravaradan et al. 2010), Text Classification (Forman 2007), Environmental Studies (Mitrovic et al. 2009) etc. Another popular application of this tool is in the field of Reinforcement Learning (RL) (Hachiya et al. 2010). Experts in the field of Machine Learning believe that the next generation evolution of algorithms would potentially need to provide added flexibility in the decision making process. When one thinks of the decision making process, it could be either as simple as the ' Q learning' technique or a more advanced form of RL with the integration of changing environmental conditions which would require a bit of dynamic programming to learn (Sutton 1998). On the whole the basic feature selection methodology could be divided into 3 main categories (Filter, Wrapper and Embedded) (Guyon and Elisseeff 2003). Figure 1 represents the process adopted by this study to analyze the effects of the proposed heuristic based feature selection method.


Figure 1: Filter based Feature Selection Method

Based on the approach mentioned above, the feature selection process could be divided into two areas of applicability. First, the applied heuristic measure could be used to identify the subset providing the maximum information gain, meaning, the applied hypothesis at this stage would be the need to identify the first ' $k$ ' attributes, where ' $k$ ' is a subset of a larger pool of attributes ' N ' $(k \in N)$. The advantage for this process is that it provides a directed sequential search, subject to the function characteristics, which could either be a maximization or minimization function. The other goal of the feature selection process could be to identify any subset of the features satisfying the objective function. This subset identified could lie anywhere in the search path. The current study is directed towards identifying the lower bound for the same set size using a heuristic based search process. The differentiating factor between this approach and prior work is explained later in the section. One of the oldest feature selection criterion, information gain has found tremendous application in the commercial data mining and machine learning industries.

Quite often the filter mentioned above is the gain-based ranking method. The ranking starts with the setup of an empty scorecard. Each attribute is independently evaluated for information gain. The attributes with the maximum gain occupy higher positions in the scorecard.

Assume that the current classification system contains 10 attributes. Based on the individual information gain values the scorecard resulted as shown in Table 3.

Table 3: Illustration of a Feature Selection Process

| J | H | E | D | A | C | B | I | G | F |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Feature Set | Classifier | Performance |
| :---: | :---: | :---: |
| $\{\mathrm{J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}, \mathrm{C}, \mathrm{B}, \mathrm{I}, \mathrm{G}, \mathrm{F}\}$ | ID3 | $98 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}, \mathrm{C}, \mathrm{B}, \mathrm{I}, \mathrm{G}\}$ | ID3 | $97 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}, \mathrm{C}, \mathrm{B}, \mathrm{I}\}$ | ID3 | $85 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}, \mathrm{C}, \mathrm{B}\}$ | ID3 | $80 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}, \mathrm{C}\}$ | ID3 | $87 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}, \mathrm{A}\}$ | ID3 | $90 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}, \mathrm{D}\}$ | ID3 | $92 \%$ |
| $\{\mathrm{~J}, \mathrm{H}, \mathrm{E}\}$ | ID3 | $89 \%$ |
| $\{\mathrm{~J}, \mathrm{H}\}$ | ID3 | $91 \%$ |
| $\{\mathrm{~J}\}$ | ID3 | $88 \%$ |

As shown above the performance of the varying sizes of the sets carrying the ranked order of the attributes being tested for a single classifier, which in this hypothetical case is the ID3 Classifier. The above illustration shows that not all attributes are required to be considered to show an improved performance. This is purely a case of pre-processing the attributes on a gain-based ranking, prior to testing them on the classifiers. As can be observed the search space for the attribute set is limited to 10 . Also, the ranking heuristic is irrespective of the nature of the attribute-value sets. There have been a number of approaches to work around with the choice of attribute sets. A few contributing to the ranking based heuristics (Duch et al. 2003), while there have been approaches to use search heuristics to identify the right set prior to classification (Vafaie 1994). The book by Liu and Motoda provides an additional insight towards the different methods used for feature selection. On the whole the contribution of different search algorithms have been towards identifying the attribute subsets.

Search Space

| A | B | C | D | E | F | G | Class |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Figure 2: Search based Feature Selection
The 'search space' indicated focuses on the attribute-value subsets and as described in the earlier sections due to the NP-hard nature of the problem needs the use of a heuristic search tool. The primary goal is to identify the maximum information gain. The current research does not provide a theoretical proof for the same but is based on the assumption; an attribute possessing a maximum information gain, would ultimately take a higher rank for the attribute selection scorecard. On a similar hypothesis, if a current attribute value performs poorly to clearly classify a current class, would combining attribute-value sets provide an increased purity for classification? Consider the example shown in Table 4.

Table 4: Illustration for the creation of the multivalued subsets

| Attribute-value sets | Class |
| :---: | :---: |
| A.1 | 1 |
| A.1 | 1 |
| A.2 | 1 |
| A.3 | 2 |
| A.3 | 2 |
| A.3 | 3 |
| A.4 | 4 |

If attribute-value 'A.1' were to be tested against the rest of the attribute-value sets, it divides the class 1 in the ratio 2:1 against the rest. Hence attribute-value 'A.1' does not provide a pure separation for the classes. But when paired up with attribute value 'A.2', the Class ' 1 ' is purely separated.

Now considering the approaches made by the ID3, GID3 and GID3* algorithms

$$
\begin{aligned}
& \eta_{u, v}=\left\|\left\{t: x^{t}=u, y^{t}=v\right\}\right\| \\
& \forall x, \xi(x)=\frac{x}{T} \log (x) \\
& \hat{H}(Y)=\log _{2}(T)
\end{aligned}
$$

## Algorithm for the Multivalued Set

For the purposes of reducing the complexity of the algorithm, readers are advised to refer to the section 'Adaptive Simulated Annealing' for the algorithm framework of the search tool used for choosing the attribute-value pairs.

Algorithm A.3: Algorithm for collecting the subsets for the maximum Information Gain.

## begin

size(dataset)
for features $=1$ :number of attributes
begin
Calculate ClassEntropy
perform 'Adaptive Simulated Annealing' with respect to collecting attribute -value pairs.
calculate Information Gain
end
end

Given below are implementations of the proposed algorithm on the datasets 'Iris' and 'Vehicles'. The data pre-processing (discretization) has been done using the CAIM based supervised discretizer. The feature subsets identified are illustrated in [Figures $4 \& 7$ ].

Dataset : Iris

Ranking Transformation:
Table 5: Iris: Comparison of the ranks (ID3 vs. MVS)

|  | ID3 | MVS Ranking |  |
| ---: | ---: | :--- | ---: |
| Attribute | Information Gain | Attribute | Information Gain |
| 3 | $1.45 \mathrm{E}+00$ | 4 | 0.618695 |
| 4 | $1.44 \mathrm{E}+00$ | 3 | 0.345634 |
| 1 | $8.77 \mathrm{E}-01$ | 1 | 0.128627 |
| 2 | $5.11 \mathrm{E}-01$ | 2 | 0.120512 |



Figure 3: Iris: Comparison of the Information Gain

| Information Gain ID3 Ranking |  | MVS |  |
| :---: | ---: | :---: | ---: |
| Attribute Set | Classifier Error |  | Attribute Sets |
| Classifier Error |  |  |  |
| $3,4,1,2$ | $22.22 \%$ |  | $4,3,1,2$ |
| $3,4,1$ | $22.22 \%$ |  | $1,4,3$ |
| 3,4 | $35.56 \%$ | 1,4 | $22.22 \%$ |
| 3 | $35.56 \%$ |  | 4 |
| $22.22 \%$ |  |  |  |

Figure 4: Figure representing the feature selection process for the dataset ' Iris'.


Figure 5: Classifier Error Performance between Information Gain (ID3 vs. MVS)

## Dataset : Vehicles

## Ranking Transformation:

Table 6: Vehicles: Comparison of the ranks (ID3 vs MVS)

| Information Gain for ID3 | Attribute | Information Gain for MVS | Attribute |
| :---: | :---: | :---: | :---: |
| $1.38 \mathrm{E}+00$ | 12 | 0.252924 | 9 |
| $8.12 \mathrm{E}-01$ | 7 | 0.212451 | 6 |
| $7.88 \mathrm{E}-01$ | 11 | 0.144444 | 8 |
| $6.14 \mathrm{E}-01$ | 4 | 0.093892 | 3 |
| $6.13 \mathrm{E}-01$ | 8 | 0.083239 | 2 |
| $5.99 \mathrm{E}-01$ | 13 | 0.066396 | 14 |
| $5.78 \mathrm{E}-01$ | 3 | 0.058774 | 10 |
| $4.83 \mathrm{E}-01$ | 9 | 0.057838 | 17 |
| $3.66 \mathrm{E}-01$ | 10 | 0.051468 | 18 |
| $3.37 \mathrm{E}-01$ | 6 | 0.048565 | 7 |
| $3.26 \mathrm{E}-01$ | 1 | 0.045457 | 11 |
| $3.08 \mathrm{E}-01$ | 2 | 0.037533 | 1 |
| $2.77 \mathrm{E}-01$ | 14 | 0.033462 | 5 |
| $2.40 \mathrm{E}-01$ | 17 | 0.033196 | 4 |
| $2.31 \mathrm{E}-01$ | 18 | 0.03098 | 13 |
| $2.12 \mathrm{E}-01$ | 5 | 0.025649 | 12 |
| $1.82 \mathrm{E}-01$ | 16 | 0.018691 | 15 |
| $1.02 \mathrm{E}-01$ | 15 | 0.015805 | 16 |



Figure 6: Vehicles: Comparison between the Information Gain Values

| Information Gain ID3 Ranking |  |  |  | MVS Ranking |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Classifier Error |  |  |  | Classifier Error |
| 12,7,11,4,8,13,3,9,10,6,1,2,14,17,18,5,16,15 | 55.32\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5,4,13,12,15,16 | 55.32\% |
| 12,7,11,4,8,13,3,9,10,6,1,2,14,17,18,5,16 | 52.07\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5,4,13,12,15 | 55.67\% |
| 12,7,11,4,8,13,3,9,10,6,1,2,14,17,18,5 | 53.31\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5,4,13,12 | 53.42\% |
| 12,7,11,4,8,13,3,9,10,6,1,2,14,17,18 | 54.26\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5,4,13 | 54.13\% |
| 12,7,11,4,8,13,3,9,10,6,1,2,14,17 | 54.25\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5,4 | 53.90\% |
| 12,7,11,4,8,13,3,9,10,6,1,2,14 | 54.14\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1,5 | 53.90\% |
| 12,7,11,4,8,13,3,9,10,6,1,2 | 54.02\% |  |  | 9,6,8,3,2,14,10,17,18,7,11,1 | 53.90\% |
| 12,7,11,4,8,13,3,9,10,6,1 | 54.37\% |  |  | 9,6,8,3,2,14,10,17,18,7,11 | 54.72\% |
| 12,7,11,4,8,13,3,9,10,6 | 54.02\% |  |  | 9,6,8,3,2,14,10,17,18,7 | 54.72\% |
| 12,7,11,4,8,13,3,9,10 | 54.02\% |  |  | 9,6,8,3,2,14,10,17,18 | 55.56\% |
| 12,7,11,4,8,13,3,9 | 57.92\% |  |  | 9,6,8,3,2,14,10,17 | 55.55\% |
| 12,7,11,4,8,13,3 | 57.45\% |  |  | 9,6,8,3,2,14,10 | 55.67\% |
| 12,7,11,4,8,13 | 59.46\% |  |  | 9,6,8,3,2,14 | 58.86\% |
| 12,7,11,4,8 | 59.22\% |  |  | 9,6,8,3,2 | 58.75\% |
| 12,7,11,4 | 59.22\% |  |  | 9,6,8,3 | 59.57\% |
| 12,7,11 | 59.22\% |  |  | 9,6,8 | 61.35\% |
| 12,7 | 60.52\% |  |  | 9,6 | 62.65\% |
| 12 | 60.52\% |  |  | 9 | 62.64\% |

Figure 7: Figure representing the feature selection model for the dataset 'Vehicles'. Refer
to Appendix A. 2 for the nomenclature for the attribute and class names.


Figure 8: Vehicles: Classifier Error Performance between Information Gain
(ID3 vs. MVS)

## Normality Testing

The key objective of this approach is to identify the right subsets, which maximize on the information gain. Hence, it is essential to ensure that the heuristic displays a normal behavior with respect to selecting the subsets. Appendix A. 1 provides the results of the normality test performed on the 'Iris' dataset. As can be observed that the normality test
fails when performed on the gain values evaluated for different set sizes. But what can be observed, that the gains are bounded by an upper value. For the sake of the feature selection evaluation mentioned above, these maximum values have been taken into consideration. At the same time the subset sizes selected under these gain values follow a normal distribution.

As can be seen the information gain value collected follows a normal distribution. With the selected number of cluster elements (identical between the ID3 and the Multivalued subsets), the algorithm did manage to find clusters of the attribute sets portraying lower classification error values. Especially, the results of the tests done on the dataset 'Iris' show that the attribute subsets collected as a result of the ranking provided by the multisubset attribute measure consistently provided a lower bound on the classifier error until the final attribute. This illustrates that, if the user were to choose an attribute subset having dimensions between the maximum and the minimum value i.e., a midsize interval, the multivalued set could provide attribute sets with lower classification error compared to the normal information gain values.

## Chapter 4.

## Discretization.

'Data Discretization' plays a crucial part for any kind of data mining or machine learning activity. This stage is quite critical especially under data pre-processing. This section introduces the user to several facets of data discretization and concludes with proposing newer heuristic-based search models for discretization. The intention is to observe the behavior of the datasets under the influence of the heuristic based discretization models. When referring to data discretization, it is the individual attributes which are subject to the discretization study. Most of the credited classifiers rely on alphanumeric data to build their prediction models. In other words, it would be easier to use a classifier on a discrete value rather than a continuous value. On the whole, most of the existing discretizing schemes could be categorized into 2 distinct classes, supervised and unsupervised discretization.

### 4.1 Unsupervised Discretization

The key discriminating factor between the two categories, as introduced in the earlier paragraph, is the relationship with the classes while carrying out the discretization. Unsupervised discretization falls in the category where the classes are not taken into account while carrying out the process. One of the key assumptions while crediting this process is that of the sorting rule. The continuous values are subjected to a sorting rule (Kotsiantis \& Kanellopoulos 2006(a)). Since the bin-size/bin-width are user defined parameters, their choice is completely independent of the way the classes would get distributed when discretized to form one category. Two of the most primitive and the
widely used methods under the unsupervised category are namely equal width and equal frequency discretization.

### 4.2 Equal Width Discretization.

As the name suggests, the main idea of this algorithm is to allocate a fixed bin width to the range of attribute-value sets. For the same, the algorithm commences on defining ' $k$ ', the width of the bin. The difference between the maximum and the minimum values of independent attribute-value sets in divided by ' $k$ '. The range of values for individual attributes is then binned into these intervals. There have been a number of views and takes on the value of ' $k$ ' to be considered while conducting the discretization process. For the purpose of this study, the Strudges' formula (Strudges 1926), $k=\log _{2}$ (number of instances $)+1$, has been utilized to identify the optimal bin width. The intention of doing so is to have equivalent bin sizes across the algorithms to make an unbiased comparison.

Figure 9 provides an illustrated view of the result of such a binning procedure. The algorithm for this discretization is as described below. Appendix C. 1 provides the information on the program used to evaluate this algorithm.

| ATT1 | ATT2 | ATT3 | ATT4 | Class | Att1 | Att2 | Att3 | Att4 | Class |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.1 | 3.5 | 1.4 | 0.2 | 1 | 2 | 6 | 1 | 1 | 1 |
| 4.9 | 3 | 1.4 | 0.2 | 1 | 2 | 4 | 1 | 1 | 1 |
| 4.7 | 3.2 | 1.3 | 0.2 | 1 | 1 | 5 | 1 | 1 | 1 |
| 4.6 | 3.1 | 1.5 | 0.2 | 1 | 1 | 4 | 1 | 1 | 1 |
| 5 | 3.6 | 1.4 | 0.2 | 1 | 2 | 6 | 1 | 1 | 1 |
| 5.4 | 3.9 | 1.7 | 0.4 | 1 | 3 | 7 | 1 | 2 | 1 |
| 4.6 | 3.4 | 1.4 | 0.3 | 1 | 1 | 5 | 1 | 1 | 1 |
| 5 | 3.4 | 1.5 | 0.2 | 1 | 2 | 5 | 1 | 1 | 1 |
| 4.4 | 2.9 | 1.4 | 0.2 | 1 | 1 | 4 | 1 | 1 | 1 |
| 4.9 | 3.1 | 1.5 | 0.1 | 1 | 2 | 4 | 1 | 1 | 1 |
| 5.4 | 3.7 | 1.5 | 0.2 | 1 | 3 | 6 | 1 | 1 | 1 |
| 4.8 | 3.4 | 1.6 | 0.2 | 1 | 2 | 5 | 1 | 1 | 1 |
| 4.8 | 3 | 1.4 | 0.1 | 1 | 2 | 4 | 1 | 1 | 1 |
| 4.3 | 3 | 1.1 | 0.1 | 1 | 1 | 4 | 1 | 1 | 1 |
| 5.8 | 4 | 1.2 | 0.2 | 1 | 4 | 7 | 1 | 1 | 1 |
| 5.7 | 4.4 | 1.5 | 0.4 | 1 | 4 | 9 | 1 | 2 | 1 |
| 5.4 | 3.9 | 1.3 | 0.4 | 1 | 3 | 7 | 1 | 2 | 1 |
| 5.1 | 3.5 | 1.4 | 0.3 | 1 | 2 | 6 | 1 | 1 | 1 |
| 5.7 | 3.8 | 1.7 | 0.3 | 1 | 4 | 7 | 1 | 1 | 1 |
| 5.1 | 3.8 | 1.5 | 0.3 | 1 | 2 | 7 | 1 | 1 | 1 |
| 5.4 | 3.4 | 1.7 | 0.2 | 1 | 3 | 5 | 1 | 1 | 1 |
| 5.1 | 3.7 | 1.5 | 0.4 | 1 | 2 | 6 | 1 | 2 | 1 |
| 4.6 | 3.6 | 1 | 0.2 | 1 | 1 | 6 | 1 | 1 | 1 |
| 5.1 | 3.3 | 1.7 | 0.5 | 1 | 2 | 5 | 1 | 2 | 1 |
| 4.8 | 3.4 | 1.9 | 0.2 | 1 | 2 | 5 | 2 | 1 | 1 |
| 5 | 3 | 1.6 | 0.2 | 1 | 2 | 4 | 1 | 1 | 1 |

Figure 9: Illustration for the Equal Width Discretization

```
begin
initialize int=zeros(number of rows -1, number of columns)
    for i=i:number of attributes (n)
    begin
        sort attribute column
        find unique
        initialize }\textrm{k}=\mp@subsup{\operatorname{log}}{2}{(number of rows +1)
        w=}\frac{\mathrm{ max_value(n)-min_value(n)}}{k
        int[1]=min_value
                for fill=2:number of rows-1
                begin
                    int[fill]=min_value+(fill-1)*w
                end
                for values=1:number of rows
                begin
                    UniqueID for values falling in the interval
                end
    end
end
```

This approach provides a 'unique identity (UniqueID)' to the clustered attribute-value sets irrespective of the unique values observed for individual attribute-value. This avoids the problem of divisibility, explained in the following section.

### 4.3 Equal Frequency Discretization

As against the previous method of equal width, wherein the intervals calculated are the function of the maximum and the minimum values of the attribute, the equal frequency discretization provides bin widths to allocate equal number of unique attribute-values per bin. This gives rise to an issue, which was raised earlier with regards to finding the right divisibility factor for the bin. Also, attribute-value sets summing to form a prime number posed a problem towards finding the right width value for the bins. This issue has been addressed in this study with providing a small bias (prime number escape) to the last unique value taking up the 'UniqueID' of the previous binning width. Figure 10 shows an
illustration of the functioning of this algorithm tested on one of the datasets. Appendix C. 5 provides the program used to perform the equal frequency discretization. Given below is the algorithm for the discretization method. For the purposes of providing a constraint on the upper bound on the values of ' $k$ ' i.e. the binning width value, a conditional heuristic has been implemented, $\log$ (number of rows) $-\log (\log$ (number of rows)) $<=19$ (Jiang et al. 2009).

| AT1 | ATT2 | AT3 | AT4 | Class |
| :---: | :---: | :---: | :---: | :---: |
| 5.1 | 3.5 | 1.4 | 0.2 | 1 |
| 4.9 | 3 | 1.4 | 0.2 | 1 |
| 4.7 | 3.2 | 1.3 | 0.2 | 1 |
| 4.6 | 3.1 | 1.5 | 0.2 | 1 |
| 5 | 3.6 | 1.4 | 0.2 | 1 |
| 5.4 | 3.9 | 1.7 | 0.4 | 1 |
| 4.6 | 3.4 | 1.4 | 0.3 | 1 |
| 5 | 3.4 | 1.5 | 0.2 | 1 |
| 4.4 | 2.9 | 1.4 | 0.2 | 1 |
| 4.9 | 3.1 | 1.5 | 0.1 | 1 |
| 5.4 | 3.7 | 1.5 | 0.2 | 1 |
| 4.8 | 3.4 | 1.6 | 0.2 | 1 |
| 4.8 | 3 | 1.4 | 0.1 | 1 |
| 4.3 | 3 | 1.1 | 0.1 | 1 |
| 5.8 | 4 | 1.2 | 0.2 | 1 |
| 5.7 | 4.4 | 1.5 | 0.4 | 1 |
| 5.4 | 3.9 | 1.3 | 0.4 | 1 |
| 5.1 | 3.5 | 1.4 | 0.3 | 1 |
| 5.7 | 3.8 | 1.7 | 0.3 | 1 |
| 5.1 | 3.8 | 1.5 | 0.3 | 1 |
| 5.4 | 3.4 | 1.7 | 0.2 | 1 |
| 5.1 | 3.7 | 1.5 | 0.4 | 1 |
| 4.6 | 3.6 | 1 | 0.2 | 1 |
| 5.1 | 3.3 | 1.7 | 0.5 | 1 |
| 4.8 | 3.4 | 1.9 | 0.2 | 1 |
| 5 | 3 | 1.6 | 0.2 | 1 |



Figure 10: Illustration for the Equal Frequency Discretization

## Algorithm A.5: Equal Frequency Discretization Algorithm

## begin

```
initialize temp=[]
    for i=i:number of attributes (n)
    begin
        sort attribute column
        find unique
        if }\operatorname{log}\mathrm{ (number of rows) }-\operatorname{log}(\operatorname{log}(\mathrm{ number of rows )})<=1
        then check divisibility function
        if number of unique values is prime
        then store the last unique value in the vector [temp]
        else divisibility =20
            for bin_cap =1:bin:size_unique
            begin
                for index=bin_cap:(bin_cap+(bin-1))
                begin
```

```
                                    provide UniqueID
                                    end
                                    end
                                    if temp is not empty then provide unique value of the last 'bin'
                                    fill in all the UniqueIDs to the row values associated with unique values
    end
end
function check divisibility
    begin
            initialize divisibility =0
            for test=2:19
            begin
                    if number is divisible by 'test' then divisibility takes the value of the
                    'test'
                    if no divisibility value is identified then divisibility returns a 'prime'
                    indication
            end
    end
```

A few points to note about the implementation procedure of the above algorithms:

- The divisibility test would necessarily take in the highest divisibility value found during the test.
- Both the algorithms have self-generated bin value ' k ', hence eliminating the need to have the users input them.
- The value of ' $k$ ' did not exceed 20, which is an important inference for the next chapter.


### 4.4 CAIM Algorithm (Kurgan 2004)

The class-attribute interdependency maximization algorithm (CAIM), is one of the popular supervised algorithms for discretization. The algorithm works on presetting the interval boundaries before being tested on for the CAIM criteria. The CAIM criteria is as defined below:

CAIM $=\frac{\sum_{r=1}^{n} \frac{\text { max }_{r}{ }^{2}}{M_{+r}}}{n}$
The values for $\max _{r}, M_{+r}$ are obtained from a Quanta Matrix. 'Quanta Matrix' is a matrix linking the number of attribute-value sets binned per interval width against the interval sizes. The algorithm iteratively adds an inner boundary to check the value of the CAIM criteria. The interval boundary providing a higher CAIM measure is retained. The algorithm runs a greedy search through all the predetermined interval boundaries.

### 4.5 Supervising the Unsupervised.

The previous section dealt with the study of the unsupervised algorithms. The performance evaluation would be showcased in the section, 'Supervised Unsupervised and Semi-supervised'. Research conducted on these algorithms has been directed towards identifying the right bin width. This study attempts to utilize a path based metaheuristic search to find the right set of instances that would improve the information gain of the discretized sets. Critics might argue about two points with regards to this approach Neighborhood.

The neighborhood is defined as the adjacency of values, chosen to form the discrete sets. For the proposed models, values might be randomly picked from the set of all points in the attribute-value subsets. This might not define a neighborhood of values to discretize into a single category.

Yes, though this might be true, at this stage the constraint of adjacency, needs to be relaxed. At the same time, the bin sizes are user defined, a lower bin size also attempts to increase the generalization of the data being considered for discretization. This is an
attempt to understand the role of a search-based approach towards different machine learning facets.

## Bounds.

Since the search is for a better discretizer, the other issue that remains to be answered is that, why wouldn't a robust solution of simply sorting the attribute values sets as per individual classes provide a better discretized set for the classifier? This idea would carry the same assumption as has been mentioned in 'Neighborhood', hence a search based heuristic approach appears to be a better solution than this one. Also, as can be observed from the normality tests for the gain values of the subsets for the dataset 'Iris' (Appendix A.1), the values observed with the current implementation environment do not cross a certain maximum, which attributes to the distribution not following a normal pattern. It can be assumed at this stage that the gain values have reached a 'maximum'. Ultimately the requirement is to have bounds on the values to be taken in for making the discretization. The research began with the initiative of finding an optimal subset of values function of increasing the information gain for the identified attribute value pairs.

### 4.5.1 Multivalued Discrete

The first set of discretization models introduced under this category take up a role of supervising the unsupervised discretizer, resulting in a heuristic based semi-supervised algorithm. These could be divided into the following two categories.

### 4.5.1.1 Multivalued Discrete Width (MDW)

As introduced in Chapters 2\&3, the multivalued algorithm creates 2 distinct partitions (discriminant/s) based on the optimal information gain achieved. This heuristic would be
utilized to initially discretize the attribute-value sets and then following the conversion to a single class as an input towards the bifurcated equal width interval. Figure 11 shows an illustrated view of the transformation from the original continuous data to the MWD followed by the algorithm representation of the same. To avoid undue complexity, the algorithm representation does not include the syntaxes for the heuristic-based searches.

| ATT1 | ATT2 | ATT3 | ATT4 | Class |
| :---: | :---: | :---: | :---: | :---: |
| 5.1 | 3.5 | 1.4 | 0.2 | 1 |
| 4.9 | 3 | 1.4 | 0.2 | 1 |
| 4.7 | 3.2 | 1.3 | 0.2 | 1 |
| 4.6 | 3.1 | 1.5 | 0.2 | 1 |
| 5 | 3.6 | 1.4 | 0.2 | 1 |
| 5.4 | 3.9 | 1.7 | 0.4 | 1 |
| 4.6 | 3.4 | 1.4 | 0.3 | 1 |
| 5 | 3.4 | 1.5 | 0.2 | 1 |
| 4.4 | 2.9 | 1.4 | 0.2 | 1 |
| 4.9 | 3.1 | 1.5 | 0.1 | 1 |
| 5.4 | 3.7 | 1.5 | 0.2 | 1 |
| 4.8 | 3.4 | 1.6 | 0.2 | 1 |
| 4.8 | 3 | 1.4 | 0.1 | 1 |
| 4.3 | 3 | 1.1 | 0.1 | 1 |
| 5.8 | 4 | 1.2 | 0.2 | 1 |
| 5.7 | 4.4 | 1.5 | 0.4 | 1 |
| 5.4 | 3.9 | 1.3 | 0.4 | 1 |
| 5.1 | 3.5 | 1.4 | 0.3 | 1 |
| 5.7 | 3.8 | 1.7 | 0.3 | 1 |
| 5.1 | 3.8 | 1.5 | 0.3 | 1 |
| 5.4 | 3.4 | 1.7 | 0.2 | 1 |
| 5.1 | 3.7 | 1.5 | 0.4 | 1 |
| 4.6 | 3.6 | 1 | 0.2 | 1 |
| 5.1 | 3.3 | 1.7 | 0.5 | 1 |
| 4.8 | 3.4 | 1.9 | 0.2 | 1 |
| 5 | 3 | 1.6 | 0.2 | 1 |



Figure 11: Illustration for the Multivalued Discrete Width Algorithm

## Algorithm A.6: Multivalued Discrete Width Algorithm

```
begin
    initiate by sizing the dataset
    for attributes=1:number of columns
    begin
        assign interval
        initiate UniqueID, check_unique,exit_loop=0
        while check_unique<=intervals or exit_loop==0
        begin
            find the number of unique elements in the column
        if number of unique }>=4\mathrm{ then identify and create dataset
        if number of unique<4 then check feasibility
        if feasibility constraints are satisfied then create dataset
        else exit_loop=1
        assign UniqueID to individual discriminant set
        update UniqueID
            end
end
```

```
function discrete
begin
    initiate by sizing the dataset
                        if number of classes for the dataset size ==1 then perform bifurcated equal
                        width
    if number of classes for the dataset size >1 then perform multidiscrete
end
```

As can be seen from the above framework of the algorithm, each time the generated discriminant dataset i.e. subset of the original data during a particular iteration instance finds a pure class, the function performs an equal width division of the dataset, creating two discriminants. The algorithm is also subjected to 'check feasibility' constraint, which would identify the existence of any subsets of unique values of at least the size of 4 . The program for this algorithm is provided in the Appendix C.6.

### 4.5.1.2 Multivalued Discrete Frequency (MDF)

The performance of the multivalued discrete frequency is similar to that of the multivalued discrete width with a slight modification of the convergence function ${ }^{\$}$. In this case, rather a bifurcation of equal frequency is being preferred. The modification algorithm function for discretization rule under the MDF is as described below.

```
function discrete
begin
            initiate by sizing the dataset
            if number of classes for the dataset size ==1 then perform bifurcated equal
frequency
            if number of classes for the dataset size >1 then perform multidiscrete
end
```

Table 7 showcases a performance of the algorithms. The performance evaluation of the algorithms is provided in the section 'Classifier Performance'.
${ }^{\text {\$ }}$ The convergence function provides an unsupervised discretization on the discriminant set providing equal classes.

Table 7: Comparison of the Classifier Errors observed between Supervised, Unsupervised and Heuristic based Semi Supervised Discretization processes.

| Classification Error |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Datasets | CAIM | Equal Frequency | Multivalued Frequency | Equal Width | Multivalued Width |
| Classifier |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| AdaBoost | Iris | 33.33\% | 15.55\% | 48.88\% | 20.00\% | 22.22\% |
| ID3 |  | 22.22\% | 0.00\% | 6.67\% | 6.66\% | 8.99\% |
| Naïve Bayes |  | 22.22\% | 0.00\% | 6.67\% | 8.88\% | 8.99\% |
| Regression |  | 22.22\% | 6.67\% | 8.89\% | 8.88\% | 8.99\% |
|  |  |  |  |  |  |  |
| AdaBoost | Glass | 50.00\% | 25.00\% | 64.06\% | 54.68\% | 56.25\% |
| ID3 |  | 48.43\% | 1.57\% | 29.68\% | 37.50\% | 29.69\% |
| Naïve Bayes |  | 45.31\% | 4.68\% | 28.13\% | 32.81\% | 26.56\% |
| Regression |  | 59.38\% | 7.81\% | 37.50\% | 31.25\% | 31.25\% |
|  |  |  |  |  |  |  |
| AdaBoost | Images | 69.02\% | 62.33\% | 66.47\% | 72.10\% | 63.15\% |
| ID3 |  | 60.52\% | 43.09\% | 19.62\% | 13.99\% | 22.11\% |
| Naïve Bayes |  | 60.68\% | 58.71\% | 18.72\% | 18.65\% | 18.05\% |
| Regression |  | 77.67\% | 43.16\% | 16.39\% | 12.03\% | 15.79\% |
|  |  |  |  |  |  |  |
| AdaBoost | PenDigits | 80.69\% | 90.26\% | 81.78\% | 79.84\% | 81.15\% |
| ID3 |  | 29.73\% | 89.88\% | 20.68\% | 7.27\% | 21.30\% |
| Naïve Bayes |  | 34.40\% | 91.80\% | 16.71\% | 16.39\% | 16.39\% |
| Regression |  | 30.50\% | 89.89\% | 13.33\% | 7.81\% | 14.10\% |
|  |  |  |  |  |  |  |
| AdaBoost | Vehicles | 61.42\% | 78.74\% | 58.66\% | 62.60\% | 59.84\% |
| ID3 |  | 54.72\% | 79.92\% | 26.77\% | 29.92\% | 33.07\% |
| Naïve Bayes |  | 57.48\% | 70.87\% | 31.10\% | 38.19\% | 35.43\% |
| Regression |  | 56.69\% | 77.56\% | 25.50\% | 31.10\% | 29.92\% |

### 4.6 Variable Intervals

The section on multivalued discrete intervals, dealt with a fixed user provided number of intervals. The next step is to analyze the effect of varying number of intervals on the developed algorithm. The objective would be to study the classifier performance on the discretization algorithms with increasing interval bin sizes. The experimental conditions specified in the section 'Testing Conditions', were maintained for every interval type. Table 8 shows the performance of such an experimental run. Discussion on the results is provided in the section 'Classifier Performance'.

Table 8: Comparison of the Classifier Errors observed on varying interval sizes

| Datasets | Classifiers | Multisubset-Frequency_4 | Multisubset-Frequency_8 | Multisubset-Frequency_16 | Multisubset-Frequency_20 | Multisubset-Width_4 | Multisubset-Width_8 | Multisubset-Width_16 | Multisubset-Width_20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | AdaBoost | 20.00\% | 20.00\% | 40.00\% | 46.66\% | 24.44\% | 22.22\% | 46.66\% | 42.22\% |
| Iris | ID3 | 17.78\% | 11.11\% | 11.11\% | 4.44\% | 15.55\% | 8.99\% | 4.44\% | 2.22\% |
|  | Naive Bayesian | 8.90\% | 4.44\% | 8.88\% | 2.22\% | 17.77\% | 8.99\% | 11.11\% | 4.44\% |
|  | Regression | 11.11\% | 2.22\% | 11.11\% | 6.67\% | 17.78\% | 8.99\% | 4.44\% | 6.67\% |
|  | AdaBoost | 60.94\% | 56.25\% | 64.06\% | 53.13\% | 60.94\% | 56.25\% | 54.69\% | 54.69\% |
| Glass | ID3 | 32.81\% | 29.24\% | 29.68\% | 26.56\% | 32.81\% | 29.69\% | 26.56\% | 20.31\% |
|  | Naive Bayesian | 43.75\% | 26.56\% | 28.13\% | 14.06\% | 43.75\% | 26.56\% | 18.75\% | 12.50\% |
|  | Regression | 42.19\% | 31.25\% | 37.50\% | 20.31\% | 42.19\% | 31.25\% | 26.56\% | 21.88\% |
|  |  |  |  |  |  |  |  |  |  |
|  | AdaBoost | 62.09\% | 60.60\% | 61.57\% | 64.29\% | 62.03\% | 60.60\% | 67.82\% | 69.54\% |
| Images | ID3 | 33.08\% | 21.06\% | 17.74\% | 18.05\% | 33.01\% | 21.06\% | 17.52\% | 16.69\% |
|  | Naïve Bayesian | 26.02\% | 19.47\% | 18.12\% | 17.82\% | 26.24\% | 19.47\% | 18.80\% | 17.14\% |
|  | Regression | 25.04\% | 18.80\% | 15.94\% | 14.21\% | 25.04\% | 18.80\% | 15.71\% | 14.29\% |
|  |  |  |  |  |  |  |  |  |  |
|  | AdaBoost | 79.00\% | 80.46\% | 81.22\% | 80.22\% | 79.00\% | 80.46\% | 81.23\% | 80.23\% |
| PenDigits | ID3 | 28.88\% | 23.44\% | 20.68\% | 19.08\% | 28.88\% | 23.70\% | 20.69\% | 19.08\% |
|  | Naïve Bayesian | 25.51\% | 18.69\% | 16.71\% | 16.62\% | 25.51\% | 18.97\% | 16.71\% | 16.63\% |
|  | Regression | 21.91\% | 17.47\% | 13.33\% | 14.40\% | 21.92\% | 17.47\% | 13.33\% | 14.41\% |
|  |  |  |  |  |  |  |  |  |  |
|  | AdaBoost | 65.35\% | 64.57\% | 69.29\% | 60.24\% | 65.35\% | 64.57\% | 69.29\% | 60.24\% |
| Vehicles | ID3 | 37.01\% | 33.85\% | 27.17\% | 22.05\% | 37.00\% | 33.85\% | 27.17\% | 22.05\% |
|  | Naive Bayesian | 38.98\% | 37.00\% | 31.10\% | 28.74\% | 38.98\% | 37.01\% | 31.10\% | 28.74\% |
|  | Regression | 31.88\% | 37.79\% | 29.14\% | 27.95\% | 31.88\% | 37.79\% | 29.14\% | 27.95\% |

### 4.7 Un-supervising the Supervised

The difficulty as identified in the previous section (Supervising the Un-supervised) is with regards to the validity of picking up the attribute-value pairs with a goal of increasing the information gain. In order to provide a more robust platform for the initial stage of discretization, an approach would be to reverse the order of which the algorithms are conjoined. In this section, the strategy is to have the initial discretization performed using the unsupervised methods followed by building up a multivalued subset discretization scheme on top. Four models are introduced, which are more of variants of the original equal frequency, equal width and multivalued subsets. In the prior section, the equal width and the equal frequency primarily providing binary divisions to the multivalued discretizer, but under the current condition, the heuristic-based multivalued discretizer would provide a binary division to the unsupervised discretizer. To avoid complexity, the part representing the search heuristic under the multivalued subsets has been left for the readers to refer to the earlier section on ASA of this document.

### 4.7.1 Frequency Multivalued Width (FMW)

In this method, the heuristic based multivalued width algorithm is built on top of the equal frequency discretization algorithm.

### 4.7.2 Frequency Multivalued Frequency (FMF)

Similar approach has been applied with the multivalued frequency algorithm.
Appendix C. 7 provides the program used to perform the FMF/FMW algorithm.

## Algorithm A.7: Frequency Multivalued Discrete Frequency / Frequency-Multivalued Discrete Width Algorithm

```
begin
    size dataset
    for features =1:number of attributes
    begin
        sort attribute column
        find unique
        perform equal frequency discretization
    end
    for perform_multidiscrete=1:number of intervals
    begin
        perform multidiscrete width or frequency discretization
        end
end
```


### 4.7.3 Frequency-Frequency (FF) / Frequency Width (FW)

In order to have a fair comparison of the algorithms mentioned above, similar approaches were applied to the robust algorithm of equal frequency and equal width algorithms. The framework of these algorithms is similar to the ones mentioned above except for the fact that the multidiscrete discretization steps are replaced with the equal frequency/equal width algorithms. Here too the equal width and equal frequency discretization algorithms
provide a binary partition. Appendix C. 8 provides the program used to perform the FF/FW algorithms. The algorithms could be explained as indicated below.

```
Algorithm A.8: Frequency-Frequency/ Frequency-Width Discretization Algorithm
begin
    size dataset
    for features =1:number of attributes
    begin
        sort attribute column
        find unique
        perform equal frequency discretization
    end
    for perform_multidiscrete=1:number of intervals
    begin
        perform equal width or equal frequency discretization
    end
end
```

Table 10 shows the result table for this approach. As can be seen, a number of instances were observed where the multivalued based heuristic managed to provide a lower classification error value. The implications of all the results identified in the above sections are summarized in the following chapter.

## Chapter 5

## Classifier Performances

The earlier sections introduced the applications of a proposed philosophy of multivalued subsets with a couple of distinct applications. Both these applications were presented in the form of a classifier error output. It would be interesting to see the performances of these classifiers with regards to the proposed and existing methods. The key classifiers as had been discussed earlier are 'AdaBoost, ID3, Naïve Bayes and Regression'.

### 5.1. Supervised, Unsupervised and Semi-supervised

This subsection highlights the performance of the individual discretization algorithms against the datasets introduced in Table 1. Since the approaches adopted by the equal width and the equal frequency discretization algorithms are quite different in terms of identifying the right interval size, the interval size needs to be manually identified for their corresponding multivalued subsets [Table 9.].

Table 9: Identifying Equal Intervals across Algorithms

|  |  | Width |  | Frequency |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Datasets | Instances | $\log _{2}$ (Instances)+1 | Interval size for <br> Multivalued sets <br> width | Observed instances <br> under interval <br> constraints | Interval size for <br> Multivalued sets <br> frequency |
| Iris | 150 | 8.22 | 8 | $\operatorname{Max}(14)$ | 14 |
| Glass | 214 | 8.34 | 8 | $\operatorname{Max}(16)$ | 16 |
| Images | 4435 | 13.114 | 13 | $\operatorname{Max}(13)$ | 13 |
| PenDigits | 4350 | 13.086 | 13 | $\operatorname{Max}(16)$ | 16 |
| Vehicles | 896 | 10.72 | 11 | $\operatorname{Max}(18)$ | 18 |

As can be observed from Table 7, the proposed algorithms provided consistent and moderate classification error rates as compared to the robust unsupervised and the
supervised algorithms. The Equal Frequency discretization algorithm provides a comparatively lower classification error rate for the dataset 'Iris', while providing a higher classification error for the dataset 'PenDigits'.

Figure 12 showcases the graphical representation of the classification errors identified across the 3 main categories of discretizers (Supervised, Unsupervised and the Heuristic based Semi-Supervised). The comparisons have been performed as per the interval sizes identified in Table 9.

Adaptive Boosting

Width


Frequency


## Frequency



## Naïve Bayesian

Width


Frequency


## Regression

Width


Frequency


Figure 12: Supervised, Unsupervised and Semi-Supervised
As can be seen, of the four classifiers identified, the 'Adaptive Boosting' displayed the most coherent behavior. The proposed algorithm showed moderate variances in the performances across the datasets as against the supervised and the unsupervised algorithms. The proposed algorithm proved to be better for the dataset 'Iris' while it suffered in performance for the dataset 'Vehicles' against the supervised algorithm. One distinguishing factors between these two datasets were in terms of a measure of class distribution. Under the observance of reduction in the unique-values caused by the discretization process, the CAIM algorithm triggered a considerably large reduction in the data dimensions for both datasets. While the reduction in the dimensions were
comparatively smaller by the multivalued-based heuristic. From the look of the graphs, the multivalued heuristic could have provided an 'overfit' discretization scheme. Or in other words the number of intervals could have been an attribute of this cause. In order to evaluate the behavior of the proposed multivalued heuristic under different interval conditions, the readers are advised to refer to the section 'Varying Interval Sizes'.

As had been mentioned in the earlier section 'Discretization', the basis for the formulation of the proposed model relies on identifying the right attribute value subsets, which are a function of an improved information gain value. This consistency could also be a result of the values the attribute randomly picks to create the discrete sets. This emphasizes the need for a proposed future work on bounds for value collection during the process and hence would be interesting to see the effect of such an action on the performance of the classifier curve.

Equal Frequency Bin Size Bounding Problem
As had introduced in the earlier section 'Discretization', the main issue with the unsupervised algorithms is the interval sizes. The variant introduced in the section provides an upper bound for the number of intervals, which need to be established. The approach has also provided a 'prime number escape', which along with the condition, log $(\mathrm{x})-\log (\log (\mathrm{x}))<=19$, provide a constraint with the number of intervals created for the equal frequency algorithm. The question that arises for this section based on the approach adopted is about the appropriate interval size for the proposed approach. With varying interval sizes across different attributes for the same dataset, the approach adopted in this case is to consider the maximum of all the interval sizes *(max).

### 5.2 Performance Evaluation for 'Varying Interval Sizes'.

This section provides a performance comparison of the multivalued discrete algorithm with varying interval sizes. Little to no contributions have been made to understand the right correspondence between a discretization scheme and the 'overfit'ting of data. The process of discretization, as explained earlier, aims at finely categorizing the continuous valued attributes into discrete categories, thus making it classifier friendly. Thus, this section aims at understanding the relation between the increase in the interval size of the multivalued subset as a discretization tool and the classification errors. These results are plotted as classification error against the number of intervals and the charts are provided in Figure 13.

## Adaptive Boosting

Width


Frequency


## Iterative Dichotomizer 3

Width


Frequency


## Naïve Bayesian



Figure 13: Illustrating the performances under varying interval sizes under different classifiers

As can be observed, most of the classifiers portrayed a reduction in the classification error as a function of the increase in the interval size except for the AdaBoost algorithm, which showed a near-constant performance of the classification error. The 'Iris' dataset showed an increase in the classification error. The boosting is attributed to increase the weights associated with the incorrect classified examples whereas it decreases the weight associated with the correctly classified examples (Kotsiantis et al. 2006(b)). In this case, with the increase in the number of intervals, the datasets are tended to be more specific than general. For the same classifier, the dataset 'PenDigits', showed consistently high
error rates for multivalue-based frequency and the width discretizer; whereas the remaining classifiers showed a gradient decent on the performance of the classifiers across increasing interval sizes. An overall comparison of the performance of the classifiers shows that the boosting-based classifier performed comparatively poorly as against the classifiers not supported with the boosting phenomenon. Data discretization/binning is attributed to one of the methods to handling noisy data. With the AdaBoost characteristics introduced in section 2.4.1 along with the overfitting phenomenon introduced earlier, the proposed multivalued discretization process (Supervising the Unsupervised) did not contribute towards lowering the classification errors under the noise reduction procedure but could be rather adherent to the phenomenon of overfitting. Hence, this section provides a mixed response to the behavior of the classifiers on the phenomenon of overfitting. It would be hard to conclude the exact influence of overfitting on the behavioral pattern observed, since no considerable amount of contributions have been made relating the two aspects.

### 5.3 Performance Evaluation for 'Un-supervising the Supervised'

As, had introduced in the section 'Un-supervising the Supervised', the key idea of this approach is to have the proposed multivalued subset algorithm to be built on top of the unsupervised algorithm. For further reasoning and details, the readers are requested to revisit the section prior to proceeding ahead. Table 10 provides results of the classification errors calculated under this approach.

Table10: Results for the section 'Un-supervising the Supervised'

| Datasets | Classifiers | FF | FMF | FMW | FW |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  | Ada_Boost | $35.55 \%$ | $33.33 \%$ | $24.44 \%$ | $35.55 \%$ |
| Iris | ID3 | $0.00 \%$ | $0.00 \%$ | $0.00 \%$ | $0.00 \%$ |
|  | Naïve Bayesian | $0.00 \%$ | $0.00 \%$ | $0.00 \%$ | $0.00 \%$ |
|  | Regression | $6.67 \%$ | $4.44 \%$ | $6.67 \%$ | $6.67 \%$ |
|  |  |  |  |  |  |
|  | Ada_Boost | $25.00 \%$ | $25.00 \%$ | $25.00 \%$ | $25.00 \%$ |
| Glass | ID3 | $1.56 \%$ | $1.56 \%$ | $1.56 \%$ | $1.56 \%$ |
|  | Naïve Bayesian | $3.13 \%$ | $3.13 \%$ | $6.25 \%$ | $3.13 \%$ |
|  | Regression | $1.56 \%$ | $1.56 \%$ | $1.56 \%$ | $1.56 \%$ |
|  |  |  |  |  |  |
|  | Ada_Boost | $69.77 \%$ | $62.63 \%$ | $62.63 \%$ | $65.93 \%$ |
| Images | ID3 | $35.41 \%$ | $36.32 \%$ | $36.16 \%$ | $35.11 \%$ |
|  | Naïve Bayesian | $58.20 \%$ | $59.25 \%$ | $59.32 \%$ | $58.87 \%$ |
|  | Regression | $37.97 \%$ | $36.39 \%$ | $36.16 \%$ | $37.22 \%$ |
|  |  |  |  |  |  |
|  | Ada_Boost | $88.97 \%$ | $90.58 \%$ | $90.58 \%$ | $88.97 \%$ |
| PenDigits | ID3 | $90.19 \%$ | $90.38 \%$ | $90.38 \%$ | $90.11 \%$ |
|  | Naïve Bayesian | $90.49 \%$ | $91.19 \%$ | $91.19 \%$ | $91.03 \%$ |
|  | Regression | $88.97 \%$ | $89.42 \%$ | $89.42 \%$ | $88.97 \%$ |
|  |  |  |  |  |  |
|  | Ada_Boost | $78.74 \%$ | $79.13 \%$ | $79.13 \%$ | $78.74 \%$ |
| Vehicles | ID3 | $77.17 \%$ | $76.77 \%$ | $76.77 \%$ | $76.77 \%$ |
|  | Naïve Bayesian | $72.83 \%$ | $71.65 \%$ | $71.65 \%$ | $72.83 \%$ |
|  | Regression | $78.74 \%$ | $79.92 \%$ | $80.71 \%$ | $77.17 \%$ |

As the results showcase, the proposed integrated model did out perform the robust built up algorithm on quite a few occasions. The approach was tested on the five datasets. The FMF approach managed to reduce the classifier error for the 'Iris' dataset under the AdaBoost classifier. The results observed were quite consistent for the 'Glass' dataset except for the Naïve Bayesian classifier where it performed poorly. The ID3 and the Naïve Bayesian classifiers managed to provide a lower bound on the classification errors for the 'Vehicles' dataset. The 'Images' dataset showcased mixed a behavior against the AdaBoost and the Regression classifiers, though the difference in performance when compared to the robust FF and FW algorithms doesn't seem to be large.

### 5.4 Statistical Analysis

With the above-mentioned approaches, the study calls for a need to statistically understand the behavior of the interaction of the algorithms with the classifiers and the datasets. For the purpose of analyzing the effect of the discretization on the datasets, a 'kurtosis'-based analysis has been provided for each section to visually understand the
shift of the 'peakedness' for data distribution for individual attributes under the act of the discretizers. 'Skewness', which is a measure of symmetrical/unsymmetrical nature of distribution, coupled with a measure of how peak the distribution is (kurtosis) would define an indicator of the effect of the discretization on the attributes in the datasets. In this case, the 'Iris' dataset (Section 2.3) has been selected as a test measure since it would be graphically feasible to show the performance of all the attributes simultaneously in a single graph. The statistical analysis could be divided into the following sections.

### 5.4.1 Analysis for Supervised, Unsupervised and Semi-Supervised

As can be observed from Figure 12, the amount of variance portrayed by the multidiscrete variants is quite lower than the variances shown by the other algorithms over the mentioned datasets. Hence the first task is to statistically conclude on the behavioral pattern of the variances observed of the three different sets of algorithms as a function of the classifiers. This comparison has been performed on the basis of the classifier results obtained as per the interval considerations observed in Table 5. Table 11 shows the variance evaluations of the 3 algorithms. Hence, conformant to the earlier statement made, the variance observed by the proposed multidiscrete algorithm is quite low compared to the variances of the other 2 algorithms. A non-parametric based ranking system has been implemented for the evaluations of the algorithms. The evaluations have been divided into three main clusters, width, frequency and combined. Each of these clusters has been divided into two main considerations. One being the evaluation of the discretization algorithms against the classifiers a function of the variability in classifier error; the other being the evaluation of the discretization algorithms against the datasets, which too being a function of the variability in classifier errors.

### 5.4.1.1 Ranking

The ranking is based on the datasets/classifiers showing a minimum variability holding a higher rank. The ranking system follows the extended non-parametric system introduced in Demsar (2006). The figure given below [Figure 14] illustrates the method.

| Datasets | Algorithm1 | Algorithm2 | Algorithm3 |
| :---: | :---: | :---: | :---: |
| Dataset1 | $(0.0013) 1$ | $(0.002) 2$ | $(0.012) 3$ |
| Dataset2 | $(0.003) 2$ | $(0.02) 3$ | $(0.001) 1$ |
| Dataset3 | $(0.01) 1$ | $(0.045) 3$ | $(0.02) 2$ |
| Dataset4 | $(0.3) 3$ | $(0.02) 2$ | $(0.001) 1$ |
| Dataset5 | $(0.001) 1$ | $(0.02) 2$ | $(0.45) 3$ |
| Dataset6 | $(0.003) 2$ | $(0.0001) 1$ | $(0.4) 3$ |
| Dataset7 | $(0.002) 3$ | $(0.001) 1$ | $(0.0015) 2$ |
| Sum of Ranks | 13 | 14 | 15 |

Figure 14: Illustration of the extended non-parametric ranking scheme

As can be seen from the figure given above, Algorithm1 posses a lower rank as compared to the other two algorithms and hence as per the scheme will be ranked higher. A similar approach has been applied for the analyzing the results for the section 'Supervised, Unsupervised and Semi-supervised'

Table 11: (Discretizer algorithm vs. Datasets) variance under the width category

| Datasets | CAIM | Equal Width | Multivalued Width | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: |
| Iris | 0.0031 | 0.0036 | 0.0044 | CAIM |
| Glass | 0.0037 | 0.0115 | 0.0187 | CAIM |
| Images | 0.0067 | 0.0826 | 0.0502 | CAIM |
| PenDigits | 0.0608 | 0.1220 | 0.1029 | CAIM |
| Vehicles | 0.0004 | 0.0231 | 0.0188 | CAIM |

Table 12: (Discretizer algorithm vs. Classifier algorithms) variance under the width category

| Algorithms | CAIM | Equal Width | Multivalued Width | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 0.0118 | 0.0052 | 0.0115 | Equal Width |
| ID3 | 0.0303 | 0.0274 | 0.0072 | Multivalued Width |
| Naïve Bayesian | 0.0207 | 0.0170 | 0.0112 | Multivalued Width |
| Regression | 0.0550 | 0.0237 | 0.0153 | Multivalued Width |

Table 13: Ranks of the variance based performance evaluation (left: Discretizer algorithm vs. Datasets) (right: Discretizer algorithms vs. Classifier algorithms) under the width category

| Rank | Algorithm |
| :---: | :---: |
| 1 | CAIM |
| 2 | Multivalued Width |
| 3 | Equal Width |


| Rank | Algorithm |
| :---: | :---: |
| 1 | Multivalued Width |
| 2 | Equal Width |
| 3 | CAIM |

Table 14: (Discretizer algorithms vs. Datasets) variance under the frequency category

| Datasets | CAIM | Equal Frequency | Multivalued Frequency | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: |
| Iris | 0.0031 | 0.0054 | 0.0431 | CAIM |
| Glass | 0.0037 | 0.0110 | 0.0278 | CAIM |
| Images | 0.0067 | 0.0103 | 0.0583 | CAIM |
| PenDigits | 0.0608 | 0.0001 | 0.1061 | Equal Frequency |
| Vehicles | 0.0008 | 0.0016 | 0.0244 | CAIM |

Table 15: (Discretizer algorithms vs. Classifier algorithms) variance performance under the frequency category

| Algorithms | CAIM | Equal Frequency | Multivalued Frequency | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 0.032915736 | 0.107873811 | 0.014482146 | Multivalued Frequency |
| ID3 | 0.027042595 | 0.178155357 | 0.007887593 | Multivalued Frequency |
| Naïve Bayesian | 0.025701939 | 0.16745357 | 0.009477698 | Multivalued Frequency |
| Regression | 0.051196374 | 0.148268321 | 0.01292317 | Multivalued Frequency |

Table 16: Ranks of the variance based evaluation (left: Discretizer algorithm vs.
Datasets) (right: Discretizer algorithms vs. Classifier algorithms) under the frequency
category

| Datasets |  |
| :---: | :---: |
| Rank | Algorithm |
| 1 | CAIM |
| 2 | Equal Frequency |
| 3 | Multivalued Frequency |


| Algorithms |  |
| :---: | :---: |
| Rank | Algorithm |
| 1 | Multivalued Frequency |
| 2 | CAIM |
| 3 | Equal Frequency |

Table 17: (Discretizer algorithms vs. Datasets) variance for cumulative analysis

| Datasets | CAIM | Equal Frequency | Multivalued Frequency | Equal Width | Multivalued Width | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iris | 0.003085803 | 0.005428421 | 0.043103543 | 0.00362601 | 0.004375823 | CAIM |
| Glass | 0.003662813 | 0.01096475 | 0.027755731 | 0.011548178 | 0.018717448 | CAIM |
| Images | 0.006663709 | 0.010310414 | 0.058316129 | 0.082608402 | 0.050196809 | CAIM |
| PenDigits | 0.060802039 | $8.34868 \mathrm{E}-05$ | 0.106117948 | 0.10345103 | 0.102936236 | Equal Frequency |
| Vehicles | 0.000789207 | 0.001641249 | 0.024399075 | 0.023129829 | 0.01878165 | CAIM |

Table 18: (Discretizer algorithms vs. Datasets) ranking scheme applied to the cumulative
variance performance

| Datasets | CAIM | Equal Frequency | Multivalued Frequency | Equal Width | Multivalued Width |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Iris | 1 | 4 | 5 | 2 | 3 |
| Glass | 1 | 2 | 5 | 3 | 4 |
| Images | 1 | 2 | 4 | 5 | 3 |
| PenDigits | 2 | 1 | 5 | 4 | 3 |
| Vehicles | 1 | 2 | 5 | 4 | 3 |
| Sum of Ranks | 6 | 11 | 24 | 18 | 16 |

Table 19: (Discretizer algorithms vs. Classifier algorithms) variance performance for cumulative analysis

| Algorithms | CAIM | Equal Frequency | Multivalued Frequency | Equal Width | Multivalued Width | Minimum Variance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 0.032915736 | 0.107873811 | 0.014482146 | 0.053797667 | 0.045954153 | Multivalued Frequency |
| ID3 | 0.027042595 | 0.178155357 | 0.007887593 | 0.019413022 | 0.008652569 | Multivalued Frequency |
| Naive Bayesian | 0.025701939 | 0.16745357 | 0.009477698 | 0.014725203 | 0.010333877 | Multivalued Frequency |
| Regression | 0.051196374 | 0.148268321 | 0.01292317 | 0.01424111 | 0.009968933 | Multivalued Width |

Table 20: (Discretizer algorithm vs. Classifier algorithms) ranking scheme applied to the cumulative variance performance

| Algorithms | CAIM | Equal Frequency | Multivalued Frequency | Equal Width | Multivalued Width |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 2 | 5 | 1 | 3 |  |
| ID3 | 4 | 5 | 1 | 3 | 2 |
| Naïre Bayesian | 4 | 5 | 1 | 3 | 2 |
| Regression | 4 | 5 | 2 | 3 | 1 |
| Sum of Ranks | 14 | 20 | 5 | 13 | 8 |

Table 21: Ranks of the variance based evaluation (left: Discretizer algorithm vs.
Datasets) (right: Discretizer algorithms vs. Classifier algorithms) under for cumulative performance analysis.

| Rank | Algorithm |
| :---: | :---: |
| 1 | CAIM |
| 2 | Equal Frequency |
| 3 | Multivalued Width |
| 4 | Equal Width |
| 5 | Multivalued Frequency |


| Rank | Algorithm |
| :---: | :---: |
| 1 | Multivalued Frequency |
| 2 | Multivalued Width |
| 3 | Equal Width |
| 4 | CAIM |
| 5 | Equal Frequency |

As can be observed from [Tables 11-13], the CAIM algorithm provided much better results for minimum variation on the classification error across the different classification algorithms contributing a consistent behavior across all the classifiers, whereas the proposed heuristic based semi-supervised 'Multivalued Width' algorithm provided much more consistency in lower classification errors across the different datasets. A similar conclusion could be inferred from [Tables 14-16], where CAIM dominated across the classifiers whereas the proposed heuristic based 'Multivalued Frequency' algorithm showcased consistent performance across the datasets. It would be interesting to understand the correlations of the paired algorithms across the two main categories, which are 'Width' and 'Frequency' [Tables 17-21]. The CAIM algorithm again proves to be a better discretizer when tested across the datasets whereas the multivalue frequency algorithm tests better across the classifiers. The kurtosis table [Table 22] and chart [Figure 15] for this section is as shown below.

Table 22: Table shows the values for the kurtosis under the section Supervised,
Unsupervised and Semi-supervised

| Algorithms | Att1 | Att2 | Att3 | Att4 |
| :---: | :---: | :---: | :---: | :---: |
| No Discretization | 2.4264 | 3.2414 | 1.6046 | 1.6648 |
| CAIM | 1.3657 | 75 | 2.1501 | 3.4317 |
| Equal Width | 2.3426 | 3.2957 | 1.5881 | 1.7267 |
| Equal Frequency | 2.1528 | 3.0215 | 1.7365 | 1.7437 |
| Multivalued Width | 3.2173 | 2.7333 | 1.7482 | 1.6261 |
| Multivalued Frequency | 1.5891 | 1.897 | 2.9581 | 1.6598 |



Figure 15: Kurtosis chart for Supervised, Unsupervised and Semi-supervised
As can be observed from the table and figure given above, 3 of the 4 attributes showed a consistent kurtosis across the different discretizer algorithms. Attribute 2 shows a high kurtosis when subjected to the CAIM algorithm.

### 5.4.2 Analysis of Varying Interval Sizes

One of the biggest concerns as addressed in the earlier chapter was the effect of the 'overfitting'. To add on to what was mentioned in the earlier chapter, overfitting would cause the performance to deteriorate. Ideally, under the phenomenon of overfitting, with the increase in number of intervals, the probability of miss-classifying the testing data should increase. An important consideration while analyzing the effect of the interval sizes /bin sizes on the data is the effect on the data distribution. [Figures 16-17] provides the charts for the kurtosis along different interval sizes based the kurtosis table [Tables 23-24]. This is an important distinguishing factor to portray the 'working' of the intervals/bin sizes since most of the classifiers except for AdaBoost show a declining trend as a function of increasing the interval sizes. The declining trend shows the adaptability of the proposed algorithm, where it shares the freedom to choose the values in an unsorted manner to improve on the function, which in this case is the information
gain. Again, the intent of the study is not to propose an, 'improved model', but rather understand the application of using multivalued subsets in different aspects of Data Mining/Machine Learning. The Multivalued frequency algorithm showed a maximum drop in classifier errors under the Naïve Bayesian classifier for the 'Glass' dataset, from an interval size of 4 to an interval size of 20. One can observe from the kurtosis chart and table given below, an effective change in the values as a function of increase in the interval bin size. Again, in this case the final values linger around the ' 2 '-value mark. While some of the attributes show a decrease in the peakedness, the others showed a moderate increase in the peak heights.

Table 23: Table showing the kurtosis for the varying interval sizes under the proposed width based multidiscrete algorithm

| Interval sizes | Att1 | Att2 | Att3 | Att4 |
| :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Bin}=0$ | 2.4264 | 3.2414 | 1.6046 | 1.6648 |
| $\operatorname{Bin}=4$ | 1.8028 | 2.1667 | 1.8687 | 2.2866 |
| $\operatorname{Bin}=8$ | 3.2173 | 2.7333 | 1.7482 | 1.6261 |
| $\operatorname{Bin}=16$ | 1.8076 | 1.6554 | 1.6528 | 1.7987 |
| $\operatorname{Bin}=20$ | 1.6289 | 1.7365 | 1.9371 | 1.9175 |



Figure 16: Figure showing the kurtosis for the varying interval sizes under the proposed Width based multidiscrete algorithm

Table 24: Table showing the kurtosis for the varying interval sizes under the proposed
frequency based multidiscrete algorithm

| Interval Sizes | Att1 | Att2 | Att3 | Att4 |
| :---: | :---: | :---: | :---: | :---: |
| Bin $=0$ | 2.4264 | 3.2414 | 1.6046 | 1.6648 |
| Bin $=4$ | 1.765 | 2.0058 | 1.5846 | 1.7139 |
| Bin $=8$ | 2.9872 | 2.3792 | 1.6984 | 2.0205 |
| Bin $=16$ | 1.9058 | 1.8364 | 2.0445 | 2.3516 |
| $\operatorname{Bin}=20$ | 1.8818 | 1.7058 | 1.8708 | 2.0015 |



Figure 17: Figure showing the kurtosis for the varying interval sizes under the proposed Frequency based multidiscrete algorithm

As can be seen from above, the data distributions for the individual attributes follows a convergence path as a function of the number of bins.

### 5.4.3 Analysis for the 'Un-supervising the Supervised'

This section analyzes the results obtained for the algorithms introduced in Section 4.7. A non-parametric approach similar to the one introduced in the section 5.4.1.1 has been applied for the statistical analysis of this section.

Table 25: (Discretizer algorithm vs. Datasets) ranking scheme applied to 'Un-supervising the Supervised'.

| Datasets | FF | FMF | FMW | FW | Minimum Varibility |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Iris | 0.02875609 | 0.02579864 | 0.01332837 | 0.02875531 | FMW |
| Glass | 0.01318464 | 0.01318568 | 0.01245108 | 0.01318568 | FMW |
| Images | 0.02717363 | 0.02034096 | 0.02070874 | 0.02384715 | FMF |
| PenDigits | $6.4555 \mathrm{E}-05$ | $5.3725 \mathrm{E}-05$ | $5.3725 \mathrm{E}-05$ | 0.00010044 | FMF/FMW |
| Vehicles | 0.0007789 | 0.00138857 | 0.001565 | 0.0006314 | FW |

Table 26: (Discretizer algorithm vs. Datasets) ranking scheme applied to the results obtained from the section 'Un-supervising the Supervised'

| Dataset | FF | FMF | FMW | FW |
| :---: | :---: | :---: | :---: | :---: |
| Iris | 4 | 2 | 1 | 3 |
| Glass | 2 | 3 | 1 | 3 |
| Images | 4 | 1 | 2 | 3 |
| PenDigits | 3 | 1 | 1 | 4 |
| Vehicles | 2 | 3 | 4 | 1 |
| Sum of Ranks | 15 | 10 | 9 | 14 |

Table 27: Ranks of the variance based evaluation (Discretizer algorithm vs. Datasets) for the section 'Un-supervising the Supervised'

| Rank | Algorithm |
| :---: | :---: |
| 1 | FMW |
| 2 | FMF |
| 3 | FW |
| 4 | FF |

Table 28: (Discretizer algorithm vs. Classifier algorithms) ranking scheme applied to
'Un-supervising the Supervised'

| Algorithms | FF | FMF | FMW | FW | Minimum Varibility |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 0.07769381 | 0.08066978 | 0.0932709 | 0.07603428 | FW |
| ID3 | 0.17488457 | 0.17441876 | 0.17444742 | 0.17406108 | FW |
| Naïve Bayesian | 0.16991792 | 0.17063069 | 0.16432071 | 0.17161072 | FMW |
| Regression | 0.16130896 | 0.16909223 | 0.16651637 | 0.15870721 | FW |

Table 29: (Discretizer algorithm vs. Classifier algorithms) ranking scheme applied to the results obtained from the section 'Un-supervising the Supervised'.

| Algorithms | FF | FMF | FMW | FW |
| :---: | :---: | :---: | :---: | :---: |
| AdaBoost | 2 | 3 | 4 | 1 |
| ID3 | 4 | 2 | 3 | 1 |
| Naïve Bayesian | 2 | 3 | 1 | 4 |
| Regression | 2 | 4 | 3 | 1 |
| Sum of Ranks | 10 | 12 | 11 | 7 |

Table 30: Ranks of the variance based evaluation (Discretizer algorithm vs. Classifier algorithms) under the section 'Un-supervising the Supervised'

| Rank | Algorithm |
| :---: | :---: |
| 1 | FW |
| 2 | FMW |
| 3 | FF |
| 4 | FMF |

As can be seen from [Tables 25-27], the Frequency Multivalued Width (FMW) provided a lower variance measure compared to other algorithms over the mentioned datasets. On a similar note the FW algorithm provided the lowest variance amongst all the classifier evaluations in the same measure [Tables 28-30]. It could be concluded at this point that
the 'equal width' heuristic played a crucial role for un-supervising the heuristic based supervised algorithm. The kurtosis table [Table 31] provided below, gives the kurtosis for the distribution transformation along these algorithms followed by the chart [Figure 18] showing that attributes stick to their corresponding distribution as a function of the different discretization algorithms introduced in this section. It can be observed that the distributions sustain their kurtosis when being subjected to the proposed discretization algorithms. Hence no significant change in the shape of the distribution can be observed following the discretization process.

Table 31: Table shows the values for the kurtosis under the section Supervised,
Unsupervised and Semi-supervised

| Algorithms | Att1 | Att2 | Att3 | Att4 |
| :---: | :---: | :---: | :---: | :---: |
| No Discretization | 2.4264 | 3.2414 | 1.6046 | 1.6648 |
| FF | 2.2782 | 2.9593 | 1.7431 | 1.7716 |
| FMF | 2.277 | 2.9593 | 1.7361 | 1.7716 |
| FMW | 2.4577 | 2.9593 | 1.7635 | 1.7716 |
| FW | 2.3148 | 2.9593 | 1.7333 | 1.7716 |



Figure 18: Figure showing the kurtosis for the varying algorithms under the section 'Unsupervising the Supervised'

## Chapter 6

## Implications of the Work

This work has successfully managed to identify subsets of attribute value pairs, which contribute towards a higher information gain. This study was divided into four major sections. The first section saw the application of the mentioned philosophy towards a feature selection process. A comparison was made on the results of the proposed methods against the standard information gain approach. Results show that though the final attribute (highest ranked attribute) for the multivalued subset method provided a slightly higher classification error, there were instances recognized where lower performance errors resulting from some of the attribute value pairs identified. The next section saw the application of the multivalued sub-setting in field of controlled discretization of data prior to subjecting it to a classifier. The two versions of the developed algorithm adherent to the width based and frequency based discretization were developed and tested against the corresponding unsupervised versions and a supervised version of CAIM. Results found that the performance of the semi supervised approach adopted by the proposed heuristic worked better attribute to uniform performance across different datasets. It did manage to provide better results comparing to its counterparts during the experimental run. The next section saw the performance of the proposed algorithm across different interval sizes. Expect for the Ada_Boost algorithm, the rest of the classification algorithms showcased a decrease in the performance errors subject to increase in the interval size. The final section introduced a reverse order procedure for mounting the heuristic based semi-
supervised algorithm. The results did manage to show a slightly better performance on certain occasion against a similar built up of the robust unsupervised discretizers.

### 6.1 Contributions to the field of Industrial Engineering

Data Mining has been viewed as a growing area of importance for its key application as a prediction tool. The above-mentioned approach provides the flexibility for the data collectors to collect real time data (continuous in nature).

## Feature Selection:

The algorithm suggested would work in identifying the right set of factors that would built a better prediction model at the cost of lowering errors on implementation samples.

Discretization:
The method suggested allows the user to collect continuous data and identify the right set of categories. This study also helps the user to judge the classification error at the cost of the interval size if the heuristic model were to be implemented.

### 6.2 Future Work

6.2.1 Establishment of bounds

As had been mentioned in the earlier section, the proposed discretization approaches relies on a heuristic search result and need not necessarily pick neighboring attributevalue sets for discretization. The lower bound for this approach could be that it discretizes all the values as per the same class distribution that exists in the current dataset, but in order for that situation to occur, the information gain needs to reach the maximum value. As can be observed from the section 'Multivalue subset based Feature Selection', the information gain values obtained for the multivalue subsets lie between a
maximum of the information gain for ID3 collected and the maximum of the GID3 value which essentially worked for this study. Hence it is essential to develop an upper bound for the selection of values to be considered while discretizing into a single category.
6.2.2. Width as a platform for the reverse model of 'Un-supervising the Supervised'. The models developed under the 'Un-supervising the Supervised' algorithms, have only considered 'Frequency' as the platform for building the heuristic based semi-supervised models. One can evaluate with having the width based platform models similar to ones presented in this study.
6.2.3 Conditional Independence Tests for the Naïve Bayesian Classifier after Discretization.

Since the Naïve Bayesian Classifier depends on the conditional independence measure between the attributes, it would be interesting to see the effect of such a discretization process on its independence measures.

## Appendices

## Appendix A. 1 . Normality Tests for 'Iris’ Dataset

Normality Tests for Informational Gain values:


Normality Tests for Subset values:


Attribute 1


Attribute 2


Attribute 3


Attribute 4

## Appendix A. 2 Nomenclature of the Attribute/Class Names

Vehicle Silkhouttes

| Attribute Number | Attribute Name |
| :---: | :---: |
| 1 | Compactness |
| 2 | Circularity |
| 3 | Distance Circularity |
| 4 | Radius Ratio |
| 5 | PR. Axis Aspect Ratio |
| 6 | Max. Length Aspect Ratio |
| 7 | Scatter Ratio |
| 8 | Elongatedness |
| 9 | PR. Axis Rectangularity |
| 10 | Max. Length Rectangularity |
| 11 | Scaled Variance Along Minor Axis |
| 12 | Scaled Radius Of Gyration |
| 13 | Skewness About Major Axis |
| 14 | Skewness About Minor Axis |
| 15 | Kurtosis About Major Axis |
| 16 | Hollows Ratio |
| Class 1 | OPEL |
| Class 2 | SAAB |
| Class 3 | BUS |
| Class 4 | VAN |

Iris

| Attribute Number | Attribute Name |
| :---: | :---: |
| 1 | Sepal Length |
| 2 | Sepal Width |
| 3 | Petal Length |
| 4 | Petal Width |
| Class 1 | Iris Setosa |
| Class 2 | Iris Versicolor |
| Class 3 | Virginica |

## Matlab Codes

## Appendix C.1.Program Code for calculating the information gain under the ID3 rule

```
% Iterative Dichotomizer 3
% This program will be used for calculating the maximum entropy of all
the
% binary partitions
close all;
clear all;
clc;
readfile = dlmread('attribute.CSV', ',' , 'A1..J214');
outfile_ID3= fopen('outfile_ID3.doc','w');
size_rows=size(readfile,l);
size_columns=size(readfile,2);
%Calculating the entropy with respect to class
unique class=unique(readfile(:, size columns));
Class_entropy = zeros(numel(unique_class),1);
for class_ent=1:numel(unique_class)
        Class_entropy(class_ent) = 0-
(length(find(readfile(:,size_columns)==(class_ent))))/size_rows.*log2((
length(find(readfile(:,size_columns)==(class_ent))))/size_rows);
end
Entropy_class=sum(Class_entropy,1);
fprintf(outfile ID3,'\n%d\n',Entropy class);
Sum_Final=zeros((size_columns-1),1);
% Loop for calculating the entropy of the temporary attributes
```

```
for attribute = 1:(size_columns-1)
    temp_value = readfile(:,attribute);
    temp_class = readfile(:,size_columns);
    assort=[temp_value temp_class];
    assort = sortrows(assort,[1]);
    value=assort(:,1);
    class=assort(:,2);
    j = unique(value);
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\unique_value.xlsx',j);
ind = ones(length(j),1);
binary_attribute= value;
for i =1:length(j)
ind(i) = length(find(value == j(i)));
end
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\AttributeValue_Count.xlsx', [j ind]);
first_value =0;
end_value =0;
Class_count = zeros(max(class),1);
Class_vector = zeros(length(j), max(class));
for i= 1:length(ind)
    first_value =end_value +1;
    end_value=first_value+(ind(i)-1);
    for window = first_value:end_value
        Class_count(class(window))= Class_count(class(window))+1;
    end
    for class_fill=1:numel(unique_class)
```

```
        Class_vector(i,class_fill)=Class_count(class_fill);
    end
    % Class vector(i,:)=[Class count(1) Class count(2) Class count(3)
Class_count(4) Class_count(5) \overline{Class_count(6) \overline{Class_count(7)];}};\mathbf{\prime}
                for m= 1:max(class)
                        Class_count(m)=0;
                end
end
Sum = [j Class_vector sum(Class_vector,2)
sum(Class_vector,2)/size_rows];
Sum_Class=sum(Class_vector,2);
RatioClass_vector = zeros(length(j), max(class));
for i=1:length(j)
        for y=1:max(class)
        RatioClass_vector(i,y)=Class_vector(i,y)/Sum_Class(i);
    end
end
Ratio = [Sum RatioClass_vector];
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\Ratio.xlsx',Ratio);
Value_Zero = RatioClass_vector;
row_cell = ones(1,length(j));
column_cell = ones(1, max(class));
Cell_EliminateZero=mat2cell(Value_Zero, row_cell, column_cell);
```

```
for p = 1: length (j)
    for t =1:max(class)
        if Cell EliminateZero{p,t} == 0
            Cel\overline{l_EliminateZero{p,t}=[];}
        end
    end
end
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\Cell_Elimination.xlsx',Cell_EliminateZero);
log_sum=ones(length(j), 1);
log_sum2=log_sum;
for i =1:leng}th(j
    a =cell2mat(Cell_EliminateZero(i,:));
    for p = 1:length(a)
        a(p)=a(p).*log2(a(p));
    end
    log_sum(i)=0-sum(a);
    log_sum2(i)=(ind(i)/length(value)).*log_sum(i);
end
%Sum_Final(attribute)=sum(log_sum2);
    Sum_Final(attribute)=sum(log_sum2);
end
%dlmwrite('',Sum_Final);
%dlmwrite('Sum.txt', Sum_Final,'precision', '%.6f','delimiter',
',','newline', 'unix');
% Entropy of the attributes-value pairs
for att_value=1:(size_columns-1)
    fprintf(outfile_ID3,'\n%d\n',Sum_Final(att_value));
fprintf(outfile_ID3,'\n\n\n\n--------%d-------------
\n\n\n\n',att_value+1);
end
```


## Appendix C.2.Program Code for calculating the information gain under the GID rule

```
% This program will be used for calculating the maximum entropy of all
the
% binary partitions
close all;
clear all;
clc;
```

readfile = dlmread('attribute_Id3.csv', ',' , 'A1..S846');
outfile= fopen('outfile.doc','w');
size_rows=size(readfile,1);
size columns=size(readfile, 2);
\%Calculating the entropy with respect to class
unique_class=unique(readfile(:,size_columns));
Class_entropy $=$ zeros(numel(unique_class),1);
for class_ent=1:numel(unique_class)
Class_entropy(class_ent) $=0-$
(length(find(readfile(:,size_columns) ==(class_ent))))/size_rows.*log2((
length(find(readfile(:,size_columns)==(class_ent))))/size_rows);
end
\% Finding the max of the unique values along the different attributes
uni_max=zeros(1,size_columns-1);
for ${ }^{-}$unique_max $=1$ :síze_columns-1
uni_max (1, unique_max)=numel(unique (readfile(:, unique_max)));
end
max_unique=max(uni_max);
Entropy_class=sum(Class_entropy,1);
\% Defining a 3D array for defining the temporary attribute
temp_attribute=zeros(length(readfile), max_unique, size_columns-1);
\% Defining the class vector across all the unique values of the
attributes
class_attribute $=$ zeros(length(readfile), size_columns-1);

```
% Creating the temporary attributes
for z = 1:size_columns-1
    readfile = sortrows(readfile,z);
    class_attribute(:,z)=readfile(:,size_columns);
    % xlswrite('temp.xlsx',readfile);
    unique_value=unique(readfile(:,z));
    for y = 1:length(unique_value)
        index = find(readfile(:,z)==unique_value(y));
            for x =1:length(index)
                temp_attribute(index(x),y,z)=1;
            end
        index =0;
    end
        unique_value=0;
end
Sum_Final=zeros(max_unique,1);
% Loop for calculating the entropy of the temporary attributes
for attribute = 1:size_columns-1
    for temp uniquevalu}\mp@subsup{]}{e = 1: length(unique(readfile(:,attribute)))}{
    temp_value = temp_attribute(:,temp_uniquevalue,attribute);
    temp_class = class_attribute(:,attribute);
    assort=[temp_value temp_class];
    assort = sortrows(assort,[1]);
    value=assort(:,1);
    class=assort(:,2);
    j = unique(value);
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\unique_value.xlsx',j);
ind = ones(length(j),1);
binary_attribute= value;
for i =1:length(j)
ind(i) = length(find(value == j(i)));
end
```

```
%xlswrite('u:\profile.cu\Desktop\Matlab Files\ID3 Datasets\Attribute
19\AttributeValue_Count.xlsx', [j ind]);
first_value =0;
end_value =0;
Class_count = zeros(max(class),1);
Class_vector = zeros(length(j), max(class));
for i= 1:length(ind)
    first_value =end_value +1;
    end_value=first_value+(ind(i)-1);
    for window = first_value:end_value
                Class_count(class(window))= Class_count(class(window))+1;
    end
    for class_fill=1:numel(unique_class)
        Class_vector(i,class_fill)=Class_count(class_fill);
    end
            for m= 1:max(class)
                                    Class_count(m)=0;
            end
end
Sum = [j Class_vector sum(Class_vector,2)
sum(Class_vectōr,2)/size_rows];
Sum Class=sum(Class vector,2);
RatioClass_vector = zeros(length(j), max(class));
for i=1:length(j)
    for y=1:max(class)
        RatioClass_vector(i,y)=Class_vector(i,y)/Sum_Class(i);
    end
end
Ratio = [Sum RatioClass_vector];
Value_Zero = RatioClass_vector;
row_cell = ones(1,length(j));
column_cell = ones(1, max(class));
Cell_EliminateZero=mat2cell(Value_Zero, row_cell, column_cell);
for p = 1: length (j)
```

```
    for t =1:max(class)
        if Cell EliminateZero{p,t} == 0
            Cel\overline{l_EliminateZero{p,t}=[];}
        end
    end
end
log_sum=ones(length(j), 1);
log_sum2=log_sum;
for i =1:leng
    a =cell2mat(Cell_EliminateZero(i,:));
    for p = 1:length(a)
        a(p)=a(p).* log2(a(p));
    end
    log_sum(i)=0-sum(a);
    log_sum2(i)=(ind(i)/length(value)).*log_sum(i);
end
    Sum_Final(temp_uniquevalue,attribute)=Entropy_class-sum(log_sum2);
    end
end
% Entropy of the attributes-value pairs
for att_value=1:size_columns-1
for r=1\overline{:length(uniquē(readfile(:,att_value)))}
    fprintf(outfile,'\n%d\n',Sum_Final(r,att_value));
end
fprintf(outfile,'\n\n\n\n--------%d-----------\n\n\n\n',att_value+1);
end
```


## Appendix C.3.Program Code for Performing the Adaptive Simulated Annealing

```
    function [Global_Best Rand_Value
NumelEl]=discretize(OriginalDataset,attribute_num)
readfile=OriginalDataset;
Dataset_Rows =size(readfile,1);
Dataset_Columns=size(readfile,2);
rand_list=rand(100000000,1);
position_rand=1;
% Class Entropy
Class_entropy = zeros(numel(unique(readfile(:,Dataset_Columns))),1);
```

```
Classes=unique(readfile(:,Dataset_Columns));
for class_count=1:numel(Classes)
    Class_entropy(Classes(class_count))=0-
(length(find(readfile(:,Dataset_Columns)==(Classes(class_count)))))/Dat
aset_Rows.*log2(length(find(readfile(:,Dataset_Columns)==(Classes(class
_count))))/Dataset_Rows);
end
Entropy_class=sum(Class_entropy,1);
    position_rand=position_rand+1;
    seed_rand=rand_list(position_rand)*1000;
    [stream
]=RandStream.create('mt19937ar','NumStreams',1,'seed',seed_rand );
random_vector=rand(stream,length(unique(readfile(:,attribute_num))),1);
constant = rand(stream,1,1);
g=random_vector>constant;
unique_value = unique(readfile(:,attribute_num));
unique_value= g.*unique(readfile(:,attribute_num));
temp_attribute=readfile(:,attribute_num);
index = find ( unique_value(:,1)>0);
j=index;
    for r=1:length(index)
        j(r,1)=unique_value(index(r),1);
    end
    for t =1:length(j)
        index_temp=find(temp_attribute==j(t));
        for temp=1:length(index_temp)
```

```
                temp_attribute(index_temp(temp),1)=1;
            end
        end
        for zeros_1=1:length(readfile(:,attribute_num))
            eliminate = find (temp_attribute(:,1)~=1);
                for k=1:length(eliminäte)
            temp_attribute(eliminate(k),1)=0;
        end
    end
    temp_value = temp_attribute;
    temp_class = readfile(:,Dataset_Columns);
    assort=[temp_value temp_class];
    assort = sortrows(assort,[1]);
    value=assort(:,1);
    class=assort(:,2);
    j = unique(value);
    Class_count = zeros(max(class), 1);
Class_vector = zeros(length(j), max(class));
% Creating a count on the classes for the
ind = ones(length(j),1);
for i =1:length(j)
ind(i) = length(find(value == j(i)));
end
% Creating a Window for maintaining the count
first_value =0;
end_value =0;
for i= 1:length(ind)
    first_value =end_value +1;
    end_value=first_value+(ind(i)-1);
    for window = first_value:end_value
```

```
            Class_count(class(window))= Class_count(class(window))+1;
    end
    for fill=1:max(Classes)
        Class_vector(i,fill)=Class_count(fill);
    end
                for m= 1:max(class)
                        Class_count (m)=0;
                end
end
Sum = [j Class_vector sum(Class_vector,2)
sum(Class_vector,2)/Dataset_Rows];
Sum_Class=sum(Class_vector,2);
RatioClass_vector = zeros(length(j), max(class));
for i=1:length(j)
    for y=1:max(class)
        RatioClass_vector(i,y)=Class_vector(i,y)/Sum_Class(i);
    end
end
Ratio = [Sum Ratioclass vector];
Value_Zero = RatioClass_vector;
row_cell = ones(1,length(j));
column_cell = ones(1, max(class));
Cell EliminateZero=mat2cell(Value Zero, row cell, column cell);
```

```
for p = 1: length (j)
    for t =1:max(class)
        if Cell_EliminateZero{p,t} == 0
            Cel\overline{l_EliminateZero{p,t}=[];}
        end
    end
end
log_sum=ones(length(j), 1);
log_sum2=log_sum;
for i =1:length(j)
    a =cell2mat(Cell_EliminateZero(i,:));
    for p = 1:length(a)
        a(p)=a(p).* log2(a(p));
    end
    log_sum(i)=0-sum(a);
    log_sum2(i)=(ind(i)/length(value)).*log_sum(i);
end
    Sum_Final=Entropy_class-sum(log_sum2);
    Fh=\overline{Sum_Final;}
    Fl=Sum_Final;
    Initial_Solution_Objective_Value=Sum_Final;
    Global_Best=Initīal_Solution_Objective_Value;
    Rand_Value=seed_rand;
    NumelEl=numel(find(g==1));
    Global_Config_Best=temp_attribute;
    Best_Solution=Sum_Final;
    Equi\overline{librium_Best=\overline{0};}
    Solution_Objective_Value=0;
% Based on the sampling the Initial Temperature is set up
T = 1000;
% Calculating the Ending Temperature
Tend = 1;
%Loop for the Temperature
while T > Tend
    loop =0;
%Loop for the Equilibrium State
Transition_L=2;
Increament = 0;
```

```
l_Transition=0;
while l_Transition<(Transition_L+Increament)
    position_rand=position_rand+1;
    seed_rand=rand_list(position_rand)*1000;
[stream ]=RandStream.create('mt19937ar','NumStreams',1,'seed',seed_rand
);
random_vector=rand(stream,length(unique(readfile(:,attribute_num))),1);
    constant = rand(stream,1,1);
    g=random_vector>constant;
    unique_value=unique(readfile(:,attribute_num));
    unique_value= g.*unique(readfile(:,attribute_num));
    temp_attribute=readfile(:,attribute_num);
    index = find ( unique_value(:,1)>0);
    j=index;
    for r=1:length(index)
        j(r,1)=unique_value(index(r),1);
    end
    for t =1:length(j)
        index_temp=find(temp_attribute(:,1)==j(t));
        for temp=1:length(index_temp)
            temp_attribute(index_temp(temp),1)=1;
        end
    end
    for zeros_1=1:length(readfile(:,attribute_num))
        eliminate = find (temp_attribute(:,1)~=1);
        for k=1:length(eliminäte)
            temp_attribute(eliminate(k),1)=0;
        end
```

```
    end
    % Calculating the objective function value for the random sets
    temp_value = temp_attribute(:,1);
    temp_class = readfile(:,Dataset_Columns);
    assort=[temp_value temp_class];
    assort = sortrows(assort,[1]);
    value=assort(:,1);
    class=assort(:,2);
    j = unique(value);
    Class_count = zeros(max(class), 1);
Class_vector = zeros(length(j), max(class));
ind = ones(length(j),1);
for i =1:length(j)
ind(i) = length(find(value == j(i)));
end
first_value =0;
end_value =0;
for i= 1:length(ind)
    first_value =end_value +1;
    end_value=first_value+(ind(i)-1);
    for window = first_value:end_value
        Class_count(class(window))= Class_count(class(window))+1;
    end
    for fill=1:max(Classes)
        Class_vector(i,fill)=Class_count(fill);
    end
```

```
        for m= 1:max(class)
            Class_count(m)=0;
                end
end
Sum = [j Class_vector sum(Class_vector,2)
sum(Class_vector,2)/Dataset_Rows];
Sum_Class=sum(Class_vector,2);
RatioClass_vector = zeros(length(j), max(class));
for i=1:length(j)
    for y=1:max(class)
        RatioClass_vector(i,y)=Class_vector(i,y)/Sum_Class(i);
        end
end
Ratio = [Sum RatioClass_vector];
Value_Zero = RatioClass_vector;
row_cell = ones(1,length(j));
column_cell = ones(1, max(class));
Cell_EliminateZero=mat2cell(Value_Zero, row_cell, column_cell);
for p = 1: length (j)
        for t =1:max(class)
            if Cell_EliminateZero{p,t} == 0
                Cel\overline{l_EliminateZero{p,t}=[];}
            end
        end
end
log_sum=ones(length(j), 1);
log_sum2=log_sum;
for i =1:length(j)
```

```
        a =cell2mat(Cell_EliminateZero(i,:));
        for p = 1:length(a)
```



```
    end
    log_sum(i)=0-sum(a);
    log_sum2(i)=(ind(i)/length(value)).*log_sum(i);
end
a= Entropy_class;
b=sum(log_sum2);
    Sum_Final=Entropy_class-sum(log_sum2);
% Keeping a note of the highest and the lowest solutions
if Fh<Sum_Final
        Fh=Sum_Final;
end
if Fl>Sum_Final
        Fl=Sum_Final;
end
New_Solution=Sum_Final;
%Defining the configurations
Best_Solution=Initial_Solution_Objective_Value;
Best_Configuration=temp_attribūte;
%Evaluating the energy difference
    if Initial_Solution_Objective_Value~=0;
    Solution_O\overline{bjective_V̄alue=Initīal_Solution_Objective_Value;}
    end
    Change_in_Energy = New_Solution- Solution_Objective_Value;
    if Change_in_Energy>0
    Solution_Objective_Value=New_Solution;
    end
```

```
    if Change_in_Energy<0
        R=rand(1);
        if exp(Change_in_Energy/T)<R
            Equilibrium_Best=Best_Solution;
            Equilibrium_Config_Best=Best_Configuration;
            Move =0;
            break;
                end
                if exp(Change_in_Energy/T)>R
                        Solution_Objective_Value=New_Solution;
                end
    end
    % Keeping a note of the Best Solution so far
    Best_Solution= Solution_Objective_Value;
    % Best_Solution
    if Equilibrium_Best<=Best_Solution
    Equilibrium_Best=Best_Solution;
    %Best_Configuration= temp_attribute;
    Equilibrium_Config_Best=temp_attribute;
    end
    Move =1;
    loop = loop+1;
    l_Transition=l_Transition + 1;
    Initial_Solutiōn_Objective_Value=0;
end
Increament =Transition_L.*(1-exp(-(Fh-Fl)/Fh));
Temp_Best=Equilibrium_Best;
Temp_Config_Best=Equilibrium_Config_Best;
% Defining Global Best Solution
if Global_Best<Temp_Best
    %Global_Best
    Global_Best=Temp_Best;
```

```
    Rand_Value=seed_rand;
    NumelEl=numel(find(g==1));
end
T=T.*0.90;
end
end
```


## Appendix C. 4 .Program Code for Performing the Equal Width Discretization

```
function [DiscreteData]= perform_equal_width(OriginalData)
%Creating a temporary Original data with dimensions same as the
Original
%Data
Temp_OriginalData=OriginalData;
Rows_Data = size(OriginalData,1);
Columns_Data=size(OriginalData,2)-1;
DiscretInterval=zeros(Rows_Data-1,Columns_Data);
Classes = unique(OriginalData(:,size(OriginalData,2)));
for feature = 1:Columns_Data
    % k=2;
k=log2(Rows_Data)+1;
        W= (max(OriginalData(:,feature))-min(OriginalData(:,feature)))/k;
        max(OriginalData(:,feature))
        min(OriginalData(:,feature))
        DiscretInterval(1,feature)=min(OriginalData(:,feature));
        for interval=2:(Rows_Data-1)
DiscretInterval(interval,feature)=min(OriginalData(:,feature)) +(interva
l-1)*W;
    end
dlmwrite('Interval.csv',DiscretInterval,',');
    for example=1:Rows_Data
```

```
    for discretize=1:numel(DiscretInterval(:,feature))-1
        if
OriginalData(example,feature)<DiscretInterval(discretize+1,feature) &&
OriginalData(example,feature)>=DiscretInterval(discretize,feature)
            Temp_OriginalData(example,feature)=discretize;
        end
    end
    end
end
DiscreteData=Temp_OriginalData;
```


## Appendix C. 5 .Program Code for Performing the Equal Frequency Discretization

```
function [Discrete_Data]=perform_equal_frequency(OriginalData)
Temp_OriginalData=OriginalData;
fprintf('\n Dimension Original Data: %d ',size(OriginalData));
Rows_Data=size(OriginalData,1);
Columns_Data=size(OriginalData,2);
remaining_unique=[];
for i=1:Columns_Data-1
    fprintf('\n Loop: %d ',i);
    OriginalData=sortrows(OriginalData,i);
    ColumnData=unique(OriginalData(:,i));
        fprintf('\n Initial Number Unique: %d',
numel(unique(ColumnData)));
    check_column=ColumnData;
```

```
        accept=0;
        if log(Rows_Data)-log(log(Rows_Data))<=19
        while accept~=1
            [div]=divisibility_equal_frequency(check_column(:,1));
            if div==21
remaining_unique=[remaining_unique;check_column(numel(check_column))];
                check_column=check_column(1:(numel(check_column)-1),:);
            end
                if div ~=21
                accept =1;
            end
        end
    else div =20;
    end
    div
    fprintf('\n Number Unique: %d \n',numel(check_column));
    size_unique=numel(check_column);
        bin=numel(check_column)/div;
        bin
        temp_assignment=zeros(numel(ColumnData),1);
        ColumnData=[temp_assignment ColumnData];
    id=1;
    bin_cap=[1:bin:size_unique];
        size_unique
        for bin_index=1:bin:size_unique
            if bin_index+(bin-1)>size_unique
                    bi\overline{n}= size_unique-bin_cap(size_unique-1);
            end
```

```
    for index=bin_index:(bin_index+(bin-1))
        ColumnData(index,1)=id;
    end
    id=id+1;
    end
    fprintf('\n Remaining Unique values: %d',size(remaining_unique));
    if size(remaining_unique) ~=0
        fprintf('\n Unique value identified id: %d ',id);
        size_rows=size(remaining_unique,1);
        fprintf('\n Remaining Unique: %d', remaining_unique);
        for fill_value=1:size_rows
ColumnData(find(ColumnData(:, 2)==remaining_unique(fill_value)),1)=id-1;
        end
    end
    for convert_discrete=1:numel(ColumnData(:,2))
Temp_OriginalData(find(OriginalData(:,i)==ColumnData(convert_discrete,2
)),i)=ColumnData(convert_discrete,1);
    end
    ColumnData
    Discrete_Data=Temp_OriginalData;
    Discrete_Data
    remaining_unique=[];
end
```


## Appendix C. 6 .Program Code for Performing the Multivalued Discrete Width Algorithm

* The main function remains constant for both the version of the Multivalued Discretization approaches, via Width and Frequency.

```
clear all
clc;
readfile=dlmread('attribute.CSv', ',', 'A1..E150');
Rows= size(readfile,1);
Columns=size(readfile,2);
Classes= unique(readfile(:,Columns));
```

```
temp_data=ones(Rows,1);
temp_data=[temp_data readfile];
for feature_attribute=1:(Columns-1)
attribute_num=feature_attribute;
intervals =16;
id=1;
check unique=0;
exit_I`op=0;
while check_unique<=intervals || exit_loop ==0;
    temp_data=sortrows(temp_data,1);
    check_num=temp_data(1,1);
    % To make sure only more than 3 values are chosen to create a
partition
        find_num=
numel(unique(temp_data(1:numel(find(temp_data(:,1)==check_num)),attribu
te_num+1)));
            if find_num>=4
            Disc=\overline{t}emp_data(find(temp_data(:,1)==check_num),2:Columns+1);
            exit_loop=2;
            end
            if find_num<4
            [break_loop temp_data check_num id] =
check_feasibili}ty_subset(temp_data,check_num,find_num,id,attribute_num
;
            if break_loop==1
                        exit_loop=1;
                        brea\overline{k};
            else
            Disc=temp_data(find(temp_data(:,1)==check_num),2:Columns+1);
            exit_loop=2;
            end
            end
    if exit_loop==2
```

```
        [Disc_Set1 Disc_Set2]=discretize(Disc,attribute_num,id);
        id = id+1;
        for index_1=1:numel(Disc_Set1)
temp_data(find(temp_data(:,attribute_num+1)==Disc_Set1(index_1)),1)=id;
    end
    id=id+1;
    for index_2=1:numel(Disc_Set2)
temp_data(find(temp_data(:,attribute_num+1)==Disc_Set2(index_2)),1)=id;
    end
    check_unique=numel(unique(temp_data(:,1)));
end
end
temp_data(:,feature_attribute+1)=temp_data(:,1);
temp_data(:,1)=1;
end
dlmwrite('Discrete_data.csv',temp_data(:,2:Columns+1),',');
```


## Appendix C.7. Program Code for Performing the Un-Supervising the Supervised

 Algorithm FMF and FMW.* The main function remains constant for both the version of the Multivalued

Discretization approaches, via Width and Frequency.

```
clear all
clc;
OriginalData=dlmread('attribute.cSv',',','A1..AK4435');
Temp_OriginalData=OriginalData;
Rows_Data=size(OriginalData,1);
Columns Data=size(OriginalData,2);
```

```
remaining_unique=[];
for i=1:Columns_Data-1
    OriginalData=sortrows(OriginalData,i);
        ColumnData=unique(OriginalData(:,i));
        numel(ColumnData)
        check_column=ColumnData;
    accept=0;
    if log(Rows_Data)-log(log(Rows_Data))}<=1
        while accept~=1
            [div]=divisibility_equal_frequency(check_column(:,1));
            if div==21
remaining_unique=[remaining_unique;check_column(numel(check_column))];
                check_column=check_column(1:(numel(check_column)-1),:);
            end
                if div ~=21
                accept =1;
            end
        end
    else div =20;
    end
    size_unique=numel(check_column);
        bin=numel(check_column)/div;
        temp_assignment=zeros(numel(ColumnData),1);
        ColumnData=[temp_assignment ColumnData];
    id=1;
    bin_cap=[1:bin:size_unique];
```

```
        % Loop for dividing under the
            for bin_index=1:bin:size_unique
                if bin_index+(bin-1)>size_unique
                bin = size_unique-bin_cap(size_unique-1);
                    end
            for index=bin_index:(bin_index+(bin-1))
                    ColumnData(index,1)=id;
        end
        id=id+1;
        end
        %_}
    % Loop to be send for the multisubset discretizer
            % Output should produce increased 'id' number
        unique_columns=unique(ColumnData(:,1));
        for insert_disc=1:numel(unique_columns)
            if
numel(unique(ColumnData(ColumnData(:,1)==unique_columns(insert_disc),2)
)) >=2
        Dataset=[];
        search_index= find(ColumnData(:,1)==unique_columns(insert_disc));
            for insert_dataset=1:numel(search_index)
            search_index(insert_dataset)
Dataset=[Dataset;OriginalData(find(OriginalData(:,i)==ColumnData(search
_index(insert_dataset),2)),:)];
            end
            [id,column]=multivalued_subset(id,Dataset,i);
ColumnData(find(ColumnData(:,1)==unique_columns(insert_disc)),1)=column
(:,1);
            end
    end
    %
```

$\qquad$

``` \%
if size(remaining_unique) ~=0
```

```
            size_rows=size(remaining_unique,1);
```

```
            size_rows=size(remaining_unique,1);
```

```
        for fill_value=1:size_rows
ColumnData(find(ColumnData(:,2)==remaining_unique(fill_value)),1)=id-1;
        end
    end
    for convert_discrete=1:numel(ColumnData(:,2))
Temp_OriginalData(find(OriginalData(:,i)==ColumnData(convert_discrete,2
)),i)=ColumnData(convert_discrete,1);
    end
    remaining_unique=[];
end
dlmwrite('discrete_data.csv',Temp_OriginalData,',');
```

Appendix C.8. Program Code for Performing the Unsupervising the Supervised Algorithm FF and FW.

* The main function remains constant for both the version of the Multivalued Discretization approaches, via Width and Frequency.

```
clear all
clc;
OriginalData=dlmread('attribute.csv',',','A1..S846');
Temp_OriginalData=OriginalData;
Rows_Data=size(OriginalData,1);
Columns_Data=size(OriginalData,2);
remaining_unique=[];
for i=1:Columns_Data-1
    OriginalData=sortrows(OriginalData,i);
    ColumnData=unique(OriginalData(:,i));
    numel(ColumnData)
    check_column=ColumnData;
    accept=0;
    if log(Rows_Data)-log(log(Rows_Data))}<=1
```

```
        while accept~=1
            [div]=divisibility_equal_frequency(check_column(:,1));
        if div==21
remaining_unique=[remaining_unique;check_column(numel(check_column))];
                        check_column=check_column(1:(numel(check_column)-1),:);
        end
            if div ~=21
                accept =1;
            end
    end
    else div =20;
    end
    size_unique=numel(check_column);
    bin=numel(check_column)/div;
    temp_assignment=zeros(numel(ColumnData),1);
    ColumnData=[temp_assignment ColumnData];
    id=1;
    bin_cap=[1:bin:size_unique];
    % Loop for dividing under the
        for bin_index=1:bin:size_unique
            if bin_index+(bin-1)>size_unique
                bin = size_unique-bin_cap(size_unique-1);
            end
    for index=bin_index:(bin_index+(bin-1))
            ColumnData(index,1)=id;
    end
    id=id+1;
    end
    % \%
```

```
    % Loop to be send for the multisubset discretizer
        % Output should produce increased 'id' number
    unique_columns=unique(ColumnData(:,1));
    for insērt_disc=1:numel(unique_columns)
        if
numel(unique(ColumnData(ColumnData(:,1)==unique_columns(insert_disc),2)
))>=2
    Dataset=[];
    search_index= find(ColumnData(:,1)==unique_columns(insert_disc));
        for insert_dataset=1:numel(search_index)
                search_index(insert_dataset)
Dataset=[Dataset;OriginalData(find(OriginalData(:,i)==ColumnData(search
_index(insert_dataset),2)),:)];
    end
    [id,column]=perform_equal_frequency(id,Dataset,i);
ColumnData(find(ColumnData(:,1)==unique_columns(insert_disc)),1)=column
(:,1);
            end
        end
    %
```

$\qquad$

``` 웅
if size(remaining_unique) ~=0
            size_rows=size(remaining_unique,1);
            for fill_value=1:size_rows
ColumnData(find(ColumnData(:,2)==remaining_unique(fill_value)),1)=id-1;
            end
        end
            for convert_discrete=1:numel(ColumnData(:,2))
Temp_OriginalData(find(OriginalData(:,i)==ColumnData(convert_discrete,2
)),i)=ColumnData(convert_discrete,1);
    end
    remaining_unique=[];
end
dlmwrite('discrete_data.csv',Temp_OriginalData,',');
```


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