# POSE ESTIMATION FOR ROBOTIC DISASSEMBLY USING RANSAC WITH LINE FEATURES 

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# POSE ESTIMATION FOR ROBOTIC DISASSEMBLY USING RANSAC WITH LINE FEATURES 

\(\left.\begin{array}{c}A Thesis <br>
Presented to <br>
the Graduate School of <br>
Clemson University <br>
of the Requirements for the Degree <br>
Master of Science <br>

Electrical Engineering\end{array}\right]\)| Aswadha Narayanan Shyam Sundar |
| :---: |
| May 2011 |

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#### Abstract

In this thesis, a new technique to recognize and estimate the pose of a given 3-D object from a single real image provided known prior knowledge of its approximate structure is proposed. Metrics to evaluate the correctness of a calculated pose are presented and analyzed. The traditional and the more recent approaches used in solving this problem are explored and the various methodologies adopted are discussed.

The first step in disassembling a given assembly from its image is to recognize the attitude and translation of each of its constituent components - a fundamental problem which is being addressed in this work. The proposed algorithm does not depend on uniquely identifiable 3D model surface features for its operation - this makes it ideally suited for object recognition for assemblies. The algorithm works well even for low-resolution occluded object images taken under variable illumination conditions and heavy shadows and performs markedly better when these factors are removed.


The algorithm uses a combination of various computer vision concepts such as segmentation, corner detection and camera calibration, and subsequently adopts a line-based object pose estimation technique (originally based on the RANSAC algorithm) to settle on the best pose estimate. The novelty of the proposed technique lies in the specific way in which the poses are evaluated in the RANSAC-like algorithm. In particular, line-based pose evaluation is adopted where the line chamfer image is used to evaluate the error distance between the projected model line and the image edges. The correctness of the computed pose is determined
based on the number of line matches computed using this error distance. As opposed to the RANSAC algorithm where the search process is pseudo-random, we do an exhaustive pose search instead. Techniques to reduce the search space by a large amount are discussed and implemented.

The algorithm was used to estimate the pose of 28 objects in 22 images, where some images contain multiple objects. The algorithm has been found to work with a 3-D mismatch error of less than 2.5 cm in $90 \%$ of the cases and less than 1 cm error in $53 \%$ of the cases in the dataset used.

## DEDICATION

To my mom who laid the very foundation of my education

To my dad who helped me build on it

## ACKNOWLEDGEMENTS

I like view this work as a crystallized version of numerous ideas inspired by a lot of people around me, directly and indirectly; wittingly and unwittingly. It just wouldn't have become what it is now without their help and encouragement.

First of all, I thank my advisor, Dr. Richard Groff for the gargantuan amount of time and effort he has taken in walking me through this long journey involving multitudes of equations, algorithms and programming techniques. In retrospect, the work would have remained just another unfulfilled effort without his crucial inputs.

I must thank my friends, particularly Vinay Gidla, Vidya Murali, Harshwardhan Karve, Aditya Sriram, Varun Prabhu and Nivedhitha Giri who provided fuel for extremely insightful discussions on topics ranging from Human psychology, Robotics, Artificial Intelligence and Theology which has had its own impact on my way of thinking, and consequently my research work. I must also thank them for putting up with my occasional bouts of insanity while I used to be engrossed in solving a sub-problem towards this work, which I have come to accept as an endemic part of a graduate student research life.

Most importantly, I thank my parents, especially my mom who goaded me to never give up on any problem, be it research or otherwise and provided me all the moral support I acutely required over this long and arduous journey. It was she who instilled in me the belief that every
problem has a definite solution and that one could find it by looking for it hard enough. I know that I cannot repay even a fraction of what I really owe her.

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## CHAPTER ONE

## INTRODUCTION

### 1.1 THE PROBLEM AND MOTIVATION

Much progress has been made in the process of automating assembly processes in the manufacturing industry. In contrast, the reverse process of automated disassembly is currently plagued by numerous practical difficulties and remains an unsolved real-world problem. Products that go through the disassembly process are most likely to be recycled and one could generally expect random deformations in them. Thus, any algorithm that is tailored for disassembly must be robust to minor object deformations.

Robotic disassembly of used products is increasingly relevant due to legal requirements placed upon manufacturers to recycle their products. However, disassembly of end-of-life products has traditionally been a manual process owing to the numerous technical difficulties associated with it. The deformed shape of waste products constitutes a common obstacle and poses a considerable challenge to the application of robotic arms in this process. A natural consequence of the difficulties associated with this process has been the adoption of recycling methodologies without any kind of inbuilt intelligence. Such recycling techniques usually involve the use of shredders, smelters and similar equipment that "blindly" convert a bunch of scrap into a heterogeneous mass of recycled material, losing much of the potential initial worth of the products.

In addition to this, a task like recycling of electronic waste usually entails dealing with toxic, hazardous substances. These factors combined with the persistent need for manual labor in this industry leads to a situation where the cost of recycling a used product often exceeds the cost of manufacturing a new, similar product. Extensive studies on the associated cost analysis have been done earlier by Yuksel and Baylakoglu [1]. In a nutshell, their study concluded that a specific, optimized disassembly plan for a given product would be economically feasible. Our work essentially constitutes an attempt to help generalize the disassembly process for any given assembly. We will proceed to discuss the various traditional approaches adopted towards solving the disassembly problem and subsequently transition to the discussion on pose estimation in the following few paragraphs.

The earliest approaches which aimed to solve the disassembly problem were almost exclusively based on Maximum Likelihood estimation (ML estimation) and Maximum a posteriori Probability estimation (MAP estimation), and have been used to estimate the relative positions of parts in a known assembly using an image. A typical ML approach is by Sanderson [2], where the assemblability of a set of assembly components in a configuration is explored in terms of ML-based constraint clearance in their vicinity. This measure of assemblability was incorporated into an AND/OR graph described by Mello and Sanderson [3] to make a decision on the sequence of steps for the disassembly procedure for an object with known construction.

The more noise-robust MAP estimate can be looked at as a variant of the Maximum Likelihood estimate in the sense that here we make use of a known a posteriori object probability distribution along with the data used for maximum likelihood estimation. Tretter [4] had earlier
used a fast multi-scale technique to compute the Sequential MAP (SMAP) of the unknown position, scale factor and 2-D rotation for each subassembly. This technique affords additional accuracy over the MAP estimate owing to the search through multiple resolution levels. The drawback with the estimation techniques is that they are extremely application-specific in terms of their success and prior knowledge of the associated assembly structure is needed for reliable pose estimation.

It is imperative to have a good way of representing assemblies prior to thinking about their disassembly. Sagerer [5] had proposed the use of context free grammars to represent assemblies. The core idea behind this representation is the reusability of the mating properties of the subassemblies. Each assembly can be looked at as a valid sentence constructed using certain predefined rules. These rules are a direct consequence of the configuration of the subassemblies. Valid sentences (an assembly, in this case) are a byproduct of substitution of a given rule in other rules in myriad ways. This is explained through a specific example in [6].

As noted earlier, there are well developed techniques to represent assemblies in terms of their subassemblies. Mello and Tretter's work [3][4] on assemblability and assembly inspection shows that if the components and configuration of an assembly are known beforehand, the problem of optimal disassembly breaks down into a simple graph search. There are standard techniques such as the Ant Colony optimization which is used for evaluating the sequence of disassembly operations with an aim of arriving at the most optimal solution and has been described by Shan [7].

However, these techniques can be applied to a practical assembly at hand for an automated disassembly process only if the individual subassemblies are first recognized and their poses computed. This would mean that one could then, for instance, construct the AND/OR assembly graph using these poses straightaway and proceed to using the Ant-colony optimization algorithm mentioned earlier. In other words, to actuate the process of disassembly of a real assembly given its image, one needs to first find out the location and orientation of each of the assembly components in order for a robot to be able to take them apart.

As mentioned earlier, most assembly objects in line for recycling need not necessarily, in practice, conform to the exact dimensions as specified by the prior object attribute information. Pose estimation can be looked at as an Artificial Intelligence problem, the solution for which seeks to emulate human intelligence in recognizing the position and orientation of a given object placed amidst a cluster of connected/disconnected objects in a scene with numerous visual constraints enumerated earlier.

Pose-estimation is an essential and a fundamental step that one must take prior to any disassembly process planning. This is because pose-estimation helps reveal some information about how the assembly components are oriented or interconnected in space and could potentially help generate directions on how to actuate the disassembly process using robotic arms or an equivalent actuator. A general disassembly algorithm is fundamentally more complex than a generic assembly algorithm. This is because a disassembly process generally may take place in a less structured environment without much a priori information about the constituent assembly object components. While there has been a significant amount of work done in designing a
disassembly plan [3] given a graph composed of nodes of connected components, such information might not be known in advance.

The object pose estimation problem is a partially unsolved problem with limited success on a real image. In this research work, an algorithm that would reliably estimate the true pose of a given object with known 3D configuration from a single image with high accuracy under variable lighting conditions and object occlusions is proposed.

In this thesis the problem of pose estimation of known objects from a single real image without having any prior information about known point or edge correspondences is solved. Given the 3-D model of a specific object represented using its corners and faces we can reliably estimate the true rotation and translation of this object in the given image despite the presence of shadows, variable lighting conditions and occlusions. In this work, the methodologies discussed are generally suited for a typical connected/disconnected assembly or its components. We also present and analyze two metrics, one discrete and another continuous, each of which give an estimate of the correctness of the calculated pose.

This thesis is organized into 5 major chapters in the following order: Introduction, The RANSAC with Line Features, Results and Discussion, Conclusions and Future work, and References. The first chapter, Introduction, is organized into two subdivisions: the first describing the problem solved and the motivation behind and the second containing the literature review involved in this research. The second chapter is divided into two parts. The first part gives the necessary background for understanding the subsequent material in the Algorithm
subsection. Owing to the complex, abstract nature of pose estimation, we further divide this part into three subdivisions: perspective transformation, pose estimation with known correspondences and image processing and 3D modeling. The second part of the chapter on Theory, describes the algorithm and its implementation. The third chapter presents the results and draws a contrast on the strengths and weakness of the proposed algorithm. The penultimate chapter discusses the potential improvements to the proposed algorithm and gives a direction for future research. The final chapter enumerates the references used over the course of this work.

### 1.2 LITERATURE SURVEY

Pose estimation is the process of determining the rotation and translation of 3-D object representation in an image with respect to a given coordinate system. Pose estimation usually involves matching the 3-D object features onto the corresponding 2-D image features. Given the correspondences between object and image features, rotation and translation can be computed. However, these correspondences cannot be easily determined given just the 3-D model and the object image and remains an unsolved problem in case of images with featureless assemblies thus far.

Pose estimation using known correspondences is a thoroughly studied problem. The solution to a fundamentally important solved problem within pose estimation is the Perspective three-point problem (also known as the P3P problem) which gives us a way to solve the pose estimation problem with known correspondences. The POSIT (Pose from Orthography and Scaling with iterations) algorithm by Dementhon [8] utilizes known correspondences for
determining the object rotation and translation matrices. This algorithm first uses the POS (Pose from Orthography) for estimating the rotation and translation matrices given a set of known correspondences. The POSIT algorithm constitutes an improvement over the POS algorithm in that it first utilizes the POS algorithm to obtain a deterministic solution for a good initial guess of the pose. Subsequently, an iterative procedure to refine the obtained pose is proposed. This algorithm, in addition to being computationally efficient, is claimed to be relatively robust to both noise and errors in correspondence set.

Dementhon subsequently proposed an iterative algorithm called the SoftPOSIT algorithm [9] which did not require known correspondences to match a set of 3-D model points to image points. The SoftPOSIT algorithm alternately estimates the correspondence probabilities given the current pose and then estimates the pose given the correspondences. This algorithm takes a set of 3-D model points and another set of 2-D image points as parameters. Note that the sets may be of different sizes. The principal idea behind this iterative algorithm is to use a doubly stochastic matrix, the entries of which signify correspondence weights in terms of probability densities, over subsequent iterations to refine the point matches. This matrix is utilized in a proposed metric to minimize the error between the set of 3-D points projected onto the image plane and the set of corresponding 2-D image points. Later work by Dementhon utilizing line features rather than corner features [9] [10] has been shown to produce results that are more robust to noise. This is expected as line features contain more information than point features.

Another prominent correspondenceless pose estimation technique is the Gravitational Pose Estimation Algorithm (GPE Algorithm) by Ugurdag [11]. This algorithm utilizes the
distance between the rays shooting out from the camera focus and passing through the object features (which are taken to be corners in this algorithm's implementation) and the image features visible in the given object image. The idea behind the formulation of this algorithm is to utilize appropriate force vectors from the image points that would make the object at an arbitrary initial pose converge to its correct location using an iterative process. In each iteration, the force vectors that act on the 3-D model points is successively refined based on the distance between the object rays and the image features. The algorithm is assumed to have converged and terminates when the change in pose metric between successive refinements is below a predefined threshold. A metric based on the rays and the image points is proposed and its performance is evaluated.

The RANSAC algorithm proposed by Fischler and Bolles [12] is by far the most relevant piece of research that meshes most closely with our work. The RANSAC algorithm in a single pass, assigns three random correspondences between 3-D object features (usually corners) and the image points. The resulting correspondence problem is then solved to obtain the associated object model point depths using the solution for the Perspective three-point problem, also known as the P3P problem. This solution is then translated to the corresponding rotation and translation matrices. It is to be noted that the P3P problem may have more than one valid solution. This will be discussed again in a greater depth in the chapter 2. There has been a scarcity of robust metrics tested on real data that could be interfaced to the RANSAC algorithm to test the goodness of an estimated pose. This is exactly where our work fits into the bigger picture. We utilize the solution to the P3P problem for each set of possible 3-D model to 2-D image three-point correspondences and deterministically calculate the pose of the object. The estimated pose is
now projected onto the image plane and its appropriateness for the image at hand is evaluated using an object-edge based metric which will be proposed in the next chapter.

The SoftPOSIT algorithm utilizing only the point features has been found to work well only on synthetic images when the initial guesses are relatively close. When the object's initial pose is kept at an arbitrary location in the presence of stray points, the algorithm converges to a false pose most of the time. The SoftPOSIT algorithm using lines is relatively more robust to the presence of false features. However, this algorithm requires a good initial guess for a given pose hypothesis in order for it to converge to the true object position. To overcome this problem, this algorithm utilizes a large number of poses and applies the iterative convergence algorithm to each of the cases. The best fit among the converged poses is the one that minimizes a predefined objective function. However, this algorithm has been tested only using accurate 3-D models and images with sharp, distinct features. Even under such circumstances, the resulting average pose estimation error computed is found to average about 20-30 degree rotation from the original pose.

The Gravitational Pose estimation suffers from similar disadvantages. The GPE, like the SoftPOSIT needs a decent initial guess for guaranteed convergence to the right pose. Further, this algorithm has been only tested on 2-D scenes viewed in 3-D and without the presence of additional noise.

The RANSAC algorithm, in its current state requires to be interfaced with techniques from computer vision and a good metric for estimating and measuring the goodness of a pose.

The proposed RANSAC-based algorithm works in conjunction with both line features and a plethora of Computer Vision techniques. We realize that this two-pronged approach is absolutely essential to recognizing the true object pose from real images taken under heavy clutter, occlusions and heavily varying lighting conditions. Further, this approach would enable us to detect the pose of the object in the image even if its 3-D model is does not match exactly with the actual object model in the image.

The fundamental difference between the line-based SoftPOSIT algorithm and the RANSAC-based algorithm we propose lies in the area of application. The line-based SoftPOSIT algorithm, by its very formulation, is more suited to pose estimation in a natural, cluttered scene. On the other hand, we try and solve the pose estimation problem for assemblies. This makes the two algorithms very different from each other as each is designed and optimized for the application it was intended for and would be inappropriate for direct comparison. SoftPOSIT with lines is more suited to scenes with an abundance of line features while our algorithm is adapted for images with very few features present. Though theoretically our algorithm would give results that are comparable or better than SoftPOSIT owing to its reliance on an exhaustive search, a major drawback would be the high computation time involved.

The other pose estimation techniques that have been proposed earlier are the genetic algorithm based EvoPose and approaches based on Neural Networks and Lookup tables.

There has been much prior work done on Genetic Algorithm based pose estimation.
Toyama's work on Model based pose estimation [13] involved searching for the most probable
poses through a six-dimensional space where the best one of the lot was picked using a fitness criterion. Kayanuma [14] had used a 3-D object model with features to determine the object pose through a single image. The pose search is done using a genetic algorithm and the number of model-image feature matches determines the fitness of the pose.

Another notable genetic algorithm based technique is the EvoPose [15]. The EvoPose is similar in operation to Toyama's method in that it searches through a six-parameter space; however, the EvoPose relies solely on points rather than edges.

Neural Network approaches have been used in the past by Langley [22] for estimating the pose of a Fixtureless assembly. However the results obtained using this approach is contingent on the presence of a sufficient number of surface features and an appropriate training data set, and has been found only moderately successful against simple assemblies.

Another computer-vision based approach for pose estimation is to have a huge dataset of an object with known poses. The camera image at hand is then compared with each of these images. The database image with the best match is singled out in the process and its pose is read from the lookup table. An example of related work is by Rother [16] who used 3-D shape priors to estimate the image appearance by projecting them down on the image plane and then computing probability of image match. While this approach could guarantee good results without much real-time computation, the necessity of having a large database for each object in a variety of different environments rules out the use of such an approach in a practical scenario.

David Lowe's seminal paper on SIFT [17] (Scale Invariant Feature Transform) is an interesting place to begin work on pose estimation. The work provides us an extremely robust way to recognize known 2-D objects in a cluttered scene containing them. Each feature is uniquely identified using local gradient information represented in a 128-bit format. The image features chosen are image points with distinct gradient vectors. This makes them invariant to rotation, translation and scaling. The drawback with this technique is that it cannot be adapted for objects with few, poor feature descriptors. Moreover, the SIFT algorithm was originally designed to operate only on 2-D object features.

The SIFT features are relevant to pose estimation because they provide a way to obtain scale and rotation invariant image features. They are particularly useful when the object under consideration has plenty of distinct identifiable surface features. In such cases, the object's pose can be computed using pose estimation techniques utilizing known correspondences.

This technique could be extended to 3-D objects by having a picture of each face of the object and recognizing them separately. Ambiguity arises when we have a general, rounded object whose faces are not clearly defined. Savarese's work [18] utilizes this idea and gets the SIFT features of the "parts" (or faces, in the paper) of a given 3-D object. The faces are recognized separately and a homography matrix to transform the priori images to match the rotated images in the scene is computed. The object is viewed as a connected graph of the images.

Correspondence-based pose estimation could be a viable research direction to proceed on provided the 3-D object model has a sufficient number of distinct features that could be readily identified from its 2-D image. This process will most likely be subject to correspondence errors in case of a real image. To overcome this problem, Haralick [19] proposed a pose estimation technique using known correspondences that would be robust for up to thirty percent of correspondence mismatches. This is in contrast to the least-squares based solution provided earlier, where such situational errors give rise to meaningless solutions. As stated earlier, this doesn't prove particularly useful in our work as we deal with images having a paucity of feature points.

## CHAPTER TWO

## THE RANSAC WITH LINE FEATURES ALGORITHM

### 2.1 BACKGROUND

### 2.1.1 PERSPECTIVE PROJECTION

The process of capturing the image of a 3D model is essentially a linear mapping of the 3D model points in the world onto a 2D image plane. This transformation, also known as perspective projection is crucial for understanding pose estimation.

For real images shot in practical scenes, a common approximate model is the pinhole camera model, which models the camera as a perspective projection. This model describes the mathematical relationship between observed 3D point in the scene and the corresponding 2D image point. In this model, the camera aperture is assumed to be a single point with multiple rays converging on it from various directions. An inverted image of the scene is formed on the plane containing the camera focus. Equivalently, one could think of the rays converging directly at the camera focus, with the image being formed at the plane passing through the aperture. The perpendicular distance of this plane from the focus is equal to the focal length of the camera.


Figure 2.1: Pinhole camera model

In Figure 2.1, I is the point of intersection of the ray OR with the image plane. The focal length is $f$ and the observed model point is $R$.

By triangle similarity of triangles OIP and ORL,

$$
\mathrm{u}=\mathrm{f} \frac{\mathrm{x}}{\mathrm{z}} \text { and } \mathrm{v}=\mathrm{f} \frac{\mathrm{y}}{\mathrm{z}}
$$

Thus, we see that the intersection coordinates are just the scaled versions of the normalized world coordinates. This leads us to the principal idea behind camera calibration.

For mapping a 3-D model observed using an image captured using a pinhole camera onto the corresponding 2-D image plane, one of the first steps one must take is to calibrate the camera. Camera calibration matrix describes the transformation between the 3-D points measured in the camera coordinate system and the 2-D coordinates using the image coordinate system.

Figure 2.2 shows the different coordinate systems we use in camera calibration. $\mathrm{O}_{\mathrm{c}}$ is origin of the camera coordinate system. Its corresponding Z -axis, $\mathrm{Z}_{\mathrm{c}}$ is perpendicular to the image plane and passes through the principal point of the image, $\left(u_{0}, v_{0}\right) . X_{c}$ and $Y_{c}$ are chosen parallel to the image coordinate axis $X_{i}$ and $Y_{i}$ as shown in the figure. $X_{o}, Y_{o}$ and $Z_{o}$ refer to the object coordinate system which is assumed to be attached to the object at hand. They are useful for expressing object dimensions in a more lucid fashion.


Figure 2.2: Representation of the various coordinate systems
$\mathrm{O}_{\mathrm{w}}$ is the center of the world coordinate system with axes $\mathrm{X}_{\mathrm{w}}, \mathrm{Y}_{\mathrm{w}}$ and $\mathrm{Z}_{\mathrm{w}}$. It is sometimes used since it is more natural to think about the rotation and translation of an object with respect to a fixed world coordinate system that is usually somewhere in the vicinity of the object as opposed to the camera coordinate system. This is because the camera coordinate system changes for different camera views and would not, consequently, provide a standard reference frame.

The camera calibration matrix, C, relates the 3-D model point, $\left[X_{i} Y_{i} Z_{i}\right]^{\mathrm{T}}$, measured with respect to the camera coordinate system with a point on the ray $\left[x_{i} y_{i} z_{i}\right]^{\mathrm{T}}$ starting from the camera focus and passing through the object model point whose position is measured with respect to the camera coordinate system with the image coordinate metric.

$$
\left[\begin{array}{l}
x_{i}  \tag{2.1}\\
y_{i} \\
z_{i}
\end{array}\right]=\left[\begin{array}{ccc}
\alpha_{u} & \gamma & u_{0} \\
0 & \alpha_{v} & v_{0} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]
$$

Equation 2.1 represents perspective projection of the 3-D model point onto the image plane. Camera calibration is the process of finding the perspective projection matrix (or camera calibration matrix) for a specific camera.


Figure 2.3: Camera coordinate system

Here, $\propto_{u}$ is the scaling factor along the X -axis and $\propto_{\mathrm{v}}$ is the scaling factor along the Y axis. $\gamma$ is the image skew factor and $\left[\mathrm{u}_{0} \mathrm{v}_{0}\right]^{\mathrm{T}}$ is the principal point and generally lies at the geometric center of the image. This point specifies the translation of the intersection point between $Z_{c}$ and the image plane with respect to the image coordinate system. If $\left[u_{0} v_{0}\right]^{T}$ were made equal to $[00]^{\mathrm{T}}$, it would imply that all the image coordinate measurements were made from the image coordinate system with an origin translated to the center of the image.

For a normal camera without distortions, the skew factor $\gamma$ is generally negligible and can be taken to be $0 . \propto_{u}$ and $\propto_{v}$ are generally in practice, equal to the focal length of the pinhole camera (A generalization which we do not assume in our case) and can be thought of the pixel scaling factors along the X and Y axes respectively. The equation 2.1 then becomes:

$$
\left[\begin{array}{l}
x_{i}  \tag{2.2}\\
y_{i} \\
z_{i}
\end{array}\right]=\left[\begin{array}{ccc}
\alpha_{u} & 0 & u_{0} \\
0 & \alpha_{v} & v_{0} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]
$$

Since $\left[x_{i} y_{i} z_{i}\right]^{\mathrm{T}}$ represents a point along a ray with coordinates measured with respect to the camera coordinates in image coordinate metric, we could obtain the actual image coordinates as measured from the screen by normalizing this vector, that is dividing $x_{i,} y_{i}$ and $z_{i}$ by $z_{i}$. The actual image coordinates of the point is $\left[x_{i} / z_{i} y_{i} / z_{i} 1\right]^{\mathrm{T}}$.

Put briefly, camera calibration is the process of finding the characteristic transformation matrix that transforms 3-D object points into the corresponding image points. One of the most popular techniques for calibrating a given camera is Zhang's method [20]. The camera
calibration matrix when multiplied with the 3-D object coordinates results in a point along the ray passing through the camera focus and the 3-D object point. On normalizing the output vector by dividing each element in the vector by the Z-value, we can get the actual image point.

Basically, camera calibration is a tool to directly transform the points on the rays passing through the image into the 3-D object points and vice-versa. This implies that if we associate three image feature points with corresponding object points, we could find out the angle subtended at the focus by each pair of the feature points. The angle subtended by any two object model points at the camera focus is computed as the cosine inverse of the dot-product of the vectors shooting out from the focus in the directions of the model points. Due to the image point and 3-D object point association, we know the true distances between the selected image feature points. Thus, the problem of pose estimation reduces to solving the perspective three-point problem which we shall discuss soon.

### 2.1.2 POSE ESTIMATION USING KNOWN CORRESPONDENCES

There are several methods of estimating the object pose, given correspondences between image points and points two of which will be presented here. The first method is based on the solution to the P3P problem. The P3P solution is used in the RANSAC algorithm while the second method used in computing the "true" pose of objects in the data set in Chapter 3.

We will now discuss the pose estimation using the solution to the perspective three-point problem. The various coordinate systems have already been described using Figures 2.2 and 2.3.

We now look at the problem of mapping a model triangle, originally in 3-D space at a known position and orientation into another an observed location through a rotation and translation.

Figure 2.4 shows the triangle ABC initially present in 3-D model being rotated and translated to a position to match with the visible triangle in the image. When this rotation and translation for a particular triangle in the 3-D model is found to match well with a triangle seen in image, we can take that rotation and translation to be the pose of the object visible in the image computed with respect to the camera coordinate system. In general, a good matching criterion would be that all the visible corner feature points that are present in the 3-D model be regenerated using the pose transformation matrix which ideally matches with the image corner features.


B

Figure 2.4: Rotation and translation of a triangle in space

The rotation and transformation of the triangle described earlier is found using the solution to the perspective three-point problem, or simply, the P3P problem. Let us now look at its problem statement and solution in greater detail.

Assume we are looking at a triangle of known dimensions using a pinhole camera. In our problem, we look at the 3-D object as being made up of triangles, one of which is under current consideration. As discussed earlier, camera calibration helps compute the angle subtended to the focus by the 3-D object points visible in the image as corner features.

The perspective 3-point problem deals with the problem of finding out the actual distances from the focus to each of the object points represented by the triangle provided the triangle dimensions and the angle subtended at the focus is known. Once these distances are known, we can compute the rotation and translation of this triangle on the object with respect to a predefined initial position with respect to the camera focus assumed originally. The perspective three-point problem can have a maximum of eight real solutions, out of which a maximum of four can be positive. This solution is presented in the appendix of [12].

Figure 2.5 shows the diagram describing the perspective three-point problem. O is the focus of the camera looking at the triangle $A B C$ with sides $R_{a b}, R_{a c}$ and $R_{b c}$. The points $A, B$ and C have the depths $\mathrm{a}, \mathrm{b}, \mathrm{c}$ with angles between $\mathrm{OA}, \mathrm{OB}$ and OC rays shooting from the camera represented as $\theta_{1}, \theta_{2}$ and $\theta_{3}$.


Figure 2.5: Perspective three-point problem

By cosine formula,

$$
\begin{aligned}
& R a b^{2}=a^{2}+b^{2}-2 a b \cos \left(\theta_{1}\right) \\
& R_{a c}{ }^{2}=a^{2}+c^{2}-2 a c \cos \left(\theta_{2}\right) \\
& R_{b c^{2}}{ }^{2}=b^{2}+c^{2}-2 b c \cos \left(\theta_{3}\right)
\end{aligned}
$$

Let us now define variables $K_{1}, K_{2}, G_{0}, G_{1}, G_{2}, G_{3}$ and $G_{4}$ as follows:

$$
\begin{aligned}
& K_{1}=\frac{R_{a b^{2}}}{R a{ }^{2}} \\
& K_{2}=\frac{R_{b c}{ }^{2}}{R_{a b^{2}}{ }^{2}}
\end{aligned}
$$

$$
\begin{aligned}
G_{4}= & \left(K_{1} K_{2}-K_{1}-K_{2}\right)^{2}-4 K_{1} K_{2} \cos ^{2}\left(\theta_{3}\right) \\
G_{3}= & 4\left(K_{1} K_{2}-K_{1}-K_{2}\right) K_{2}\left(1-K_{1}\right) \cos \left(\theta_{1}\right)+4 K_{1} \cos \left(\theta_{3}\right)\left[\left(K_{1} K_{2}+K_{2}-K_{1}\right) \cos \left(\theta_{2}\right)+\right. \\
& \left.2 K_{2} \cos \left(\theta_{1}\right) \cos \left(\theta_{3}\right)\right] \\
G_{2}= & {\left[2 K_{2}\left(1-K_{1}\right) \cos \left(\theta_{1}\right)\right]^{2}+2\left[K_{1} K_{2}+K_{1}-K_{2}\right]\left[K_{1} K_{2}-K_{1}-K_{2}\right]+} \\
& 4 K_{1}\left[\left(K_{1}-K_{2}\right) \cos ^{2}\left(\theta_{3}\right)+\left(1-K_{2}\right) K_{1} \cos ^{2}\left(\theta_{2}\right)-2 K_{2}\left(1+K_{1}\right) \cos \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \cos \left(\theta_{3}\right)\right] \\
G_{1}= & 4\left(K_{1} K_{2}+K_{1}-K_{2}\right) K_{2}\left(1-K_{1}\right) \cos \left(\theta_{1}\right)+4 K_{1}\left[\left(K_{1} K_{2}-K_{1}+K_{2}\right) \cos \left(\theta_{2}\right) \cos \left(\theta_{3}\right)+\right. \\
& \left.2 K_{1} K_{2} \cos \left(\theta_{1}\right) \cos ^{2}\left(\theta_{2}\right)\right] \\
G_{0}= & \left(K_{1} K_{2}+K_{1}-K_{2}\right)^{2}-4 K_{1}^{2} K_{2} \cos ^{2}\left(\theta_{2}\right)
\end{aligned}
$$

The solution to the P3P problem is obtained from the roots of the following quartic equation:

$$
G_{4} x^{4}+G_{3} x^{3}+G_{2} x^{2}+G_{1} x+G_{0}=0
$$

Now,

$$
\begin{aligned}
& a=\frac{R a b}{\sqrt{x^{2}-2 x \cos \left(\theta_{1}\right)+1}} \\
& b=a \cdot x \\
& y=\cos \left(\theta_{2}\right) \pm \sqrt{\cos ^{2}\left(\theta_{2}\right)+\frac{R a c^{2}-a^{2}}{a^{2}}} \\
& c=y \cdot a
\end{aligned}
$$

The $a, b$ and $c$ values must be verified by the cosine equations we started out with before they can be accepted. There can be a maximum of 8 real solutions and a maximum of 4 positive real roots.

The direct solution of the 3-point perspective problem yields distances rather than the rotation and translation matrices for the pose of an object. The distances can be transformed into the rotation and translation matrices using as follows.

Let $P_{1}, P_{2}$ and $P_{3}$ be the three model points chosen from the 3-D object model initially placed at the camera focus. The distances of the three feature points from the camera focus are detected assuming the feature points in the image correspond to the model points. Since the vectors from O to $\mathrm{A}, \mathrm{B}$ and C are known (From the camera calibration matrix formula discussed earlier), the 3-D position of the feature points are also known. Let us represent them by variables $\mathrm{P}_{1}{ }^{\prime}, \mathrm{P}_{2}{ }^{\prime}$ and $\mathrm{P}_{3}{ }^{\prime}$.

Let us now define matrices $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ containing vectors describing the triangle object coordinate system as follows:

$$
\begin{aligned}
& M_{1}=\left[\begin{array}{lll}
\left(P_{2}-P_{1}\right) & \left(P_{3}-P_{1}\right) & \left(P_{2}-P_{1}\right) \times\left(P_{3}-P_{1}\right)
\end{array}\right] \\
& M_{2}=\left[\begin{array}{lll}
\left(P_{2}^{\prime}-P_{1}^{\prime}\right) & \left(P_{3}^{\prime}-P_{1}^{\prime}\right) & \left(P_{2}^{\prime}-P_{1}^{\prime}\right) \times\left(P_{3^{\prime}}-P_{1}^{\prime}\right)
\end{array}\right]
\end{aligned}
$$

The approximate rotation matrix, $\mathrm{R}_{\text {approx }}$ is obtained using the following equations:

$$
\begin{aligned}
& M_{2}=R_{\text {approx }} M_{1} \\
& R_{\text {approx }}=M_{2} M_{1}{ }^{-1}
\end{aligned}
$$

$\mathrm{R}_{\text {approx }}$ is now subjected to singular value decomposition and is forced into a true rotation matrix of determinant equal to 1 using the following two equations. If the determinant of the resulting matrix is found to be -1 instead, the matrix is subject to scalar multiplication by -1 to make the determinant equal to 1 . Now, the rotation matrix, $\mathrm{R}_{\text {approx }}$ is calculated as a product of the orthogonal matrices U and $\mathrm{V}^{\mathrm{T}}$ :

$$
\begin{aligned}
& U \Sigma V^{T}=R_{\text {approx }} \\
& R_{\text {approx }}=U V^{T}
\end{aligned}
$$

The corresponding translation is found using the traditional pose-transformation formula, where $P$ is a general point from the original pose and $P^{\prime}$ is the transformed point after rotation and translation:

$$
\begin{aligned}
& P^{\prime}=\left[\begin{array}{ll}
R & T \\
0 & 1
\end{array}\right]\left[\begin{array}{l}
P \\
1
\end{array}\right] \\
& T_{1}=P_{1}^{\prime}-R P_{1} \\
& T_{2}=P_{2}^{\prime}-R P_{2} \\
& T_{3}=P_{3}^{\prime}-R P_{3}
\end{aligned}
$$

The translation, which must ideally have $\mathrm{T}_{1}=\mathrm{T}_{2}=\mathrm{T}_{3}$, is now subject to error minimization by taking the average of the three vectors $\mathrm{T}_{1}, \mathrm{~T}_{2}$ and $\mathrm{T}_{3}$.

$$
T=\frac{T_{1}+T_{2}+T_{3}}{3}
$$

We will now discuss the second pose estimation technique using sets of known correspondences mentioned in the beginning of this section. It would be quite useful to discuss the mathematics behind the computation of pose given a set of 3-D object points with respect to a known world coordinate and the corresponding 2-D image points. This technique is used for refining the estimated pose and obtaining the true pose by superimposing the estimated pose over the image of the object under consideration. This pose estimation technique differs slightly from the P3P pose estimation technique. In the P3P pose estimation technique, we have known correspondences between just three model points and three image features. In contrast, in this
pose estimation technique, we have known correspondences between at least six image points which are chosen manually and the associated object model points. In other words, we do not depend on the features detected by the corner detector, on the contrary, we hand-pick the image points corresponding to the object corners. An outline of this technique is described in [9].

Let $X_{i}, Y_{i}$ and $Z_{i}$ be the 3-D model coordinates with respect to the camera coordinate system and $x_{i}$ and $y_{i}$ be the corresponding image coordinates of depth s . The relationship between the two can be expressed as:

$$
s\left[\begin{array}{c}
x_{i} \\
y_{i} \\
1
\end{array}\right]=\left[\begin{array}{cccc}
\alpha_{u} & 0 & u_{0} & 0 \\
0 & \alpha_{v} & v_{0} & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i} \\
1
\end{array}\right]
$$

If $X_{i}, Y_{i}$ and $Z_{i}$ are in terms of the object coordinate system, the object rotation and translation with respect to the camera coordinate system is given by:

$$
s\left[\begin{array}{c}
x_{i}  \tag{2.3}\\
y_{i} \\
1
\end{array}\right]=\left[\begin{array}{cccc}
\alpha_{u} & 0 & u_{0} & 0 \\
0 & \alpha_{v} & v_{0} & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{cc}
r_{1} & t_{1} \\
r_{2} & t_{2} \\
r_{3} & t_{3} \\
\odot & 1
\end{array}\right]\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i} \\
1
\end{array}\right]
$$

Here, $r_{1}, r_{2}$ and $r_{3}$ are the horizontal rows of the rotation matrix and $t_{1}, t_{2}$ and $t_{3}$ are the scalar values present in the translation vector.

Thus, $R=\left[\begin{array}{l}r_{1} \\ r_{2} \\ r_{3}\end{array}\right] \in \mathrm{R}^{3 \times 3}$ and, $t=\left[\begin{array}{l}t_{1} \\ t_{2} \\ t_{3}\end{array}\right] \in \mathrm{R}^{3 \times 1}$
From equation 2.3, multiplying by the inverse of the camera calibration matrix, C , on both sides, we get:

$$
s C^{-1}\left[\begin{array}{c}
x_{i} \\
y_{i} \\
1
\end{array}\right]=C^{-1}\left[\begin{array}{ll}
C & \bigodot_{3 \times 1}
\end{array}\right]\left[\begin{array}{cc}
R & t \\
\bigodot_{1 \times 3} & 1
\end{array}\right]\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i} \\
1
\end{array}\right]
$$

Multiplying the inverse of the camera calibration matrix with the image coordinates yields a 3D point $\left[x_{i}{ }^{\prime} y_{i}{ }^{\prime} z_{i}\right]^{\mathrm{T}}$ along the ray that emanates from the camera focus and passes through the 3D object corner:

$$
\left[\begin{array}{l}
x_{i}{ }^{\prime}  \tag{2.4}\\
y_{i}{ }^{\prime} \\
z_{i^{\prime}}
\end{array}\right]=C^{-1}\left[\begin{array}{l}
x_{i} \\
y_{i} \\
1
\end{array}\right]
$$

Note that point $\left[x_{i} y_{i}^{\prime} z_{i}\right]^{\mathrm{T}}$ has units based on the camera coordinate system while point $\left[x_{i} y_{i}\right]^{\mathrm{T}}$ is in image coordinate system and with values typically measured in pixels.

For 3D object model points rotated and translated from the camera focus to somewhere out in space by an appropriate rotation and translation matrix, the equation becomes:

$$
s\left[\begin{array}{c}
x_{i}{ }^{\prime} \\
y_{i}{ }^{\prime} \\
z_{i^{\prime}}
\end{array}\right]=\left[\begin{array}{ll}
R & t
\end{array}\right]\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i} \\
1
\end{array}\right]
$$

Expanding and rewriting the first two entries in the matrix on the left, we get:

$$
\begin{align*}
& s x_{i}^{\prime}=r_{1}\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]+t_{1}  \tag{2.5}\\
& s y_{i^{\prime}}=r_{2}\left[\begin{array}{l}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]+t_{2} \tag{2.6}
\end{align*}
$$

The depth can be obtained directly from equation 2.3:

$$
s=r_{3}\left[\begin{array}{c}
X_{i}  \tag{2.7}\\
Y_{i} \\
Z_{i}
\end{array}\right]+t_{3}
$$

Combining equations 2.5, 2.6 and 2.7,

$$
\begin{align*}
& r_{3}\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right] x_{i}{ }^{\prime}+t_{3} x_{i}^{\prime}-r_{1}\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]-t_{1}=0  \tag{2.8}\\
& r_{3}\left[\begin{array}{l}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right] y_{i}+t_{3} y_{i}^{\prime}-r_{1}\left[\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right]-t_{2}=0 \tag{2.9}
\end{align*}
$$

Let us now define a column vector, Q , containing 12 elements - the entries of the rotation and the translation matrices as:

$$
Q=\left[\begin{array}{l}
r_{1}^{\prime} \\
r_{2}^{\prime} \\
r_{3}^{\prime} \\
t_{1} \\
t_{2} \\
t_{3}
\end{array}\right]
$$

And another matrix J:

$$
J=\left[\right]
$$

We notice that we could rewrite equations 2.8 and 2.9 in matrix form as:

$$
J Q=\odot_{2 N \times 12}
$$

Here, N is the number of $3 \mathrm{D}-2 \mathrm{D}$ point correspondences given to us. The J matrix contains 2 N rows and 12 columns. Since we have the 3D and 2D point correspondences and points themselves, we could construct the J matrix as described above. We must be careful to use the $\left[x_{i}{ }^{\prime} y_{i}{ }^{\prime}\right]^{\mathrm{T}}$ values rather than $\left[x_{i} y_{i}\right]$ in the matrix. $\left[x_{i}{ }^{\prime} y_{i}{ }^{\prime}\right]$ is obtained from $\left[x_{i} y_{i}\right]$ as given by equation 2.4.

To solve for the best non-trivial solution for Q , we use the singular value decomposition (SVD) technique on matrix $J: J=\mathrm{U} \Sigma \mathrm{V}^{T}$. The best solution for Q is the last column of the V matrix, as it minimizes the norm of the matrix JQ. Values for the rotation and translation matrices are now extracted from Q .

The obtained rotation matrix $\hat{R}$ is again subjected to singular value decomposition to obtain its $U, \Sigma$ and $V$ matrices:

$$
\widehat{R}=U \Sigma V^{T}
$$

This matrix is forced to give an orthogonal matrix $\hat{\hat{R}}$ by multiplying $U$ and $V^{T}$ matrices:

$$
\hat{\hat{R}}=U V^{T}
$$

If the $\hat{\hat{R}}$ matrix is found to have a determinant equal to -1 , each element in the matrix is multiplied with -1 to force the determinant to be 1 . Thus, we now obtain a true orthogonal rotation matrix satisfying:

$$
\hat{\hat{R}}^{T} R=R \hat{\hat{\widehat{R}}}^{T}=I
$$

The associated translation matrix is given by:

$$
\left.\hat{t}=\left[\begin{array}{l}
t_{1} \\
t_{2} \\
t_{3}
\end{array}\right] \right\rvert\, \frac{|\hat{\hat{R}}|}{|\hat{R}|}
$$

These steps constitute the solution for pose estimation using known correspondences.

### 2.1.3 IMAGE PROCESSING AND 3D MODELING

This section describes the different image processing concepts used by the proposed algorithm. The algorithm uses a corner detector, an edge chamfer image and incorporates a color-based segmentation algorithm. This section also presents an overview of the representation of the 3D object model in the algorithm and associated rotation formulae.

The proposed algorithm uses corners as image feature points. Some of the corners detected in the image will most probably correspond to the true object corners which are also the 3D model points. There is always a question of detecting the best and the most probable corners in a given image. Xiao and Yung [21] proposed an adaptive algorithm to detect corners based on the local and global curvature properties of the image edges. The edges are detected using the conventional canny edge detector.

The distance of the projected lines from the image lines is computed using the chamfer distance image generated from the edge image. The distance data from chamfer images is directly used in the error metric proposed in the algorithm. A chamfer edge image can be thought of as a lookup table generated from a binary edge image that contains the distance at each pixel to the nearest edge. A pre-computed chamfer edge image helps reduce the computation time in the algorithm we are about to propose. There are three different kinds of norms used prominently: 1-norm (Also known as the Manhattan distance), 2-norm or the Euclidean which we use and the infinite norm (or the Chessboard distance).

We estimate the pose of the model to lie largely in the segmented region of the image that separates the object from its background. Segmenting out the desired object in the image is particularly useful in avoiding a lot of potential false positives in pose estimation, in addition to decreasing the computation time. In our case, we assume that objects in the image can be one of the following three colors: red, green and yellow.

A reliable way to segment out the different objects based on color is to use the ratio of the color channels. For example, the red object in the image would have a ratio of red to green and red to blue considerably higher at the image pixels that represent the object. Similarly, the green object in the image would have a higher ratio of green to red and green to blue channels. The segmentation of the yellow object is slightly different. It is common knowledge that the yellow color is made up of an equal combination of the red and green components with no blue ideally. If the red and green channels are of comparable intensity and the ratios of red-blue and greenblue is comparatively high, we may classify the pixel to be on the yellow object in the image.

The segmented out region is dilated by a small amount to make sure that the edges of the object which contain the most relevant corners are not left out of consideration. Analyzing the clustered patches of segmented regions using connected components gives us a way to identify multiple physically well-separated similar objects in the same image.

Prior to pose estimation of an object, one needs a reasonably general method to represent a 3D object. This aids the process of projecting the model onto the image plane and computing the set of visible edges used in the algorithm.

A 3-D convex polyhedron can be looked at as a set of surfaces, each of which is characterized by a set of vertices. The vertices are ordered so that the application of the righthand thumb rule to the ordered vertices will result in a vector that is parallel to the outward surface normal. The advantage of using this representation is the easy way it provides in determining whether a surface is visible in the image or not. The normal of a visible surface when projected onto the image will have a negative- $Z$ component as it faces the camera. The camera coordinate system chosen is just as described earlier in camera calibration.


Figure 2.6: Representation of a 3D model

The cuboid shown in Figure 2.6 can be represented by two matrices: One matrix holding the set of all the vertices with each column holding vertex coordinates and another holding the set of ordered vertices of each surface in successive rows. The set of vertices for this cuboid is given by $\mathrm{V}=\{1,2,3,4,5,6,7,8\}$ while the set of surfaces is given by: $\mathrm{S}=\{(1,3,4,2),(2,4,8,6)$, $(5,7,3,1),(5,1,2,6),(3,7,8,4),(6,8,7,5)\}$.

A general polyhedron made up of polygons, in particular, have certain interesting properties that make them an ideal place to start formulating an algorithm for general pose estimation. To begin with, we can be sure that for any perspective camera view of the object, we can see at least one surface of the object fully provided the object is fully covered by the image. If we were to pick out any three vertices of a convex polyhedron, there is always a camera view that exists that can capture all three vertices simultaneously. These principles are indirectly used in the algorithm we will be formulating in the next section.

Another formula worth noting would be the Rodrigues' rotation formula. According to this theorem, any rotation matrix can be expressed as a rotation about a particular vector, h and is given by the formula $R=e^{\hat{h} \theta}$.

Equivalently the rotation matrix can be expressed as,
$R=I+\hat{h} \sin (\theta)+\left(h h^{\prime}-I\right) \cdot(1-\cos (\theta))$

Here,
$h=\left[\begin{array}{l}h 1 \\ h 2 \\ h 3\end{array}\right]$

And,

$$
\hat{h}=\left[\begin{array}{ccc}
0 & -h 3 & h 2 \\
h 3 & 0 & -h 1 \\
-h 2 & h 1 & 0
\end{array}\right]
$$

### 2.2 ALGORITHM

The proposed RANSAC-like algorithm, in a given loop iteration, hypothesizes correspondences between a triplet of model points and another triplet of detected image corner feature points. Over time, the algorithm exhausts all such possible hypotheses and settles on a best pose of this population which is then output. The implementation of this algorithm takes object model and its image as parameters and estimates the pose of the object using just this information without any knowledge of point correspondences. It must be noted that this technique explodes rapidly when the number of corner points is large. However, in the experiments done here, a number of techniques are used in parallel to offset this problem. This is in contrast to a conventional RANSAC implementation where a fixed number of poses is selected at random to overcome this problem. Two metrics, one discrete and another continuous to evaluate the suitability of each of the poses is presented and evaluated against the real image. The metrics proposed are based on two parameters: 1.) Average visible line mismatch error 2.) Total mismatch error for all the visible lines.

This RANSAC-like line-based algorithm is novel not because of the exhaustive pose search adopted, but rather because of the specific way of evaluating each pose. The proposed algorithm uses data from the chamfer image constructed using image edge information in the
metrics to estimate the correctness of a pose. The data read off the chamfer image corresponding to the visible lines gives an idea of how close the projected line is actually present in the image. This is the primary contributor to the mathematical soundness of the algorithm. Additional techniques employed by the algorithm include color-based object segmentation and local/global ratios which eliminate potential false positives.

The algorithm can be summarized in the following steps:
1.) Segment the image based on the red, green and yellow colors as described earlier. We get three corresponding segmented images.
2.) For each of the segmented images, we find the various segmented connected components.
3.) Each of the connected components of the segmented region is dilated by a few pixels separately to make sure the object in the image lies completely within the segmented region. For each of the connected components, we identify the image corners in the region.


Figure 2.7: Segmented object
4.) For each of the connected components, we have a chamfer edge image that corresponds to the image edges present within the segmented region. The data read off the pixels of this image corresponding to the visible projected model edges is used to compute the error distance between the image edges and the projected model edges. This information is crucial in computing the average line mismatch error and total line mismatch error used in the proposed metric.


Figure 2.8: Edge image


Figure 2.9: Chamfer edge image
5.) In our database, we have a set of 3-D object models which are present in the image. We try and fit each of the models to each of the connected components of the segmented region of image successively.
6.) To do this, we pick out all permutations of three image corner-point sets and associate them with the corners of each of the unique triangles that make up the 3-D model.


Figure 2.10: Prominent corners in the segmented region
7.) For each such association, we get a perspective three-point problem with multiple solutions.
8.) Each of the solutions is translated into the appropriate rotation and translation matrices.
9.) The rotation and translation matrices combined with the camera calibration matrix helps find the get the positions of the new 3-D coordinates and project them onto the image plane.
10.) Since the new 3-D model pose is known, the sets of visible edges can be computed. These edges are now compared against the edge-based chamfer distance image. The
visibility of each of the surfaces is calculated by computing the surface normal of each of the surfaces with ordered vertices after projection onto the image plane. If the surface normal has a negative Z component, we say that the surface faces the camera and is hence visible. In this process, care must be taken to make sure that all the projected vertices lie within the image boundaries. If any of the vertices overshoot these boundaries, the pose can be directly discarded.
11.) Now we compute the average pixel mismatch error for each of the visible lines. If the average pixel error for a line is below a predefined threshold, we consider that line a match.
12.) We also compute the total mismatch error for all the matched lines.
13.) The discrete metric ranks the poses in a lexicographical order, with the poses with higher line matches and then lower total mismatch error for all the visible given a higher priority. Thus, the pose with the minimum total line mismatch error among the poses with the maximum number of line matches is considered the best estimate.
14.) The continuous metric, on the other hand, is formulated as a summation of a function of average line mismatch error and total mismatch error for each of the lines. This gives us a single number whose magnitude gives a quantitative representation of the appropriateness of the pose.

## Elimination of false positives:

For each of the object 3-D poses projected onto the image plane, we define two ratios and make sure they are above a predefined threshold:
1.) Global ratio: Ratio of the area of the convex hull of the projected model and the segmented out region.
2.) Local ratio: Ratio of the area of the intersection of segmented region and the convex hull and the area of the convex hull itself.

Optionally, we might employ a constraint on the translation vector to make sure the object's estimated pose does not go any further beyond a predefined value.


Figure 2.11: Computing Average and Total Line Mismatch Error

Let $A_{i}$ be the average mismatch error and $T_{i}$ be the total mismatch error for line i. The average mismatch error of line i is defined as the average value of the pixels in the chamfer image that correspond to the $\mathrm{i}^{\text {th }}$ visible model line projected onto the image plane. The total mismatch error of line $i$ is defined as the total value of the pixels in the chamfer image that correspond to the $\mathrm{i}^{\text {th }}$ visible model line projected onto the image plane.

$$
\begin{aligned}
& A_{i}=\frac{\text { Sum of all corresponding pixel values }}{\text { Total number of line pixels }} \\
& T_{i}=\text { Sum of all corresponding pixel values }
\end{aligned}
$$

The average and total mismatch error for red line 1 shown in Figure 2.11 is lesser than that of line 2 due to a better match with the darker regions of the chamfer image.

Let the mismatch threshold be $\mathrm{T}_{\mathrm{m}}$. Let the total number of visible lines be N . The continuous metric $\mathrm{M}_{\mathrm{c}}$ is defined by the equation:

$$
M_{c}=\sum_{i=1}^{N}-T_{i} e^{A i}
$$

A pose with a higher value of Mc is said to be a better pose estimate.

For evaluating the discrete metric, we consider two parameters:
1.) Count - it refers to the number of visible lines with an average mismatch error lesser than or equal to $\mathrm{T}_{\mathrm{m}}$.
2.) Total - it refers to the sum of the mismatch error of all the visible lines which have an average mismatch error lesser than or equal to $\mathrm{T}_{\mathrm{m}}$.

The possible poses are arranged in a lexicographical order with the poses arranged in order of decreasing "count" values. Among the poses with equal values of "count", the poses are rearranged in the order of increasing "total" values. Thus, the poses with the highest value of
"count" are considered to be the better poses. Among them the pose with the least value of "total" is taken to be the best pose.


Figure 2.12: Global and Local ratios used in the algorithm

In Figure 2.11, the ellipse represents the segmented out region while the rectangle represents the convex hull of the pose of the 3-D object projected onto the image plane.

The global ratio, GR is defined as:

$$
\mathrm{GR}=\frac{\text { Area of yellow region }+ \text { Area of green region }}{\text { Area of red region }+ \text { Area of yellow region }}
$$

The local ratio, LR is defined as:

$$
\mathrm{LR}=\frac{\text { Area of yellow region }}{\text { Area of red region }+ \text { Area of yellow region }}
$$

## CHAPTER THREE

## RESULTS AND DISCUSSION

### 3.1 DATA USED AND METHODOLOGIES

The experiment can be summarized as follows. A USB web camera was mounted on a camera stand and was positioned to look down at different assembly scenes. The images of the assemblies were then stored in a standard format; the jpeg image format was used in this experiment. A measure of the true pose of the assembly components was done manually - this will be described later in this section. The images were fed as a parameter to the algorithm implemented in Matlab to produce the estimated poses of the assembly objects.

The images of the object were taken under typical uneven home illumination conditions under an overhead CFL light source. The objects used were Screw Blocks with rounded corners and holes with little or no surface texture information. The camera used was a Logitech C250 webcam which can afford a maximum resolution of $640 \times 480$. We use a more typical webcam resolution of $320 \times 240$ in our experiments to put to test the strength of the algorithm. The webcam was fixed in a camera stand of height of about 20 cm with an angle of descent of about 45 degrees.

About 22 images of the Lego blocks were taken under varying lighting conditions and a variety of poses. 17 of these images included the red Lego cuboid in the actual image, 13 included the green Lego cube and 17 included stray yellow cube(s) for providing clutter and
noise which is precisely the conditions under which the algorithm is designed to work robustly. The cuboid used was of the dimensions $6 \times 3 \times 3 \mathrm{~cm}$ while the cube was of the dimensions $3 \times 3 \times 3 \mathrm{~cm}$.

The algorithm was implemented in Matlab version R2007 under the Windows Vista (Service pack 1) operating system. The OpenCV 2.0 implementation for camera calibration was used to get the camera calibration matrix while the Matlab corner detector implementation by He and Yung was used to detect the prominent corners in the images.

Implementation details:

Prominent Matlab function routines developed:
1.) CheckBlob.m: This function is the top-level function which calls all the other functions. This function takes the image file, 3D object model, and threshold parameters for segmented image dilation, corner detector thresholds and discrete metric threshold.
2.) SegmentCC.m: It is called by CheckBlob.m recursively to return the discrete connected components of blobs of each color separately by calling the function implemented in Segment.m file.
3.) Controlpointgen.m: This function generates the set of unique triplets of 3-D model points (Which we call control point sets) which by themselves are enough to exhaustively search the entire possible pose mapping from the object model to 2D image points for a symmetric object like a cube or a cuboid.
4.) P3P_solve.m: This function solves the Perspective 3-point pose estimation problem mentioned earlier and returns the depths of the image points.
5.) RT.m: This function converts the solution to the P3P problem into the corresponding rotation and translation matrices.
6.) Edge_distance_compare_match.m: It takes the chamfer edge distance image of the edge image, the projected image points, the set of visible lines for the estimated pose and the threshold for computing the discrete metric. It returns a measure of pose suitability computed both using the discrete and the continuous metrics discussed earlier.
7.) Convhullparea.m: This function computes the Local and Global ratios described earlier. It takes the projected image model points and the segmented image as parameters.

We use two threshold values for generating the corners and the corresponding edges using the corner detector. The first threshold which is used for obtaining the set of corners is made moderately high so that only the prominent image corners are chosen. The second threshold which is used for specifying the percentage of edges detected is made low so that a sufficiently high amount of edge information is used for pose estimation. This approach increases the reliability of the pose estimated while maintaining relatively low pose computation times.

The true pose of the object was estimated by carefully selecting the corresponding image points for the visible 3-D model points and solving the pose estimation problem with known correspondences discussed earlier. Further pose refinement was done using a GUI developed for fine-tuning pose estimates through close visual inspection. This could have been done in an
alternate way: by physically measuring the rotation and translation with respect to a world coordinate system. Physical measurements are not, in practice, completely error free and the measurements might prove quite unrepeatable in cases where we might need to come back to a particular exact test setup in the future for additional measurements. Since we would be finetuning these estimates through close visual inspection using the Matlab GUI developed for this purpose anyway, one might argue that there would be little difference between the two estimates.

The values of the standard tuning parameters used in the algorithm implementation are:
Global ratio: 50 percent

Local Ratio: 50 percent
Extent of dilation of segmented region: 15 pixels
Discrete metric threshold: 2 pixels
Translation constraint: 40 cm from camera focus
Threshold used to detect prominent corners: 0.5

Threshold used to detect prominent edges: 0.1

### 3.2 RESULTS

This section presents the typical results got from the program run. The blue lines represent the measured pose, which is also assumed to be the true pose of the object. The red lines are obtained by plotting the 3D model on the image plane using the best pose computed during the program run.

The following abbreviations have been used in the results section and have been defined in the subsequent paragraphs:

MSPE - Mean squared pixel error
NE - Norm error
RME - Rotation mismatch error

TME - Translation mismatch error
3DME - 3D mismatch error

(a) MSPE: 4.7114; NE: 0.8748; RME: 0.0724; TME: $0.8718 ; 3$ DME: 0.6990

(b) MSPE: 19.6079; NE: 0.2198; RME: 0.1041; TME: 0.2076; 3DME: 0.3854

(c) MSPE: 17.1985; NE: 1.4975; RME: 0.0833; TME: 1.4952; 3DME: 1.3547

(d) MSPE: 36.5514; NE: 1.1882; RME: 0.4335; TME: 1.1063; 3DME: 1.3139

(e) MSPE: 64.1339; NE: 0.7009; RME: 0.1870 ; TME: 0.6768; 3DME: 0.6475

(f) MSPE: 338.306; NE: 1.8328; RME: 0.8299; TME: 1.7586; 3DME: 2.2175

(g) MSPE: 18.0037; NE: 0.7518; RME: 0.0524;

TME: 0.7500; 3DME: 0.8114
Figure 3.1 (a-g) - True and estimated poses of the red cuboid

(a) MSPE: 0.7773; NE: 0.5490; RME: 0.6323; TME: 0.0390; 3DME: 0.6323

(b) MSPE: 16.8409 ; NE: 0.5464; RME: 0.0780; TME: 0.5408; 3DME: 0.5263

(c) MSPE: 74.9824; NE: 2.3954; RME: 1.5019; TME: 2.3715; 3DME: 2.0771

Figure 3.2 (a-c) - True and estimated poses of the green cuboid


Figure 3.3: 3D Mismatch Error Performance plot


Figure 3.4: Translation Mismatch Error Performance plot

### 3.3 DISCUSSION

The performance plot shown in Figure 3.3 summarizes the accuracy of the algorithm which was tested on a 22-image dataset with 28 assembly component poses considered. As observed from the graph, the algorithm works with a 3D mismatch error of less than 2.5 cm error in $90 \%$ of the cases (Groups 1 and 2 together) and less than 1 cm error in $53 \%$ of the cases (Group 1).

The proposed algorithm works quite well in cases in relatively uniform lighting conditions where the corner features appear more distinct. In cases where the features are
indiscernible owing to poor lighting, occlusions due to surrounding objects or the actual pose of the object itself with few surfaces visible, the detected pose, as is evident from the images is less accurate. As evinced by the results, the algorithm works pretty well for well-lit objects much of which is visible in the image. The algorithm is hampered in a few cases owing to the presence of heavy shadows by the surrounding objects which affect the estimated orientation of the target object.

The strength of the algorithm lies in its ability to work despite all these conditions and give a result that is not off from the true one by a large margin. The segmentation approaches make sure that the object translation is close enough to the true one. The discrete line-match based metric tries to make sure that most of the object edges are matched as closely as possible making the estimate as close as possible to the true one. The continuous metric is found to be more prone to errors compared to the discrete metric in converging to the true pose of the object. The reason for this could be attributed to the nature of the continuous metric which is designed to take the global metrics into consideration much more than the local metrics. This is in sharp contrast with the discrete metric which gives a higher preference to local matching thresholds.

The discrete metric has been found to give consistently better results compared to the continuous metric. Thus, we take the pose computed by the discrete metric as the final output of the implementation and list the match computed by the continuous metric along for reference.

To adapt this algorithm for a disassembly process, a good approach would be to find out the subassemblies with the best metric and try removing them from the assembly first as their
pose is more reliable. This process is repeated until there are no more subassemblies left in the scene.

The results presented in the previous section illustrate the typical outputs generated by the algorithm implementation. The mean squared pixel error is defined as the average of the squared distances between the corresponding projected image model points and the image points. The norm error refers to the minimum norm of the difference of the measured object pose and the estimated poses (with all the possible poses generated by equivalent local rotations considered).

The norm error gives a better picture of the actual difference in poses as it works for an N -dimensional space (In our case, $\mathrm{N}=3$ ). On the other hand, Mean square pixel error is highly sensitive to slight changes in local orientation of the object and comparatively static to translation, especially along the Z-axis.

The drawback with the norm error is that it does not distinguish between the rotation and translation mismatches. To present a clearer picture, we compute and present these mismatch values separately. The translation mismatch is defined as the norm of the difference between the vectors TM and TC. The rotation mismatch, on the other hand, is defined as the norm of the matrix $\left[I-R M . R C^{-1}\right]$, or equivalently, norm of the matrix $\left[I-R C \cdot R M^{-1}\right]$. Here, RM and TM are the rotation and translation matrices obtained from the actual measurement and are assumed to represent the true pose of the object. RC and TC are the rotation and translation matrices computed during the program run and represent the estimated object pose. Since the norm error
involves computing the norm of the matrix containing difference of both the rotation and translation vectors, it is hard to intuitively understand the physical significance of its mathematical value.

The most natural choice of error measurement metric would be the 3D mismatch error metric. This metric is defined as the average 3D distance error between the corresponding corners of the true and the estimated object poses. This metric, unlike the others, is quite intuitive as it gives us a way to visualize the mismatch errors between the poses in 3D space. It is for this reason that this error metric is preferred over the others for the Mismatch error-Population plot.

The performance plot gives the relation between the percentages of population with the corresponding 3D error metric below a given value. As mentioned before, in ninety percent of the cases, the 3 D error was less than 2.5 cm on an average; and in fifty three percent of the cases, the 3D error was less than 1 cm . Close observation of the plot gives rise to the hypothesis of the existence of three discrete groups of object images, each having a distinct range of error. Further investigation reveals that group 1, with the lowest error; consists primarily of the cases where the object remains relatively unaffected by occlusions or shadows. Group 2 , on the other hand, is moderately affected by both occlusions and shadows, while in cases in group 3, are severely occluded and subject to heavy shadows. This suggests a clear metric-based demarcation between the various cases considered.

The adopted exhaustive search technique, on a higher level looks computationally expensive making its implementation appear infeasible for any practical application. This is
particularly true in our case where we iterate through each of the connected components of each color segmented out and subsequently try matching every model in the database to the target region. On closer observation, we realize this problem has a huge search space that can easily be reduced to a much smaller one. This is evident from the practical implementation of the algorithm, where the search is done in typically less than 3 minutes for the given images. Thus, the drawback of having a high computation order and time is partly offset by filtering out the majority of possible poses which turn out to be false positives for the most part using specific techniques incorporated in the algorithm which have been discussed earlier.

The algorithm primarily derives its reasonably reliable performance from the fact that reliable, repeatable color-based segmentation can be performed on objects with most of the false positives can be done away with using the Global and Local ratios discussed earlier. We further reduce the computational time by using specific triplets of 3-D model points which are chosen in such a way that at least one valid mapping between a triplet and an identified set of three valid object image corner features exist. In other words, this set of triplets is sufficient to generate all the possible valid poses while simultaneously eliminating any pose redundancies along the way. In addition to this, feeding two appropriate thresholds to the corner detector, one kept low to just get a set of corner points that have at least three valid object corners for pose estimation, and another kept high to get a sufficient number of edges to enhance reliability of the estimated pose. Since we iterate through the corners alone, the computation time is considerably reduced. While these approaches do not change the order of computation as such, but the overall number of iterations that one has to take to come up with a reasonable pose estimate is considerably reduced.

The algorithm, in its current state, places a greater focus on arriving at a more reliable solution rather than an optimal, computationally less expensive solution and consequentially takes a disproportionate amount of time for utility in a fast-paced, real time scenario. We will proceed to discuss how this handicap could be potentially eliminated in the next chapter.

## CHAPTER FOUR

## CONCLUSIONS AND FUTURE WORK

The results obtained indicate that the algorithm is reasonably suited for toy problems involving disassembly of small assemblies with few features. The experiments performed were in a largely structured environment under relatively uneven illumination conditions and moderate occlusions. However, the proposed algorithm can be expected to work in most cases even in an unstructured environment, provided reliable segmentation is achieved.

A major source of error in results produced by the algorithm arises from the modules used to detect the corners and edges. The 3D object is modeled as a set of edges forming faces with sharp corners. The actual object model, on the other hand, possesses rounded corners which are often not detected by the corner detector used. The algorithm remains relatively resistant to this weakness since it requires just three true object corners be detected in the image with negligible error. Much of this drawback with corner detection could be overcome if we employed an alternate approach where edge detection is based on regions with similar local color/texture rather than on local gradients. Once we have a reliable method to detect edges, the problem of obtaining reliable corners will be a natural consequence. This approach would enable us to have a reliable pose estimate even when similar-looking objects are stacked together in the image. However, this implies we cannot employ this technique on a largely texture-free object. Since most real-world assemblies can be assumed to have surface-texture in some form, this idea might be useful to consider.

Another notable weakness with the algorithm implementation stems from the color-based segmentation technique. While the color-based segmentation technique is extremely reliable for uniformly colored objects, problems arise when similar colored objects are placed very closely and their dilated segmented regions in the image overlap. This calls for an intelligent, contentaware segmentation technique.

The algorithm has been found to produce a higher pose-estimate error in cases where heavy shadows are predominant, leading to false edges. As discussed earlier, while one might argue that this problem could be eliminated using color/texture-based edge detection to arrive at a set of true edges, we also could view this problem a little differently. High pose estimate errors arise when the number of line matches is high owing to multiple object model line matches on the same image line. To offset this, we could potentially introduce a constraint where 3D object model edges which map a little too close on the image plane be appropriately penalized. Again, the drawback with this approach is that such cases are quite possible in real images and errors may arise in certain cases due to the penalization itself.

The algorithm, at present, works well for objects with corners (both sharp and rounded) and well defined sides. However, the algorithm is not expected to work on a surface devoid any noticeable corners and straight edges. This could be viewed as a drawback in for an algorithm like this as and can be largely expected to fail while attempting to find a solution for the pose of a smooth object like a sphere or an ellipsoid in an image given an approximate 3-D model constructed using a Delaunay triangulation. Approximating a rounded object usually entails having a large number of object model vertices. Thus, even if at least three false corners are
detected on the smooth object, they could still be used to determine the object pose. An implementation that requires permutations of all the possible corner points in the model would be highly impractical and would lead to very high implementation runtime. It is imperative that an alternate formal approach to deal with such cases be formulated to obtain an algorithm that works on a bigger class of objects.

An alternate approach would be to have an automated technique to map features on 3D objects onto an image plane and find out the appropriate correspondences between the synthetic and the real images. This is closely related to the method discussed in literature survey section where a given database is searched for the best matching pose. Unfortunately, such an approach would not be feasible for objects with little or no identifiable visual surface features - much like the ones we have tested our algorithm on. Also, we now have an additional correspondencedetermination problem at hand.

To overcome the problem of pose estimation of occluded objects, one could first find out the poses of objects with the highest metric in 3-D space. The poses of objects with a lower metric could subsequently be corrected based on the poses we place a greater trust on (The poses with a higher metric).

Looking at the problem of occlusion with the disassembly process in perspective, we can expect this to pose an insignificant practical impediment to the real problem we are trying to solve. This is because we can build a system that would actuate the removal of objects in the
order of their decreasing metric. This would subsequently ensure the visibility of the onceoccluded objects leading to the dismantling of the entire assembly over time.

A second look at the algorithm would reveal that the search process is really independent of the previous state that was evaluated. This implies that the whole algorithm could be parallelized and made to run in real-time using a cluster of GPU-enhanced machines. This arrangement would be another step in building a machine to emulate the human brain in figuring out the best way in taking apart a given assembly without much delay.

Concurrently, one could adopt a line-based version of an iterative algorithm like the SoftPOSIT. Starting out with initial random pose guesses from every local region of the camera view volume and letting them converge independently would result in a population of potentially viable poses. Each of these poses could then be subject to evaluation using the techniques discussed in this thesis.

## CHAPTER FIVE

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