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REVENUE AND ORDER MANAGEMENT UNDER DEMAND UNCERTAINTY

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Abstract

We consider a firm that delivers its products across several customers or markets, each with unique revenue and uncertain demand size for a single selling season. Given that the firm experiences a long procurement lead time, the firm must decide, far in advance of the selling season not only the markets to be pursued but also the procurement quantity. In this dissertation, we present several operational scenarios in which the firm must decide which customer demands to satisfy, at what level to satisfy each customer demand, and how much to produce (or order) in total.

Traditionally, a newsvendor approach to the single period problem assumes the use of an expected profit objective. However, maximizing expected profit would not be appropriate for firms that cannot afford successive losses or negligible profits over several consecutive selling seasons. Such a setting would most likely require minimizing the downside risk of accepting uncertain demands into the production plan. We consider the implications of such competing objectives.

We also investigate the impact that various forms of demand can have on the flexibility of a firm in their customer/market selection process. a firm may face a small set of unconfirmed orders, and each order will often either come in at a predefined level, or it will not come in at all. We explore optimization solution methods for this all-or-nothing demand case with risk-averse objective utilizing conditional value at risk (CVaR) concept from portfolio management.

Finally, in this research, we explore extensions of the market selection problem. First, we consider the impact of incorporating market-specific expediting costs into the demand selection and procurement decisions. Using a lost sales assumption instead of an expediting assumption, we perform a similar analysis using market-specific lost sales costs. For each extension we investigate two different approaches: i) Greedy approach: here we allocate order quantity to market with lowest expediting cost (lowest expected revenue) first. ii) Rationing approach: here we find the shortage (lost sale) then ration it across all the markets. We present ideas and approaches for each of these extensions to the selective newsvendor problem.

Dedication

I dedicate this fine piece of work to my beloved husband, Kulvir, and my family for being infinitely supportive.

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Chapter 1

Introduction

In this fast paced environment it is important for a firm to be flexible and to quickly adapt to changes. In these uncertain conditions the firm has to be proactive in deciding who gets its product. Thus, strategies and theories related to managing revenue have become increasingly important given the fact that it's extremely difficult to change the available limited resources. Everyone talks about revenue management and its revolutionizing impacts on the hospitality and service industry. Revenue management combined with order management has also remarkably affected the inventory and production planning systems. It can be said that revenue and order management is a systematic process that lets a firm decide to allocate right amount of its product to right customers. The revenue and order management for inventory control systems leads to a more efficient and effective distribution of available resources. In this dissertation we consider a supplier that offers a product with stochastic demand over a single selling season and is concerned with revenue and order management decisions. The supplier has to decide in advance what demands/orders to reject and accept in order to maximize its revenue. In inventory control and production planning area, the classical newsvendor problem is one of the mathematical models to find an optimal procurement policy for a product with random demand during a single selling season.

The newsvendor problem is a well-researched area of stochastic inventory control. There are many generalized mathematical models that characterize the newsvendor problem with more than one solution approach or algorithm to solve each of these models. However, we cannot always use a single model beneficially whenever an instance of newsvendor problem occurs. This might depend on the nature of the problem a concerned firm is dealing with. A firm may choose one of these generalized models or it might need to formulate a new model, depending on the firm's goal and operating conditions. Once the model formulation is complete, the second step would be to develop a solution approach to solve the model. Depending on factors like the stochastic nature of the consumer demand, it might be necessary to develop a new tailored solution approach to solve the formulated model.

Porteus [39] provides a nice literature overview in the area of stochastic inventory control. Tsay, Nahmias, and Agarwal [58] and Cachon [10] provide reviews on more recent research developments that focus on inventory management in supply chains with multiple demands that are applicable to the newsvendor problem. Moon and Silver [35] present heuristic approaches for solving the multi-item newsvendor problem with a budget constraint. In this multi-product model requires fixed demand distributions for every product.

There is a growing base of knowledge concerning the flexibility in selecting markets and orders or demand sources for production. For deterministic demand selection models that address economic ordering decisions, multi-period lot-sizing decisions with production capacity constraints, and lead-time flexibility of producers, see Geunes, Shen and Romeijn [25], Taaffe and Geunes [51], and Charnsirisakskul, Griffin and Keskinocak [13], respectively. These models allow a supplier to choose which markets to serve, which orders to fulfill, and when to fulfill each order, in contrast to the typical product ordering-based decisions that do not consider the unique characteristics of the customer base. Integrating the pricing decision into demand selection, Geunes, Romeijn and Taaffe [24] study a lot-sizing problem that addresses the relationship between product pricing and order acceptance/rejection decisions. Through these pricing decisions, the production planning model implicitly decides what demand levels the firm should satisfy in order to maximize contribution to profit after production.

Some researchers suggest methods for dealing with a product that is offered at several price (or demand) levels, as well as across multiple periods. These selected papers frame the problem as a multi-product or multi-period newsvendor in their modeling approach. In one paper, the firm must purchase its capacity for each demand level before the first period and cannot request any further replenishment (see Shumsky and Zhang [47]). They offer flexibility by incorporating product substitution, which allows product to be shifted from one demand class to another. Other papers allow additional quantities to be produced or procured during the selling season. As demand information is revealed, the manufacturer can make procurement decisions for the next period. Some notable work in this area is found in Sen and Zhang [45], Monahan, Petruzzi and Zhao [34], and Kouvelis and Gutierrez [30].

Some of the more closely-related research on stochastic demand and order selection are Petruzzi and Monahan [37], Carr and Duenyas [11], Carr and Lovejoy [12] and Taaffe, Geunes and Romeijn [52]. Petruzzi and Monahan [37] address ways of selecting between two sources of demands, the primary market and the secondary (or outlet store) market. While these demands might occur simultaneously, the firm must decide the preferred time to move the product to the outlet store market. Carr and Duenyas [11] consider a sequential production system that receives demand for both make-to-stock and make-to-order products. A contractual obligation exists to produce make-to-stock demand, and the firm can supplement its production by accepting (and sequencing) additional make-to-order jobs in the production system.

Carr and Lovejoy [12] study inverse newsvendor problem that chooses a consumer demand distribution based on a pre-defined order quantity, and hence there is no decision to make in setting the order quantity. Based on a set of demand portfolios (which may contain several customer classes), they determine the amount of demand to satisfy within each portfolio while not exceeding the pre-defined capacity. In addition to this, they assume all customer demands have already been ranked and high priority demands are filled completely before low priority ones are considered. Since the optimal choice of markets may change based on the available funds for marketing, we cannot provide an a priori ranking of demands, but allow the model to implicitly determine the most attractive set of markets.

Our work in this dissertation builds upon a fundamental result for the selective newsvendor problem first introduced in Taaffe, Geunes, and Romeijn [53]. All potential markets have unique contributions to the profit, as well as the uncertainty in the size of each market. Using the Decreasing Expected Net Revenue to Uncertainty (DERU) ratio, they implicitly determine the most profitable markets to select as well as the appropriate quantity to order from the firm's supplier. In effect, they consider how a firm can "shape" the best demand distribution for a single product by selecting from different potential markets.

Our research study also allows for demand flexibility by modeling the stochastic demand consisting of a set of potential customer orders. We further assume that firm can obtain unique revenues in each demand source (or customer order). Similar to Taaffe, Geunes, and Romeijn [53] the firm has to make decisions by simultaneously selecting the most desirable markets as well as determining the appropriate total order quantity before demand is actually realized and we also assume that once the supplier/firm knows the actual materialized demand, it must satisfy all pursued demands. We further assume that if a market is not selected then the related demand is essentially lost. In the case of constrained production with a single-period setting, the supplier can have an underage cost consisting of either expediting cost or outsourcing cost. Whereas, overage in a single-period setting is considered product to salvage. Demand flexibility allows the supplier to decide among the highly profitable, yet risky, orders or less profitable, but possibly more stable, orders. In contrast with the classical newsvendor problem, expected profit is now influenced by both the procurement quantity and the selected markets. Recent research on profit maximizing models providing integrated demand selection and ordering decisions for this so-called "selective newsvendor problem" (SNP) has been studied by Carr and Lovejoy [12], Petruzzi and Monahan [37], Taaffe and Romeijn [54] and Taaffe, Geunes, and Romeijn [53], and Taaffe, Romeijn, and Tirumalasetty [55].

The selective newsvendor problem evaluates how each market contributes to the overall expected profit of the firm. As each market has an expected revenue, as well as uncertainty in how large the market will actually be, there are obvious trade-offs between achievable revenues and associated demand risks. This relationship can be viewed similarly to the concepts introduced in mean-variance optimization in portfolio management (see the seminal work by Markowitz [33]). Many mean-variance applications for inventory problems use a modified objective function (expected profit or expected cost) that includes a penalty term for demand-variance (or risk). Here, the risk minimization depends on the magnitude of the penalty used. Chen and Federgruen [14] re-visit a number of basic inventory models using the mean-variance approach. They exhibit how a systematic mean-variance trade-off analysis can be carried out efficiently, and how the resulting strategies differ from those obtained in standard analyses. Tsan-Ming, Duan and Yan [17] also formulate the newsvendor problem with no demand selection flexibility as a mean-variance model and showed that if a firm tries to maximize the expected profit such that the variance of profit is constrained, the optimal order quantity is always less than the classical (by maximizing the expected profit) order quantity. It also does numerical studies substantiating the above claim. For excellent reviews of mean-variance analysis, portfolio optimization and risk aversion, see Steinbach [49] and Brealy and Myers [9].

Most of the previous research work considers only one kind of objective to optimize while assuming that the stochastic nature of the consumer demands is known and they are normally distributed. However, this might not be the case and there may be a need to develop new models and solution approaches to address other critical objectives and different demand distributions. We address these gaps in the literature and provide the main contribution of this research. We identify some drawbacks of the research done on the newsvendor problem so far and present the outline and intended contribution of this research work. This dissertation mainly focuses on the effect of risk on decision making under demand uncertainty for revenue and order management.

1.1 Contribution 1: Demand Selection with Risk

In most of the research articles on the newsvendor problem, the typical objective employed is to maximize the expected profit or to minimize the expected cost. However, it is not always sensible to use such an objective, as it depends mostly on the concerned firm's goal and its requirements. For a firm that operates on a tight budget and cannot afford to record several successive financial losses spanning consecutive periods, it is likely that their objective is not only to maximize the expected profit, but to minimize the variance from that goal. If the risk (variability) associated is too high it may prefer to minimize the risk or variability instead of minimizing the expected cost or maximizing the expected profit. Additionally, a firm may place an upper bound on the risk it is willing to absorb and choose to minimize the expected cost. Another firm with different goals may place a lower bound on the expected profit and choose to minimize the associated risk or variability. There are many such possible mathematical models but only some of them may be beneficial depending on a particular firm's situation. Hence, in this dissertation we investigate several mathematical models that can accommodate the needs or issues unique to individual firms.

The mean-variance approach is a trade-off analysis that attempts to achieve a desired rate of return while minimizing the risk involved with obtaining that return. In our approach, we determine an optimal set of markets based on their expected revenues (or returns) and the associated demand uncertainties (return risk). As a result of this contribution we simultaneously incorporate the risk and the profitability into the demand selection and the ordering policy. We introduce the risk averse SNP model where customer/market demands are normally distributed. This contribution is explained in greater detail in Chapter 2.

1.2 Contribution 2: Alternative Demand Types and Risk Considerations

Previous work on the selective newsvendor problem, or for that matter, stochastic demand selection, has been limited to normally distributed demands (for one period) or Poisson distributed demands (for time-based models). While there are many applications for models that contain such types of demands, a firm could be facing a smaller set of unconfirmed orders, where each order will either come in at a pre-defined level, or it will not come in at all. Such demands are called Bernoulli or All-or-Nothing (AON) demands. Though realistic, demands of this nature have not been studied for demand selection problems in the past research. These being discrete distributions, using a standard mixed-integer programming (MIP) approach can be extremely difficult. This approach requires scenario analysis, where the scenarios are exponential in number and the resulting problem size explodes. Taaffe, Romeijn and Tirumalasetty [55] have presented the solution approach to this problem as a cutting plane algorithm and is based on the idea of the so-called L-shaped method (LSM) (see, e.g., Birge and Louveaux [8]), which applies Benders decomposition to a suitable reformulation of a linear two-stage stochastic programming problem with fixed recourse. Stochastic linear and integer programming have been widely studied, especially in recent years. For relevant references to books and survey articles on the subject, see Prekopa [40], Birge and Louveaux [8], and Sen and Higle [46].

In this chapter we will extend the line of research presented in contribution 1. There we introduce the risk averse SNP model where customer/market demands are normally distributed. In this chapter of the dissertation, we model the risk analysis with demand selection where customer orders follow AON and uniform distribution. This work addresses the impact that these various types of demand distributions would have on both the demand selection, procurement policies and the applicability of the heuristic approach presented as a result of contribution 1. We also use the conditional value at risk (CVaR) approach for developing an optimization model for Bernoulli distributed demands. CVaR has been previously used in many settings, most notably portfolio management studied by Rockafellar and Uryasev [41], Taaffe [50] and Gotoh and Takano [26].

As a result of this contribution, we analyze the effect that various forms of demand (namely, Bernoulli/all-or-nothing (AON) and uniform) have on profitability and selection decisions using a risk averse environment and also provide insights into the effect that these demand distributions have on minimizing the potential worst case losses. We discuss different approaches (namely, CVaR and simulation) for risk averse SNP with these demand distributions in Chapter 3.

1.3 Contribution 3: Generalizations of the Selective Newsvendor Problem

In the selective newsvendor problem unique demands can be pursued or rejected as part of the procurement policy. Here we consider generalizations to the SNP model. In this part we present ideas and approaches for various extensions to the expected profit approach for the SNP. In generalized SNP modeling, we first consider the impact of incorporating market-specific expediting costs into the demand selection and procurement decisions. Secondly, we consider using a lost sales assumption instead of an expediting assumption. We consider two different approaches for both types of generalizations to the SNP: greedy approach and rationing approach. Given the set of selected markets, in the greedy approach, we start allocating the order quantity to market with the highest expediting cost (or the highest per unit revenue). Thus, the shortage (or the lost sale) will be observed for the least expensive market in the set of selected markets. However, in the rationing approach, we ration this shortage (or lost sale) equally across all selected markets.

As a result, in this contribution we present a detailed discussion, the problem formulation and various solution approaches (exact mathematical optimal solution approach and simulation based heuristic approach) for each generalization to the

SNP. This contribution is detailed in Chapter 4.

Chapter 2

Risk-Averse Selective Newsvendor Problem

2.1 Abstract

Consider a firm that offers a product during a single selling season. The firm has the flexibility of choosing which demand sources to serve, but these decisions must be made prior to knowing the actual demand that will materialize in each market. Moreover, we assume the firm operates on a tight budget and cannot afford to record several successive financial losses spanning consecutive periods. In this case, it is likely that their objective is not only to maximize expected profit, but to minimize the variance from that goal. We provide insights into the tradeoff between expected profit and demand uncertainty using a mean variance approach. We also present a solution approach, via simulation, to determine a market set (and total order quantity) when the firm's objective is to minimize the probability of receiving a profit below a critical threshold value.

2.2 Background and Literature Review

As product lives continue to decrease with technological advances and fashion trends, and the efficiency of manufacturing processes offer less room for improvement, a supplier or manufacturing firm is constantly trying to identify other ways to improve profitability. In the classic newsvendor problem, the firm seeks an optimal procurement policy for a product with random demand during a single selling season. There is extensive literature on this topic, and we refer the reader to Porteus [39], Tsay, Nahmias and Agrawal [58], Cachon [10], and Petruzzi and Dada [36] for reviews and research in this area.

If the firm can obtain unique revenues in each demand source (or market), then the problem becomes one of simultaneously selecting the most desirable *markets* as well as determining the appropriate total *order quantity* before demand is actually realized. Recent research has offered profit maximizing models that provide integrated demand selection and ordering decisions for this so-called "selective newsvendor" problem (SNP). Forms of the SNP have been studied recently by Carr and Lovejoy [12], Petruzzi and Monahan [37], Taaffe, Geunes and Romeijn [53], and Taaffe, Romeijn, and Tirumalasetty [55].

In both categories of the aforementioned problems, the typical objective is to maximize expected profit or minimize expected cost, which would be appropriate for a risk-neutral firm. However, not all (in fact, very few) firms have the luxury of operating in a risk-neutral environment Schweitzer and Cachon [44]. The actual profit (or loss) may be quite different than expected profit for a particular selling season, and many firms could be more concerned with this variability. Therefore, we consider a firm that cannot afford successive losses or negligible profits spanning several selling seasons. For such a firm, we will evaluate two risk models. In one approach, we still assume that the firm's objective is to minimize demand variance while achieving a desired expected profit or revenue. This approach is commonly referred to as mean-variance analysis. In the second approach, while the firm's desire may be to maximize expected profit, their objective will be to minimize the number of outcomes that could result in profits below their budgeted or minimum acceptable profit level. An introduction to the selective newsvendor models with risk was presented in Taaffe and Tirumalasetty [56]. We build on the research presented in Taaffe and Tirumalasetty [56], now addressing a more thorough set of computational tests on the two specific cases listed above. In addition, many insights into efficiently running the simulation experiments are presented in this chapter.

Various aspects of risk aversion in newsvendor problems have been considered in past work. Lau [31] is the first paper to directly study the effect that risk has on the newsvendor problem. The paper considers two objectives, maximizing expected utility, and maximizing the probability of achieving a budgeted profit, which is quite similar to the focus of our work. However, we have the added complexity of simultaneously selecting the most attractive markets while determining the appropriately-sized order quantity. Lau [31] depicts two demand points beyond which the firm will no longer achieve the desired profit level, and then solves for the quantity that maximizes the probability that the profit level will be achieved. The paper concludes that analytical solutions can be obtained if the underlying demand distribution is normal or exponential. This approach works for a standard newsvendor when there is only one demand distribution for which all demands generate the same per-unit revenues. Applying this methodology to our problem breaks down due to our unique revenues in individual markets.

Eeckhoudt, Gollier and Schlesinger [21] also studies a risk averse newsvendor for which any demands not met by the original order can be satisfied through a high-cost local supplier. This paper also concluded that the optimal risk-averse order quantity is less than the amount ordered in the expected value solution. More recently, Collins [19] offers some results that counter these previous papers.

More recently, Li [32] has presented a supplier's risk aversion while determining the optimal time for production. This paper considers the risk attitude of the supplier and the updating of the demand arrival time distribution. This study concluded that the optimal policy remains the same, while the critical time to produce depends on the risk attitude of the supplier. In another risk averse paper, Keren and Pliskin [28] have derived first order conditions for optimality of the risk-averse newsvendor problem with an objective of maximizing expected-utility. This paper presents the closed form solution for the case of uniformly distributed demand.

Finally, Collins [19] conjectures that there is a class of problems for which the risk averse and expected value solutions are identical, that there are many problems for which the expected solution provides a good approximation to the risk averse solution, and that in most problems in practice, the risk averse solution would actually be to order *more* than the expected value solution. Finally, the reader can turn to Chen et al. [16] and Van Mieghem [60] for additional risk aversion research.

In this chapter, we investigate how a selective newsvendor can integrate risk into its demand selection and ordering policy. While we maintain some similar assumptions to those in Lau [31] and Eeckhoudt et al. [21], we also have the added complexity of market selection, which can result in different procurement policies. In Section 2.3, we introduce the general profit equation for the selective newsvendor problem and discuss the form of the distribution for profit. Then, in Section 2.4, we present two demand selection models, each identifying a unique method for quantifying risk. Section 2.5 provides a detailed description of the solution approach necessary to solve the more difficult of the two models. In Section 2.6, we present computational tests and findings for each model. Finally, we summarize our findings and suggest directions for future work in Section 2.7.

2.3 Quantifying Profit for the Selective Newsvendor

We begin by defining c as the per-unit cost of obtaining or procuring the product to be sold. The product can be sold in market i at a per-unit price of r_i. If realized demand D_i is less than the quantity ordered, the firm can salvage each remaining unit for a value of v. If demand exceeds the order quantity, there is a shortage cost of e per unit. However, we assume that the demand is still met through expediting via a local supplier or single-period backlogging whereby a second order can be placed with the firm's regular supplier. In either case, the unit cost is still e.

Recall that, in the selective newsvendor framework, the firm must decide its market selections prior to placing the order for Q units. Let $y_i = 1$ if the firm decides to satisfy demand in market i, and 0 if the firm rejects market i's demand. Also assume that S_i represents the entry or fixed cost of choosing market i. We present the following expression for the total realized profit, based on the order quantity, market selection decisions, and realized demand.

$$H(Q,y) = \begin{cases} \sum\limits_{i=1}^{n}(r_iD_i - S_i)y_i - cQ + v(Q - \sum\limits_{i=1}^{n} D_iy_i) & Q > \sum_{i=1}^n D_iy_i \\ \sum\limits_{i=1}^{n}(r_iD_i - S_i)y_i - cQ - e(\sum\limits_{i=1}^{n} D_iy_i - Q) & Q \le \sum_{i=1}^n D_iy_i \end{cases}.$$

Given a binary vector of market selection variables y, and letting $D^y = \sum_{i=1}^n D_iy_i$ represent the total demand of the selected markets, the mean and variance of this total selected demand are $E(D^y) = \sum_{i=1}^n \mu_iy_i$ and $\text{Var}(D^y) = \sum_{i=1}^n \sigma_i^2 y_i$, respectively. We can then express the firm's expected profit as a function $G(Q, y)$ of the order quantity Q and the binary vector y:

$$\begin{aligned} G(Q,y) = \ & \textstyle\sum_{i=1}^{n}(r_i\mu_i - S_i)y_i - cQ + vE\left[\max\left(0, Q - \sum_{i=1}^{n} D_i y_i\right)\right] \\ & -eE\left[\max\left(0, \textstyle\sum_{i=1}^{n} D_i y_i - Q\right)\right]. \end{aligned}$$

The general *selective newsvendor problem* [SNP] is now given by

$$\begin{aligned} \textbf{[SNP]} \quad & \text{maximize} \quad && G(Q,y) && \\ & \text{subject to:} && Q \geq 0 && \quad (2.1) \\ & && y_i \in \{0,1\} \qquad i = 1,\ldots,n. && \quad (2.2) \end{aligned}$$

2.3.1 SNP with Normal Demands

In this chapter, we investigate several risk models where the size of each demand source i is normally distributed, such as when each market's demand consists of many individual orders. (The normal distributions we consider have parameters such that the probability of negative demand is negligible.) Even if individual order sizes are not normally distributed, total market demand can be accurately represented by a normal distribution (using the central limit theorem). We refer to Eppen [22], Carr and Lovejoy [12], Aviv [4],and Dong and Rudi [20] for other examples of situations where demand normality applies.

For a given vector y, the expected profit function $G(Q, y)$ is concave, and maximizing the expected profit is equivalent to minimizing the cost in the associated newsvendor problem. This leads to an optimal order quantity of $Q^*_y = F_y^{-1}(\rho)$, where $\rho = \frac{e-c}{e-v}$. Moreover, the total demand satisfied (i.e., $D^y = \sum_{i=1}^{n} D_i y_i$) is also a normal random variable, and using the standard normal loss function, the expected profit equation reduces to

$$G(Q, y) = \sum_{i=1}^{n} \bar{r}_i y_i - K(c, v, e)\sqrt{\sum_{i=1}^{n} \sigma_i^2 y_i}, \tag{2.3}$$

where $\bar{r}_i = ((r_i - c)\mu_i - S_i)$, and $K(c, v, e) = \{(c - v)z(\rho) + (e - v)L(z(\rho))\}$, for further details refer to Taaffe et al. [53]. Thus, the expected profit equation depends solely on market selection variables, and the optimal order quantity is simply a function of y, given by $Q_y^* = \sum_{i=1}^{n} \mu_i y_i + z(\rho)\sqrt{\sum_{i=1}^{n} \sigma_i y_i}$. To maximize the firm's expected profit, we must solve the following selective newsvendor problem (SNP-N):

$$\begin{array}{llll} \textbf{[SNP-N]} & \text{maximize} & \sum_{i=1}^{n} \bar{r}_i y_i - K(c, v, e)\sqrt{\sum_{i=1}^{n} \sigma_i^2 y_i} & \\ & \text{subject to:} & y_i \in \{0, 1\} & i = 1, \ldots, n. \end{array} \tag{2.4}$$

Taaffe et al. [53] provide an optimal sorting scheme and selection algorithm, called the Decreasing Expected Revenue to Uncertainty (DERU) Ratio Property. We re-introduce this property here for the purpose of completeness.

Property 2.3.1. ***Decreasing Expected Revenue to Uncertainty (DERU) Ratio Property*** *(cf. Taaffe et al. [53]): After indexing markets in decreasing order of expected net revenue to uncertainty, an optimal solution to [SNP-N] exists such that if we select customer l, we also select customers $1, 2, \ldots, l - 1$.*

Romeijn, Geunes and Taaffe [43] also provide a sorting and selection algorithm for a capacity-constrained case.

2.3.2 The Profit Distribution

We make a key observation here. We previously stated that the random variable corresponding to total demand satisfied is normally distributed, since it is the convolution of normally distributed market demands. However, the profit function $G(Q, y)$ is not normally distributed. We simulated 10,000 profit realizations of $G(Q, y)$ in order to approximate the shape of the profit distribution, and Figure 2.1 depicts those results. Regardless of how many simulated tests were conducted, the profit distribution is skewed left, with a pronounced tail of outcomes with very low probability of occurrence. Since there are penalties for underages (e) as well as overages (v), extremely low or high demand realizations will result in lower profit (or possibly a loss). These extreme conditions contribute to the left tail of the profit distribution.

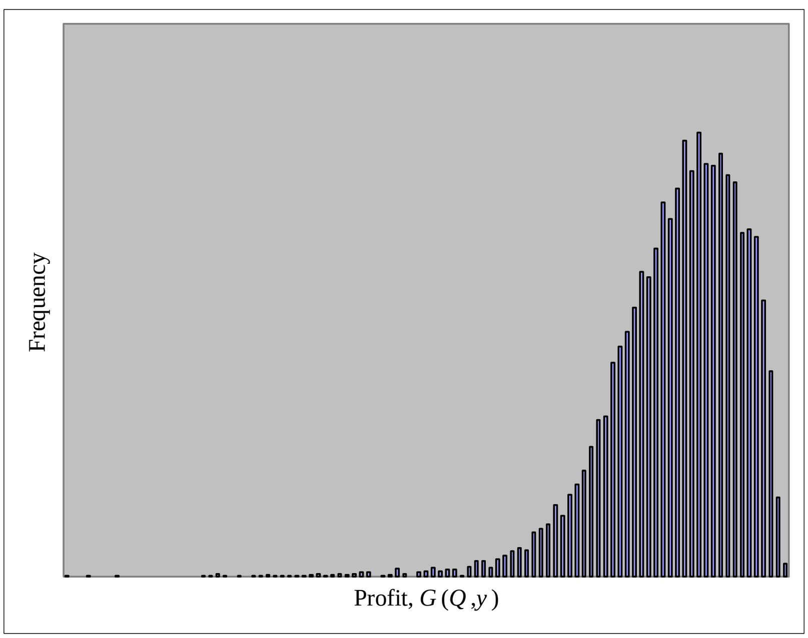

Figure 2.1: Distribution of Profit for (Q, y) - Normal demands.

Notice that the maximum achievable profit does not greatly exceed the expected profit (i.e., it does not have a similar right tail on the distribution). In the newsvendor problem, the critical fractile defines the point in the demand distribution for D^y at which we maximize expected profit. As realized demand moves away from the demand quantity associated with this point, the firm's profit will decrease. However, in our selective newsvendor framework, we also have market-specific revenues r_i associated with each market i. Thus, the maximum profit that the firm can achieve occurs when all realized demand occurs in the market(s) with the highest revenue, and total realized demand still equals the order quantity. Thus, the maximum profit shown in the profit distribution is more well-defined than the maximum loss.

Now consider that the firm would like to minimize the worst-case set of profits (losses). Since the profit distribution is not normally distributed, this complicates the solution approach. In the next section, we show how we utilize the fact that the demands are normal in solving selective newsvendor problems with risk. Another data-driven approach can be found in Bertsimas and Thiele [7], whereby they build upon the sample of available data instead of estimating the probability distributions. However, the risk policy they develop, along with the underlying model, are fundamentally different than those presented in this chapter.

2.4 Selective Newsvendor Models with Risk

The objective of maximizing expected profit is applicable in risk-neutral operating conditions. However, the actual realized profit may be quite different from the expected profit. Consider the case where a firm cannot afford to record a huge loss, possibly due to poor performance in previous selling seasons or limited available capital. In order to stay in business, it is likely that the firm's objective is not just to maximize expected profit, but also to minimize the variance from that goal or the associated risk. When a firm is concerned about the risk of potential losses, there are many ways in which the firm can actually quantify this risk into a model. In Section 2.4.1, we assume that risk is measured in terms of demand uncertainty. Then, the model in Section 2.4.2 assumes that risk is measured in terms of expected profit, where our goal is to minimize *worst-case losses or profits*.

2.4.1 Minimizing Demand Uncertainty - Model [MV]

The SNP is based largely on the relationship between expected revenue (or profit) and demand uncertainty, which lends itself nicely to solution approaches similar to those used in portfolio optimization. The seminal work done by Markowitz [33] over 50 years ago, followed by a large number of articles on this topic, provide extensive discussion on mean-variance optimization.

The mean-variance approach focuses on minimizing the risk involved with obtaining a desired return. Here, we place a target level of expected profit and focus on minimizing demand variability. We present the user with an efficient frontier of expected profit versus the minimum demand variability that can be achieved with that expected profit. While maximum profit is desirable, a firm will not sacrifice the entire stability of its operation to achieve a small incremental profit. The efficient frontier would enable the firm to make suitable market selections, providing insight into this tradeoff between expected profit, expected net revenue and demand uncertainty.

We first present a mean-variance formulation based on minimizing demand variance while achieving a desired expected profit value:

$$\begin{aligned}
\textbf{[MV-G]} \qquad & \text{minimize} \qquad && \textstyle\sum_{i=1}^{n} \sigma_i^2 y_i \\
& \text{subject to:} && \textstyle\sum_{i=1}^{n} \bar{r}_i y_i - K(c,v,e)\sqrt{\sum_{i=1}^{n} \sigma_i^2 y_i} \geq G^L \qquad (2.5) \\
& && y_i \in \{0,1\} \qquad\qquad i = 1,\ldots,n,
\end{aligned}$$

where $\bar{r}_i = (r_i - c)\mu_i - S_i$ is the total expected net revenue from serving market i, σ_i^2 is the variance of demand from market i, $y_i, i = 1, \ldots, n$ are the binary market selection variables and G^L is the target lower bound for the expected profit. If we allow y_i's to take non-integer values then expected profit equal to G^L can be achieved, so the constraint in the formulation [MV-G] can be an equality. When we enforce the integer restrictions, G^L acts as a lower bound. We can obtain a solution frontier by setting the target value G^L at different levels. In formulation [MV-G] we minimize a term (total demand variance) that also appears as part of the constraint for the expected profit setting. Instead, we could minimize demand uncertainty while achieving a particular total expected net revenue level. Therefore we introduce the [MV-R] formulation, in which we provide a target lower bound for total expected net revenue and enforce integer restrictions on y_i's. We now present the [MV-R] formulation with a target for expected net revenue:

$$
\begin{array}{lll}
\textbf{[MV-R]} & \text{minimize} & \sum_{i=1}^{n} \sigma_i^2 y_i \\
& \text{subject to:} & \sum_{i=1}^{n} \bar{r}_i y_i \geq R^L \\
& & y_i \in \{0,1\} \qquad i = 1, \ldots, n.
\end{array}
\tag{2.6}
$$

In this case, we define the acceptable net revenue level R^L and solve for the minimum demand variance. An efficient frontier can be obtained by considering many net revenue values. If we are able to identify a set of k values for R_L and G_L corresponding exactly to a set of potential solutions $\hat{y}^1, \hat{y}^2, \ldots, \hat{y}^k$, then the solution front obtained by one model will contain the same solutions obtained using the other model.

Property 2.4.1. *Using realizable values for R_L based on the solution vector y, the discrete solution frontier generated using [MV-R] will represent the same set of solu-*

tions as the discrete solution frontier generated using [MV-G] based on the realizable values of G_L corresponding to R_L.

Proof. Let the solution for [MV-R] with a target level of R^{L_1} be $(\tilde{Q}^R, \tilde{y}^R)$. The expected profit for this solution can be calculated as $G = \sum_{i=1}^n \bar{r}_i \tilde{y}_i^R - K(c,v,e)\sqrt{\sum_{i=1}^n \sigma_i^2 \tilde{y}_i^R}$. Clearly $G \geq R^{L_1} - K(c,v,e)\sqrt{\sum_{i=1}^n \sigma_i^2 \tilde{y}_i^R}$ because $\sum_{i=1}^n \bar{r}_i \tilde{y}_i^R \geq R^{L_1}$. Letting $R^{L_1} - K(c,v,e)\sqrt{\sum_{i=1}^n \sigma_i^2 \tilde{y}_i^R} = G^{L_1}$, the solution $(\tilde{Q}^R, \tilde{y}^R)$ holds for [MV-G] with a target level of G^{L_1} on the expected profit. □

In this special case, for different values of R^{L_1} we obtain solutions to [MV-G] with corresponding target level values of G^{L_1}. Hence we can find the solution frontier for either [MV-G] or [MV-R] and get the frontier for its counterpart.

However, in general, the two formulations are not equivalent, and it is possible to observe different solution fronts. As it would require 2^n observations to account for each unique solution, our goal is not to construct the frontier in this fashion. A more logical approach would be to include several test values for R_L or G_L at common intervals to depict the trend and shape of the frontier. Nonetheless, using [MV-R] is certainly preferred over [MV-G], since it is quite easy to solve, even with the integer restrictions. Moreover, [MV-G] has a nonlinear constraint.

2.4.2 Minimizing Worst-case Profits or Losses - Model [RM]

While some firms may be quite satisfied with analyzing tradeoffs between expected profits and demand uncertainty, other firms may be more focused on the risk element rather than the profit element. We consider our firm to be "risk minimizing," whereby the firm minimizes the percentage of potential profits (or losses) below a predefined value. We will refer to this value as a profit level throughout the remainder of the chapter, although a negative value would obviously represent a loss. We present the risk minimizing selective newsvendor as

$$
\begin{array}{llll}
\textbf{[RM]} & \text{minimize} & F_G(P) & \\
 & \text{subject to:} & Q \geq 0, & \\
 & & y_i \in \{0,1\} & i = 1, \ldots, n,
\end{array}
$$

where P represents the critical profit value and F_G denotes the cumulative distribution of the profit equation $G(Q, y)$. Recall that by adding markets we may be able to increase expected revenue and profit, but not necessarily reduce the overall risk. While this tradeoff may be desirable using [MV-G] or [MV-R], it is not desirable under model [RM]. The critical factor in determining the preferred market selection set is now P. Also note that the firm must set P such that some markets will actually be selected. For $P \leq 0$ and $y_i = 0$ for $i = 1, \ldots, n$, we have $F_G(P) = 0$, an optimal solution with no markets selected. By selecting a value of $P > 0$, however, $F_G(P) = 1$ when no markets are selected, so the model would attempt to add markets to lower this percentage.

2.5 Solution Approach to [RM]

This section introduces the solution approaches to [RM]. In the first subsection we calculate the worst-case profits, or $F_G(P)$. We show that the unique revenue r_i for each demand source i results in several profit values from a single demand value $D_{\hat{y}}$. For this reason it does not have a closed form solution and leads us to use the simulation analysis. In section 4.2 we present a simulation approach for finding $F_G(P)$ and the optimal order quantity. Section 4.3 explains the constructive heuristic solution via simulation to [RM].

2.5.1 Calculating Worst-case Profits, or $F_G(P)$

For model [RM], we must determine $F_G(P)$ for a given value of P and candidate solution ($\hat{Q}$,$\hat{y}$), despite the fact that F_G is not normally distributed. The following discussion describes the difficulty in performing this calculation. Consider that we can write the profit equation as

$$\begin{aligned} G(Q,y) &= \sum_{i=1}^{n}(r_iD_i - S_i)y_i - cQ + v[\max(0, Q - \sum_{i=1}^{n} D_iy_i] \\ &\quad -e[\max(0, \sum_{i=1}^{n} D_iy_i - Q)] \\ &= \sum_{i=1}^{n} ((r_i - e)D_i - S_i))\, y_i - (c-v)Q + (e-v)\min(Q, \sum_{i=1}^{n} D_iy_i). \end{aligned}$$

Given a solution ($\hat{Q}$,$\hat{y}$), our main interest is to determine the proportion of outcomes from $\sum_{i=1}^{n} D_i\hat{y}_i$ in which $G(\hat{Q}, \hat{y}) \leq P$. Conditioning on the realization of demands, and letting $D^{\hat{y}} = \sum_{i=1}^{n} D_i\hat{y}_i$, we have the following:

$$\begin{aligned} Pr(G(\hat{Q},\hat{y}) \leq P \mid D^{\hat{y}} > \hat{Q}) &= Pr(\sum_{i=1}^{n}((r_i-e)D_i - S_i)\hat{y}_i - (c-v)\hat{Q} + \\ &\quad (e-v)\hat{Q} \leq P \mid D^{\hat{y}} > \hat{Q}) \\ &= Pr(\sum_{i=1}^{n}((r_i-e)D_i - S_i)\hat{y}_i - (c-e)\hat{Q} \leq P \mid D^{\hat{y}} > \hat{Q}) \\ &= Pr(X_1 \leq P \mid D^{\hat{y}} > \hat{Q}), \end{aligned}$$

where X_1 denotes a normal random variable for profit. Likewise, we also have that

$$\begin{aligned} Pr(G(\hat{Q},\hat{y}) \leq P \mid D^{\hat{y}} \leq \hat{Q}) &= Pr(\sum_{i=1}^{n}((r_i-e)D_i - S_i)\hat{y}_i - (c-v)\hat{Q} + \\ &\quad (e-v)D^{y} \leq P \mid D^{\hat{y}} \leq \hat{Q}) \\ &= Pr(X_2 \leq P \mid D^{\hat{y}} \leq \hat{Q}), \end{aligned}$$

where X_2 denotes a different normal random variable for profit. The total probability of outcomes below P, or worst-case profits, is now given by

$$\begin{aligned} F_G(P) = Pr(G(\hat{Q},\hat{y}) \le P) &= Pr(X_1 \le P \mid D^{\hat{y}} > \hat{Q}) * Pr(D^{\hat{y}} > \hat{Q}) + \\ &\quad Pr(X_2 \le P \mid D^{\hat{y}} \le \hat{Q}) * Pr(D^{\hat{y}} \le \hat{Q}). \end{aligned} \tag{2.7}$$

Due to the normality of $D^{\hat{y}}$, we conclude that

$$Pr\left(D^{\hat{y}} > \hat{Q}\right) = 1 - Pr\left(Z \le \frac{\hat{Q} - \mu_{D^{\hat{y}}}}{\sigma_{D^{\hat{y}}}}\right); \quad Pr\left(D^{\hat{y}} \le \hat{Q}\right) = Pr\left(Z \le \frac{\hat{Q}-\mu_{D^{\hat{y}}}}{\sigma_{D^{\hat{y}}}}\right),$$

where $\mu_{D^{\hat{y}}}$ and $\sigma_{D^{\hat{y}}}$ denote the mean and standard deviation for the underlying demand distribution $D_{\hat{y}}$, and Z is the standard normal random variable. The above quantities can be easily calculated since $D^{\hat{y}}$ is normally distributed. Letting X_1^T and X_2^T denote the truncated normal distribution for X_1 and X_2, respectively, the conditional probabilities in (2.7) are calculated as:

$$\begin{aligned} Pr(X_1 \le P \mid D^{\hat{y}} > \hat{Q}) = \quad F_{X_1^T}(P) &= \frac{Pr\left(Z \le \frac{P-\mu_{X_1}}{\sigma_{X_1}}\right)}{Pr\left(Z \le \frac{P_{\hat{Q}_{X_1}}-\mu_{X_1}}{\sigma_{X_1}}\right)} \\ Pr(X_2 \le P \mid D^{\hat{y}} \le \hat{Q}) = \quad F_{X_2^T}(P) &= \frac{Pr\left(Z \le \frac{P-\mu_{X_2}}{\sigma_{X_2}}\right)}{Pr\left(Z \le \frac{P_{\hat{Q}_{X_2}}-\mu_{X_1}}{\sigma_{X_2}}\right)}, \end{aligned}$$

where $\mu_{X_1}, \mu_{X_2}, \sigma_{X_2}$, and σ_{X_2} denote the mean and standard deviation for X_1 and X_2, respectively. In order to obey the conditional probabilities in (2.7), we must only consider a truncated portion of $D_{\hat{y}}$ for each random variable X_1 and X_2, defined by $P_{\hat{Q}_{X_1}}$ and $P_{\hat{Q}_{X_2}}$ above. Unfortunately, there is no well-defined profit truncation point for each case that corresponds to the demand truncation point $\hat{Q}$, i.e., there is not a one-to-one correspondence between $D_{\hat{y}}$ and X_1 or X_2. Each demand source i can have a unique revenue r_i, resulting in several profit values from a single demand value $D_{\hat{y}}$. For this reason, we will use simulation analysis to populate the profit distribution $G(Q, y)$ and calculate worst-case profits, $F_G(P)$.

2.5.2 A Simulation Approach

Using a candidate solution ($\hat{Q}$,$\hat{y}$), we have the ability to describe $F_G(P)$ through simulation replications. We also show that simulation can be used to select an appropriate value for $\hat{Q}$, once the market selection vector y has been fixed for a particular solution.

2.5.2.1 Finding $F_G(P)$ Using Simulation

In this section, we will approximate the value of $F_G(P)$ using simulation. Given a market selection vector $\hat{y}$, an associated order quantity $\hat{Q}$, and a pre-defined critical profit level P, we can estimate $F_G(P)$. As stated previously, Figure 2.1 presents the form of the distribution $G(\hat{Q}, \hat{y})$. Here, we now specify the critical P, and by simulating demand realizations, we can then determine how many of these realizations (or occurrences) will result in a profit below P.

In order to evaluate model [RM], we require this $F_G(P)$ value for every market selection and order quantity tested. For every call to simulation, there will be an associated expense in computational time. Thus, we will limit the number of replications performed and still provide an adequate answer in a reasonable amount of simulation time. We note that the demands for each replication are only simulated once. Then, with these demands "fixed," we determine an appropriate order quantity (Section 2.5.2.2) or market selection (Section 2.5.3).

2.5.2.2 The Optimal Order Quantity

Let $Q_1 = F_y^{-1}(\frac{e-c}{e-v})$, the optimal order quantity for the SNP with an expected profit objective. For models involving risk, it is not clear that Q_1 will remain optimal. Consider the following example with 40 markets. Using simulation to generate profit realizations for increasing values of Q, Figure 2.2 presents the relationship between the value of Q and the percent of observations not meeting a critical profit level P (i.e., probability that realized profit does not meet the threshold profit level).

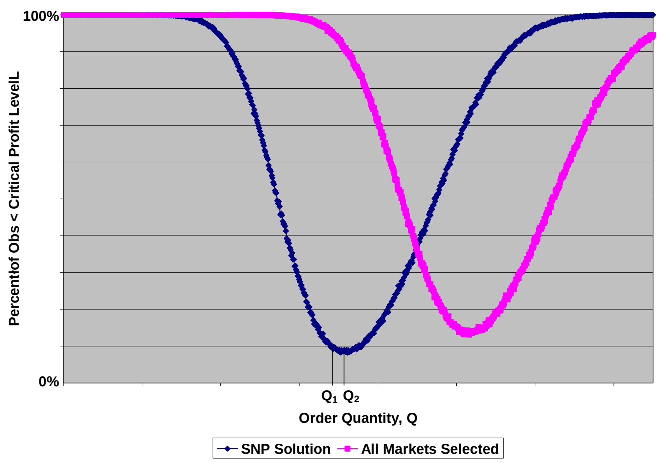

Figure 2.2: Order Quantity vs. Profit Realized.

Note that the value of Q_1 given by the selective newsvendor does not coincide with Q_2, the order quantity that provides the highest probability of meeting our value for P.

Based on the figure, we can also observe that the function describing the relationship of Q and $F_G(P)$ certainly appears to be unimodal. This implies that we can use a line search technique to converge on $\hat{Q}$ for a given market selection vector $\hat{y}$. We have chosen to use the golden section technique Bazaraa, Sherali and Shetty [6] for our approach. As with the number of replications performed in a simulation, the stopping criterion will have a direct effect on the overall solution time. If small improvements in $F_G(P)$ require another iteration (and subsequent update in the value for Q), the required number of iterations for convergence will, of course, increase. At each iteration in the line search process, we are re-evaluating the scenarios, which leads to a longer overall solution time.

2.5.3 Solution Approach with Simulation

For every new solution ($G(\hat{Q}, \hat{y})$) tested, we must perform two main tasks: 1) re-evaluate the set of simulation replications to appropriately represent the distribution of profit; and 2) implement a line search technique to locate a preferred order quantity.

Given some set of selected markets $\hat{y}$, if the addition of market i into the solution reduces the frequency of profits below P, we would expect this market to be beneficial. We desire such shifts in the profit distribution that reduce the location and size of the left tail of the profit distribution (see Figure 2.1).

With this in mind, we propose the following constructive heuristic to find approximate or near optimal solutions to model [RM]. The procedure is actually independent of the underlying demand distribution, although we will focus on markets in which the demand data are normally distributed.

2.5.3.1 Solving Problem [RM]

In developing a solution approach, we tested the ability to find high-quality solutions based on a constructive heuristic approach. First, we evaluate $F_G(P)$ for every potential market i when i is the only selected market. That is, for every i, we set $y_i = 1$ and $y_j = 0$ for all $j \neq i$, and determine the value of the distribution function of profit, denoted as $F_{G(Q,i)}(P)$. We then re-index all markets $i = 1, \ldots, n$ in non-decreasing order of the value $F_{G(Q,i)}(P)$. Then, starting with re-ordered market [1], we systematically add each market to the solution (i.e., $Q, \hat{y}$), testing for each iteration whether the value of $F_{G(Q,\hat{y})}(P)$ decreases further. The final solution will contain the markets for which a minimum value of $F_{G(Q,\hat{y})}(P)$ is achieved. We present the solution approach to problem [RM]:

Constructive Heuristic Solution to [RM]

0) Set $j = 1$.

1) Select only market j and find the optimal order quantity Q^j for this market selection. Find Q^j based on the line search method proposed in Section 2.5.2.2. During the procedure for finding Q^j, we also populate the profit distribution associated with solution vector (Q^j, y^j) using simulation. Then calculate the percentage of worst-case profits for this market selection, or $F_{G(Q,j)}(P)$.

2) Update $j = j + 1$; Repeat Step 1 until $j > n$.

3) Sort the markets in non-decreasing order of $F_{G(Q,j)}(P)$ values to obtain the sorted market order [1],[2],[3],. . . ,[n]. Set j to $j = 1$.

4) Select markets [1],[2],. . . ,[j] and estimate $F_G(P)[j]$ by populating the profit distribution using simulation. Again, determine $Q[j]$ using the line search method proposed in Section 2.5.2.2.

5) Update $j = j + 1$; Repeat Step 4 until $j > n$.

6) We calculate n such potential solutions to problem [RM]. From the set $S = \{F_G(P)[j], j = 1, \ldots, n\}$, the solution to [RM] is such that $F_G^*(P) = \min \{F_G(P)[j], j = 1, \ldots, n\}$.

This solution approach does not require evaluating all 2^n possible market selections, which would be computationally prohibitive, as illustrated in our computational tests in Section 2.6.

2.6 Computational Tests

Throughout this section, we will be using sample test instances from which we can draw our conclusions. The following paragraph describes the parameters used in greater detail. We varied the size of the market pool between 5 and 50 markets, depending on the experiments being conducted. Every market has unit revenue in the range U[$200,$240], while the unit production cost is set at $200. Expected demand and demand variance for each market are distributed according to U[500,1000] units and U[50000,100000], respectively. The fixed cost for market entry are drawn from U[$2500,$7500]. Finally, the salvage value is $150 per unit, and the expediting or shortage cost $500 per unit, respectively. All computational tests were conducted on Dell desktop machines with a 3.0 GHz processor and 1 GB of RAM.

2.6.1 Mean-variance Results - Models [MV-R] and [MV-G]

In mean-variance analysis, one main goal is to provide the decision maker with insight into the tradeoff between increased profit and increased risk. As discussed in Section 2.4.1, the two proposed models use revenue and profit as the desired outcomes, with demand uncertainty as the risk.

By minimizing demand variance with a lower bound on the expected net revenue (or expected profit), we can identify the boundary or frontier of the feasible space of market solutions. Recall that Q is not a decision variable for [MV-R] and [MV-G], and its value will not be affected by the objective of minimizing demand uncertainty. Thus, Q can be calculated after a set of markets are selected (see Section 2.3.1). (This is not the case for the risk minimizing model results in Section 2.6.2.)

The efficient solution frontiers obtained from [MV-R] and [MV-G] depend on the production cost (c), salvage value (v) and expediting cost (e). In the Appendix, Figures 2.5, 2.6 and 2.7 show how the two frontiers change with respect to c, v and e respectively.

For any of the solution frontiers generated, once a firm determines an acceptable expected profit level, the optimal risk level (demand uncertainty, in this case) and specific market selection vector at that point can be obtained easily. In fact, it is also interesting to note that as the minimum expected profit level is increased, the optimal market selection vector may remain unchanged for several iterations, which results in the staggered appearance of the frontiers in each of the figures.

2.6.2 Risk Minimizing Results - Model [RM]

We present the results that describe the performance of the algorithm, as well as the change in solution values from the original SNP solution. We also test four distinct critical profit levels to gauge the effect this has on markets selected and overall order quantity. These four profit levels are calculated as 25%, 10%, 5% and 1% of the expected profit given by the SNP solution approach, denoted as G_{SNP}.

We created a set of 20 test instances for each size of the potential market pool: 5, 10, 15, 20, 30, 40, and 50. For each simulation replication or demand realization, we calculate demand based on the market selection variables y_i for that particular solution.

2.6.2.1 Simulation Replications and Order Quantity Calcuation

One critical decision in conducting the simulations is setting the required number of simulation replications. Once the output is considered reliable, it is important to stop adding replications and proceed with the next potential solution vector. Using a 10-market set as an example, Figure 2.3 displays the minimum percent of worst-case profits found at various replication settings, when line search is included in the solution approach. The figure presents an average across 20 test problems. The objective function values ($F_G(P)$) are fairly stable, only showing a slight increase as replications are increased. This indicates that we can approximate $F_G(P)$ even at 1000 or 5000 replications. However, we may miss some extreme demand (and thus profit) realizations that cause the percent of worst-case profits to increase.

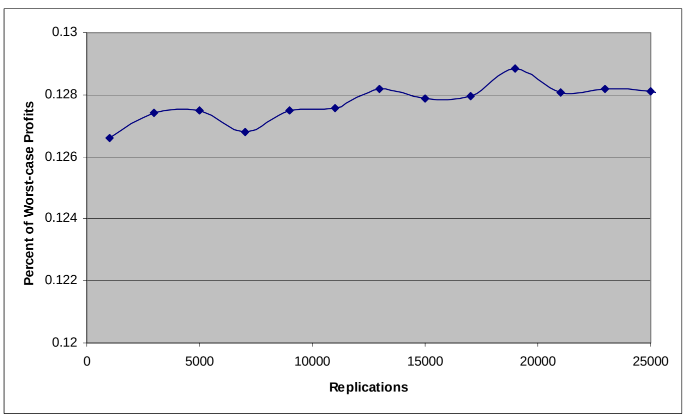

Figure 2.3: Solution value vs. # of replications (line search, 10-market case).

The golden section line search technique Bazaraa et al. [6] evaluated order quantities in a range of 0 to the maximum total demand if all markets were included (and demand in each market was realized at its highest level). The procedure would converge on an order quantity once the current best quantity produced less than a 1% improvement from the prior iteration's order quantity value. This process proved to be more computationally expensive than adding simulation replications. In order to obtain solutions for larger problems, we conducted experiments in which the line search technique was not used. In its place, we used the preferred order quantity generated via the standard SNP approach, or $Q^y = \sum_{i=1}^{n} \mu_i y_i + z(\rho)\sqrt{\sum_{i=1}^{n} \sigma_i^2 y_i}$.

Again using a 10-market set as an example, Figure 2.4 displays the minimum percent of worst-case profits found at various replication settings, when line search is not included in the solution approach. The figure presents an average over 20 test problems.

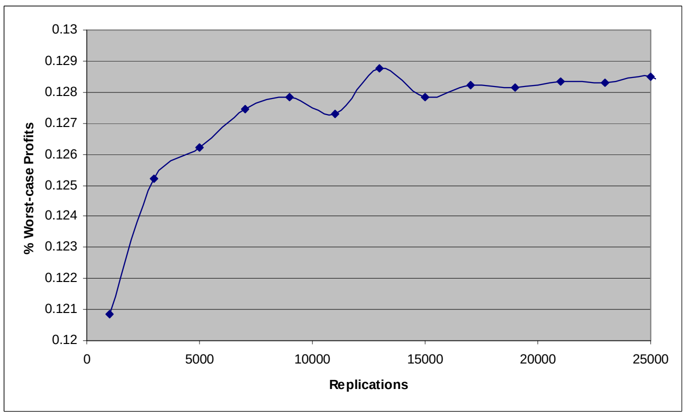

Figure 2.4: Solution value vs. # of replications (no line search, 10-market case).

We opted to run 10,000 replications for each solution tested since there was not a significant increase in reliability beyond this level. We also used 10,000 replications for the tests conducted with line search as well.

2.6.2.2 Analysis

In order to benchmark the quality of the constructive heuristic, we solved problem [RM] by enumerating all possible solutions and evaluating them using the simulation solution approach from Section 2.5.3. We point out that full enumeration does not guarantee an optimal solution since the underlying solution approach is an approximation via simulation. Moreover, for larger test instances, full enumeration is not possible. Still, this does provide an important comparison for the smaller problems.

Tables 2.1 and 2.2 present the critical performance measures using full enumeration and the constructive heuristic, respectively. To address long run times within each approach, we could either reduce the number of simulation replications or eliminate the line search. We implemented the respective algorithms with line search (LS) and without line search (NLS) to determine a preferred order quantity, and for the four levels of profit previously mentioned. Also note that we record the expected profit and order quantity for the standard SNP approach with no risk in the final column. We simulated each potential solution 10,000 times except when noted differently.

Table 2.1: Enumeration Results for [RM]

	Solution Approach - Enumeration								
Scenario/	P=0.25(G_{SNP})		P=0.1(G_{SNP})		P=0.05(G_{SNP})		P=0.01(G_{SNP})		SNP
Measurement	LS	NLS	LS	NLS	LS	NLS	LS	NLS	Solutions
5 Markets									
CPU Time	1 sec	<1 sec	1 sec	<1 sec	1 sec	<1 sec	1 sec	<1 sec	G_{SNP}=19150
G	17713	17713	17520	17520	17520	17212	17307	17212	
$F_G(P)$	0.3018	0.3101	0.2806	0.2883	0.2741	0.2813	0.2690	0.2761	Q_{SNP}=3013
Q	3089	3177	2999	3081	3006	3021	2996	3021	
10 Markets									
CPU Time	71 sec	3 sec	71 sec	3 sec	70 sec	3 sec	70 sec	3 sec	G_{SNP}=61533
G	60904	60787	59562	58723	58572	58723	58505	58447	
$F_G(P)$	0.1573	0.1591	0.1264	0.1274	0.1169	0.1183	0.1102	0.1118	Q_{SNP}=6824
Q	6188	6220	5835	5681	5630	5681	5591	5602	
15 Markets	**		**		**		**		
CPU Time	5 min	2 min	5 min	2 min	5 min	140 sec	5 min	140 sec	G_{SNP}=100669
G	96452	96345	96326	96773	93200	95855	93994	94805	
$F_G(P)$	0.1026	0.1053	0.0740	0.0817	0.0658	0.0742	0.0596	0.0687	Q_{SNP}=10025
Q	9021	8942	8940	8777	8757	8583	8842	8400	
20 Markets									
CPU Time	n/a	91 min	n/a	89 min	n/a	89 min	n/a	78 min	G_{SNP}=151445
G		146277		140566		137657		136464	
$F_G(P)$		0.0659		0.0447		0.0393		0.0353	Q_{SNP}=13844
Q		12047		11124		10742		10660	

** Results from 1000 replications

Solving each problem using full enumeration was very time consuming. Combining the requirement of simulation and line search, the solution time for a 15-market

Table 2.2: Heuristic Results for [RM]

Scenario/ Measurement	Solution Approach - Heuristics								SNP Solutions
	P=0.25(G_{SNP})		P=0.1(G_{SNP})		P=0.05(G_{SNP})		P=0.01(G_{SNP})		
	LS	NLS	LS	NLS	LS	NLS	LS	NLS	
5 Markets									
CPU Time	<1 sec	<1 sec	<1 sec	<1 sec	<1 sec	<1 sec	<1 sec	<1 sec	G_{SNP}=19150
G	17713	17713	17520	17520	17520	17212	17307	17212	
$F_G(P)$	0.3017	0.3099	0.2805	0.2882	0.2740	0.2812	0.2689	0.2760	Q_{SNP}=3013
Q	3089	3177	2999	3081	3006	3021	2996	3021	
10 Markets									
CPU Time	1 sec	<1 sec	1 sec	<1 sec	1 sec	<1 sec	1 sec	<1 sec	G_{SNP}=61533
G	59663	58877	58473	58551	58645	58723	58407	58521	
$F_G(P)$	0.1591	0.1636	0.1264	0.1273	0.1169	0.1182	0.1102	0.1117	Q_{SNP}=6824
Q	5987	6017	5566	5615	5656	5681	5550	5626	
15 Markets									
CPU Time	3 sec	<1 sec	3 sec	<1 sec	3 sec	<1 sec	2 sec	<1 sec	G_{SNP}=100669
G	93390	93268	96695	96406	95907	95619	95807	94530	
$F_G(P)$	0.1232	0.1307	0.0791	0.0825	0.0709	0.0747	0.0649	0.0689	Q_{SNP}=10025
Q	10021	10850	8818	8623	8721	8438	8694	8333	
20 Markets									
CPU Time	7 sec	<1 sec	5 sec	<1 sec	5 sec	<1 sec	4 sec	<1 sec	G_{SNP}=151445
G	144547	144867	141947	131959	145715	141031	143383	138982	
$F_G(P)$	0.0817	0.0878	0.0141	0.0475	0.0337	0.0398	0.0295	0.0358	Q_{SNP}=13844
Q	15416	15993	11614	9934	12101	11015	11798	10798	
30 Markets									
CPU Time	17 sec	<1 sec	13 sec	<1 sec	11 sec	<1 sec	10 sec	<1 sec	G_{SNP}=248725
G	239696	239946	220513	216355	226866	218819	225794	216521	
$F_G(P)$	0.0383	0.0490	0.017	0.030	0.0088	0.0179	0.0067	0.0156	Q_{SNP}=21453
Q	24596	24176	18480	18416	16407	14786	16357	14657	
40 Markets									
CPU Time	30 sec	1 sec	24 sec	1 sec	20 sec	1 sec	17 sec	1 sec	G_{SNP}=349532
G	320676	338779	323776	329999	306846	267303	317445	275687	
$F_G(P)$	0.0245	0.0331	0.0080	0.0229	0.0029	0.0011	0.0016	0.0090	Q_{SNP}=28827
Q	31413	31883	30069	30440	21252	15900	22409	16662	
50 Markets									
CPU Time	51 sec	1 sec	45 sec	1 sec	36 sec	1 sec	29 sec	1 sec	G_{SNP}=453313
G	318733	438689	331254	436090	375595	294069	413434	345540	
$F_G(P)$	0.0350	0.0263	0.0095	0.0180	0.0010	0.0088	0.0003	0.0057	Q_{SNP}=35846
Q	30453	39056	31299	38805	24863	16616	28243	19613	

problem exceeded three hours per test problem. In order to still provide a comparison at the 15-market level, we chose to use 1000 simulation replications. The constructive heuristic was quite fast in comparison, producing results for the 50-market problems within one minute.

Overall, we achieve similar quality solutions using our heuristic approach as compared to the enumerative approach, with the noise in solution quality due to the simulations required to develop the profit distribution. The heuristic approach actually achieves a lower $F_G(P)$ than the enumerative approach in several cases. (Recall that the enumerative procedure is still a heuristic itself, since we must use simulation to construct the profit distribution for every potential market selection assignment.) Thus, it is important to note that we are not giving up much in the way of solution quality for a significant reduction in solution time. We also point out that when minimizing risk, the resulting expected profit values (G) are always less than those generated for G_{SNP}, since G_{SNP} represents a risk-neutral approach.

We discuss more specific results for critical profit levels of 25 %, 10%, 5% and 1% of G_{SNP} value. In these cases, we observe that the order quantity is consistently below the corresponding Q_{SNP}. Based on the problem data used, in risk averse settings, minimizing worst-case profits (or losses) results in ordering less. Another key result is that $F_G(P)$ for line search is consistently smaller than the "no line search" approach. Moreover, the difference in solution quality (line search vs. no line search) increases as additional markets are added to the problem, so the need to perform line search becomes increasingly important for the 40- and 50-market scenarios. For the 10%, 5%, and 1% cases, the order quantity calculation without line search typically underestimated the Q that produced minimal risk, further supporting the use of line search in the solution method.

Again, with the exception of 50-market line search problem for 25%, for the

25%, 10%, 5%, and 1% cases, we also observe that $F_G(P)$ decreases as number of markets is increased. This is mainly due to the the shift in location of the profit distribution. With an increase in the number of markets, the new 10% critical profit level is much smaller in relation to the expected profit value. Thus, fewer profit observations will occur below the new P.

2.7 Conclusions

In this chapter, we offer multiple approaches for assessing and evaluating the risk associated with a particular solution to the selective newsvendor problem. For the risk minimizing model, we introduced a constructive heuristic that provides high quality solutions at a fraction of the time of an enumerative approach. With the data sets tested, the selective newsvendor with risk orders less than the risk-neutral selective newsvendor, especially for cases in which only the extreme worst-case profits (or losses) are being minimized. The solution approach with line search provides much better results than simply using the order quantity based on the expected value approach of the selective newsvendor. Both the line search and simulation replications require significant computing time, and these items must be considered as problem size increases.

We point out that obtaining solutions to probabilistic risk models can be quite cumbersome, and we offer approaches that firms dealing with risk issues can implement. When there is no closed-form solution approach available for defining the profit distribution (and worst-case profits), we must resort to an approach using simulation as described in this chapter.

In future work, we would like to consider the benefit of including a local search algorithm to improve the constructive heuristic solution. This would become increasingly important as the number of markets is increased. It would also be worthwhile to provide a large testbed of problems and observe how the solutions change across the problems. We also plan to address the impact that various types of demand distributions (such as all-or-nothing or Bernoulli demands, and uniform demands) would have on the resulting solutions and solution approaches. Another area of interest would be the multi-period market or order selection problem with risk. This is a very rich area of research with lots of opportunity.

2.8 Appendix

The following figures depict the sensitivity of the solution frontiers to production cost, salvage value, and expediting cost for models [MV-G] and [MV-R].

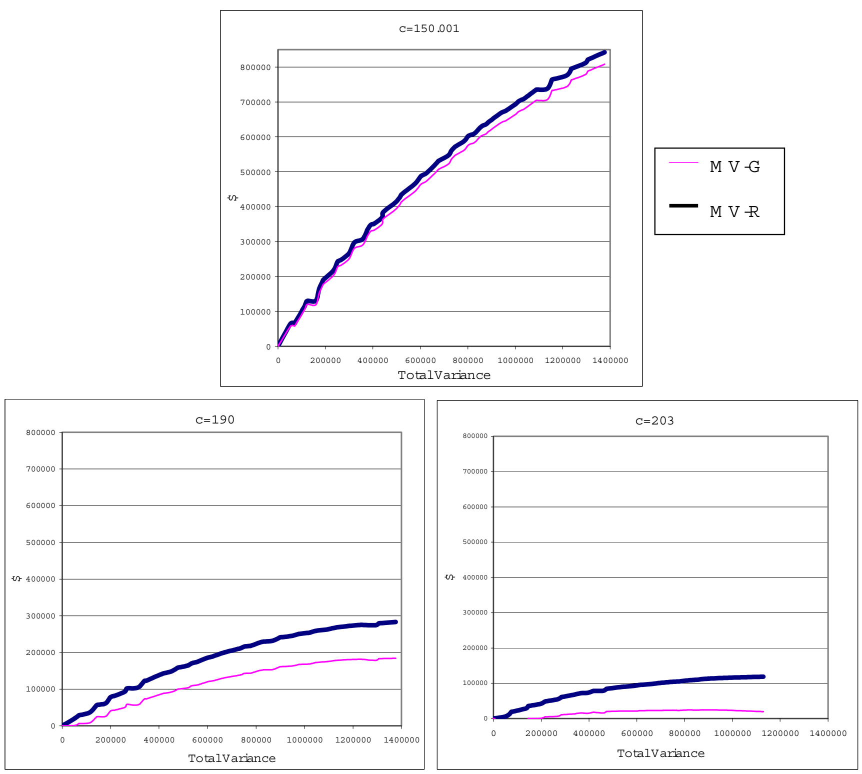

Figure 2.5: Solution Frontier Sensitivity to Production Cost (c)

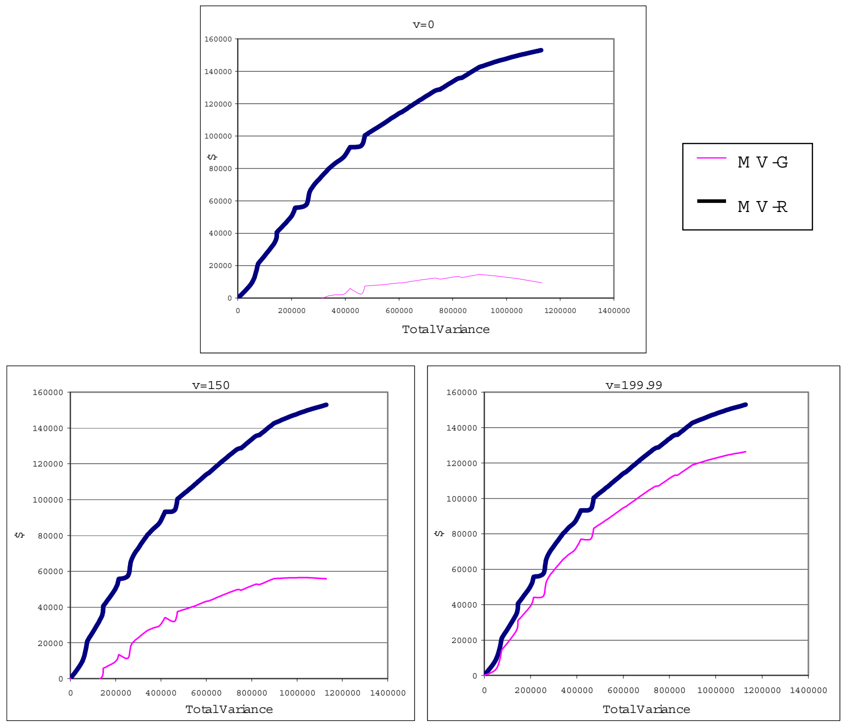

Figure 2.6: Solution Frontier Sensitivity to Salvage Value (v)

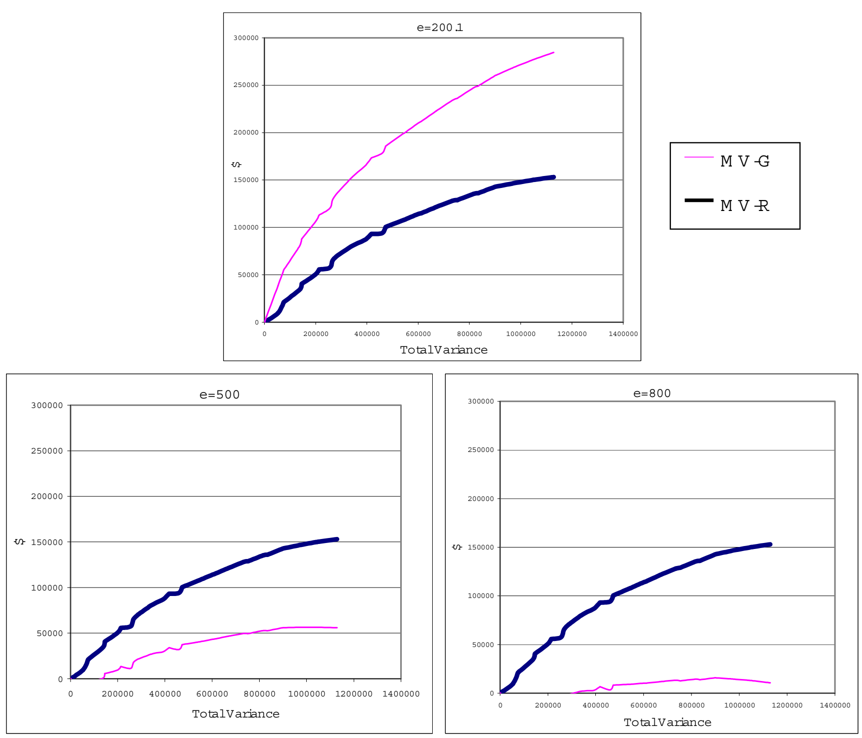

Figure 2.7: Solution Frontier Sensitivity to Expediting Cost (e)

Chapter 3

Alternate Demand Distributions for the Risk-Averse SNP

3.1 Abstract

Consider a firm that operates in consecutive single selling seasons, delivering its products across several markets with unique revenue and uncertain demands in each market. Using a profit maximization approach based on a newsvendor-type model, the firm may still incur several losses across consecutive periods in the short run. Risk analysis with demand selection has been modeled where customer/market demands follow normal distributions. Often a firm faces a set of potential unconfirmed orders, where each order will either come in at a predefined level or it will not come in at all. In this chapter, we consider these All-or-Nothing (AON) demands and provide insights into their effect on expected profit and the frequency of extremely costly procurement policies. Instead of solely identifying the market/demand set and procurement quantity that maximizes expected profit, we use a Conditional Value-at-Risk approach that allows a decision maker to control the number of profitable

but risky demands to consider in the overall procurement policy. This approach is compared against an expected profit objective, and several managerial insights are provided.

3.2 Background and Literature Review

This is an era resulting in ever-decreasing product lives and, in some cases, an increasing customer influence over a firm's production decisions. In an effort to deliver to customer demands, producers are often at the mercy of a very unpredictable demand stream (or order base). Our work is motivated by observations at a large manufacturer whose sales teams attempt to secure orders for low-volume telecommunications infrastructure equipment. Unsecured orders are scheduled for a specific period of time into the future, with limited knowledge about whether or not they will actually materialize. These orders are customized, but only at a certain (relatively late) point in the production process. So the manufacturer can, in fact, begin procuring and assembling materials that are non-customized, and then customize the product once orders come in. When the customer is the dominant supply chain player, he can actually influence the manufacturer's production timeline, and the manufacturer may allow customer orders/due dates within the procurement lead times for the product (in the hopes of securing the contract).

Each customer has unique qualities, and some customers will invariably play a more dominant role in the industry. Therefore, the negotiated price of the product will be unique for each customer, and the salesforce allocation to each customer will also be unique. When all markets or orders that materialize must be satisfied, this problem becomes a classical newsvendor problem. For further research on various types of newsvendor problems, see Khouja [29], Petruzzi and Dada [36], Bertsimas and Thiele [7], Chen and Chuang [15], and Chung and Flynn [18].

As explained in the previous chapter, in our setting we permit demand selection flexibility, which allows the firm to essentially choose which orders it wants to pursue and satisfy. Recent research on various forms of demand selection has considered both deterministic and stochastic demand models (see, e.g., Carr and Lovejoy [12], Petruzzi and Monahan [37] and Taaffe and Geunes [51]). Specifically, Carr and Lovejoy [12] study an inverse newsvendor problem by optimally choosing a demand distribution from a set of feasible demand portfolios, which may include several customer classes. Given deterministic demands, Geunes, Shen, and Romeijn [25] address ordering policies with customer selection for generalized classical EOQ models. When selecting customers to generate a demand portfolio, Bakal, Geunes and Romeijn [5] consider that each market/customer observes price-sensitive demands. Also, Yang, Yang and Abdel-Malek [63] have studied a supplier selection problem, where a buyer faces an uncertain demand and has to decide ordering quantities from a set of suppliers with different yields and prices.

In stochastic demand settings, demand selection allows the firm to decide whether it should pursue highly profitable, yet risky, orders over less profitable, but possibly more stable orders. Thus, expected profit is now influenced by the size and uncertainty of the order as well as the set of markets that the firm decides to satisfy. Simultaneously selecting the most desirable market set and setting the appropriate total order quantity has been commonly referred to as the "selective newsvendor" problem (SNP) (see, e.g., Taaffe, Geunes, and Romeijn [53]).

All previously-cited research employs an objective of maximizing expected profit or minimizing expected cost. However, all firms do not enjoy risk-neutral operating conditions. In fact, many firms operate on limited operating budgets that do not allow for incurring several losses over consecutive selling seasons. Research focusing on non-risk-neutral decision makers with newsvendor operating conditions began with Lau [31] and has grown in recent years (see, e.g., Eeckhoudt et al. [21], Agrawal and Seshadri [2], Schweitzer and Cachon [44], Chen et al. [16], Wang and Webster [61], and Wu et al. [62]). An introduction to the risk averse *SNP* is presented in Chapter 2. We presented a risk averse model where the firm's objective is to minimize the number of outcomes that could occur below their targeted profit level. In particular, we studied the effect of incorporating risk into the SNP, presenting the analysis based on either minimizing demand uncertainty or worst-case profits. We also demonstrated that embedding simulation into a constructive heuristic provides high-quality market selections. This approach, as well as the risk-neutral approach found in Taaffe, Geunes and Romeijn [53], provide solution approaches and insights for the SNP with *normally distributed demands*.

In our motivating example described earlier, our firm faces a set of unconfirmed demands or orders, and each order will either come in at a predefined level or not at all (i.e., demands are *not* normally distributed). This behavior of customer demand is modeled using a Bernoulli distribution or what we denote as all-or-nothing (AON) demands. Taaffe and Romeijn [54] and Taaffe, Romeijn, and Tirumalasetty [55] have addressed the AON problem with an objective of maximizing expected profit. In this chapter, we account for the risk associated with each demand, and offer several methods for providing the firm with more information about the relationship between expected profit and the risk of single-period losses.

In order to incorporate risk in the AON problem, we utilize the concept of Conditional Value-at-Risk, or CVaR. CVaR has been widely used in the field of portfolio management, and it is rapidly gaining influence in the insurance industry as well. Rockafellar and Uryasev [41] introduced a new approach to optimize a portfolio in order to reduce the risk of high losses. This approach mainly tried to optimize the portfolio by calculating Value-at-Risk (VaR) and optimizing CVaR simultaneously. This approach is of particular interest to our work as the technique applied here can be combined with analytical or scenario-based methods. Given that our problem has unique demand realization points (i.e., all-or-nothing) and, thus, has a finite number of outcomes or scenarios, CVaR allows the evaluation of worst-case profits and shaping of the resulting profit distribution through careful demand selection and procurement decisions.

In summary, this chapter is an extension of both risk-averse and AON problems in single-period settings. Past work in Chapter 2 has considered risk-averse producers with normally distributed demands, and risk-neutral producers with either normally distributed demands or all-or-nothing demands ([53],[55]), however these two characteristics have not been addressed simultaneously. In this chapter, we model risk analysis with demand selection where customer orders are AON, and we study its effect on demand selection, procurement policies, and the minimization of the potential worst-case profits or losses.

Often, the firm may know a demand range for a customer and nothing more. Considering equally likely observations in that range, a uniform distribution would provide a good approximation. In this chapter we also model the risk analysis with demand selection where customer orders follow uniform distribution and will study its effect on minimizing the potential worst case losses using the simulation with local search algorithm. This work addresses the impact that these various types of demand distributions would have on both demand selection and procurement policies and the applicability of the heuristic approach presented in the previous chapter.

The rest of the chapter is organized as follows. In Section 3.3 we review the expected profit approach for all-or-nothing demands and discuss the form of the profit distribution. In Section 3.4, we present the optimization model for all-or-nothing demands under risk averse conditions. Section 3.4.3 discusses the various managerial insights discovered via sensitivity analysis. In Section 3.6, we present the form of profit distributions for AON and uniform distribution by using heuristics presented in Chapter 2. Section 3.7 provides our conclusions and directions for future work.

3.3 Expected Profit Approach: AON Demands

The SNP with All-or-nothing (SNP-AON) demands has been previously studied in Taaffe and Romeijn [54] and Taaffe, Romeijn, and Tirumalasetty [55]. We consider a set of n potential orders that a supplier can serve. Let D_i denote the random variable for demand source i ($i = 1, \ldots, n$) with probability distribution F_i, and assume that these demands are statistically independent. Most prior models with selection flexibility assumed that each random variable for demand is normally distributed. We consider that a firm may face a set of unconfirmed orders, and each order will either come in at a predefined level, or it will not come in at all. Let p_i be the probability of an order being realized at a pre-defined level d_i for market i:

$$\Pr(D_i = x) = \begin{cases} 1 - p_i & \text{if } x = 0 \\ p_i & \text{if } x = d_i. \end{cases}$$

The firm must decide far in advance of the selling season both the actual markets it will serve and the total quantity Q to be procured. We define the various parameters as follows: let c denote a per-unit procurement cost, r_i denote the revenue associated with market i, S_i denote the fixed cost for entering market i, v denote the salvage value and e denote the shortage cost or expediting cost. Without loss of generality we assume that $r_i > c$, otherwise we could immediately eliminate market i from consideration and also $e > c$ and $c > v$. Let $y_i (i = 1, \ldots, n)$ represent the binary demand selection variables representing the firms choice to select or reject order i.

We present the following expression for the total realized profit, based on a function $G(Q, y)$ of the order quantity Q, the binary vector y, and the random variable for demand D_i:

$$G(Q, y) = \begin{cases} \sum_{i=1}^{n}(r_i D_i - S_i)y_i - cQ + v(Q - \sum_{i=1}^{n} D_i y_i) & Q > \sum_{i=1}^{n} D_i y_i \\ \sum_{i=1}^{n}(r_i D_i - S_i)y_i - cQ - e(\sum_{i=1}^{n} D_i y_i - Q) & Q \leq \sum_{i=1}^{n} D_i y_i \end{cases}, \quad (3.1)$$

where $\sum_{i=1}^{n} D_i y_i$ represents the total demand from the selected markets. Since $E(\sum_{i=1}^{n} D_i) = \sum_{i=1}^{n} d_i p_i$, we can then express the firm's *expected profit* as:

$$\begin{aligned} H(Q, y) = \sum_{i=1}^{n}(r_i d_i p_i - S_i)y_i - cQ + vE\left[\max\left(0, Q - \sum_{i=1}^{n} D_i y_i\right)\right] \\ - eE\left[\max\left(0, \sum_{i=1}^{n} D_i y_i - Q\right)\right]. \end{aligned} \quad (3.2)$$

The formulation for the selective newsvendor problem for AON demands is now given by:

$$\begin{aligned} \text{maximize} \quad & H(Q, y) \\ \text{subject to:} \quad & Q \geq 0 \\ & y_i \in \{0, 1\} \quad i = 1, \ldots, n. \end{aligned}$$

A more explicit formulation of the profit function can be written by describing the unique *demand scenarios*, where each scenario w is comprised of a set $I_w \subseteq \{1, \ldots, n\}$ that contains the orders whose demands are realized. Let P_w denote the probability that demand scenario w is realized and is obtained by:

$$P_w = \prod_{i \in I_w} p_i \cdot \prod_{i \notin I_w} (1 - p_i), \quad w = 1, \ldots, W.$$

Note that there are a total of $W \equiv 2^n$ potential scenarios. By introducing the artificial variables u_w representing the shortage in *scenario* w, we can restate the AON optimization problem as the following mixed-integer linear programming problem:

[AON-EP]

$$\begin{array}{lll} \text{maximize} & \sum_{i=1}^{n} \left((r_i - v)d_i p_i - S_i\right) y_i - (c - v)Q - (e - v) \sum_{w=1}^{W} P_w u_w & \\ \text{subject to:} & u_w \geq \sum_{i \in I_w} d_i y_i - Q & w = 1, \ldots, W, \\ & Q \geq 0 & \\ & u_w \geq 0 & w = 1, \ldots, W, \\ & y_i \in \{0, 1\} & i = 1, \ldots, n. \end{array}$$

Taaffe, Romeijn, and Tirumalasetty [55] introduce an exact solution approach (based on the L-shaped method) for solving this stochastic programming problem. Their tailored algorithm can solve problems with three times as many selected orders as a state-of-the-art commercial solver. For larger problem instances, due to scenario explosion, an alternative heuristic solution is provided, and it was demonstrated to work quite well. In the next section, we will discuss the optimization approach (namely, CVaR) for incorporating risk into SNP with AON demands.

3.4 Conditional Value at Risk (CVaR) Models

Risk management is a crucial topic for researchers and market practitioners, with Value-at-Risk (VaR) established as a benchmark measure to evaluate risk within the financial literature. It measures the maximum loss associated with a specified confidence level of outcomes. Although VaR has been extensively applied in risk management, researchers have criticized this risk measure. Artzner, Delbaen, Eber and Heath [3] pointed out that VaR is not a coherent measure of risk since it fails to hold the subadditivity property. Moreover, VaR does not explain the magnitude of the loss when the VaR limit is exceeded. Furthermore, it is difficult to optimize when calculated using scenarios, and this has led to the use of an alternative measure – Conditional Value-at-Risk (CVaR). Pflug [38] proved that CVaR is a consistent measure of risk for its sub-additivity and convexity properties, and Uryasev [59] presented a description of both (1) an approach for minimizing CVaR and (2) optimization problems with CVaR constraints. Additional examples of the use of CVaR can be found in Rockafellar and Uryasev [41, 42], Tomlin and Wang [57] and Gotoh and Takano [26].

3.4.1 CVaR Formulation

In this section, we plan to incorporate risk into the AON-EP model from Section 3.3. We apply a conditional value-at-risk (CVaR) approach for developing an optimization model. CVaR, also known as expected shortfall, is a widely applied concept in financial risk management to evaluate risk of the market portfolio. Figure 3.1 depicts a typical loss distribution for some set of instruments in a portfolio. One would use CVaR to evaluate risk by focusing on the set of portfolio outcomes where losses exceed the value-at-risk, or VaR. In the figure, CVaR at a $100\alpha\%$ level is the expected return on the portfolio in the worst $100(1-\alpha)\%$ of the cases. CVaR can be optimized using linear programming and non-smooth optimization algorithms, and these techniques can effectively address large numbers of scenarios.

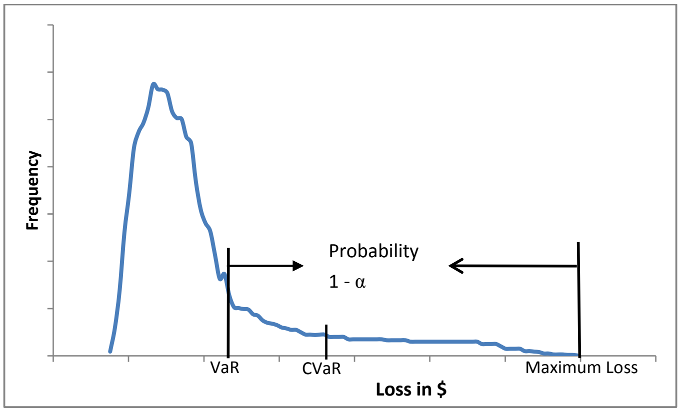

Figure 3.1: Typical CVaR approach used in portfolio management.

When using CVaR to minimize worst-case losses (as is the case for portfolio management), CVaR is always greater than or equal to VaR. CVaR is used in conjunction with VaR and is applied for estimating the risk with non-symmetric return distributions. Our approach will be to maximize CVaR in order to minimize the scenarios with losses below some targeted profit level (which will be VaR in our case). Given that our demands are AON, the scenario-based approach of CVaR can be applied, and exact solutions (based on specific problem parameters) can be obtained. CVaR can identify demands and a procurement quantity to maximize the worst-case set of resulting profits (which may actually be losses) from the distribution of possible profit scenarios. Figure 3.2 depicts how CVaR would be applied to our selective newsvendor setting in the presence of a profit distribution. The graph denotes the critical profits below VaR, as well as the average of the worst-case profits or CVaR.

In setting up the formulation, we define ζ as a decision variable denoting the VaR. Let α represent the significance level for the total profit distribution across all scenarios. In other words, ζ or VaR is the targeted profit level below which we want to

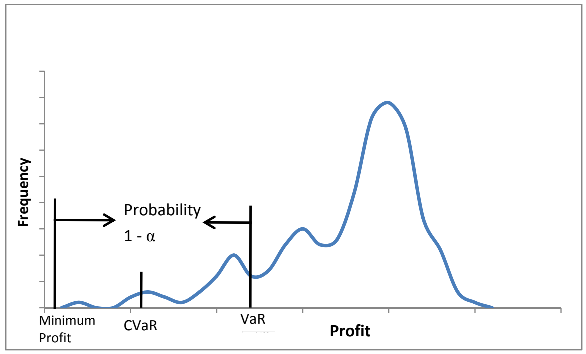

Figure 3.2: CVaR in selective newsvendor setting with profit maximization.

minimize the number of outcomes or realizations. We can also say that ζ is a decision variable based on the α-percentile of profits, i.e., in 100(1 - α)% of the scenarios, the outcome will not exceed ζ. Finally, CVaR is a weighted measure of ζ and the profits below ζ (which, again, may be extremely small profits or actually losses). We define τ_w as the tail loss for scenario w, where tail loss is defined as the amount by which losses in scenario w exceed ζ. The CVaR model for the risk averse SNP with AON demands can now be presented as:

[AON-CV]

$$\text{maximize} \quad \zeta - (1-\alpha)^{-1}\left(\textstyle\sum_{w=1}^{W} P_w \tau_w\right) \tag{3.3}$$

$$\text{subject to:} \quad \tau_w \geq \zeta - \textstyle\sum_{i=1}^{n} \left((r_i - v)d_i p_i - S_i\right) y_i + (c - v)Q$$

$$+(e - v)u_w \qquad w = 1, \dots, W, \tag{3.4}$$

$$u_w \geq \textstyle\sum_{i \in I_w} d_i y_i - Q \qquad w = 1, \dots, W, \tag{3.5}$$

$$u_w \geq 0 \qquad w = 1, \dots, W, \tag{3.6}$$

$$\tau_w \geq 0, \qquad w = 1, \dots, W, \tag{3.7}$$

$$Q \geq 0, \tag{3.8}$$

$$y_i \in \{0, 1\} \qquad i = 1, \dots, n. \tag{3.9}$$

In our production setting, the manager of a risk-averse firm wants to maximize CVaR, as shown in the objective function of [AON-CV]. Note that as we have imposed a constraint of τ_w being positive, the model tries to increase VaR and hence positively impact the objective function. However, large increases in VaR will result in more scenarios with tail losses, counterbalancing this effect. By measuring CVaR, we consider the magnitude of the tail losses to achieve a more accurate estimate of the risks of maximizing profit.

3.4.2 Observations and Data Analysis

In this section, we demonstrate the power of our CVaR approach in solving the MIP formulation of the risk averse SNP with AON demands, or the AON-CV model. All tests were conducted on a Dell desktop with a 3.0 GHz processor and 1 GB of RAM. For the implementation of our AON-CV model, we used CPLEX 10 with Concert Technology to solve mixed-integer linear problems. We considered problem instances ranging in size from 5 to 15 potential orders. Unit revenue for the orders were drawn independently from the uniform distribution on [275;325], denoted by U[275;325], and the production cost and salvage values were set to be \$200 and \$150, respectively. The expediting cost for the initial set of tests was set to \$250. Potential order sizes (or demands) were generated from a U[100;200] distribution, while the associated probabilities of realization were drawn from U[0;1]. We generated 10 random problem instances for each market size. Table 3.1 summarizes the results that compare the solutions to the expected profit and risk-averse selective newsvendor problems with AON demands. Also note that the significance level α is set to 0.75 for the AON-CV model.

Table 3.1: Comparison between solutions to [AON-EP] and [AON-CV] models.

Parameters	5 Markets		10 Markets		15 Markets	
	EP	CV	EP	CV	EP	CV
Expected Profit	14427	8282	27887	21914	43147	36483
VaR	1746	7402	15760	18678	28600	31607
CVaR	-1370	2321	3625	10614	8620	20458
Q	333	125	599	350	975	634
Avg # of Orders Selected	2.9	2.0	6.0	4.6	9.4	7.7

The expected profit obtained for AON-CV model is always less than that for AON-EP model. Under the 10-market scenario, for example, the increase in expected profit for AON-EP model is 21% over the AON-CV model. The firm orders 42% more in the AON-EP model as compared to the AON-CV model. While the decrease in number of markets selected for the AON-CV model is 23%, we can expect to have far fewer "worst-case losses" than if we chose the markets and set the order quantity as in the AON-EP model. We observe that in order to remove these scenarios, the AON-CV model orders less by selecting fewer markets as compared to the AON-EP model. The VaR and CVaR values for AON-CV are consistently higher than those for AON-EP in each market scenario. For problems with 10 markets, VaR for AON-CV is approximately 15% more than AON-EP. Also, CVaR for AON-CV is approximately 65% higher than CVaR for AON-EP. The AON-CV model attempts to improve the worst-case outcomes, thus affecting the value of CVAR.

Another main observation to mention here is that as we increase the size of the market pool either for AON-EP or AON-CV model, we tend to satisfy more demands. The effect of risk pooling tends to reduce the demand variability and results in selecting more markets and thus, ordering more. Also, the percent change in markets selected decreases as we have more markets in the starting pool.

To further understand how the CVaR approach affects the solution, we chose a particular potential order size of 10 markets to present our observations for comparing the AON-EP and AON-CV models. Figure 3.3 presents the differences in profit generated from each model, at $\alpha = 0.75$. G_{CVaR} represents expected profit for [AON-CV] and G_{AON} represents expected profit for [AON-EP]. In addition, we indicate the VaR and CVaR points on the AON-CV profit distribution.

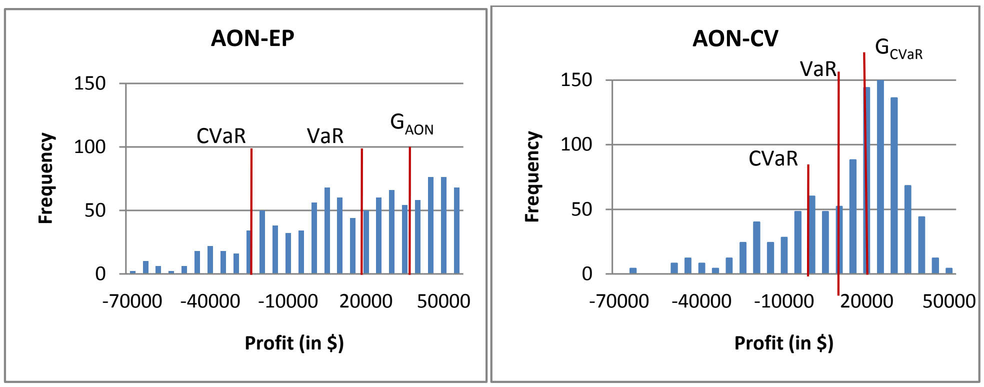

Figure 3.3: AON-CV model vs AON-EP model ($\alpha = 0.75$).

Notice the clustering of outcomes around the expected profit value for the AON-CV profit distributions. We avoid extreme outcomes (both in terms of unprofitable outcomes and highly profitable outcomes).

It is observed that the AON-CV model has fewer worst-case profit scenarios as compared to the AON-EP model. However, we also observed a reduction in expected profit for the AON-CV model, which is precisely the effect of incorporating risk in the order selection decision. Notice that the distribution becomes more peaked in the middle, indicating a tightening of the distribution.

Taaffe et al. [55] identified specific order characteristics that are most important in the acceptance/rejection decision. While unit revenue, fixed sales costs, demand size, and order likelihood all play a role, the results indicate that the probability that the order will materialize (or *order likelihood*) is the key determinant.

When using a CVaR objective, we are interested in observing how the set of markets selected changes. Figure 3.4 presents an accept/reject classification of all orders for one test instance using both the CVaR and AON-EP models, where order likelihood is plotted against unit revenue for each order. We observe that the market selection vector is different for the AON-EP and AON-CV models. Three markets, each with low probability of occurrence, were initially selected in the AON-EP model are now in the set of non-selected markets for the AON-CV model. It is interesting to know that two non-selected markets for AON-EP model are now in set of selected markets for the AON-CV model. Notice that in some cases orders with low unit revenue will

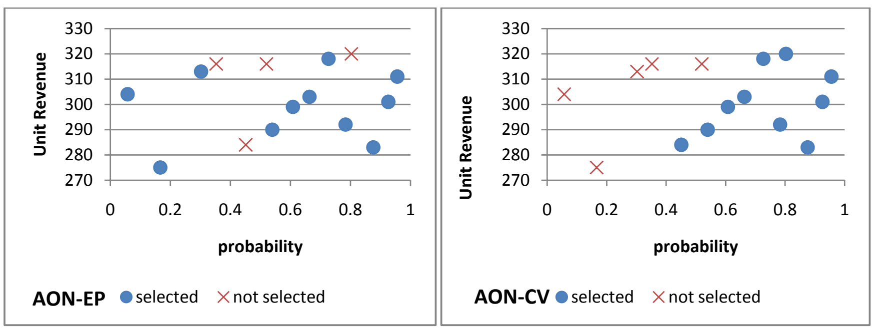

Figure 3.4: Orders selected: [AON-CV] vs. [AON-EP].

still be selected if the order likelihood is high.

3.4.3 Sensitivity Analysis and Insights

The AON-CV model produces an optimal solution to the the risk averse SNP with AON demands. In this section, we provide insights and observations to assist a decision maker in determining how certain problem parameters influence the outcomes for expected profit, total markets selected and total order quantity in the procurement policy. We investigate the effect of each of the following:

- Varying the significance level, α

- Varying the expediting cost, e
- Varying the salvage value, v
- Varying the material cost, c

In the first test, the significance level α is set at four levels of 0.5, 0.75, and 0.85, and 0.95, where the implication is that we focus on maximizing the smallest 50%, 25%, 15%, or 5% of all scenario outcomes. For the second scenario, to learn the effects of varying expediting cost we set e at 250, 375 and 500. We then vary the material cost and salvage cost respectively at four different settings to learn more about sensitivity of expected profit due to changes in these costs.

3.4.3.1 Effect of Varying Significance Level α

In this section we provide our insights into the effect of varying the significance level, α. By increasing α, we are placing additional weight on the average of the profit outcomes below ζ, and less weight on the actual value of ζ. This has the effect of increasing our risk aversity, since we are increasing our focus on worst-case outcomes (or those below ζ). We selected four levels for this parameter: 0.50, 0.75 and 0.85, and 0.95. Table 3.2 presents the comparison results for several parameters at each significance level (when the number of markets is ten).

As we increase the value of α, the model restricts the number of outcomes that exist below VaR. We also note that the number of selected markets decreases quite slowly with associated increases to α. In fact, it is not until we increase from $\alpha = 0.85$ to $\alpha = 0.95$ that we see a significant reduction in markets selected. We also observed that VaR becomes greater than expected profit when $\alpha = 0.5$. Similarly, $F_G(\zeta) = 0.414$ indicates that over 40% of the profit outcomes are below ζ in this case. With increases to α, we essentially focus on a smaller set of outcomes, and we

Table 3.2: Effect of varying α

10 Markets, e = 250				
	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.85$	$\alpha = 0.95$
Expected Profit	25562	21914	18840	10829
VaR	28068	18678	13117	5527
CVaR	17255	10615	7072	2437
Q	445	350	276	129
# of Markets Selected	5.0	4.6	3.9	2.0
$F_G(\zeta)$	0.414	0.197	0.094	0.021

increase the likelihood of having worst-case profits closer to VaR. Of course, this is at the expense of expected profit.

In Figure 3.5, we plot the results of one particular instance at all significance levels for a 10-market test instance. This setting would help us in better understanding the effect of α on the AON-CV model.

Notice how there is a tightening of the profit outcomes with higher values of α. As the decision maker becomes more risk-averse and is most concerned about only extremely poor outcomes, all effort is placed on maximizing profit in these worst-case scenarios. Thus, the left-hand tail of the profit distribution will be *maximized* at the cost of reducing expected profit. Also worth mentioning is that there is significant variation in the profit outcomes that define each distribution, and it is not clear how operating at one value of α is not necessarily better than operating at another value of α. Such a presentation of information, however, will allow a manager to make a more informed demand management decision.

3.4.3.2 Effect of Varying Expediting Cost, e

To study the effect of expediting cost in the AON-CV model, we employed three different levels for expediting costs as \$250, \$375 and \$500. Using the same

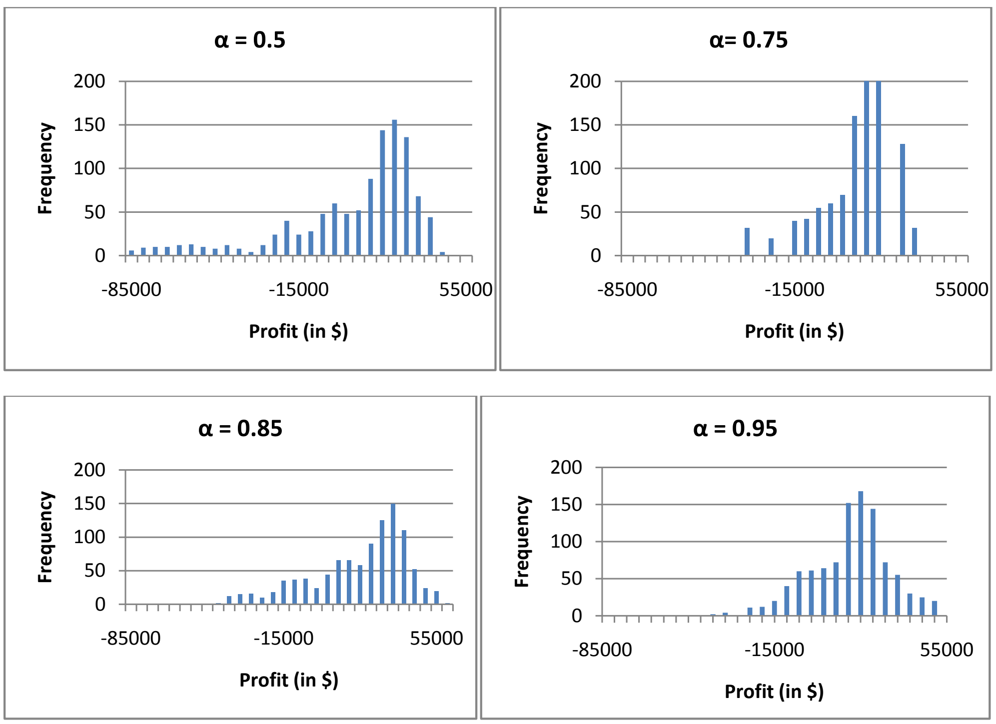

Figure 3.5: Effect of α.

10-market problems as previously described, Table 3.3 presents our findings when $\alpha = 0.75\%$. As the expediting cost is increased from \$250 to \$500, the number of

Table 3.3: Effect of varying expediting cost

10 Markets, $\alpha = 0.75$			
	$e = \$250$	$e = \$375$	$e = \$500$
Expected Profit	21914	14478	12395
VaR	18678	13849	10299
CVaR	10614	6733	5294
Q	350	347	271
# of Markets Selected	4.6	3.2	2.0
$F_G(\zeta)$	0.197	0.187	0.130

markets to be pursued by the firm decreases and, as a result, the order quantity decreases. In this case, the reduction in number of markets selected is fairly steady, while expected profit drops significantly from $e = 250$ to $e = 375$, as compared to the drop from $e = 375$ to $e = 500$.

Higher expediting costs lead the firm to be more selective in how many orders to accept and resulted in decreased expected profit. It is clear that competing firms may find themselves in a marketplace with varying expediting costs, and knowing how to quantify the reduction in expected profits and associated market riskiness would be extremely beneficial during the production planning and procurement phase. The percent of scenarios below ζ is decreasing in e, but since VaR and CVaR are both decision variables, and since we are not directly changing α, this change in outcomes below ζ is more difficult to interpret.

In Section 3.4.3.1, recall that an average of 2.0 markets were selected with $e = 250$ and $\alpha = 0.95$ in Table 3.2. Similarly, we observe that 2.0 markets were also selected with $e = 500$ and $\alpha = 0.75$ in Table 3.3. However, a much higher order quantity and expected profit (on average) are associated with this result. Since

$\alpha = 0.75$ with this higher expediting cost, we are allowing more risk into the objective. Thus, it is not surprising that the expected profit from this particular setting is higher than the expected profit at $\alpha = 0.95$ in Table 3.2.

3.4.3.3 Effect of Varying the Salvage Value, v

Using the same 10-market problems as previously described, we used different settings for salvage cost as follows:

- base case level as \$150 (used in all test scenarios),
- salvage cost less than base case, \$100,
- salvage cost more than base case but less than material cost c, \$180,
- salvage cost equal to material cost c, \$200.

Table 3.4 presents our findings when $\alpha = 0.75\%$. As the salvage cost is increased

Table 3.4: Effect of varying salvage value

10 Markets, c= 200, e = 250				
	$v = 100$	$v = 150$	$v = 180$	$v = 200$
Expected Profit	19780	21914	24268	31674
VaR	16700	18678	20222	22767
CVaR	9349	10615	11946	13443
Q	295	350	386	720
# of Markets Selected	4.4	4.6	4.7	4.9
$F_G(\zeta)$	0.19	0.197	0.22	0.22

from \$100 to \$200, the number of markets to be pursued by the firm slowly increases, whereas the order quantity and expected profit increase dramatically. Moreover, the increase in order quantity and expected profit is most significant when salvage cost is increased from $v = 180$ to $v = 200$. This is due to a special structure of the newsvendor problem which lets the model to order more as both salvage cost and material cost being equal. We also note that the percent of scenarios below ζ is increasing in v.

3.4.3.4 Effect of Varying Material Cost, c

Using the same 10-market problems as previously described, we used different levels for material cost as follows:

- base case level as \$200 (used in all test scenarios),
- material cost equal to salvage cost v, \$150,
- material cost more than base case but less than expediting cost e, \$230,
- material cost equal to expediting cost e, \$250.

Table 3.5 presents our findings when $\alpha = 0.75\%$.

Table 3.5: Effect of varying material cost

10 Markets, v= 150, e = 250				
	$c = 150$	$c = 200$	$c = 230$	$c = 250$
Expected Profit	64255	21914	8787	4369
VaR	48207	18678	7289	4767
CVaR	33003	10615	3247	1434
Q	958	350	146	0
# of Markets Selected	6.5	4.6	4.7	1.0
$F_G(\zeta)$	0.23	0.197	0.13	0.09

Increasing material cost resulted in decreasing expected profit, VaR, CVaR, Q and number of markets selected. This is due to a basic fundamental result that increasing cost results in decreased demand. If we decrease material cost to a value equal to the salvage value, as expected there is over a 65% increase in profit. However, when we increased unit cost from \$200 to a value equal to expediting cost, it resulted 0 units being ordered. Essentially, there is no advantage to purchasing in advance of realizing actual demand. Thus, all demand will be satisfied through expediting. As the material cost is increased from \$150 to \$250, the number of markets to be pursued by the firm decreases and, as a result, the order quantity decreases. In this case, the reduction in number of markets selected is fairly steady except when we change c from 230 to 250, while expected profit drops significantly across each change in material cost.

We introduce Figure 3.6 to depict the change in expected profit based on changes to c, v, or e. We observe that expected profit is most sensitive to changes in material cost, followed by expedite cost and then salvage value.

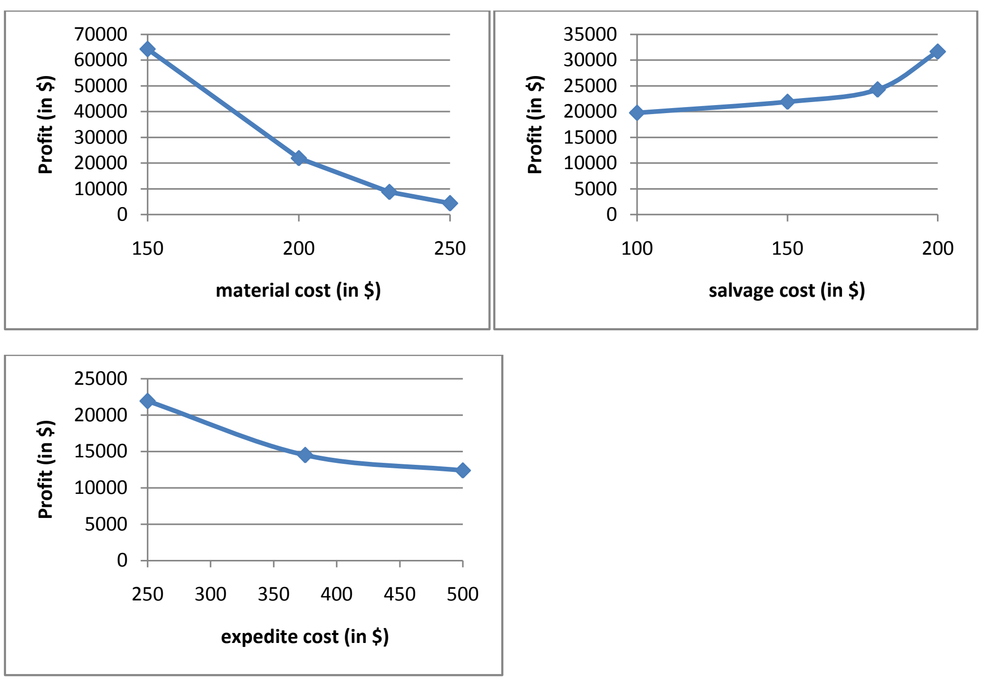

Figure 3.6: Sensitivity of expected profit due to various costs.

3.5 Mean-CVaR Model Considerations

Section 3.4 provided a comparison of using a risk-based objective against a reward-based objective. The value of risk vs. reward has been a subject of extensive research since the early works of Markowitz [33], and it deserves treatment here as well. While a firm would like to believe it could choose to manage its procurement and production operation using only one of these objectives, it is often most desirable (and realistic) to consider some (convex) combination of these conflicting objectives. We provide several mean-CVaR models to address this concern.

3.5.1 A Weighted Objective Formulation: Expected Profit and CVaR

A firm might have conflicting desires at one time of either maximizing profit or reducing worst-case outcomes. For these purposes, it is worthwhile to study different forms of risk aversion models. If we want to reduce minimum worst-case profit outcomes, we can use the AON-CV model. For other cases, we can employ a combination of these AON-EP and AON-CV according to the criticality of each competing objective.

One such logical extension to the AON-CV model would be the inclusion of total expected profit in the objective function. This formulation now maximizes both the total expected profit, AON-EP model's objective, and the objective of AON-CV model. All of the constraints for the AON-CV model remain, however the objective function will now be a convex combination of the CVaR and AON objective functions.

We state the problem as follows:

[WOBJ]

$$\text{maximize} \quad \lambda[\sum_{i=1}^{n}((r_i - v)d_i p_i - S_i)\, y_i - (c - v)Q -$$

$$(e - v)\sum_{w=1}^{W} P_w u_w] + (1 - \lambda)[\zeta - (1 - \alpha)^{-1}(\sum_{w=1}^{W} P_w \tau_w)] \quad (3.10)$$

$$\text{subject to :} \quad 0 \leq \lambda \ \leq 1, \quad (3.11)$$

$$\text{Constraints (3.4)-(3.9),}$$

where λ is the weighting parameter. In order to study the behavior of this model, we chose five different values of λ as 0, 0.25, 0.5, 0.75, and 1. We provide an illustration of the effect of using the [WOBJ] formulation to help the decision maker in setting the optimal policy according to the firm's importance of profit vs. risk. Table 3.6 summarizes the results for the 10-market case with $e = 250$ and $\alpha = 0.75$.

Table 3.6: The effect of λ on model results (e=250, α = 0.75)

Model	Expected Profit	Q	Average # Selected	VaR	CVaR	$F_G(VaR)$	$F_G(20000)$
AON-CV	21914	350	4.6	18678	10614	0.221	0.271
WOBJ ($\lambda = 0.00$)	21914	350	4.6	18678	10614	0.221	0.271
WOBJ ($\lambda = 0.25$)	23968	388	5.0	20166	10400	0.208	0.272
WOBJ ($\lambda = 0.50$)	24996	422	5.0	21530	9751	0.239	0.273
WOBJ ($\lambda = 0.75$)	27231	517	5.8	21850	5706	0.235	0.277
WOBJ ($\lambda = 0.90$)	27817	572	5.9	20032	3077	0.233	0.285
WOBJ ($\lambda = 0.99$)	27887	599	6.0	18139	1968	0.232	0.302
WOBJ ($\lambda = 1.00$)	27887	599	6.0	15760	3625	0.249	0.302
AON-EP	27887	599	6.0	15760	3625	0.249	0.302

The weighted objective with $\lambda = 0$ is the same model as [AON-CV] and thus we have the identical results for these cases. Similarly, [WOBJ] with $\lambda = 1$ produced the same results as [AON-EP]. Among the objective weighting options available to the manager, the AON-EP model provides an upper bound on the expected profit value.

This is explained due to no involvement of risk and the objective being purely that of maximizing expected profit. On the other hand, the AON-CV model incorporates the risk factor and thus acts as a lower bound on expected profit. Thus, the firm can control the level of risk allowed into the solution by setting the value of λ appropriately. We note that the effect of introducing a weighting parameter λ through the new model [WOBJ] has comparisons to adjusting the value of α in Section 3.4.3.1, however in that case we only consider the expected profit equation implicitly through changes to α.

The value of $F_G(x)$ represents the probability of outcomes with worst-case profits below x, where $G()$ is the profit equation shown in equation (4.3). In this table, we provide $F_G(VaR)$ as well as $F_G(20000)$. As the value of λ increases, the value of VaR is also changing. This results in no pattern or trend in how $F_G(VaR)$ changes. However, when we consider a fixed profit value (in this case, \$20,000), we can clearly see a pattern. With increases to λ, $F_G(20000)$ also increases. In other words, as we move towards an expected profit objective, the percentage of more extreme scenarios being introduced into the solution also increases.

Figure 3.7 illustrates the trend of expected profit, VaR, and CVaR as a function of λ. It appears that we observe expected profit increasing linearly with increases in λ. However, VaR and CVaR do not follow this pattern.

3.5.2 Minimum acceptable CVaR level

Another mean-CVaR modeling approach introduces a restriction (or constraint) on allowable worst-case profits. We use the objective of maximizing total expected profit, while satisfying a constraint requiring the percentile of worst-case profits to be no less than some parameter v. In other words, we use an AON-EP objective function with the AON-CV constraints, but we also include a bound on the value of the

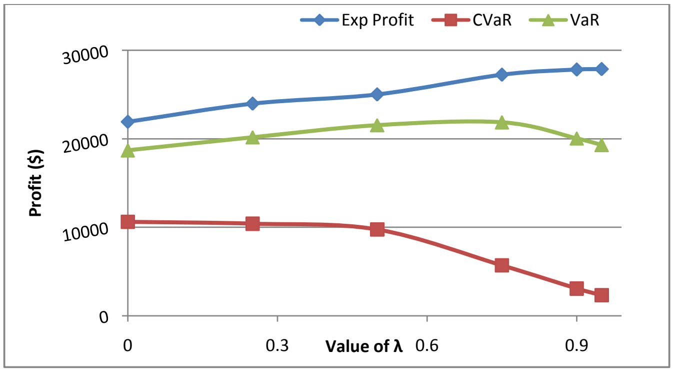

Figure 3.7: Expected profit and "at-risk" measures vs. λ.

AON-CV objective as a constraint in this new model. We introduce this formulation as:

[AON-MinCV]

$$
\begin{array}{lll}
\text{maximize} & \sum_{i=1}^{n} \left((r_i - v) d_i p_i - S_i \right) y_i - (c - v)Q - (e - v) \sum_{w=1}^{W} P_w u_w & \\
\text{subject to :} & \zeta - (1-\alpha)^{-1} (\sum_{w=1}^{W} P_w \tau_w) \geq v & w = 1, \ldots, W, \\
& \tau_w \geq \zeta - \sum_{i=1}^{n} \left((r_i - v) d_i p_i - S_i \right) y_i & \\
& -(c - v)Q - (e - v) u_w & w = 1, \ldots, W, \\
& u_w \geq \sum_{i \in I_w} d_i y_i - Q & w = 1, \ldots, W, \\
& u_w \geq 0 & w = 1, \ldots, W, \\
& \tau_w \geq 0, & w = 1, \ldots, W, \\
& Q \geq 0, & \\
& y_i \in \{0, 1\} & i = 1, \ldots, n.
\end{array}
$$

In setting the value of v, we must take care in not imposing such a strict bound that the result is always an infeasible solution. One approach would be to solve [AON-CV] first, and then try several values of v, none of which should exceed the observed VaR from the AON-CV model. To find an appropriate lower bound value on v, one could solve [AON-CV] with a large α. We use this approach in presenting results in Table 3.7. This table provides the sensitivity of several key parameters to the value of v, at a value of $\alpha = 0.75$.

Table 3.7: AON-MinCV Model results at several CVaR levels

CVaR Bound	Total Expected profit	VaR	CVaR	$F_G(\zeta)$	Q	y
v = 3000	28435	12855	3000	0.168	544	5.1
v = 6000	27473	15557	6000	0.159	498	4.8
v = 9000	27166	17227	9000	0.143	463	4.2
v = 12000	19120	17067	12000	0.072	296	2.0

Notice how there is little change in expected profit and percent of outcomes below ζ for v at 3000, 6000, and 9000. This is followed by a significant reduction in profit and the number of markets selected when $v = 12000$. To understand this, consider the results presented in Table 3.1. The maximum CVaR attained in the [AON-CV] was found to be 10614 on average. If we attempt to maximize expected profit with a bound on CVaR that is lower than 10614 (on average for the problems tested), the expected profit objective is not significantly affected. As we increase the CVaR bound beyond 10614, we notice much larger shifts in the characteristics in the solution.

3.5.3 Risk Minimization with a Profit Constraint

Firms may have specific knowledge about a target minimum profit that they require to stay in business, remain profitable, or otherwise must meet for the current selling season. Under such a setting, the firm could be considered a “risk minimizer” and would focus solely on minimizing worst-case profits (potentially losses) below some critical level. In the previous chapter we present our work on this approach to the so-called risk averse selective newsvendor problem RA-SNP for normally distributed demands. A simple constructive heuristic was proposed, using simulation, to identify markets to accept and an associated procurement policy.

We introduce the RA-SNP model and objective for comparison purposes with the CVaR approach in this research. The RA-SNP model is presented as follows:

$$
\begin{array}{llll}
\textbf{[RA-SNP]} & \text{minimize} & F_G(P) & \\
 & \text{subject to:} & Q \geq 0, & \\
 & & y_i \in \{0, 1\} & i = 1, \ldots, n.
\end{array}
$$

In this model, F_G is the cumulative distribution of profit and P denotes the critical profit level, below which the firm wants to minimize the potential outcomes or scenarios.

While both [RA-SNP] and [AON-CV] are risk-minimizing models, minimizing $F_G(P)$ is not synonymous with maximizing CVaR. The AON-CV model approach does not fix the value below which the percent of outcomes is minimized. Rather, we allow the threshold value (or VaR) to be increased or decreased in order to achieve the maximum value for CVaR. The CVaR approach indirectly results in reducing the number of worst-case profit scenarios, and we include $F_G(\zeta)$ in Table 3.6 for this reason. Moreover, it is an *exact approach* to incorporate risk. The difficulty in applying CVaR when demands are normally distributed provided motivation for the RA-SNP modeling approach. Figure 3.8 illustrates the difference between the two approaches. Where CVaR represented a single point on the x-axis, we have $F_G(P)$ representing the probability of experiencing outcomes below a critical profit threshold of P.

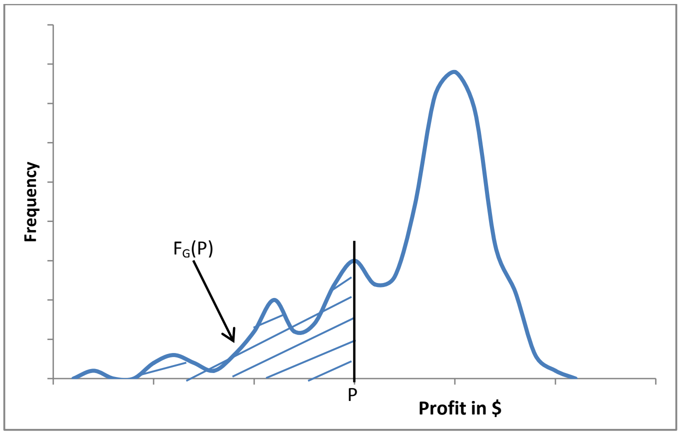

Figure 3.8: Illustration of RA-SNP approach.

3.6 Simulation Approach for Risk Averse SNP

Using simulation we will estimate the distribution for $F_G(\zeta)$ for the Bernoulli and Uniform demand distributions. A key assumption in this body of work is that the heuristic approach developed in Chapter 2 can also be applied here. The heuristic was based on normal demands, so it is unclear whether this will provide robust results for uniform and Bernoulli demands. We plan to offer insights into the behavior of these distributions when subjected to a risk averse objective. However, the purpose of this work was to demonstrate the shape of the resulting profit distribution as well as to observe the change in expected profit if we restrict the percent of allowable losses or worst-case profits below the critical/acceptable profit level P.

For the case in which demands are normally distributed, expected demand and demand variance for each market are drawn from the uniform distribution on [500, 1000] and [50000, 100000], denoted as U[500, 1000] and U[50000, 100000]. We simulated 10,000 profit realizations of G(Q, y) in order to estimate the shape of the distribution for the profit equation. Unit revenue for the orders were drawn independently from the uniform distribution on [275; 325], denoted by U[275; 325], and the production cost, expediting cost, and salvage values were set to be 200, 500, and 150, respectively. The potential order sizes (or demands) were generated from a U[100; 200] distribution, while the associated probabilities of realization were drawn from U[0; 1].

3.6.1 Simulation Approach for Risk Averse SNP with AON Demands

The heuristic adopted from previous chapter is used to minimize percent of worst case losses, and to find the associated expected profit, and order quantity based on the markets chosen. We offer insights into the behavior of the Bernoulli and Uniform distributions when subjected to a risk averse objective. We have considered the 10 test instances created for each size of market pool: 10, 20, and 30. Every market has unit revenue in the range U[$275; $325], while unit production cost is $200. The fixed cost for market entry are drawn from U[$2500, $7500], and salvage value is set as $150 per unit. Finally, the expediting or shortage cost is $500 per unit. We also tested three distinct critical profit levels to observe the effect on market selection, $F_G(P)$ and overall order quantity. These profit levels are calculated as 25%, 10% and 5% of the expected profit given by the SNP approach without risk, denoted as $\hat{G}_{SNP}$, refer to Taaffe and Tirumalasetty [56]. All computational tests were conducted on Dell desktop machines with a 3.0 GHz processor and 1 GB of RAM.

To obtain the estimated SNP values of G and Q for these distributions we have applied the same formulation of SNP presented in last chapter with the exception that mean and variance of demand were used of the respective distributions.

We consider a situation where an order may either come in at a predefined level or not at all, i.e., demand for market i is governed by a Bernoulli distribution:

$$\Pr(D_i = x) = \begin{cases} 1 - p_i & \text{if } x = 0 \\ p_i & \text{if } x = d_i. \end{cases}$$

Here p_i is drawn from U[0, 1]. Parameters required for performing simulation tests are described as follows: expected demand for each market is independently drawn from U[500, 1000]. Now, to correctly represent the expected demand for Bernoulli distribution, expected demand and demand variance for each market are given by:

$$\mu_i = p_i * D_i$$
$$\sigma^2 = (1 - p_i) * p_i * D_i$$

We used market pool sizes of 10, 20 and 30 and created a set of 10 test instances for each potential market pool. We ran several similar simulation tests for various test instances each with 10,000 realizations and obtained the consistent result for each test instance regarding the shape of the distribution for profit equation as shown in Figure 3.9. This figure shows the profit distribution for Bernoulli demands for a sample test instance. It is an estimated shape of the profit equation distribution for the risk averse AON-SNP (i.e., with demand flexibility allowing for market selection by the firm) and the scenario when all markets have been selected for a sample test instance. Due to the all-or-nothing demand realizations, the resulting variance is much higher. We can still say it is left-skewed to some extent.

The heuristic adopted in chapter 2 is used to minimize number of worst-case profit scenarios, and to find the associated expected profit, and order quantity based on the markets chosen. In Bernoulli distribution there is probability p_i associated

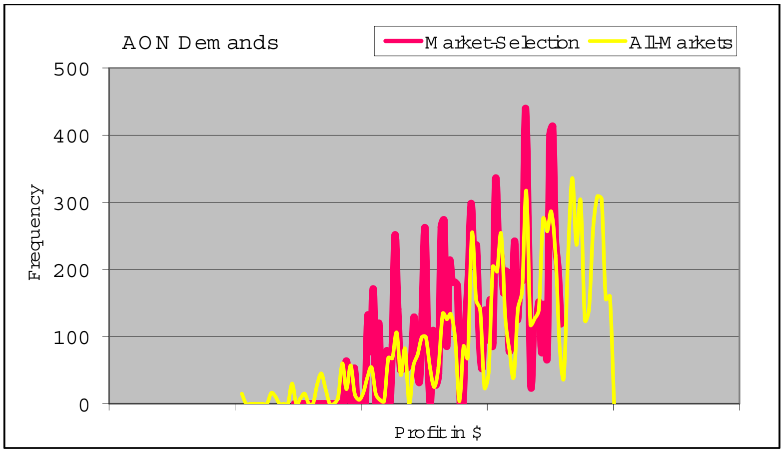

Figure 3.9: Profit Distribution for AON.

with each demand to materialize. Table 3.8 indicates that the effect of combination of heuristic and [RA-SNP] is minimal. There is negligible difference between the expected values of expected profit approach and that of risk averse SNP.

Table 3.8: Procurement policies for AON demands

Parameters	Bernoulli Distribution			Estimated SNP values
	P=0.25$\hat{G}_{AON}$	P=0.10$\hat{G}_{AON}$	P=0.05$\hat{G}_{AON}$	
10 Markets				
G	335421	338467	321620	G = 346428
$F_G(P)$	0.132	0.048	0.032	
Q	2809	2714	2565	Q = 3944
20 Markets				
G	671765	663370	664786	G = 679655
$F_G(P)$	0.047	0.008	0.004	
Q	5536	5289	5231	Q = 7781
30 Markets				
G	1028426	1026289	1016237	G = 1036447
$F_G(P)$	0.022	0.002	0.000605	
Q	8363	8071	7878	Q = 11892

3.6.2 Simulation Approach for Risk Averse SNP with Uniform Demands

Here the assumption is that the market demands are independently drawn from uniform distributions. We applied the basic properties of uniform distribution to come up with the range for market demands. We begin by assuming that we still have μ_i and σ^2 for each market in the range of U[500, 1000] and U[50000, 100000] similar to the normal distribution. With known μ_i and $(\sigma_i)^2$ values, we now calculate U[a_i, b_i] for each market demand:

$$(\sigma_i)^2 = (b_i - a_i)^2/12 \text{ , which implies } (b_i - a_i) = \sqrt{12}\sigma_i.$$

Now, we can find a_i and b_i as follows:

$$\begin{aligned} a_i = &\quad \mu_i - \sqrt{12}\sigma_i/2 \\ b_i = &\quad \mu_i + \sqrt{12}\sigma_i/2 \end{aligned}$$

We ran the same type of simulation test for 10,000 profit realizations for various test instances for each size of market pool: 10, 20, and 30. Figure 3.10 provides the profit distribution for uniformly distributed demands for a sample test instance. We show the risk averse SNP with market selection flexibility versus the case when all markets have been selected.

Each time we obtained a similar shape of the distribution for the profit equation. The profit distribution is again left-skewed. As expected we observed that when more markets have been rejected by the heuristic, then the corresponding graph has fewer extreme losses and fewer extreme profits. The heuristic method resulted in different market selections as compared to AON demands in above section or normal

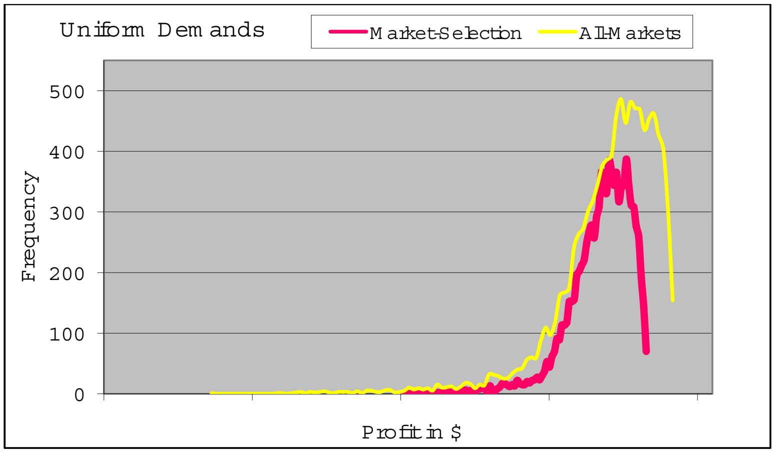

Figure 3.10: Profit Distribution for Uniform.

demands of previous chapter. For instance, extreme values in either tail of the normal demands distribution are less probable as compared to extreme values of the Uniform distribution where all values are equally likely. In Table 3.9 (Uniform distribution),

Table 3.9: Procurement policies for Uniform demands

Parameters	Uniform Distribution			Estimated SNP values
	P=0.25$\hat{G}_{AON}$	P=0.10$\hat{G}_{AON}$	P=0.05$\hat{G}_{AON}$	
10 Markets				
G	78167	77930	77153	G = 82242
$F_G(P)$	0.127	0.089	0.081	
Q	6921	6876	6927	Q = 7865
20 Markets				
G	187592	184195	180168	G = 192121
$F_G(P)$	0.053	0.024	0.017	
Q	18286	12831	13209	Q = 16285
30 Markets				
G	296360	294869	285853	G = 308908
$F_G(P)$	0.022	0.008	0.003	
Q	27205	25995	18259	Q = 25022

for the specific market pool we observed less than 10% reduction in expected profit values while minimizing the worst-case losses. Also, the difference between expected profit for SNP approach and that of risk averse SNP is again minute. We observed all these results are more for the reason how the heuristic behaves rather than the influence of demand distribution.

3.7 Conclusions

We offer several alternate market selection and product ordering models that vary in their inclusion of risk. We introduced an exact solution approach to incorporate risk that uses Conditional Value-at-Risk via scenario analysis. We showed that increase in the size of market selection pool increases the expected profit. Through understanding how market selection is affected by competing objectives of expected profit and profit risk, we provide the decision maker in any firm with a tool set for assessing how much risk the firm can take. This exchange between expected profit and risk has been presented in several ways, via a combined objective model, alternate values for the significance level in the CVaR model, and minimum acceptable values for CVaR. Depending on the direction governed by any project team, the decision maker can assess the value of profit and risk to the team, and choose how to present the recommended market selection and procurement quantity decisions to the team.

Similar to Eeckhoudt et al [21], the optimal risk averse order quantity is less than the amount ordered by reward based model. The risk averse model also results in fewer worst case profit scenarios as compared to the expected value solution. We also found that the probability of materializing an order is also a key determinant in order acceptance/ rejection decision. In risk averse model, an order will be selected if it likelihood is high even if it has low unit revenue. We also observed that, for risk averse setting, higher expediting and material costs resulted in decreased expected profit and to be more selective in how many orders to accept.

We have tested potential market sizes from 5 to 15. While we can analyze these cases in quite thorough detail, there is clearly motivation for studying larger problem sizes. However, the problem suffers from the curse of dimensionality, and the exact solution approach proposed in the current work becomes intractable using CPLEX and Concert Technology. We plan to consider heuristic approaches to solving such problem sizes, determining upper bounds that can help identify the quality of solutions produced by a heuristic.

We analyzed the simulation based heuristic developed in Chapter 2 for Bernoulli and Uniform demands. Simulation analysis helped us to investigate the resulting profit distributions for each of the demand distributions, which are again used in determining respective procurement policies. As mentioned earlier, all the results were more influenced by the behavior of the heuristic (presented in chapter 2) as compared to demand distributions.

3.8 Appendix

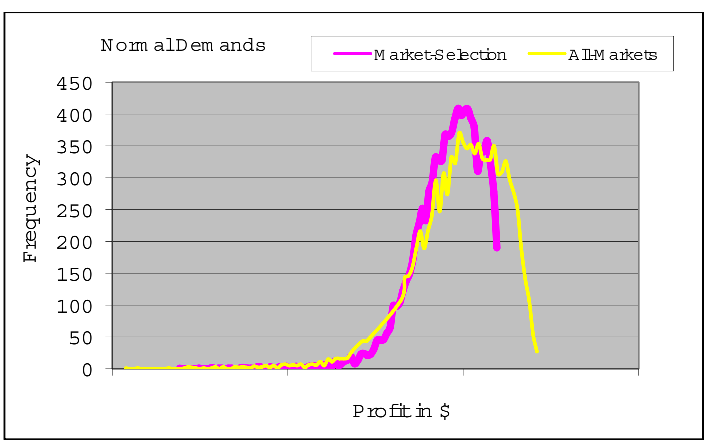

Figure 3.11: Profit Distribution for Normal.

Chapter 4

Extensions to the Selective Newsvendor Problem

4.1 Abstract

Consider a newsvendor problem in which unique demands can be pursued or rejected as part of the procurement plan. This has been known as the selective newsvendor problem (SNP). In this research, we examine several extensions to the SNP. First, we consider the impact of incorporating market-specific expediting costs into the demand selection and procurement decisions. Second, using a lost sales assumption instead of an expediting assumption, we perform a similar analysis by allocating a penalty or cost for the lost sales. We present various ideas and approaches for each of these extensions to the SNP.

4.2 Background and Literature Review

The changing fashion trends and technological advances results in ever decreasing product lives and allows very little room for improvement in efficiency of the manufacturing processes. In an effort to stay competitive, firms constantly seek alternative ways to improve profitability. In this chapter also, we consider a supplier or a firm that offers a product for a single selling season. The firm can obtain unique revenues in each demand source, and it can select demand sources it wants to pursue. To maximize the expected profit, the firm must simultaneously select the most desirable demand sources as well as determine the appropriate total order quantity before demand is actually realized. This type of demand flexibility has been previously described and modeled by Taaffe, Geunes and Romeijn [53] and Bakal, Geunes, and Romeijn [5]. They refer to this type of problem as the "selective newsvendor problem" or SNP. In Taaffe et al. [53], the authors show how individual demand sizes (and the selection of those demands) can be influenced through targeted advertising or marketing. This is introduced for operations in which marketing resources are either unlimited or constrained. One assumption made in their research is the requirement that demand in all "selected" markets be completely fulfilled, which may require expediting from local high-cost suppliers. This is a reasonable requirement because if the firm does not satisfy the selected markets fully, these markets may not want to purchase again from the firm in subsequent periods. In Bakal et al. [5], they introduce the effect of pricing considerations on market selection. In a market-specific expediting cost extension of the SNP, we also assume that once a supplier/firm knows the actual materialized demand, it still must satisfy all pursued demands by expediting (the portion of demand which is in excess of the total order quantity) from local high-cost suppliers. However, we relax this assumption in a lost sales extension of the

SNP. We assume that if realized demand for selected markets is more than the total order quantity, then the remaining unsatisfied portion of the realized demand is lost and we associate a fixed penalty with this portion of the lost sales/demand.

There is an extensive literature on stochastic inventory control and more specifically, the newsvendor problem, some of which are Porteus [39], Tsay, Nahmias, and Agarwal [58], Cachon [10], and Petruzzi and Dada [36]. The classical newsvendor problem is to find an optimal procurement policy for a product with random demand during a single selling season and it has been widely studied by researchers. In its simplest form there is a single item with stochastic demand observed for one period. Again, a wide number of extensions have also been studied in the literature. Khouja [29] presents an extensive literature review on the newsvendor problem and its various possible extensions. One of the basic extensions is studying a multi-item newsvendor case. In this extension, every item has its own unique and independent demand and unique revenue associated with an item. Similar to a multi-item newsvendor problem, we have different markets each with an uncertain demand and an unique revenue associated with it. The multi-item newsvendor problem still assumes all demands are included in the cumulative demand distribution, whereas, the idea of selecting profitable (or the most desirable set of) markets differentiates our research from the multi-item newsvendor problem. However, we review the literature to explore the existing techniques for identifying an appropriate order quantity in a multi-item newsvendor setting. We study the possibility of blending one such technique with our research problem based on the set of selected markets.

Hadley and Whitin [27] study the multi-item constrained newsvendor problem and presented policies to obtain the optimal order quantity. They develop two algorithms. The first is based on a search for Lagrangian multipliers which is suitable when order quantities are large, and results of this algorithm will have to be rounded to integers. They present a second algorithm called marginal analysis to find an integer solution for the case when optimal order quantities are small and rounding may have a significant impact on expected value of profits. Silver, Pyke and Peterson [48] provide detailed mathematical analysis of the constrained multi-item single period newsvendor problem and a review of many extensions related to it. Moon and Silver [35] revisit the study by Hadley and Whitin [27] on multi-item newsvendor problem with budget constraints and fixed ordering costs. They present dynamic programming procedures settings in which i) demands follow a normal distribution or ii) demands are distribution-free. They also present a simple heuristic approach for solving these problems. Abdel-Malek, Montanari and Morales [1] consider a similar problem with budget constraints. They present an exact solution method when the demand probability density function for each item is assumed to be uniform, as well as a general iterative method yielding near optimal solutions for general continuous density demand functions. Erlebacher [23] also presents optimal and heuristic solutions for solving a capacitated newsvendor problem with multiple items.

Motivated from the literature presented on a multi-item newsvendor problem and assumptions made in Taaffe et al. [53], in this chapter we present various extensions to the selective newsvendor problem. For each extension to the SNP we present two approaches: a rationing approach and a greedy approach. In the rationing approach we ration the total shortages or lost sales across all of the selected markets and present an exact mathematical formulation as a mixed integer nonlinear programming problem (MINLP). We use GAMS software to solve this MINLP. For the greedy approach, we try to satisfy the demand of most attractive markets and allocate the shortage to a least attractive market. We show that a reasonable MINLP cannot be formulated and solved. Instead, we introduce simulation-based heuristics that incorporate ideas from past work with alternatives for finding the order quantity. First, we consider the impact of incorporating market-specific expediting costs into the demand selection and procurement decisions. Then, using a lost sales assumption instead of an expediting assumption, we perform a similar analysis using a single lost sales cost (but, in effect, this case still has market-specific characteristics due to unique revenues in each market). The structure of our problem illustrates similarities to that of a multi-item newsvendor problem, and literature has provided some insights into allocating capacity to satisfy the demand of individual items. For a given set of selected markets, we can also treat each market as an independent newsvendor problem and find market-specific order quantities. The cumulative sum of all individual order quantities then provides the total order quantity for the whole set of selected markets.

In the remaining sections we explain these ideas and approaches in detail for each of these extensions to the SNP. The rest of this chapter is organized as follows. For a brief review of the SNP we begin with a brief introduction to SNP in Section 4.3. In Section 4.4 we present a discussion and the problem formulation to incorporate the effect of a market-specific expediting cost in SNP. An assumption of lost-sales is explained in Section 4.6. Section 4.7 concludes this chapter with our remarks and future work directions.

4.3 Selective Newsvendor Problem

Our work in this chapter builds upon the research first introduced in Taaffe et al. [53]. To maintain consistency with prior chapters, we define D_i as the random variable representing demand in market i, $(i = 1, \ldots, n)$. Denote c as the per-unit cost of obtaining or procuring the product to be sold. The product can be sold in market i at a per-unit price of r_i. If realized demand is less than the quantity ordered, the firm can salvage each remaining unit for a value of v. If demand exceeds the order quantity, there is a shortage cost/expedite cost of e per unit in market i.

Let $y_i = 1$ if the firm decides to satisfy demand in market i, and 0 if the firm rejects market i's demand. Also assume that S_i represents the entry or fixed cost of choosing market i. The total realized profit, based on the order quantity, market selection decisions, and realized demand, is expressed as follows:

$$H(Q,y) = \begin{cases} \sum\limits_{i=1}^{n}(r_iD_i - S_i)y_i - cQ + v(Q - \sum\limits_{i=1}^{n} D_iy_i) & Q > \sum_{i=1}^{n} D_iy_i \\ \sum\limits_{i=1}^{n}(r_iD_i - S_i)y_i - cQ - e(\sum\limits_{i=1}^{n} D_iy_i - Q) & Q \leq \sum_{i=1}^{n} D_iy_i \end{cases}.$$

Given a binary vector of market selection variables y, and letting $D^y = \sum_{i=1}^{n} D_iy_i$ represent the total demand of the selected markets, the mean and variance of total demand are $E(D^y) = \sum_{i=1}^{n} \mu_iy_i$ and $\text{Var}(D^y) = \sum_{i=1}^{n} \sigma_i^2 y_i$, respectively. Then the firm's expected profit is expressed as a function $G(Q,y)$ of the order quantity Q and the binary vector y:

$$\begin{aligned} G(Q,y) = \sum_{i=1}^{n}(r_i\mu_i - S_i)y_i - cQ + vE\left[\max\left(0, Q - \sum_{i=1}^{n} D_iy_i\right)\right] \\ - eE\left[\max\left(0, \sum_{i=1}^{n} D_iy_i - Q\right)\right]. \end{aligned} \tag{4.1}$$

The general *selective newsvendor problem* [SNP] is now given by

$$\begin{aligned} \textbf{[SNP]} \quad & \text{maximize} \quad G(Q,y) \\ & \text{subject to:} \quad Q \geq 0 & (4.2) \\ & \qquad y_i \in \{0,1\} \qquad i = 1,\ldots,n. & (4.3) \end{aligned}$$

As shown in Section 2.3.1 there exists an optimal solution for the case of normal demands using sorting mechanism defined by DERU property.

4.4 Market-Specific Expediting Cost

We now suppose that the cost to expedite units after demand realization is market-specific, i.e., we let e_i denote the cost per unit for expediting units to market i after demand is realized. We would like to mention that in some cases expediting costs can exceed the revenue generated for a particular market, thus leading to reasons that we would not select a market. However, demand uncertainty also plays a large role in defining the attractiveness of a market. Thus, the less predictable that demand is, the more likely it is that we will not select that market. We define different sorting mechanisms to select markets based on various key indicators such as e_i, r_i, μ_i and σ_i^2. We must now define a strategy for allocating the original Q units received from the overseas supplier, thus determining the amount of "shortage" each selected market faces upon demand realization. We present two allocation approaches: rationing and greedy.

4.4.1 Rationing the Shortages

In this approach we ration shortages equally among all markets. Consider I_1 to represent the set of selected markets. Then, for a given vector y, the shortage in market i now becomes:

$$\frac{(D^y - Q)^+}{|I_1|},$$

where $|I_1| = \sum_{i \in I_1} y_i$, which is simply the number of selected markets. The expected shortage cost for market i can then be written as

$$\frac{e_i}{|I_1|} \int_Q^{\infty} (D^y - Q) f(D^y) dD^y = \frac{e_i}{|I_1|} \Lambda_{D^y}(Q).$$

where we define $\Lambda_{D^y}(Q)$ as the loss function for a given order quantity Q and market selection vector y. The expected total profit equation now becomes

$$G(Q, y) = \sum_{i \in I} (r_i \mu_i - S_i) y_i - cQ + v \int_0^Q (Q - D^y) f(D^y) dD^y - \sum_{i \in I_1} \frac{e_i}{|I_1|} \Lambda_{D^y}(Q). \quad (4.4)$$

Given the vector y then, our optimal order quantity is determined by the following equation:

$$F_{D^y}(Q_y^*) = \left(\frac{\sum_{i \in I_1} e_i}{\sum_{i \in I_1} y_i} - c \right) / \left(\frac{\sum_{i \in I_1} e_i}{\sum_{i \in I_1} y_i} - v \right) = \rho, \quad (4.5)$$

where ρ denotes the critical fractile based on the markets selected. Note that we can actually find an optimal Q_y^* assuming prior knowledge of market selection vector y, however our focus is concerning those situations where market selection is not known. In this case, the critical fractile is a function of the markets selected. We attempt to make a special note here that rationing shortages would only be prudent if a particular market would actually notice the shortage (i.e., the shortage would cause a portion of the market's order to arrive late). Otherwise, we should always allocate the overseas product to the markets with the highest shortage costs first when the market is not aware that any shortage actually occurred. Without a late arrival of product to a market, there would appear to be no motivation for rationing shortages.

Since D^y is normally distributed, we can write the optimal order quantity as

$Q_y^* = \sum_{i=1}^n \mu_i y_i + z(\rho)\sqrt{\sum_{i=1}^n \sigma_i y_i}$, where $z(\rho) = \frac{Q_y^* - \mu_y}{\sigma_y} = \Phi^{-1}(\rho)$ is the standard normal variate value associated with the fractile ρ. We can write the loss function $\Lambda_{D^y}(Q_y^*)$ in terms of the standard normal loss function $L(z) = \int_z^\infty (u-z)\phi(u)du$, where $\phi(u)$ is the p.d.f of the standard normal distribution, with c.d.f $\Phi(u)$. In particular, $\Lambda_{D^y}(Q_y^*) = \sigma_y L(z(\rho))$. We can write the expected profit equation as follows:

$$\begin{aligned}
G(Q,y) &= \sum_{i\in I}(r_i\mu_i - S_i)y_i - cQ - v\int_0^Q (D^y - Q)f(D^y)dD^y - \sum_{i\in I_1}\frac{e_i}{|I_1|}\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}(r_i\mu_i - S_i)y_i - cQ - v\int_0^Q (D^y - Q)f(D^y)dD^y \\
&\quad - v\int_Q^\infty (D^y - Q)f(D^y)dD^y + v\int_Q^\infty (D^y - Q)f(D^y)dD^y - \sum_{i\in I_1}\frac{e_i}{|I_1|}\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}(r_i\mu_i - S_i)y_i - cQ - v\mu_i y_i + vQ + v\Lambda_{D^y}(Q) - \sum_{i\in I_1}\frac{e_i}{|I_1|}\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}((r_i - v)\mu_i - S_i)y_i - (c-v)Q + v\Lambda_{D^y}(Q) - \sum_{i\in I_1}\frac{e_i}{|I_1|}\Lambda_{D^y}(Q).
\end{aligned}$$

We rewrite the expected profit equation as:

$$G(Q,y) = \sum_{i\in I}((r_i - v)\mu_i - S_i)y_i - (c-v)Q + v\Lambda_{D^y}(Q) - \sum_{i\in I_1}\frac{e_i}{|I_1|}\Lambda_{D^y}(Q). \quad (4.6)$$

We formulate our mixed integer programming problem for the case of rationing shortages as:

$$\begin{aligned}
\textbf{[R-SNP]} \qquad & \text{maximize} \quad G(Q,y) & \\
& \text{subject to:} \quad Q \geq 0, & (4.7)\\
& \qquad\qquad y_i \in \{0,1\} \qquad i = 1,\ldots,n. & (4.8)
\end{aligned}$$

where $G(Q,y)$ is defined by (4.6). The following paragraph describes the parameters used in greater detail. We varied the size of the market pool between 5 and 50 markets, depending on the experiments being conducted. Every market has unit revenue in the range U[$212,$252], while the unit production cost is set at $200. Expected demand and demand variance for each market are distributed according to U[800,1200] units and U[30000,60000], respectively. The fixed cost for market entry are drawn from U[$10000,$15000]. The expediting cost for each market is drawn from U[$400, $500]. Finally, the salvage value is $150 per unit. We use this same data set for all the computational tests through out this chapter. We use GAMS software to optimally solve [R-SNP]. We utilize Lindoglobal solver to optimally solve this MINLP problem. Lindoglobal in GAMS reports the global optimal solution to the problem on hand. We present the average results from ten test problems for each market pool in Table 4.1.

Table 4.1: Base case: Rationing the Shortages

Markets	Expected Profit	Order Quantity	Markets Selected	fractile
5	64525	4908	4.5	0.85
10	137735	9974	9.3	0.85
15	215756	14407	14	0.85
20	300071	19010	18	0.86
30	479812	28782	28	0.85
40	634630	38627	37	0.85
50	834311	48390	47	0.85

We observe that as the number of markets under consideration for selection increases, risk pooling allows the selection of more markets and hence allows for ordering more, resulting in an increase in average profit. The critical fractile remains nearly constant over all market sets. This is intuitive as we can control the markets selected and order quantity, while the input cost and revenue parameters remain unchanged. To provide various managerial insights, we perform the sensitivity analysis by varying the different cost parameters and study their impact on the structure of the SNP. For the 10-market problem setting, the unit cost c is varied at three levels: \$170 which is near the salvage cost of \$150, \$188 which is below base unit cost of \$200 and \$212 which is higher than base unit cost and near to per unit revenue. Similarly, expediting cost is then varied at three levels: U[\$250,\$350], U[\$350,\$450] and U[\$450,\$550]. We conclude by studying the impact of varying per unit revenue on the procurement policies. We start the range of per unit revenue equal to the unit cost U[\$200,\$240], slightly greater than unit cost U[\$202,\$242] and a larger increase over unit cost U[\$225,\$265], which is also higher than the base case. Table 4.2 summarizes all the results.

Table 4.2: Varying the parameters: Rationing the shortages

Parameters	Level 1	Level 2	Level 3
Unit Cost	c=\$170	c=\$188	c=\$212
10 Markets			
Expected Profit	461253	264920	44914
Order Quantity	11037	10797	6175
Markets Selected	10	10	5.7
ρ	0.93	0.87	0.90
Expediting Cost	e_i=U[\$250,\$350]	e_i=U[\$350,\$450]	e_i=U[\$450,\$550]
10 Markets			
Expected Profit	151456	141044	135088
Order Quantity	9704	9983	10040
Markets Selected	9.4	9.4	9.4
ρ	0.69	0.81	0.87
Revenue	r_i=U[\$200,\$240]	r_i=U[\$202,\$242]	r_i=U[\$225,\$265]
10 Markets			
Expected Profit	50246	62450	266620
Order Quantity	6342	6778	10679
Markets Selected	5.8	6.2	10
ρ	0.92	0.83	0.91

We observe that if the unit cost gets closer to the salvage cost, the model tends to order more and has the highest expected profit. The expected profit decreases as we move $c = \$170$ to $c = \$212$. When the unit cost is set equal to the lower end of the range for the per unit revenue, the model tends to order less due to fewer profitable markets. This tends to decrease the number of selected markets and hence the expected profit. When the lower end of the expediting cost is changed from \$250 to \$450 the decrease in expected profit is about 10%. By changing the revenue's lower end from \$200 to \$202, there is around 6% increase in the order quantity resulting in 20% increase in the expected profit. Again, by increasing the revenue from \$212 to \$225, the expected profit almost gets doubled. We also observed that ρ is also changing for each parameter variation. ρ is calculated using expediting cost and material cost. Thus changes in these also result in change in value of ρ . However, change in ρ is not directly explained by change in values of revenue. As ρ is the point in the demand distribution (ordering decision) corresponding to the maximum expected profit. Also increase in revenue results in ordering more amount which indirectly incorporates the corresponding adjustments in ρ. From this sensitivity analysis we infer that the expected profit obtained by this model is comparatively more sensitive to the changes in the unit material cost and the per unit revenue as compared to the expediting or salvage cost.

4.4.2 Greedy Approach

In the greedy approach, we allocate a shortage to the least attractive market in a set of selected markets. The market attractiveness is defined by the expediting cost associated with each market. Suppose that if total demand from all markets selected exceeds Q we then allocate the Q units in decreasing order of market shortage cost, i.e., to the highest shortage cost market first. By re-sorting markets in decreasing order of shortage cost, we have the following analysis. Clearly, we only incur a shortage cost for market 1 if $Q < D_1$, and in this case, the amount of the shortage equals $D_1 - Q$; similarly, we incur some shortage cost in market 2 if $Q < D_1 + D_2$, and in this case the amount of the shortage equals D_2 if $D_1 > Q$, and $D_2 + D_1 - Q$ otherwise. In general, given that $\sum_{j=1}^{i} D_j > Q$, the amount short in market i equals D_i if $\sum_{j=1}^{i-1} D_j > Q$ and $\sum_{j=1}^{i} D_j - Q$ otherwise.

To streamline our notation, let $D_{[i]} = \sum_{j=1}^{i} D_j$, and let $f_{[i]}(D_{[i]})$ and $F_{[i]}(D_{[i]})$ denote the pdf and cdf of $D_{[i]}$. The shortage cost for market i equals

$$\begin{aligned} & e_i \left\{ D_i \times \Pr[D_{[i-1]} \geq Q] + (D_{[i]} - Q)^{+} \Pr[(D_{[i-1]} \leq Q)] \right\} \\ = \quad & e_i D_i (1 - F_{[i-1]}(Q)) + e_i (D_{[i]} - Q)^{+} F_{[i-1]}(Q). \end{aligned}$$

The expected shortage cost for market i, which we denote by $\Gamma(i)$, therefore equals

$$\Gamma(i) = e_i \mu_i (1 - F_{[i-1]}(Q)) + e_i F_{[i-1]}(Q) \int_{Q}^{\infty} (D_{[i]} - Q) f_{[i]}(D_{[i]}) dD_{[i]}.$$

For a given selection of markets (which we denote by $\hat{y}$), our expected profit equation (4.1) now becomes

$$G(Q, \hat{y}) = \sum_{i \in I} (r_i \mu_i - S_i) y_i - cQ + v \int_{0}^{Q} (Q - D^y) f(D^y) dD^y - \sum_{i \in I} \Gamma(i), \qquad (4.9)$$

The problem here appears to be that we must know what markets we select to even compute the $\Gamma(i)$ values, and our critical fractile value will be a function of our market selection decisions. Since the $\Gamma(i)$ values and the critical fractile value are functions of our market selection decisions, we cannot arrive at a formulation that removes the selection variables from consideration (as is the case in [SNP-N]). We may draw some conclusions based on the conditions for which (4.9) is concave, and this may shed some light on any special conditions in which this may be true. We first proceed with the partial derivative of $\Gamma(i)$ with respect to Q. We show both the first and second partial derivatives.

$$\begin{aligned}
\frac{\partial \Gamma(i)}{\partial Q} &= e_i f_{[i-1]}(Q)\left[\int_Q^\infty (D_{[i]} - Q) f_{[i]}(D_{[i]}) dD_{[i]} - \mu_i\right] + e_i F_{[i-1]}(Q)\left[F_{[i]}(Q) - 1\right] \\
\frac{\partial^2 \Gamma(i)}{\partial Q^2} &= 2e_i f_{[i-1]}(Q)\left[F_{[i]}(Q) - 1\right] + e_i F_{[i-1]}(Q)\left[f_{[i]}(Q)\right]
\end{aligned}$$

Now, taking the second derivative of the entire expected profit equation (4.9) results in:

$$\frac{\partial^2 G(Q,y)}{\partial Q^2} = \underbrace{v f_{D^y}(Q)}_{\geq 0} - \sum_{i \in I} e_i \left(\underbrace{2 f_{[i-1]}(Q)\left[F_{[i]}(Q) - 1\right]}_{\leq 0} + \underbrace{F_{[i-1]}(Q) f_{[i]}(Q)}_{\geq 0} \right)$$

This has not led to identifying any conditions for concavity. Let's consider only a 2-market scenario. Then, the test for concavity is determined by

$$\begin{aligned}
\tfrac{\partial^2 G(Q,y)}{\partial Q^2} &= v f_{D^y}(Q) - e_1\left(2 f_{[0]}(Q)\left[F_{[1]}(Q) - 1\right] + F_{[0]}(Q) f_{[1]}(Q)\right) \\
&\quad - e_2\left(2 f_{[1]}(Q)\left[F_{[2]}(Q) - 1\right] + F_{[1]}(Q) f_{[2]}(Q)\right) \\
&= v f_{D^y}(Q) - e_2\left(2 f_{[1]}(Q)\left[F_{[2]}(Q) - 1\right] + F_{[1]}(Q) f_{[2]}(Q)\right)
\end{aligned}$$

And, again, identifying conditions when $\frac{\partial^2 G(Q,y)}{\partial Q^2} < 0$ will be difficult to determine. One potential approach is to develop a heuristic to assess market attractiveness based on e_i, r_i, μ_i, and σ_i^2. Using a heuristic we enumerate the "high quality" permutations, and find the preferred selection and procurement decisions for each ordering. This heuristic is explained in detail in the following section.

4.5 Construction of Heuristic

We developed a heuristic approach to obtain the procurement plans for the market-specific expediting cost generalization to the SNP. To find both the preferred market selections as well as an associated order quantity; we propose an approach that examines solutions resulting from each of several market rankings. We use a similar approach to the DERU described in chapter 2, in that we add one market at a time and compare against the incumbent (or best-known) solution. We summarize the main steps necessary in developing the heuristic and solving this problem as below:

I Sort the markets. To begin with we need to find the set of selected markets ($\bar{y}$). We develop various sorting schemes to define the attractiveness of a market.

II Determine the order quantity. For this given set of selected markets $\bar{y}$, we now determine the order quantity Q.

III Select markets and create a secondary sort for allocating shortages. Once we know Q, the next decision is to allocate this order quantity among the selected markets to obtain maximum expected profit. After the markets have been sorted, to allocate the total order quantity as explained in Section 4.4.2,we now sort the markets based on non-increasing expedite cost, but this time we only sort markets that are part of the current selection vector.

IV Simulation: Repeat the process for many demand realizations. Repeat the whole process for many demand realizations. Find the maximum average profit over all the simulation replications.

I. Sort the Markets. We use various sorting schemes to rank markets and obtain solutions that maximize expected profit where expected profit is reported as the average of thousands of simulation outcomes. We list the tested rankings below:

1 Non-increasing expedite cost, [EC]. We sort the markets based on non-increasing values of expedite cost. In this way we give preference to the most expensive markets in terms of expediting cost.

2 DERU ratio. Index markets in decreasing order of expected net revenue to uncertainty (demand variance).This ratio ordering is intuitive, as a higher net revenue makes a market more attractive, while increases in the market's uncertainty lead to a less attractive market. DERU is shown to be optimal for SNP in (cf. Taaffe et al. [53]).

3 Non-increasing expected net revenue, [REV]. Rank markets in non-increasing order of their expected net revenue. This way we give first preference to the most lucrative markets followed by less profitable ones.

4 Non-decreasing uncertainty (or demand variance), [VAR]. Sort the markets based on increasing variance in demand. Market with least demand variance is first in sorted array and is given the first preference to be selected in the solution vector. This way we try to first satisfy more stable markets/customers over highly risky ones.

5 DERU and expediting cost, [DERU*EC]. i.e. Non-increasing order of expected net revenue to the product of expedite cost & demand variance. This ratio is the combination of the first two sorting schemes providing with the most attractive and more stable market at the top of rankings.

II. Determine Order Quantity. Once sorting mechanism is complete, the next step is to determine the total order quantity. We adopt the following various options to find the total order quantity.

A We consider each market as an independent single item newsvendor problem. In an one period single item newsvendor setting an optimal order quantity is $Q* = F^{-1}(\rho)$, where $\rho = \frac{e-c}{e-v}$. Here we find $\rho_i = \frac{e_i-c}{e_i-v}$ and subsequently find Q_i for each market. We then add Q_i's only for the selected markets.

B For each potential market selection find Q based on the rationing approach, i.e., blending the expediting costs of the selected markets and then calculating the critical fractile ρ.

C Use the above option (option B), but place more weight on those markets with lower expediting costs, as these would be the ones where shortages would possibly occur. We multiply the expediting cost of the sorted market with the rank of the market in a sorted array. If W_i be the rank of the market i in the sorted market selection vector, then we represent the critical fractile ρ as follows:

$$F_{D^y}(Q_y^*) = \left(\frac{\sum_{i\in I_1} W_i * e_i}{\sum_{i\in I_1} W_i * y_i} - c\right) / \left(\frac{\sum_{i\in I_1} W_i * e_i}{\sum_{i\in I_1} W_i * y_i} - v\right) = \rho, \tag{4.10}$$

III. Select Markets, Sort and Allocate. Next step is to allocate the total order quantity in order to maximize the average profit for a firm. If the total demand from all markets selected exceeds Q we then allocate the Q units in decreasing order of market shortage cost, i.e., to the highest shortage cost market first.

IV. Repeat For Many Demand Realizations. We save a solution for each ranking, where solution k of a given ranking will contain all markets $1...k$ of that ranking. This solution approach does not require evaluating all 2^n possible market selections, which would be computationally prohibitive. While we may miss certain solutions, testing five unique rankings alleviates this problem to some extent.

The implementation of the heuristic is described as follows:

1 Generate the realized demand for each market and store these realized (or simulated) demands for all simulation runs.

2 As y_i is a binary variable, we put zero for all i in initial market selection vector.

3 Sort markets based on one of five above mentioned ranking mechanisms.

4 Find the total order quantity using any one of three previously mentioned options for the given market selection.

5 For each simulation run, add markets one by one to the selection vector, calculating realized profit after each market addition. When a shortage occurs:

- Greedy approach: distribute available product to markets with highest expediting cost markets first.
- Rationing approach: distribute available product equally across chosen markets.

6 Tabulate realized profits for all market selection vectors across all simulation runs.

7 Calculate the average profit for each ranking, and select the market selection vector with the highest average profit across all simulation runs. We added the corresponding order quantity for this solution.

4.5.1 Rationing Approach

We used the heuristic as mentioned in Section 4.5 to test a rationing approach for several market selection alternatives. We evaluate all of the previously mentioned ranking schemes. The only difference here is that we find the order quantity, Q, using the critical fractile as given by the option B. We tested the quality of solutions obtained from the heuristic against the solutions by the exact optimization formulation using GAMS. For one particular test instance for 5 and 10 markets, we obtain solutions from the heuristic for all five rankings and compare it against the corresponding solutions by solving MINLP using GAMS. For the purpose of studying the impact of the randomness due to the simulation based heuristic approach, we test the heuristic for 10,000, 100,000 and 1,000,000 replications. We tabulate our observations in the Table 4.3. We notice that the heuristic provided approximately similar quality results as by GAMS for 100,000 replications. However, running 100,000 replications is computationally prohibitive, thus leading us to use 10,000 replications to reduce the complexity of the heuristic to obtain the good quality procurement policies even for big market size problems.

4.5.2 Greedy Approach

We employ simulation to generate demand realizations of all markets, in order to generate the average profit across all simulations (or demand scenarios). Using the same data as previously explained in the rationing approach, we run 10,000 simulation replications and 100 test instances for every market pool size and report the average values for each ranking. we employ this heuristic using all three options to determine the order quantity. Table 4.4 summarizes results for the heuristic employing option A to find the order quantity for the selected markets. After finding procurement policies based on these rankings, we provide the best and worst rankings in terms of average profit. In this table, we observe that the ranking based on combination of DERU and expediting cost performs best for five out of seven market selections. As this ranking tries to first satisfy the demand of the market with highest revenue,

Table 4.3: Comparison: Heuristic vs GAMS

Paramters	[EC]	[DERU]	[VAR]	[NR]	$[DERU * EC]$	GAMS
5 Markets	Solution - Heuristic-10,000 replications					
Average Profit	88386	89386	88469	89386	89386	89474
Q	3874	4557	5492	4557	4557	4557
Markets Selected	3.0	4.0	5.0	4.0	4.0	4.0
5 Markets	Solution - Heuristic-100,000 replications					
Average Profit	88119	89304	88235	89304	89304	89474
Q	3874	4557	5492	4557	4557	4557
Markets Selected	3.0	4.0	5.0	4.0	4.0	4.0
5 Markets	Solution - Heuristic-1,000,000 replications					
Average Profit	88119	89304	88235	89304	89304	89474
Q	3874	4557	5492	4557	4557	4557
Markets Selected	3.0	4.0	5.0	4.0	4.0	4.0
5 Markets	Solution - Heuristic-10,000 replications					
Average Profit	72668	72926	72668	72926	72926	72551
Q	5000	4061	5000	4061	4061	4061
Markets Selected	5.0	4.0	5.0	4.0	4.0	4.0
5 Markets	Solution - Heuristic-100,000 replications					
Average Profit	72443	72537	72443	72537	72537	72551
Q	5000	4061	5000	4061	4061	4061
Markets Selected	5.0	4.0	5.0	4.0	4.0	4.0
5 Markets	Solution - Heuristic-1,000,000 replications					
Average Profit	72443	72537	72443	72537	72537	72551
Q	5000	4061	5000	4061	4061	4061
Markets Selected	5.0	4.0	5.0	4.0	4.0	4.0

highest expediting cost and most stable market in terms of demand variance. Hence resulting in the highest average profit from the selected set of markets. However, the ranking using expected net revenue also provided approximately as good results as the best ranking, because this ranking allocates the shortages to the least profitable market in terms of the expected net revenue. DERU ranking performed little less than these two rankings. Whereas, the ranking based on demand variance performed worst among all rankings. This ranking mainly focuses on allocating shortages to most unstable market with least expediting cost while ignoring the associated revenue parameter. The best ranking result always outperformed worst result by 100% gain in average profit except for the case of 50 markets, where the gain is around 90%. Table 4.5 provides average results by the heuristic using option B to find the order quantity for the markets in the selection set $\bar{y}$. In this heuristic, the ranking based on expected net revenue and the combination of DERU and expediting cost ranking provided the best results. However, all the rankings performed almost equally good except for the 5 market scenario. Here, the ranking based on demand variance again provided worse results. Average profit, the total order quantity and the number of markets selected increases with the increase in the size of the market explained by the risk pooling factor. Table 4.6 tabulates all the results from the heuristic using option C to find the total order quantity for $\bar{y}$. Similar to the previous heuristic, this one also provides approximately similar results for all five rankings except for 5 market scenario. This heuristic however performed slightly better than the heuristic using option B for determining the order quantity for the selected markets. Whereas, the heuristic using option A (treating each market as an independent newsvendor problem) performed best in 20% cases among three heuristics. Whereas, the heuristic based on option C performed best for the rest 80% of the cases. The reason might be the way Q has been calculated by assigning higher weights to low expediting costs.

Table 4.4: Greedy approach for the e_i case using option A for Q

Paramters	Expedite Cost	DERU Ratio	Net Revenue	Uncertainty	[DERU*EC]
5 Markets					
Average Profit	44956	73671	88383	44317	91792
Q	5385	3892	4407	5833	4248
Markets Selected	4.5	3.2	3.7	4.8	3.5
10 Markets					
Average Profit	90462	152880	187278	86255	183926
Q	11335	8171	8599	11908	8782
Markets Selected	9.4	6.8	7.1	9.9	7.3
15 Markets					
Average Profit	142568	263007	296125	140908	262165
Q	17332	12131	12615	17911	13084
Markets Selected	14.3	10.0	10.4	14.9	10.8
20 Markets					
Average Profit	186559	352268	366061	178545	395411
Q	23302	15894	16030	23642	15817
Markets Selected	19.3	13.1	13.2	19.6	13.0
30 Markets					
Average Profit	276748	541356	553961	276119	560512
Q	35364	23570	23945	35833	25062
Markets Selected	29.3	19.4	19.7	29.7	20.7
40 Markets					
Average Profit	368356	655366	693595	364600	734242
Q	47407	32121	32648	48246	32912
Markets Selected	39.2	26.4	26.9	40	27.1
50 Markets					
Average Profit	453309	808921	814139	459181	874506
Q	59507	40364	40495	60287	41256
Markets Selected	49.3	33.2	33.3	50	33.9

Table 4.5: Greedy approach for the e_i case using option B for Q

Paramters	Expedite Cost	DERU Ratio	Net Revenue	Uncertainty	[DERU*EC]
5 Markets					
Average Profit	60453	63641	80158	59814	80753
Q	5244	4056	4688	5327	4520
Markets Selected	4.8	3.7	4.3	4.9	4.1
10 Markets					
Average Profit	144972	147163	178068	140873	174901
Q	10576	8571	9095	10651	9130
Markets Selected	9.9	8.0	8.5	10	8.5
15 Markets					
Average Profit	242990	239689	270277	242363	264136
Q	15795	13026	13979	15846	14041
Markets Selected	14.9	12.2	13.2	15	13.2
20 Markets					
Average Profit	336252	332946	359645	327256	359002
Q	20936	18568	18657	20771	18722
Markets Selected	19.9	17.6	17.7	19.8	17.8
30 Markets					
Average Profit	528854	512370	541425	527234	569939
Q	31189	28232	28718	30918	28725
Markets Selected	29.9	27.0	27.5	29.7	27.5
40 Markets					
Average Profit	724783	701500	741690	720366	729561
Q	41371	37758	39110	41416	38108
Markets Selected	39.9	36.3	37.7	40	36.7
50 Markets					
Average Profit	914675	894617	921474	917082	921966
Q	51517	47383	48310	51564	48642
Markets Selected	49.9	45.8	46.7	50	47.0

Table 4.6: Greedy approach for the e_i case using option C for Q

Paramters	Expedite Cost	DERU Ratio	Net Revenue	Uncertainty	[DERU*EC]
5 Markets					
Average Profit	60510	63672	80298	59818	80954
Q	5234	4065	4680	5326	4516
Markets Selected	4.8	3.7	4.3	4.9	4.1
10 Markets					
Average Profit	145284	147393	178337	140873	175138
Q	10557	8558	9091	10648	9113
Markets Selected	9.9	8.0	8.5	10	8.5
15 Markets					
Average Profit	243811	240411	270839	242695	264579
Q	15768	13010	13972	15842	14034
Markets Selected	14.9	12.2	13.2	15	13.2
20 Markets					
Average Profit	337615	333589	360196	327669	359650
Q	20904	18564	18624	20764	18653
Markets Selected	19.9	17.6	17.7	19.8	17.7
30 Markets					
Average Profit	531256	513267	542342	527793	569959
Q	31148	28146	28673	30910	28617
Markets Selected	29.9	27.0	27.5	29.7	27.4
40 Markets					
Average Profit	728061	702441	742587	721061	730692
Q	41322	37692	39090	41407	38102
Markets Selected	39.9	36.3	37.6	40	36.7
50 Markets					
Average Profit	918772	895716	922450	917762	923470
Q	51462	47285	48298	51554	48541
Markets Selected	49.9	45.7	46.7	50	46.9

4.6 Lost Sales Case

Under the case of lost sales, we assume that any realized market demand not satisfied through the procurement quantity will be lost. Assume the cost per unit of a lost sale is l, the same across all markets, and that the individual markets will not know if their requests will be satisfied until all demand has been realized. The question is, can we set a policy and determine mathematically the optimal procurement quantity Q^* and market selection vector y such that we maximize our expected profit?

There are actually two decisions to make. First, how much product should we purchase from the overseas supplier? Then, once the individual market demand is realized, how should we allocate our supply? Let's assume that the procurement quantity from the supplier has been set. Since our assumption is that the per-unit lost sale cost is identical for each market, then the only discriminating factor across all markets is the market-specific revenue of r_i per unit. Therefore, an optimal allocation strategy would simply be to arrange the markets in non-increasing order of per-unit revenue and allocate the supply until it has been consumed. This assumes, as stated earlier, that we do not need to commit to any market orders prior to demand realization. For the sake of the following discussion, we will place all markets in non-increasing order of per-unit revenue.

To determine the appropriate expected profit equation under the lost sales case, we include the salvage amount based on excess supply purchased, the lost sale amount based on the demand not satisfied, the total material or purchase cost for the procurement quantity, and the revenue achieved through our allocation policy. We state this expected profit equation as

$$G(Q,y) = -cQ + v\int_0^Q (Q - D^y)f(D^y)dD^y - l\int_Q^\infty (D^y - Q)f(D^y)dD^y + \overline{R} \quad (4.11)$$

where $\overline{R}$ is the expected revenue.

4.6.1 Rationing the Lost Sales

In this section we try to ration the lost sales equally across all markets. In case of rationing the lost sales, we have consider its impact at two places. At one place, we pay the penalty/cost "l" on the lost amount of sales. We include this as a shortage term in expected profit equation. At another place, we consider this while calculating the expected revenue term. Rationing of lost sales decreases the per unit revenue for each selected market by the rationed amount of lost sales. Thus, we subtract the rationed amount of lost sales from the mean demand for each selected market so that we do not earn revenue on full mean demand as in other cases of SNP where we assume to expedite any shortage amount. Thus, we are entitled to earn revenue only on the portion of mean demand left after subtracting the lost sales. We present the new net revenue term for individual market as:

$$\rho(i) = r_i(\mu_i - \frac{\Lambda_{D^y}(Q)}{|I_1|})y_i.$$

where $\frac{\Lambda_{D^y}(Q)}{|I_1|}$ presents the average shortage in each market and is previously explained in detail in section for a market-specific expediting cost extension to the SNP. The expected profit equation is:

$$\begin{aligned}
G(Q,y) &= S_i y_i - cQ + v\int_0^Q (Q-D^y)f(D^y)dD^y - l\int_Q^\infty (D^y-Q)f(D^y)dD^y \\
&\quad + \sum_{i\in I}\rho(i) \\
&= \sum_{i\in I}\{r_i[\mu_i - \frac{\Lambda_{D^y}(Q)}{|I_1|}] - S_i\}y_i - cQ + v\int_0^Q (Q-D^y)f(D^y)d(D^y) \\
&\quad - l\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}\{r_i[\mu_i - \frac{\Lambda_{D^y}(Q)}{|I_1|}] - S_i\}y_i - cQ - v\int_0^Q (D^y-Q)f(D^y)dD^y \\
&\quad - v\int_Q^\infty (D^y-Q)f(D^y)dD^y + v\int_Q^\infty (D^y-Q)f(D^y)dD^y - l\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}\{r_i[\mu_i - \frac{\Lambda_{D^y}(Q)}{|I_1|}] - S_i\}y_i - cQ - l\Lambda_{D^y}(Q) - v[\mu_y - Q] \\
&\quad + v\Lambda_{D^y}(Q) \\
&= \sum_{i\in I}\{r_i[\mu_i - \frac{\Lambda_{D^y}(Q)}{|I_1|}] - S_i\}y_i - (c-v)Q - (l-v)\Lambda_{D^y}(Q) - v\mu_y \qquad (4.12)
\end{aligned}$$

Notice that Q is dependent on the markets selection, as this will affect the revenue for market selection, as well as the lost sales. Given the vector y, the optimal order quantity is given by:

$$F_{D^y}(Q^*_y) = \left(\frac{\sum_{i\in I_1}(l+r_i)}{\sum_{i\in I_1} y_i} - c\right) / \left(\frac{\sum_{i\in I_1}(l+r_i)}{\sum_{i\in I_1} y_i} - v\right) \qquad (4.13)$$

We rewrite our mixed integer nonlinear programming problem for the case of rationing lost sales as follows:

$$
\begin{aligned}
\textbf{[L-SNP]} \qquad & \text{maximize} \quad && G(Q, y) && \\
& \text{subject to:} \quad && Q \geq 0, && \qquad (4.14) \\
& && y_i \in \{0, 1\} \qquad i = 1, \ldots, n. && \qquad (4.15)
\end{aligned}
$$

We again make us of "lindoglobal" solver available with GAMS and the previously described data set for optimally solving our "MINLP" for this case. In addition, we consider the lost sales cost l as \$40. The average results over ten test problems are presented in the following table. As the size of market pool increases, the risk

Table 4.7: Base case: Rationing the lost sales

Markets	Expected Profit	Order Quantity	Markets Selected	fractile
5	76718	4936	4.9	0.61
10	155277	9711	9.5	0.63
15	237182	14387	14	0.64
20	324883	18992	19	0.64
30	510309	28504	28	0.64
40	670235	38085	38	0.63
50	874050	47740	47.4	0.63

pooling allows to select more number of markets and hence allows for ordering more quantity. There is also increase in average profit as the the size of market selection increases from 5 to 50. However, the critical fractile remains constant over all cases. Similar to the marker-specific expediting case, here also we vary the different cost parameters to investigate their impact on the structure of this generalization to the SNP. In addition to the previously mentioned data for sensitivity analysis, we also vary lost sales cost at three different levels: \$40, \$80 and \$120. Table 4.8 summarizes all the results for sensitivity analysis. As the unit cost gets closer to the salvage cost,

Table 4.8: Varying the parameters: Rationing the lost sales

Parameters	Level 1	Level 2	Level 3
Unit Cost	c=\$170	c=\$188	c=\$212
10 Markets			
Expected Profit	466968	277638	62079
Order Quantity	10691	10365	6192
Markets Selected	10	10	6
ρ	0.84	0.69	0.78
Expediting Cost	l=\$40	l=\$80	e_i=U\$120
10 Markets			
Expected Profit	155277	149038	144627
Order Quantity	9711	9771	9889
Markets Selected	9.5	9	9
ρ	0.63	0.72	0.77
Revenue	r_i=U[\$200,\$240]	r_i=U[\$202,\$242]	r_i=U[\$225,\$265]
10 Markets			
Expected Profit	64938	77180	282465
Order Quantity	6262	6560	10258
Markets Selected	6	6	10
ρ	0.81	0.8	0.63

the model tends to order more and achieves the highest expected profit. This is due to the special structure inherited from the newsvendor problem. The expected profit decreases as c changes from $c = \$170$ to $c = \$212$. When the unit cost becomes equal to the starting point of range for the per unit revenue, due to fewer available profitable markets the model tends to order less. This results in decreasing the number of selected markets and hence the expected profit. The decrease in expected profit in this case is approximately 60%. When we change the lost sales cost from \$40 to \$120 the decrease in expected profit is about 7%. Thus, change in lost sales value does not drastically impact the expected profit. By changing revenue's lower range from \$200 to \$202, there is around 5% increase in the order quantity resulting in 19% increase in the expected profit. Whereas, the fractile ρ and number of selected markets remain the same. Again, when we increase the revenue from \$212 to \$225, the expected profit almost gets doubled. Similar to the previous analysis in the market-specific expediting case, this model is also very sensitive to changes in the unit cost and the per unit revenue.

4.6.2 Greedy Approach

In the greedy approach, addressing the revenue achieved is slightly more complicated than in the case in which all demand for selected markets is ultimately satisfied. We cannot directly use the expected demand term as before, since we are not guaranteed to satisfy all demand per market in the markets we select. This leads to the following approach. A market will be satisfied if all markets with higher per-unit revenue have been satisfied completely and there is still available supply. Using the notation described in Section 4.4, let $D_{[i]} = \sum_{j=1}^{i} D_j$, and let $f_{[i]}(D_{[i]})$ and $F_{[i]}(D_{[i]})$ denote the pdf and cdf of $D_{[i]}$. Then, the revenue achieved for market i is

$$
\begin{aligned}
&= r_i \Big(D_i \cdot \Pr\left[D_{[i]} \leq Q \right] + \left(Q - D_{[i-1]} \right)^{+} \cdot \Pr\left[D_{[i]} \leq Q \right] \Big) \\
&= r_i D_i F_{[i]}(Q) + r_i \left(Q - D_{[i-1]} \right)^{+} F_{[i]}(Q)
\end{aligned}
$$

The expected revenue for market i, denoted as $\rho(i)$, therefore equals

$$
\rho(i) = r_i \mu_i F_{[i]}(Q) + r_i F_{[i]}(Q) \int_{Q}^{\infty} \left(Q - D_{[i-1]} \right) f_{[i-1]}(D_{[i-1]}) dD_{[i-1]}
$$

And by selecting all markets, the expected profit equation with lost sales will be We state this expected profit equation as

$$G(Q,y) = -cQ + v\int_0^Q (Q - D^y) f(D^y) dD^y - l\int_Q^\infty (D^y - Q) f(D^y) dD^y + \sum_{i \in I} \rho(i) \quad (4.16)$$

Again, we must know the selected markets prior to computing the expected revenue and profit. Finding any conditions in which $G(Q,y)$ is concave will most likely be similar to the expediting case. Alternatively, we may introduce a heuristic approach.

4.6.3 Constructive Heuristic

We adopt a similar heuristic as described in Section 4.5. We consider all market selections to be independent of each other. Then we apply one of three options to determine the order quantity and use the heuristic proposed in the Section 4.5, to identify (Q, y) solutions of high quality. We have a fixed lost sales cost for all markets, thus we can only test three of the following previously mentioned ranking schemes (excluding the ones involving market-specific expediting costs).

I. Sort the Markets

1 DERU. Index markets in decreasing order of expected net revenue to uncertainty (demand variance).

2 Non-increasing per unit revenue, [REV. Rank markets in non-increasing order of the per unit revenue.

3 Non-decreasing uncertainty (or demand variance), [VAR]. Sort the markets based on increasing variance in demand.

II. Determine Order Quantity. We use all the three options mentioned in the Section 4.5.

III. Select markets, Sort and Allocate. After the markets have been selected, next decision is to allocate the total order quantity. We again sort the selected markets $\bar{y}$ based on non-increasing per unit revenue, but this time we sort for only the markets that are part of the current selection vector.

IV. Simulation. We repeat the whole process for many demand realizations using simulation and find the average profit for each replication.

We experimented the previously mentioned data set to evaluate the performance of the heuristic based on these three rankings. Results obtained for the heuristic using option A for determining the total order quantity are summarized in the Table 4.9. For this heuristic, as the size of market selection increases from 5 to 50, the average profit, the order quantity and the number of markets selected also increase. Table 4.10 and Table 4.11 provide results for the heuristic using option B and option C for Q respectively.

Performance Metrics: We observed that, for all three heuristics, all three rankings are equally competitive providing results approximately of similar quality. However, we noted that in all cases, the ranking based on the per unit revenue [REV] still always provided best values for the average profit across all market selections. As explained earlier, this again tries to allocate lost sales to the market with the least per unit revenue. We also noticed that the number of markets selected and the total order quantity increases as the size of the market pool increases explained by the effect of risk pooling. The heuristic using option C for finding total order quantity performs best among all three heuristics.

Table 4.9: Greedy approach for the lost sales case using option A for Q

Parameters	DERU ratio	r_i	Uncertainty
5 Markets			
Average Profit	97399	111826	93585
Q	4887	5173	4511
Markets Selected	3.2	4.9	4.3
10 Markets			
Average Profit	181372	187185	180575
Q	9846	10346	9309
Markets Selected	9.3	9.8	8.9
15 Markets			
Average Profit	274838	286059	266205
Q	15024	15496	14574
Markets Selected	14.1	14.7	13.9
20 Markets			
Average Profit	360148	375255	352614
Q	19888	20534	19582
Markets Selected	19.2	19.5	18.6
30 Markets			
Average Profit	536247	545679	524363
Q	30938	30659	29976
Markets Selected	28.9	29.2	28.6
40 Markets			
Average Profit	714733	726021	693587
Q	40770	40870	40210
Markets Selected	38.0	38.9	38.4
50 Markets			
Average Profit	882635	888508	863171
Q	50611	51052	50526
Markets Selected	48.4	48.6	48.2

Table 4.10: Greedy approach for the lost sales case using option B for Q

Parameters	DERU ratio	r_i	Uncertainty
5 Markets			
Average Profit	111399	118531	104735
Q	5052	5116	4515
Markets Selected	4.5	4.9	4.4
10 Markets			
Average Profit	208772	209008	208250
Q	9994	10191	9532
Markets Selected	9.6	10	9.3
15 Markets			
Average Profit	31938	324115	316620
Q	15167	15225	14609
Markets Selected	14.5	14.9	14.3
20 Markets			
Average Profit	426148	433370	421946
Q	20103	20247	19674
Markets Selected	19.8	19.9	19.3
30 Markets			
Average Profit	642942	644707	641315
Q	29997	30234	29694
Markets Selected	29.4	29.9	29.3
40 Markets			
Average Profit	858733	859726	856280
Q	40177	40295	39741
Markets Selected	39.7	39.9	39.3
50 Markets			
Average Profit	1063841	1070500	1056475
Q	50134	50301	49883
Markets Selected	49.7	49.8	49.4

Table 4.11: Greedy approach for the lost sales case using option C for Q

Parameters	DERU ratio	r_i	Uncertainty
5 Markets			
Average Profit	112635	116400	109976
Q	4986	5113	4501
Markets Selected	4.2	4.9	4.4
10 Markets			
Average Profit	211001	212650	210967
Q	9972	10157	9518
Markets Selected	9.6	10	9.3
15 Markets			
Average Profit	320901	329188	318829
Q	15124	15215	14608
Markets Selected	14.5	14.9	14.4
20 Markets			
Average Profit	429480	436785	426387
Q	19975	20246	19635
Markets Selected	19.8	19.9	19.3
30 Markets			
Average Profit	651132	653144	648849
Q	29938	30212	29646
Markets Selected	29.4	29.9	29.3
40 Markets			
Average Profit	866935	869784	865534
Q	39730	40260	39685
Markets Selected	39.7	39.9	39.3
50 Markets			
Average Profit	1069353	1081899	1067153
Q	49911	50287	49820
Markets Selected	49.8	49.9	49.4

4.7 Conclusions

In this chapter we have presented various extensions to the so called "selective newsvendor problem". We have extended the horizon of research for studying the SNP by including the market-specific expediting cost and the lost sales scenario. As a conclusion, we have described our potential implementation approaches for each of the extension. We developed three heuristic solution approaches for finding the total order quantity for both the market-specific expediting cost and the lost sales scenario. To find both the preferred market selections (as well as an associated order quantity): We proposed an approach that examines solutions resulting from each of several market rankings. We contribute to the research in the field of demand selection by providing an optimal solution method for the rationing approach for distributing shortages. We help managers to make more informed decisions by educating them the sensitivity of the expected profit related to the changes in the different parameters like costs and revenue.

The heuristic method for rationing provides solutions comparable to those obtained when optimally solving with GAMS. The best solution by the greedy approach for the market-specific expediting cost or for the lost sales scenario provides higher expected profit as compared to the rationing approach. For the market-specific expediting cost, the rankings based on net revenue when provided best results for the greedy approach. Similarly for the lost sales case, ranking based on per unit revenue always provided best solutions for the greedy approach. However, ranking based on demand variance always provided worst procurement policies among all the rankings. For both extensions to the SNP, the heuristic based on the option when the total order quantity is calculated using weighted critical fractile from the rationing approach. In conclusion we provided a high quality constructive heuristic for distributing the shortages via greedy approach.

One of the future work directions would be to consider the effect of correlated demands. The correlation could be either negative or positive. Another future work directions would be to study the impact on procurement policy if we have a limit on Q in the expediting case, i.e., $Q \leq B$. One can also study whether there are other demand distributions such that this analysis is tractable (e.g., Uniform)?

Chapter 5

Conclusions

In this dissertation we present the various operational scenarios for the revenue and order management under demand uncertainty. We contribute in the field of demand selection by providing the decision maker in any firm with a tool set for assessing the value of profit and risk, when obtaining solutions for the probabilistic risk models can be quite cumbersome. Chapter 1 extensively explores the existing literature in the field of stochastic demand management to review some fundamental ideas. In Chapter 2, we present the risk averse selective newsvendor problem for normally distributed demands. We continued our study on the risk averse selective newsvendor problem in Chapter 3 for the various forms of demand distributions. In Chapter 4, we revert back to basic selective newsvendor problem and present some generalizations to this and open the new horizons of research in this particular field.

In Chapter 2, for the risk minimizing model, we introduced a constructive heuristic that provides high quality solutions at a fraction of the time of an enumerative approach. We point out that obtaining solutions to probabilistic risk models can be quite cumbersome, and we offer approaches that firms dealing with risk issues can implement. When there is no closed-form solution approach available for defining the profit distribution (and worst-case profits), we must resort to an approach using simulation as described in the Chapter 2. One future area of interest would be the multi-period market or order selection problem with risk. This is a very rich area of research with lots of opportunity.

In Chapter 3, we offer several alternate market selection and product ordering models that vary in their inclusion of risk. We introduced an exact solution approach to incorporate risk that uses Conditional Value-at-Risk via scenario analysis for AON demands. We have tested potential market sizes from 5 to 15 for AON demands. While we can analyze these cases in quite thorough detail, there is clearly motivation for studying larger problem sizes. However, the problem suffers from the curse of dimensionality, and the exact solution approach proposed in the current work becomes intractable using CPLEX and Concert Technology. We have to consider heuristic approaches to solving such problem sizes, determining upper bounds that can help identify the quality of solutions produced by a heuristic.

Simulation analysis helped us to investigate the resulting profit distributions for each of the demand distributions, which are again used in determining respective procurement policies. As mentioned in the Chapter 3, all the results were more influenced by the behavior of the heuristic (presented in Chapter 2) as compared to demand distributions.

In Chapter 4, we extend the area of research for studying the SNP by including the market-specific expediting cost and the lost sales scenario. As a conclusion, we describe our potential implementation approaches for each of the extension. We present an exact closed form solution for rationing approach for both generalizations to the SNP. For both generalizations we formulated an optimization problem in the form of mixed integer nonlinear problem and solved it using the lindoglobal solver provided by GAMS software. In a greedy allocation approach, we develop the simulation based heuristic solution approach for finding the optimal order quantity for both the market-specific expediting cost and the lost sales scenario. For this heuristic, we offer three different options to determine the total order quantity for the selected set of markets. Thus, in effect we offer three heuristics. We also provide insights into the quality of solutions from the heuristic against the ones obtained by solving the exact optimization problem. One of the future work directions from here would be to study the impact on procurement policy if we have a limit on Q in the expediting case, i.e., $Q \leq B$. One can also study whether there are other demand distributions such that this analysis is tractable (e.g., Uniform)?

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