5-2012

The Development of Freshman College Calculus Students' Mathematics Identity and How it Predicts Students' Career Choice

Jennifer Cribbs
Clemson University, jdshipl@clemson.edu

Follow this and additional works at: https://tigerprints.clemson.edu/all_dissertations

Part of the Education Commons

THE DEVELOPMENT OF FRESHMAN COLLEGE CALCULUS STUDENTS'
MATHEMATICS IDENTITY AND HOW IT PREDICTS STUDENTS' CAREER
CHOICE

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Curriculum and Instruction

By
Jennifer Dawn Cribbs
May 2012

Accepted by:
Dr. Robert Horton, Committee Co-chair
Dr. Zahra Hazari, Committee Co-chair
Dr. Megan Che
Dr. Cassie Quigley

ABSTRACT

There is a need for research to explore the connections between students' self-perceptions and their goals and future engagement with mathematics. This is particularly the case when considering that student interest declines as they transition through K-12 and gender differences continue to persist in mathematics related careers. Knowing how students identify with mathematics might provide insight into students' self-perceptions of mathematics and how these perceptions relate to students' career choices.

This quantitative study uses a mathematics identity framework based upon students' self-perceptions related to mathematics. Specifically, students' self-perceptions relating to mathematics interest, recognition by others in mathematics, and mathematical competence and performance were explored. Data were drawn from the Factors Influencing College Success in Mathematics (FICS-Math) project, which was a national survey of college students enrolled in a single-variable calculus course at 2- and 4- year institutions across the United States. This survey yielded a total of 10,492 surveys from students attending 336 college calculus courses/sections at 134 institutions.

The results highlight the salience of the mathematics identity framework, indicating that mathematics interest, being recognized by others in mathematics, and beliefs about their ability to perform and understand mathematics were directly related to students' mathematics identity. This led to the construction of a structural equation model for the mathematics identity framework detailing the relationship between the sub-constructs of mathematics identity. Results also indicated that gender differences in

students' self-perceptions still exist though effect sizes were small. In addition, self-perceptions as seen through a mathematics identity proxy were shown to be a strong predictor of students' career choice as a mathematician, as a science/math teacher, and in STEM fields.

This study establishes an explanatory framework for mathematics identity that provides insight into gender differences and students' career choices in mathematics related fields. Implications of this study are that students' self-perceptions might provide insight into why students persist in areas related to mathematics, how teachers might help students develop a positive sense of affiliation with mathematics, and how this mathematics identity framework might provide a lens for future research.

DEDICATION

This dissertation is dedicated to the memory of my Aunt, Laura Giles, whose

generosity and love of books lives on through the lives of everyone she touched.

ACKNOWLEDGEMENTS

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

In this changing world, those who understand and can do mathematics will

have significantly enhanced opportunities and options for shaping their

futures. Mathematical competence opens doors to productive futures. A

lack of mathematical competence keeps those doors closed. NCTM

challenges the assumption that mathematics is only for the select few. On

the contrary, everyone needs to understand mathematics. All students

should have the opportunity and the support necessary to learn significant

mathematics with depth and understanding. There is no conflict between

equity and excellence (NCTM, 2000, p. 5).

This statement was written as part of the vision of the National Council of Teachers of

Mathematics (NCTM) for school mathematics in the *Principles and Standards for*

Mathematics. It emphasizes the importance of students' education and experiences with

mathematics and the influence these experiences have on students' futures. As NCTM

stated, all students should have opportunities in the classroom to see themselves as

knowers and doers of mathematics. In this way, all students might see the value of

mathematics for their futures.

In order to establish a strong rationale for this study, this chapter (1) summarizes

how the National Council of Teachers of Mathematics (NCTM) vision for mathematical

competence is being addressed in research through student performance, (2) details the

importance of research on persistence toward mathematics, (3) discusses relevant identity

research, (4) provides a list of research questions, (5) states the limitations of the study, and (6) defines key terms.

Despite the vision of NCTM for mathematical competence for all students, there is evidence that student mathematics performance in the United States is weak relative to other countries and that student performance declines when comparing 4th and 8th grade students (TIMSS, 2007). This weak performance in mathematics continues to affect students as they transition from high school to college. Strong American Schools (2008) reported that over one-third of college students need remediation, which is an indication of the inadequacy of American high schools in preparing students for higher education. In addition, research shows that the need for remediation is greater for mathematics (22%) than for writing (14%) or reading (11%) when looking at college freshman (Parsad, Lewis, Greene, 2003). As troubling as that statistic is for the state of mathematics education in the U.S., the continued gaps in students' performance when looking at gender and race is even more troubling (TIMSS, 2007, NCES, 2010). The stability of this gap is evident when comparing students' overall average mathematics scores between 1992, 2005, and 2009 using National Assessment of Educational Progress (NAEP) data. The existence of these gaps highlights the fact that mathematics education in the U.S. is not providing mathematical competence for all students and this is effectively limiting some students' opportunities.

Persistence in Mathematics

These trends in students' mathematics performance as detailed by various research are cause for concern. However, other factors need to be considered as possible influences on students' mathematical competence and persistence in mathematics. Previous research gives evidence for connecting students' motivation and beliefs with students' choices (Simpkins & Davis-Kean, 2005; Simpkins, Davis-Kean, & Eccles, 2006). In a study exploring longitudinal associations conducted by Simpkins and Davis-Kean (2006), mathematics and science activity in the 5th grade was predictive of students' expectancies and values (as measured in 6th and 10th grade through math and science self-concept, interest, and perception of importance). That study also indicated that students' expectancies and values were more predictive of the number of high school mathematics and science courses students took than their grades (Simpkins & Dean-Kean, 2006). This finding stresses the importance of research focusing on student experiences and how these experiences influence students' attitudes towards mathematics. In another study looking at student attitudes, the level at which students valued mathematics declined as they transitioned from 2nd to 12th grade (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). Since research has shown that value toward mathematics influences students' choices, such as the number of high school mathematics classes they take, this decline could have implications for students' activity and persistence toward mathematics. This decline in the value of mathematics also runs contrary to the NCTM's call for students to see mathematics as important to their lives.

Research efforts not only need to focus on understanding students' attitudes toward mathematics better but how these attitudes influence persistence. Further, examining students' career choices is one way of investigating student persistence. Specifically, the National Science and Technology Council (NSTC, 2006) stated that research needs to include efforts to "explore linkages between STEM workforce research and education research in curriculum and instructional practices, equity, and student cognition and learning" that would help to better understand factors for why students are not persisting in science, technology, engineering, and mathematics (STEM) fields (NSTC, 2006, p. 6). The continued underrepresentation of female students intending to enroll in STEM when they enter college only emphasizes the importance of better understanding these linkages (National Science Foundation, 2011).

Therefore, the focus on mathematics is not only important for students pursuing STEM careers, but also for everyday life and the workplace because of the changing world and enhanced opportunities for "those who understand and can do mathematics" (NCTM, 2000, p. 5). According to NCTM's vision, learning mathematics can provide an opportunity to empower students. This vision is especially important to consider because of the high percentage of students entering college needing remediation in mathematics. Educators and researchers must question why there is a declining interest in mathematics as students transition through K-12 and why there is a continued underrepresentation of females in STEM fields. This research is focused on understanding the factors influencing students' self-perceptions about mathematic through a mathematics identity

framework. In conjunction, this study investigates how students' mathematics identity influences their career choices in mathematics related fields.

Identity Research

The construct of identity provides researchers with the opportunity to explore the connection between students' self-perceptions and persistence in mathematics. Specifically, mathematics identity research can explore the complex nature of the mathematics classroom, the broader context of mathematics education, and what it means to be a mathematics learner (Lester, 2007). This, along with Gee's (2001) contention that identity can be used as an analytic lens for research in education and Sfard and Prusak's (2005) statement that the application of identity could be "the missing link" between learning and its sociocultural context, provides a strong rationale for continuing to examine identity in relation to mathematics. Despite the potential of mathematics identity to examine these complex connections and better understand students' experiences and persistence in mathematics, Cobb (2004) stated that identity research in mathematics is underdeveloped as an explanatory construct. He elaborated by stating that a "central issue for mathematics educators concerns the process by which students' emerging identities in the mathematics classroom might, over time, involve changes in their more enduring sense of who they are and who they want to become" (Cobb, 2004, p. 336). Research on the construct of identity in relation to mathematics has begun this work of creating an explanatory framework (Holland & Lave, 2001; Sfard & Prusak, 2005; Solomon, 2007), but these research efforts have been mostly confined to a micro-identity approach

(moment-to-moment) versus a macro-identity approach (global view) for examining student identity. Lichtwarck-Aschoff and her colleagues (2008) refer to the micro-level as "the level where concrete experiences take place, actions and interactions are carried out, and which involves minutes to hours to days" (p. 374). They also refer to macro-level as an aggregated time level, "which describes changes across long-time intervals involving years and decades" (p. 374). This aggregated time level would represent summaries across time of different contexts rather than a daily record of the phenomena of interest (Lichtwarck-Aschoff, van Geert, Bosma, & Kunnen, 2008).

In addition, Nasir, Hand, and Taylor (2008) stated that much of the research relating culture, race, mathematics learning, and identity has taken a qualitative approach. They expand on this by stating that it is important to consider these concepts in relation to students' experiences on a broader scale (Nasir, Hand, Taylor, 2008). In order to understand how students' self-perceptions concerning mathematics influence students on a broader scale, an explanatory model for mathematics identity must first be hypothesized (based on prior research) and tested.

The purpose of this study is to investigate students' self-perceptions concerning mathematics and how these self-perceptions influence students' career choice. By using a mathematics identity framework, a better understanding of how students' self-perceptions are influencing their long-term goals is developed. In particular, specific factors related to students' self-perceptions toward mathematics have been discussed in prior research, which might be viable for a mathematics identity framework. Interest is one of these factors as it has been discussed as context specific and has been linked to students'

motivation and engagement with mathematics (Frenzel, Goetz, Pekrun, & Watt, 2010; Silvia, 2006). Another factor that has been discussed in literature is recognition. Research has found that how students' perceived their parents and teachers seen them in relation to mathematics influenced students' academic competence and performance in mathematics (Bouchey & Harter, 2005). In addition, competency beliefs and students' beliefs about their ability to perform have been shown to influence the activities in which students participate (Bandura, 1997; Bussey & Bandura, 1999). Those studies provide evidence for their inclusion in a mathematics identity framework and have been included in prior research investigating science and physics identity (Carlone & Johnson, 2007; Hazari, Sadler, Sonnert, Shanahan, 2010). This mathematics identity framework also allows for the relationship between gender and mathematics identity to be investigated. In this way, this study adds to the body of research on mathematics identity. The list of research questions guiding this study are given below.

1) How well do the empirical data support the sub-constructs of interest, recognition, competence, and performance for composing the construct of mathematics identity?

2) a) To what extent do the data measure the sub-constructs of interest, recognition, competence, and performance and these sub-constructs measure the construct of mathematics identity?

b) What is the relationship between the sub-constructs of mathematics identity and gender?

3) a) How strongly does mathematics identity predict career choice as a mathematician?

b) How strongly does mathematics identity predict career choice as a mathematics or science teacher?

c) How strongly does mathematics identity predict career choice in a STEM field?

Limitations

There are several limitations to this study. The first limitation comes from the sample, which consisted of students enrolled in single-variable college calculus courses across the United States. Because these students were enrolled in college calculus, the sample could have an over-representation of students pursuing a degree in a STEM field. This might also mean that there is an over-representation of students who positively relate to mathematics. It is possible that a different population of students (e.g. students enrolled in freshman level college English courses) might have yielded different results in the analysis.

A second limitation for this study is that many of the variables used in the analysis were dichotomous. Though appropriate analysis methods were conducted to account for this, these items still provided limited variability. Because of this, it may be more difficult to see differences between groups of students and how they identify with mathematics. There are also some issues with non-centrality that could not be completely overcome even when using non-parametric methods of analysis. This was evident in the confirmatory factor analysis fit indices, which had a root-mean-square error of

approximation (RMSEA) value that was greater than the recommended level. RMSEA is a measure of centrality and this value being larger than recommended indicated non-centrality in the data. Because many of the variables were dichotomous in the study, it is possible that it was causing this non-centrality issue in analysis.

Definitions of Key Terms

Academic Self-concept – an individual's perceptions of self with respect to achievement in school" (Reyes, 1984, p. 559) and "confidence in learning mathematics" (Reyes, 1984, p. 560)

Competence (identity sub-construct) – people's beliefs about their ability to understand mathematics

Identity – how individuals see themselves based on their perceptions and navigation of everyday experiences in a given context

Interest (identity sub-construct) – a person's desire or curiosity to think and learn about mathematics

Latent Variable (Construct or Factor) – a variable that is not directly measured or observable, meaning that it is inferred from a set of variables such as mathematics identity (Schumacker & Lomax, 2010)

Mathematics Identity - how students see themselves in relation to mathematics based upon their perceptions and navigation of everyday experiences with mathematics

Observed Variable – a variable that can be directly measured or observed such as students' grades (Schumacker & Lomax, 2010)

Performance (identity sub-construct) – people's beliefs about their ability to perform in mathematics

Recognition (identity sub-construct) - how people perceive others view them in relation to mathematics

Structural Equation Modeling (SEM) – a combination of confirmatory factor analysis (CFA), regression, and path analysis to investigate observed and latent variables (Schumacker & Lomax, 2010)

CHAPTER TWO

LITERATURE REVIEW

Chapter Two is a detailed literature review highlighting relevant research and theoretical perspectives guiding this research study. This chapter is divided into two major sections: a literature review on identity research and the theoretical framework. The literature review of identity research is further divided into the sections (1) trends of affect in mathematics research, (2) identity development, (3) mathematics identity, (4) gender differences, and (5) student perceptions of interest, recognition, competence, and performance. The theoretical framework presented in this study is based on both theoretical and empirical research: Specifically, the following literature provided guidance for this study: (1) Gee's (2001) theory of identity, (2) Carlone and Johnson's (2007) research on science identity, and (3) Hazari, Sonnert, Sadler, and Shanahan's (2010) research on physics identity. This literature review supports the theoretical framework discussed for mathematics identity and guides the methods used in the analysis.

Trends of Affect in Mathematics Research

The history of research on affect in mathematics involves taking into account how research paradigms have shifted. In McLeod's (1992) review of literature on affect, he stated that previous reviews of literature were based on the traditional paradigm, which focused on "quantitative methods, paper-and-pencil tests, and the positivistic perspective of behaviorist or differential psychology" (p. 577). This makes sense when considering

qualitative research did not become popular until the 1980's, when researchers became

discontent with the methods being used and began to search for a way to address the

deeper questions of interest including description and meaning (Osborne, 1994; Laverty,

2003). This was also accompanied by the paradigm shift from behaviorism to

constructivism in mathematics education (Steffe & Kieren, 1994).

Reyes (1984) discussed the need for research looking at affective variables

because of their potential to influence persistence and attitudes toward mathematics. She

also stated that research with affective variables needed to have a strong theoretical basis

that took into account the previous literature, both in mathematics and psychology

(Reyes, 1984). There was criticism of research on affective variables being theoretically

weak and lacking a clear direction of influence, needing refined measurement

instruments, and having conflicting and weak correlations when looking at genders (Zan,

Brown, Evans, & Hannula, 2005). While the previous literature reviews on affect have

focused on attitude, McLeod (1992) discussed and expanded the topic of affect to include

beliefs, attitudes, and emotions. His work on affective variables took into account some

of the criticisms by making stronger connections and explanations of related theory.

DeBellis and Goldin (1997) expanded McLeod's discussion of affect by including a

fourth concept, values, along with a different method to compare the four concepts.

There have been substantial changes in research on affect since McLeod's (1992)

review of literature. Philipp (2007) listed five occurrences that have influenced these

changes: (1) the "acceptance and infusion" by the educational system of the ideas

expressed in the NCTM standards; (2) the increase of publication outlets; (3) the

increased political involvement and its influence on the education system in the U.S. and on educational research; (4) the advancements in technology that have provided easier access and reporting of research and have aided in collecting and analyzing data; and (5) "the emergence of sociocultural and participatory theories of learning" (p. 264) These changes, along with the increased discussion among researchers of the relational nature of mathematics, created a growing interest in the topic of affect as it relates to mathematics. This relational nature of teaching involves teachers, students, content, and the multidimensional relationship among all of these components (Franke, Kazemi, & Battey, 2007). Franke, Kazemi, and Battey (2007) also stated that learning can be "seen as social and shared, where teachers and students bring histories and identities to the interactions, where participation is the focus" (p. 228) One way to address the complexity of this type of research is through affective variables. Research that includes affective variables is important for understanding student decisions and learning. The current push in research looks to connect affective variables such as the four discussed above with cognitive factors. Another important move in research efforts is the sociocultural approach, which focuses on social practices and positions within communities (Zan, Brown, Evans, & Hannula, 2005). This has led to an increased focus on the construct of identity.

Self-concept is another factor that has been studied extensively in mathematics education. Though there may be correlations between self-concept and identity, further exploration of the constructs reveals distinct differences. Reyes (1984) defined academic self-concept as consisting of "an individual's perceptions of self with respect to

achievement in school" (p. 559) and further stated that mathematics self-concept is "confidence in learning mathematics" (p. 560). Michaelides (2008) also stated that self-concept "combines diverse beliefs about self-worth, whether an individual respects and accepts himself/herself" (p. 6). Much research on mathematics self-concept has focused on looking at student achievement (Crosswhite, 1972; Fennema & Sherman, 1978; Armstrong, 1981; Liu & Meng, 2010). Studies on self-concept tend to confine the construct as being composed of competency and interest, such as with a study conducted with 416 (9th and 10th grade) high school students in Australia by Pietsch, Walker, and Chapman (2003). The social comparison component in that study was considered as separate from the construct of self-concept, where recognition (from parents, relatives, peers, or teachers) was not considered. Research such as that focuses on more of a micro-level of student beliefs as it takes a picture of students at one moment or several moments in time. Identity research takes more of a macro-level approach by looking at the accumulation of students' experiences and perceptions.

Because of the more stable picture of students' beliefs concerning mathematics, identity takes into account a wider array of sub-constructs that not only incorporates student interest but also the social aspect of what students perceive it means to be a "mathematics person." Wenger (1998) stated that "identity serves as the pivot between the social and the individual" (p. 145). She also stated that focusing on identity within the social learning theory, specifically communities of practice, extends the framework to (1) narrow "the focus onto the person, but from a social perspective" and (2) "expands the focus beyond communities of practice, calling attention to broader processes of

14

identification and social structures" (Wenger, 1998, p. 145). In addition to this perspective, researchers exploring identity have the potential to address the relational nature of mathematics. Research in mathematics education often uses this perspective when examining identity, acknowledging the importance that students' community, culture, background, and other social interactions play on learning (Holland, Lachicotte, Skinner, & Cain, 1998; Boaler, 2000). With mathematics being increasingly discussed in a relational manner and the need for frameworks that can better explain the complex nature of these relationships, mathematics identity becomes an important and unique construct for researchers to investigate.

Identity development

Identity development has been a topic of research and discussion in the field of psychology that has expanded into other research areas such as education. From Erickson's foundational work on identity formation in the 1950's and 1960's (stages of development based on age) to the development of social identity theory (based on membership in a social group), identity research has been used as a lens for researchers who are trying to better understand learning and student experiences inside and outside of the classroom (Erickson, 1968; Ashforth & Mael, 1989). Erickson's (1968) work discussed stages of identity development that individuals passed through as they transitioned from birth to adulthood. His theory discussed the influence that external factors had on individuals' identity development such as parents and society. Marcia's (1966) work added to the understanding of identity development. His work questioned

the rigid transitions in stages of development (as discussed by Erickson) with specific

endpoints. Marcia's (1966) research resulted in four general assumptions:

> (1) adolescents can remain stable in any of the four statuses; (2)
>
> adolescents can move not only from lower to higher statuses, but also
>
> from higher to lower; (3) identity achievement is thus not necessarily the
>
> endpoint of development; and (4) a developmental pathway can comprise
>
> a variable number of status transitions (Meeus, Iedema, Helsen, &
>
> Vollebergh, 1999, p. 421).

This is an important transition in the theory of identity development because it highlights

the idea of identity continually changing, which stresses the importance of providing

students with opportunities to identify positively with a particular content area, such as

mathematics. Another important development in identity research was the social identity

theory discussed by Henri Tajfel and John Turner, which further expanded psychology

research on identity to include social aspects of identity formation (Ashforth & Mael,

1989). This theory is based on the idea that people identify with various social categories

such as gender, age, or organizational affiliation. According to this theory, social

classifications serve two purposes. The first is to order the environment, providing a way

for an individual to define others, and the second helps individuals define themselves in

relation to the social environment (Ashforth & Mael, 1989). This theory is more in line

with the current theoretical perspectives, such as situated learning, that stress social

aspects for learning in the classroom and how students situate themselves in social

environments.

Lave and Wenger's (1991) theory of situated learning stated that learning is a social practice that involves the process of legitimate peripheral participation. This concerns the relationships between "newcomers" and "old-timers" as individuals negotiate what it means to be a member of a community (Lave & Wenger, 1991, p. 29). According to this theory, identity, knowing, and social membership are interconnected (Lave & Wenger, 1991). Further expanding on this work, Wenger (1998) examined learning and identity through her theory of communities of practice, which posits identity theories as a branch of social learning theories. Identity, when examined as a component of learning, is defined as "learning as becoming" (Wenger, 1998, p. 5). Wenger also stated that social theories of learning focus on participation where participation is the process of "being active participants in the practices of social communities and constructing identities in relation to these communities" (Wenger, 1998, p. 4). It is through this perspective that the complexity of social interactions and what it means to belong in a community can be discussed. Much research related to identity examines students' learning through this perspective where learning is a process of negotiating meaning and participation in the classroom. When considering what it means to be a member of a community or be considered a certain kind of person, the social interactions and ways of participating with the social environment are vital.

While Wenger's (1998) communities of practice theory has provided a lens for understanding the importance of social interactions and membership, Gee's (2001) work has significantly influenced the development of identity theory and how identity can be used as an analytic lens in education. His framework discussed the relationship between

identity to historical, societal, and situational influences. Gee's (2001) theory of identity

also emphasizes the idea that all people have multiple identities that are based on how

they interact with society. The complexity of identity is further expanded when

considering that a person's identity can be viewed through a given context, moment-to-

moment interactions, or be situation-based (Gee, 2001). This is important to consider

since this perspective suggests that identity can be viewed from multiple perspectives

based on how it is being investigated. It also stresses the importance of identity research

to consider the complex interactions of individuals, taking into consideration the multiple

influences on a person's identity. A limitation to Gee's (2001) work is that it was not

context specific though his work does emphasize that identity is context specific.

Cobb and Hodge (2011) elaborated on Gee's work by discussing the differences

between normative, core, and personal identities and how they are connected to research

in mathematics. They stated that normative identity is focused on how students

developed a sense of affiliation with what it means to be a mathematical person in the

classroom setting. Understanding how students develop this normative identity in a

mathematics context involves observing the interactions and activities that are part of a

particular classroom. Core identity involves understanding how students develop a "more

enduring sense of who they are and who they want to become" (Cobb & Hodge, 2011, p.

189). Research in this area of identity development involves exploring students' long-

term goals and commitments as well as how their experiences and perceptions have

influenced them. In contrast to the other two types of identity, personal identity "is

concerned with who students are becoming in particular mathematics classrooms" (Cobb

18

& Hodge, 2011, p. 190). Exploring this would entail understanding how students

reconcile their core and normative identities in relation to their personal identity and how

students develop understanding and mathematical competence in the classroom (Cobb &

Hodge, 2011). Since this study is exploring students' experiences and perceptions of

mathematics to better understand their long-term goals and persistence in mathematics, it

is focused on students' core identity development. By investigating students' previous

experiences and attitudes concerning mathematics, a framework for mathematics identity

was constructed. This fills the gap in literature on mathematics identity, which has

focused on moment-to-moment interactions instead of a global view when considering

students' identity development. Though this prior research has been mostly concerned

with moment-to-moment identity development as seen in a classroom setting, it does

provide further insight for this study and the establishment of an explanatory framework

for mathematics identity.

Mathematics Identity

Research in the area of mathematics identity has focused on a narrative approach.

For example, Sfard and Prusak (2005) equate identity-building to storytelling, which is

similar to the discussion by Holland et. al. (1998) of figured worlds using a narrative

perspective of identity. They stated that identity is "improvised" and based on "specific

social situations – from the cultural resources at hand" (p. 4). In addition to taking a

narrative approach, research in mathematics identity has primarily been done on a micro-

level, which looks at the moment-to-moment interactions in the classroom (Lichtwarck-

Aschoff, van Geert, Bosma, & Kunnen, 2008). This study takes a macro-level approach (global view) for investigating mathematics identity.

Despite a growing interest in this area of research in mathematics education, there is still no agreed upon working definition for identity (Lester, 2007; Sfard & Prusak, 2005). Sfard and Prusak (2005) stated that in order for a concept to be operational and thus applicable in research it needs to meet three criteria based on Blumer's test of admissibility: (1) descriptions should specify what one should look at with a concept; (2) what should not be considered needs to be included in the description of the concept; and (3) it needs to "enable accumulation of knowledge" (p. 15). They also state that they chose to "equate identities with stories about persons" in their discussion of identity research in education (Sfard & Prusak, 2005, p. 14). Holland et. al. (1998) have a similar view of identity, defining it as "self-understandings, especially those with strong emotional resonance for the teller" (p. 3). These definitions take into account a qualitative approach that has been used when investigating mathematics identity, and highlight the role that a person's self-perceptions has when examining mathematics identity.

Philipp (2007) posits a broader definition for mathematics identity as "the embodiment of an individual's knowledge, beliefs, values, commitments, intentions, and affect as they relate to one's participation within a particular community of practice; the ways one has learned to think, act, and interact" (p. 259). That definition expands on the role a person's self-perceptions plays when considering how students see themselves in relation to the communities around them and the ways that they participate within those communities. The definition that is eventually agreed upon needs to incorporate the

complexities of identity such as individuals' perception of themselves, perceptions that individuals believe others have about them, individuals' perception of their social position in particular contexts, and the multiple identities of individuals (Philipp, 2007). It is through this lens that the definition of mathematics identity has been developed in this study.

This study defines mathematics identity as how students see themselves in relation to mathematics based upon their perceptions and navigation of everyday experiences with mathematics. This definition of mathematics identity focuses on students' beliefs about themselves in relation to mathematics and how their experiences with mathematics have influenced their perceptions. With this global view of mathematics identity, this study takes up the call by other researchers to look at the broader influence that students' beliefs and experiences have on their mathematics identity (Nasir, Hand, Taylor, 2008) and put forth an explanatory model that investigates how these experiences and beliefs help them to develop an "enduring sense of who they are and who they want to become" (Cobb, 2004, p. 336). It is also important to note that this perspective of mathematics identity takes into consideration the sociocultural perspective and focuses on the influence of students' experiences and perceptions on their choices, beyond performance outcomes. This is particularly important when considering the social nature of why some individuals or groups of individuals are not perceived, by others or themselves, as legitimate members of a group. For example, mathematics identity is an area of research that has the potential to help researchers explore gender stereotyping or gender differences in mathematics.

Gender Differences

To better understand the underrepresentation of females in STEM fields, a mathematics identity framework can be used. This is because mathematics identity takes into account a person's perceptions and sense of affiliation within the mathematics community. Societal influences can also be reflected in how students identify with mathematics and the future choices that they make in relation to mathematics. Prior research provides insight into how this underrepresentation and gender stereotyping is present in the mathematics community. Research investigating mathematics and gender differences has considered participation rates (Windshuttle, 1988; Dekkers, de Laeter, & Malone, 1986; Meyer, 1989), performance (Hyde, Fennema, & Lamon, 1990; Hedges & Nowell, 1995; Lindberg, Hyde, & Peterson, 2010), interest (Marsh & Yeung, 1998, Jacobs et. al., 2002; Fouad, 1999; Einarsdottir & Rounds, 2009), career choice (Eccles, 1994; Parsons, Adler, Meece, 1984; Su, Rounds, & Armstrong, 2009), and competency beliefs (Hyde, Fennema, Ryan, Frost, Hopp, 1990; Watt, 2004; Lindberg, Hyde, & Hirsch, 2008, Else-Quest, Hyde, & Linn, 2010). This research has found that despite changes in the culture of the United States and the reduction of some of the gender gaps, there is continued evidence of gender gaps in mathematics. This is particularly true when looking at choice of career in a science, technology, engineering, and mathematics (STEM) field. Based on data reported by the National Science Foundation (NSF, 2011), females remain underrepresented in STEM fields and this underrepresentation is more prevalent in some areas than others. For example, the NSF (2011) reported that females

22

make up only 10.7% of the employed engineers. They also reported that the number of

females employed as mathematical or computer scientists has declined from 31% to

24.8% from 1983 to 2009. In addition, this trend can be seen when looking at the decline

in the percent of degrees awarded in mathematical sciences to females from 48% in 2001

to 43% in 2009. Other fields such as engineering and computer science have also had a

decrease in the percentage of females awarded a degree between 2001 and 2009 (NSF,

2011). Though these results provide insight into current gender gaps, they do not explain

why these trends are occurring. It is important to explore reasons why this

underrepresentation is still persisting and why gender gaps are increasing in some cases.

Fennema and Sherman (1977) conducted a pivotal research study that explored

gender differences in mathematics education using the Mathematics Attitude Scales

(FSMAS) instrument. That study included $9^{th} – 12^{th}$ grade students (589 females and 644

males) who were enrolled in high school mathematics courses at four schools. The

FSMAS instrument introduced nine scales including students' attitudes toward success in

mathematics, students' confidence in learning mathematics, effectance motivation

(students' motivational preferences) in mathematics, and students' beliefs about the

usefulness of mathematics. Results of their study indicated only small gender differences

when looking at mathematics achievement and spatial ability. Fennema and Sherman

(1977) further stated that the differences found were likely to be the influence of socio-

cultural factors such as role stereotyping. Though that study considered many factors

related to affective variables and gender differences related to mathematics, the

interconnected nature of these variables was not explored in depth.

Other research continues to support the results from Fennema and Sherman's (1977) study. Lindberg, Hyde, Peterson, and Linn (2010) conducted a meta-analysis to explore mathematics performance and gender. Their meta-analysis included 242 studies published from 1990 to 2007 with results indicating that males and females perform similarly in that the average effect size (computed using Cohen's d) of reported on studies including a total sample of 1,286,350 persons was d = +0.05. There was evidence of differences between males and females when considering depth of knowledge, though the authors cautioned that this evidence was based on only three studies due to limited studies taking this variable into account, and the effect was small. This is still important to consider since the depth of knowledge that was discussed is a skill required in high-level STEM careers, and the differences found were in favor of males. Regardless of that finding, the authors concluded that even when considering variability, differences in performance between males and females are small and should be considered as evidence against gender stereotyping in mathematics (Lindberg, Hyde, Peterson, & Linn, 2010).

Since research has continued to indicate that there are no differences or small differences in performance between males and females, other factors need to be considered to explain the underrepresentation of females in STEM fields. Identity research has the potential to investigate cultural, situational, and personal aspects on an individual and is an avenue of research that lends itself to exploring gender differences. This study explores other factors related to students' perceptions of mathematics through a mathematics identity framework, which provides insight into this underrepresentation of female students in STEM fields.

Student Perceptions

In order to determine what factors need to be considered in the mathematics

identity framework, prior research on affective measures that relate to student perceptions

needs to be considered. Students' perceptions of mathematics are important as they can

influence how students identify with mathematics and the choices they make in relation

to mathematics. There are four distinct factors that are summarized in this section of the

literature review, which provides insight into students' mathematics identity: (a) interest,

(b) recognition, (c) competence, and (d) performance. These factors and how they are

viable for this study are discussed in the remainder of the literature review. Discussion of

research that addresses the factors in relation to gender and students' career choice are

also included.

Interest

Interest "refers to an individual's engagement with particular classes of objects

and activities" (Frenzel, Goetz, Pekrun, & Watt, 2010, p. 509). Research on interest

began in the area of psychology and is attributed to the work of Herbart in the early

1800's (Schiefele, 1991). He believed that interest was closely associated with learning in

that it "allowed for correct and complete recognition of an object, leads to meaningful

learning, promotes long-term storage of knowledge, and provides motivation for further

learning" (Schiefele, 1991, p. 300). Dewey (1913) is also noted for his work on the

construct of interest, which explored interest-based learning as opposed to effort-based

learning. Dewey's work has provided insight into the conceptualization of interest in education research (Schiefele, 1991). Kintsch is cited for being the first to discuss the relationship between interest and learning in his work published in 1980 looking at student prior knowledge (Schraw, Flowerday, & Lehman, 2001). Research has since emphasized the role that interest plays in student motivation and engagement with activities (Deci & Ryan, 1985; Hidi, 2000; Silvia, 2006). It has also helped researchers better understand how to conceptualize interest and establish theoretical perspectives for how researchers can use interest as an explanatory factor.

Frenzel, Goetz, Pekrun, and Watt (2010) stated that there are three important aspects to consider for the construct of interest: (1) it is both a state and trait character (meaning that interest can be situationally activated moment to moment or stable based on a person's individual interest in a topic or activity); (2) it is content-specific; (3) it is considered to be closely related to the concepts of value and enjoyment. The first aspect that was discussed mentions the two types of interest that have been generally agreed upon by researchers: situational interest and individual interest (Hidi, 2001). Situational interest is based on attention holding such as students being presented with a novel activity in class (Hidi, 2001). This type of interest can be positive or negative, where the effect of an event or activity fades with time (Stevens & Oliveraz, 2005). In contrast to situational interest, individual interest (dispositional interest) is a reflection of an individual's preferences and an enduring sense of who the individual is based on her/his experiences, knowledge, values, and emotions toward a specific domain or activity (Hidi & Harachkiewicz, 2000; Rounds, 1995). In this study, interest refers to individual

interest, which has been associated with research investigating career choice (Su, Rounds, & Armstrong, 2009). That study was a meta-analytic review using technical manuals of vocational interest inventories, which resulted in 108 inventories. Though that study included a large sample size with individuals ranging in age 16 to 42, the broad scope of the study limited the depth of conclusions that could be made. Individual sample analysis might have eliminated some of the confounding variables. In addition to a connection to career choice, it has been theorized that interest is associated with an individual's identity (Hogan & Blake, 1999). Other research has focused on motivation theory such as attainment value and intrinsic/interest value in examining the connections between individual interest, motivation, and career choices (Eccles-Parsons, 1983; Meece, Parsons, Kaczala, Goff, & Futterman,1982). These theories and research studies have provided support in that interest can be used as an explanatory factor and could be a good indicator of student persistence and career choice.

The second aspect of interest that Frenzel et. al. (2010) discussed was that interest was considered to be content-specific. A particular concern in mathematics education has been the decline in an individual's interest in mathematics from childhood to adulthood (Eccles, Wigfield, Flanagan, Miller, Reuman, & Yee, 1989; Gottfried, Fleming, & Gottfried, 2001). This becomes even more troubling when considering that the decline in interest seems to increase in magnitude later in adolescence (Fredrickes & Eccles, 2002; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Watt, 2004).

In addition to considering students' declining interest in mathematics, research on the construct of interest has focused on gender differences. These differences have been

seen at the elementary level (Lichtenfeld, Frenzel, & Pekrun, 2007) as well as the secondary level (OECD, 2004). Eccles (1994) began to discuss these differences with her expectancy-value model of achievement related choices. According to that theory and other related research, differences in occupational choices are attributed to differences in individuals' expectations for success and subjective task value. Eccles further theorized that these differences in expectations and task value are due to females having less confidence in their ability than males and gendered socialization (Eccles, 1994). This stresses the importance of self-perceptions on students' career choices. Other research has further expanded on that theoretical perspective indicating an influence of social interactions and experiences on students' interest and career choices (Jacobs, Davis-Kean, Bleeker, Eccles, & Malanchuk, 2005). These studies support the idea that interest is an especially applicable factor to consider in the area of mathematics and could provide insight into student choices in relation to mathematics. Interest also has the potential to highlight gender differences as seen in prior research but is not the only factor that contributes to students' self-perceptions.

Recognition

Recognition is an important construct when looking at how people perceive themselves because it takes into account the social aspect to identity construction. Holland and Lave (2001) stated that "because the self is the nexus of an ongoing flow of social activity and necessarily participates in this activity, it cannot be finalized or defined in itself, in its own terms" (p. 11). The development of mathematics identity is

28

influenced by how individuals participate and interact with the people and communities around them. This means it is important to take into consideration how individuals perceive others view them in relation to mathematics. Wenger (1998) supported the recognition component of identity in her book on "communities of practice." She stressed that communities of practice are a social theory of learning that takes into account human beings as social creatures who participate with the world (Wenger, 1998). This theory of learning considers what it means to belong and have a sense of community membership, which is an integral part of individuals' sense of affiliation or identification with certain communities (such as the mathematics community). Other theories support the important role that recognition plays for students.

Social cognitive career theory considers being recognized by others as important, such as the relationship between parents' expectations and students' career interests. Research using this theory has shown parent support does influence students' career interest (Ferry, Fouad, &Smith, 2000; Lapan, Hinkelman, Adams, & Turner, 1999) and self-efficacy (Turner, Steward, Lapan, 2004). For example, Bleeker and Jacobs (2004) conducted a study of parents' expectations of their children in comparison with career choice. That study was a follow-up to a previous study (Jacobs & Eccles, 1992), including a sample of 354 mothers and their children. The authors determined that mothers' beliefs about their children's abilities to succeed in mathematics were significantly related to the career choices that the children made. Though that study found that mother's self-perception was predictive of their children's career choices, the interconnected nature of other self-perceptions was not included in the analysis.

Those studies emphasize the role that parents' expectations play on students' choices. Other research has continued to investigate this role by looking at how students' perceptions of their ability influenced their own perception of their ability (Bouchey & Harter, 2005; Eccles-Parsons, Adler, & Kaczala, 1982; Felson, 1989). Bouchey and Harter (2005) conducted a study of 378 middle school students' perceptions of their mother's and father's beliefs about their competence in math and science and the importance of math and science. Findings indicated a positive and direct effect of those perceptions on students' perceived academic competence as well as their grades. This supports the inclusion of students' perceptions of how their parents view them in relation to mathematics when considering students' mathematics identity.

In addition, research has indicated that parents are not the only influence on students' perceptions of themselves in mathematics; teachers also play an important role in how students perceive themselves. Bouchey and Harter (2005) found that perceptions of teachers' beliefs and behavior were positively correlated with students' self-perceptions about their academic competence and grades. Furthermore, results from a meta-analysis conducted including a sample of 136 manuscripts by Harris and Rosenthal (1985) found that teachers with positive expectations for their students exhibit specific behaviors "display a warmer socioemotional climate, express a more positive use of feedback, provide more input in terms of amount and difficulty of material that is taught, and increase the amount of student output by supplying more response opportunities and interacting more frequently with the student" (p. 377). Teachers influence students' perceptions by the instructional practices and sociomathematical norms that are

established in the classroom. Though that study considered many teacher behavior variables such as praise, positive climate, and eye contact, how students perceived their teachers viewed them in relation to mathematics was not explored. This means that inferences might be made from the results but direct correlations or effect could not.

Other research has shown that teacher beliefs about mathematics and their pedagogical beliefs in relation to mathematics have influenced their instructional practices and student achievement (Peterson, 1990; Putnam, Heaton, Prawat and Remillard, 1992; Thompson, 1992), but teachers also influence their students based on their expectations. Buckley (2010) conducted a case study on a department-wide curriculum redesign of a mathematics department that was attempting to address a high failure rate in the low-level courses at the school. The teachers' expectations of what students in these low-level classes were capable of were listed as one of the reasons why the redesign of the curriculum ended up perpetuating inequalities of equal access to high level mathematics for all students (Buckley, 2010). Teachers' expectations of students' abilities and teachers' views about mathematics could also influence the rigor and opportunities that students are presented within the classroom. In turn, students' perceptions of how their teachers view them in relation to mathematics influence students' perceptions of themselves. This supports the inclusion of teacher recognition when investigating students' mathematics identity.

Because students are influenced by how they perceive their parents and teachers view them in relation to mathematics, gender stereotyping could become particularly problematic for females. Previous research has shown that parents hold different beliefs

about the mathematical abilities of their sons than they do about their daughters (Furnham, Reeves, Budhani, 2002; Frome & Eccles, 1998). Furnham, Reeves, and Budhani (2002) conducted a study asking parents (N=156) to rate their sons' and daughters' intelligence (verbal, mathematical, and spatial). Results indicated that parents rated their sons significantly higher than their daughters. Though this study did highlight potential stereotyping, the influence of parents' beliefs on students' perceptions or subsequent performance or competence with mathematics was not explored.

Beyer (1990, 1995, 1998, 1999) has confronted the idea that females commonly underestimate themselves as a demonstration of modesty, stating that females underestimate their ability in areas that have been commonly considered to be masculine domains. This means that other influences need to be considered for how students perceive themselves in relation to mathematics. While research has indicated that parents underestimate their daughters' abilities in areas such as mathematics (Beyer, 1999), other research has also indicated that teachers exhibit this type of gender stereotyping (Li, 1999; Helwig, Anderson, Tindal, 2001). In a review of relevant literature on teacher beliefs and gender differences, Li (1999) was not able to find conclusive evidence of teachers having different beliefs about males and females. However, she did report evidence of teachers stereotyping mathematics as a male domain as seen through overrating male students' mathematics ability and having higher expectations for male students (Li, 1999). These studies provide evidence that females' self-perceptions are being influenced by the gender stereotyping that they are observing from others, particularly in the area of mathematics, and support the inclusion of recognition by others

(mother, father, and teachers) as important for students' perceptions in relation to mathematics. It is also evidence that gender differences and students' career choices might be explored through recognition.

Competence

In addition to self-perceptions related to students' interest and being recognized in the area of mathematics, research has investigated students' competency beliefs. Specifically, students' perceived competence has been the subject of research in the area of motivation, learning, and achievement. This research was sparked by theories such as Bandura's (1977) theory of self-efficacy and social cognitive theory as well as Eccles and Wigfield's (2002) work in motivation using the expectancy-value theory. Individuals' competency beliefs influence their choices such as the activities in which they participate (Bandura, 1997; Bussey & Bandura, 1999). Research has supported this, indicating that students with high scores for self-perceived academic competence are "more persistent, more likely to adopt master and/or performance approach goals, less anxious, process the learning material at a deeper level, and achieve better study results" (Ferla, Valcke, Schuyten, 2010, p. 519). Because of the connection between competency beliefs and student choices and goals, this construct is viable for exploring student persistence. In another study conducted by Bouchey and Harter (2005), competency beliefs were investigated for 378 middle school students. The study found that students' competency beliefs influenced their scholastic behavior and performance in those content areas. The researchers in that study stated that the rationale for the study was the limited research

that explored content specific competency beliefs (Bouchey & Harter, 2005). Though that study added to literature on the topic in mathematics, it was focused on the influence competency beliefs have on student performance rather than other outcomes or goals such as career choice. Because beliefs about competence can influence persistence as well as goals, it is relevant to consider as part of the mathematics identity construct. Other research and theory supports the inclusion of competency beliefs when looking at persistence because it can influence student engagement, anxiety, and ability (Miserandino, 1996; Frome & Eccles, 1998).

Competency beliefs are also important to consider because they have the potential to distinguish gender differences in students' mathematics identity. Solomon (2007) conducted a study with twelve first-year undergraduate mathematics students that provides insight into competency beliefs. He stated that some of the female students' comments in interviews indicated that a lack of understanding concerning mathematics concepts was threatening and left students feeling as if mathematics was unattainable. Though females made statements in interviews that expressed identities of exclusion, males did express some level of marginalization as well. The contrast between males and females was that males did not express concern about their state of belonging related to learning mathematics, but females expressed a desire to pursue practices that would involve imagination and engagement. In essence, males associated functional identity with mathematics on speed and performance, while females associated functional identity with mathematics on speed and understanding, though there were exceptions to this for some of the students. Solomon (2007) also stated that his research supported the concepts

that had already been discussed by other researchers (Boaler, 2002; Burton, 1999; Fennema & Romberg, 1999) in that mathematics as it is currently taught often "treats students as powerless and unimportant 'outsiders', permanently marginalizing many" (p. 92). His research indicated that mathematics needs to focus on a participatory pedagogy that encourages exploration, negotiation, and ownership of knowledge so that it is accessible to all students. This can in turn help students to develop an inclusive rather than exclusive identity with the mathematics community. Though Solomon's (2007) study did emphasize some differences between males' and females' sense of belonging with mathematics, the study was limited in scope with only 5 females in the study, and one female student expressed an inclusive identity with mathematics. She was the only student out of both males and females that was reported to express this sense of belonging, while the rest of her peers expressed some level of marginalization (Solomon, 2007).

Other research supports conclusions from Solomon's (2007) study with findings that males report higher levels of mathematics competence than females (Else-Quest, Hyde, Linn, 2010; Lindberg, Hyde, & Hirsch, 2008; Watt, 2004). Those prior studies emphasize the importance of including competency beliefs when investigating mathematics identity and how gender stereotyping might be seen when exploring this factor in relation to mathematics. It also highlights how students' competency beliefs might influence their engagement and participation in mathematics, such as future participation seen through choice of career.

Performance

Beliefs about performance and competence are closely related, with many of the same foundational supporting theories. For example, researchers using Bandura's (1986) social cognitive theory have the potential explore students' beliefs about their ability to do academic tasks such as problem solving as seen through self-efficacy. Research has shown a connection between various affective measures such as self-efficacy, anxiety, and self-concept to students' performance. For example, a study was conducted by Pajaras and Graham (1999) to determine the influence of motivation variables on students' mathematics performance. Results indicated that students' self-efficacy was the sole motivation variable that predicted students' performance when also looking at anxiety, self-concept, and self-regulation in a sample of 273 first year middle school students. These findings stress the importance of considering students' perceptions of performance as they can influence their actual performance, but there were limitations to the study that might have influenced the results. The first was that mathematics performance was based on two end-of-unit exams created by a mathematics department chair and teaching team, but these tests, though similar, were not identical. Reliability between tests was reported, but the test items were not discussed so the level of conceptual understanding needed to complete the assessment is unclear. Also, when gender differences are being investigated, it might be worth noting whether items on the tests took into account gender bias.

Research indicates that gender differences in students' confidence in their mathematics ability do not appear until middle school, where males tend to rate

themselves higher than females (Pintrich & De Groot, 1990; Seegers & Boekaerts, 1996). In Pajaras and Graham's (1999) study of middle school students, there were no significant differences between males and females. However, other research has shown differences do exist between middle school students where males rate themselves more positively than females in relation to their ability to perform in mathematics (Seegers & Boekaerts, 1996). There is also evidence that gender differences exist when looking at students' confidence about their math abilities at the high school level (Wigfield, Eccles, Pintrich, 1996). Because there is still debate about the extent of differences that exist between male and female students when looking at beliefs about performance, this construct is important to investigate further and consider in a mathematics identity framework.

Summary

Research exploring identity has highlighted the complex nature of identity development. This includes the importance of considering historical, societal, and situational influences on individuals' development of identity as well as the interconnected nature of these factors. Individuals are continually influenced by the environment and relationships they are a part of; this stresses a need to investigate mathematics identity development through a lens that considers these interactions and influences. It is also important to understand that individuals have multiple, overlapping identities such as mathematics identity and gender identity that influence each other. Students' mathematics identity is also influenced by multiple factors, which are also

inter-correlated with each other. Using a framework that takes these complex

relationships into account can add to the understanding of students' mathematics identity

development and possibly their career choices in mathematics related fields. It can also

stress the relationship between gender and affective variables such as students' beliefs in

relation to interest, recognition, competence, and performance in the area of mathematics.

The way that these relationships might be viewed is further detailed in the theoretical

framework.

Theoretical Framework

A mathematics identity framework is used in this study because it gives

researchers the potential to explore the complex interactions that relate to how students

develop a sense of affiliation and membership with the mathematics community. By

using a mathematics identity framework, this study is accounting for the sociocultural

link that Sfard and Prusak (2005) stated to be an important component to identity

research. This approach also provides a way to explore how other identities (such as

gender) influence students' content identity (such as mathematics). In this way, students'

enculturation into the community of mathematics can be explored, including students'

affiliation or alienation with this community based on their perceptions. The inclusion of

the four constructs (interest, recognition, competence, and performance) in this study

provides a richer lens for investigating students' mathematics identity than considering

only one of these constructs and helps to establish a more global view of how students

identify with mathematics. It is also important to consider identity as it has been

connected to students' persistence and engagement (Boaler & Greeno, 2000; Carlone & Johnson, 2007; Hazari, Sonnert, Sadler, & Shanahan, 2010). This makes mathematics identity viable for investigating students' career choices. In this way, how students have developed a more enduring sense of who they are and who they want to be in relation to mathematics can be explored. The theoretical framework for mathematics identity in this study is informed by Gee's (2001) theoretical work on identity, Carlone and Johnson's (2007) research investigating science identity, and Hazari, Sonnert, Sadler, and Shanahan's (2010) research investigating physics identity. It is a synthesis of this prior research that guides the current study.

Gee's (2001) work on identity established the theoretical perspective on how identity can be used as an analytic lens in education. One of the key ideas that were presented in that work was that people have multiple identities. This idea as it pertains to mathematics identity is illustrated in Figure 2.1.
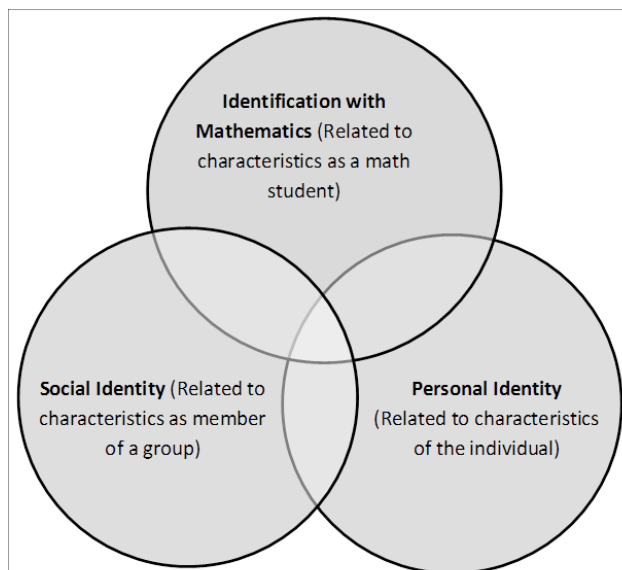
Figure 2.1: Interconnected Nature of Students' Identities

Figure 2.1 emphasizes how a person's multiple identities overlap and influence each other. Though the way in which these multiple identities are interconnected is not explicitly discussed in this study, the figure shows social identity and personal identity are both interconnected with mathematics identity. Social identity relates to the characteristics as a member of a group, and personal identity relates to a person's individual characteristics. Mathematics identity is both being influenced and influencing a person's social and personal identities. In this way, mathematics identity is seen as both a context specific and socially oriented construct. This means that students develop a sense of self in relation to mathematics based on their experiences with and perceptions of mathematics.

Further, mathematics identity is seen as being composed of multiple components. It is the combination of these components that provides a picture of a person's mathematics identity. It is also a way to conceptualize how students develop a more enduring sense of identification with mathematics. The mathematics identity framework used in this study draws from previous research in science and physics identity (Carlone & Johnson, 2007; Hazari, Sonnert, Sadler & Shanahan, 2010). Carlone and Johnson (2007) conducted a qualitative study investigating identity development in women of color as they transitioned through undergraduate, graduate, and science-related careers. That study put forth a model of science identity that included the sub-constructs of recognition, competence, and performance. Results from that study validated the relevance of these components for looking at science identity and provided a better understanding of how gendered, ethnic, and racial factors influence experiences and

career trajectories (Carlone & Johnson, 2007). Hazari et. al. (2010) expanded on Carlone and Johnson's (2007) research by conducting a quantitative study looking at students' physics identity. That study surveyed college students enrolled in introductory English classes across the United States. Because the survey investigated students' experiences in high school, the theoretical framework was expanded to include a fourth component of interest for physics identity. Results from that study validated the theoretical framework being used and found that physics identity was a strong predictor for the choice of a physics career. It also highlighted gender differences when looking at physics identity (Hazari, Sonnert, Sadler & Shanahan, 2010). Building on previous research, this study hypothesizes that mathematics identity is composed of the sub-constructs interest, recognition, competence, and performance. The conceptualization of how these sub-constructs are related is illustrated in Figure 2.2.
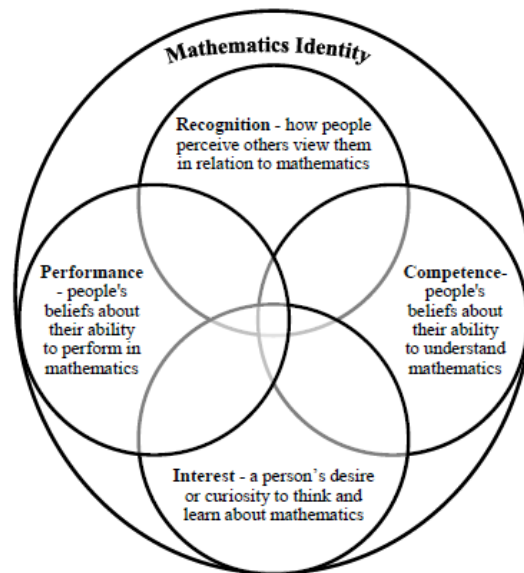
Figure 2.2: Framework for Mathematics Identity

Recognition is defined as how people perceive others view them in relation to mathematics. This sub-construct is investigated using variables related to how students perceive their parents, relatives, peers, and mathematics teachers see them in relation to mathematics. This is important to point out because Philipp (2007) stated that the definition for mathematics identity need to not only include how individuals perceive themselves but also how they perceive others view them. Interest is also an important sub-construct considered in the framework and is defined as a person's desire or curiosity to think and learn about mathematics. Interest has the potential to explore students' value toward mathematics and subsequent mathematics related career choices. The connection between motivation and student interest has been shown in prior research (Bandura, 1986; Fouad, Smith, Zao, 2002), which makes it a viable sub-construct to consider for students' identification and future engagement with mathematics. Both the sub-constructs of competence and performance are closely related though there is evidence that they should be considered as separate (Carlone & Johnson, 2007). Competence is defined as people's beliefs about their ability to understand mathematics, and performance is defined as peoples' beliefs about their ability to perform in mathematics. Students' perceptions of their ability to perform in or understand mathematics are influenced by their experiences and could influence how they choose to participate in mathematics. It is by exploring these sub-constructs together that students' emerging mathematics identity can be better understood. In addition to understanding students' persistence in mathematics, this framework for mathematics identity could provide insight into the continued gender gap in STEM fields.

Because one of the purposes of this study is to create an explanatory framework for mathematics identity in order to add to current research in this area, it was important to consider a framework that could be tested through quantitative methods. The prior research that this study builds on provides a framework that is developed enough to be tested in this manner. The purposes of this study guide the methods used, which are discussed in the next chapter.

CHAPTER THREE

METHODS

Chapter Three describes the methods used in this study. This chapter is divided into two sections: (1) study design and (2) quantitative analysis. The study design details the FICS-Math study, survey development, survey validity and reliability, and sample. The quantitative analysis discusses the analysis used for each of the three research questions in this study. The specific methods used in this analysis were (1) exploratory factor analysis, (2) structural equation modeling, and (3) logistic regression. Quantitative analysis methods used were conducted using R statistical software, which is a "free, open-source, cooperatively developed implementation of the S statistical programming language and computer environment" (Fox, 2006, p. 465; R Development Core Team, 2011).

FICS-Math Study

The Factors Influencing College Success in Mathematics (FICS-Math) study was a national study that sampled single-variable calculus classes at 2- and 4- year colleges and universities across the U.S. The purpose of the study was to collect retrospective data concerning students' experiences in high school mathematics, students' background information, students' perceptions and career goals, as well as performance in their college calculus classes. The FICS-Math survey is composed of 61 items divided into 9 sections. The FICS-Math study was funded by the National Science Foundation (NSF #F15226-105) with Dr. Phil Sadler acting as the principal investigator. The study was a

collaborative effort between researchers at the Smithsonian Center for Astrophysics at Harvard University and the Department of Engineering and Science Education at Clemson University. The method adopted for the FICS-Math study was modeled after the Factors Influencing College Success in Science (FICSS) study conducted in 2002 and the Persistence Research in Science and Engineering (PRiSE) study conducted in 2006. This type of large-scale study can gather more generalizable data than small-scale studies, and FICS-Math, in particular, is the first nationwide study of this type to look at factors influencing college calculus performance.

Survey Development

Development of the FICS-Math survey entailed four major components. The first was a comprehensive literature review of mathematics education journals from the past ten years focusing on factors that influence college calculus performance. The second component involved information gathered from the previous FICSS (Factors Influencing College Success in Science) and PRiSE (Persistence Research in Science and Engineering) surveys such as the use of prior pedagogical or math-related questions that were found to be stable and valid. The third component entailed asking college calculus students to respond to open-ended questions asking them to identify factors that helped them prepare for college calculus. The last component was an online survey sent to mathematics teachers and professors across the nation. This survey asked professors "What can high school teachers do to prepare students for success in college calculus

courses?" and asked teachers "What do you do, as a mathematics teacher, that you think make a positive difference in helping our students succeed in college calculus?"

Validity and Reliability

Content validity for the FICS-Math survey was established through a synthesis of the components given above, a pilot test of the survey, and a focus group discussing the survey with experts in science and mathematics education. The pilot study was conducted with 47 students at two separate institutions. The pilot test indicated that the FICS-Math survey was valid and established an average time of 15 to 20 minutes to complete the survey.

A test re-test study was conducted to examine the stability (a form of reliability) of the survey. This entailed administering the survey to the same sample with a delay between administrations to determine if there were significant differences between responses. The FICS-Math survey was administered by researchers in the college calculus classes of four different universities at a two week interval. This phase of the research project was done in the fall of 2009 yielding 148 completed surveys. Results from the test re-test study indicated an overall reliability with a correlation coefficient of 0.71 for linear variables and 94 percent agreement for dichotomous and categorical variables for the FICS-Math survey items. These results indicate a degree of reliability, especially when considering Thorndike's (1997) analysis. He found that a reliability coefficient of 0.5 corresponds to a 0.04% likelihood of a reversal in the direction of an effect for a sample of 100 (Thorndike, 1997).

Sample

A list of degree-granting postsecondary institutions in the United States was obtained from the National Center for Education Statistics (NCES) for recruiting purposes. The table of institutions was composed of 1,668 two-year and 2,637 four-year schools for a total of 4,305 institutions and contained fall 2007 enrollment numbers for 2-year institutions and fall 2006 enrollment numbers for 4-year institutions. In order to ensure that the sample collected was representative of the national sample of students enrolled in 2-year and 4-year institutions, overall undergraduate enrollment numbers (full-time and part-time) were used to set goals for recruitment. This analysis determined that approximately a third of the national undergraduate population attended schools with fewer than 5,400 undergraduates, approximately a third of these students attended schools between 5,400 and 14,800 undergraduates, and the final third of the sample of students attended schools with more than 14,800. These cut-off points were used to separate the schools into small, medium, and large lists. This list was randomized and stratified by size (small, medium, and large) and type (4-year and 2-year). The resulting six lists contained the following number of institutions: 2,089 small 4-year colleges, 348 medium 4-year colleges, 200 large 4-year colleges, 1,279 small 2-year colleges, 289 medium 2-year colleges, and 100 large 2-year colleges.

Recruiting was conducted using these randomized lists by the heads of mathematics departments. Correspondence was initiated and maintained through email and phone until a sufficient number of participants was attained for each bin. Of the 276 institutions contacted, 182 (65.9%) agreed to participate and 113 (48.6%) returned usable

student surveys. Surveys were administered in the fall of 2009 and returned to Harvard University yielding a total of 10,492 surveys from students attending 336 college calculus courses/sections at 134 institutions. Table 3.1 details the population and sample along with corresponding response rates.

Table 3.1: Population and Sample Response Rates

Population and Sample					
		small	medium	large	total
2 year	population estimate	2932	19342	16783	39057
	percent of overall population	*1.8*	*11.6*	*10.1*	*23.5*
	sample size	188	1460	1812	3460
	percent of overall sample	*1.8*	*14.0*	*17.4*	*33.2*
4 year	population estimate	12140	66357	48698	127195
	percent of overall population	*7.3*	*39.9*	*29.3*	*76.5*
	sample size	870	2401	3706	6977
	percent of overall sample	*8.3*	*23.0*	*35.5*	*66.8*

Response Rate					
		small	medium	large	total
2 year	institutions contacted	15	97	49	161
	institutions returning surveys	10	38	25	73
	percent returning/contacted	*66.7*	*39.2*	*51.0*	*45.3*
4 year	institutions contacted	52	40	23	115
	institutions returning surveys	21	27	13	61
	percent returning/contacted	*40.4*	*67.5*	*56.5*	*53.0*
Overall	institutions contacted	276			
	institutions returning surveys	134			
	percent returning/contacted	*48.6*			

Table 3.1 provides the overall population estimate and sample size for small, medium, and large size schools for each type of institution (2- and 4-year). The

percentages are also reported in the table so that comparisons can be made between the

population estimate and sample size as well as an overall response rate for the sample.

The response rate was also included in Table 3.1 for the six different lists used in

recruiting. Figure 3.1 illustrates the distribution of survey responses across the nation.

Legend: Red=2-year small schools, Blue=2-year medium schools, Purple=2-year large schools.
Green=4-year small schools, Yellow=4-year medium schools, Orange=4-year large schools

Figure 3.1: FICS-Math Sample Distribution

Figure 3.1 distinguishes between the six lists that were used in recruiting. The

distribution of respondents by gender was 60% male and 34% female, with 6% not

reporting their gender. The race and ethnicity distribution was as follows: 66.7% White,

4.6% African-American, 10.7% Asian, 8.9% Hispanic, and 0.4% American

Indian/Alaskan Native.

Analysis for Research Question1

  In order to answer the first research question (How well do the empirical data support the sub-constructs of interest, recognition, competence, and performance for composing the construct of mathematics identity) exploratory factor analysis (EFA) was used. The purpose of factor analysis is to "reveal any latent variables that cause the manifest variables to covary" (Osborne & Costello, 2009, p. 133). While EFA is often used in instrument development, it was used in this study to determine if the factors extracted would support the theoretical model presented. Specifically, this analysis was selected to determine if the sub-constructs of mathematics identity (interest, recognition, competence, and performance) are distinct concepts. The method used for factor analysis was maximum likelihood. Because the items being used from the FICS-Math survey are dichotomous variables, Spearman correlations were used. Promax rotation was also used for the analysis because this is an oblique method of rotation, which is appropriate because the factors in this study were hypothesized to be strongly correlated with one another.

Analysis for Research Question 2

  Once the theorized construct of mathematics identity was tested using EFA, structural equation modeling (SEM) was used to investigate question number two.

  a) To what extent do the data measure the sub-constructs of interest, recognition, competence, and performance and these sub-constructs measure the construct of mathematics identity?

b) What is the relationship between the sub-constructs of mathematics

identity and gender?

SEM is an analysis method that combines confirmatory factor analysis (CFA), regression, and path analysis to investigate observed and latent variables. A latent variable (construct or factor) is not directly observable or measured, which means it is inferred from a set of observed variables (Schumacker & Lomax, 2010). These latent variables are measured using observed variables (indicators). SEM was an appropriate analysis method for this study because it has the potential to explore the complex relationships of interest in the theoretical framework, and it addresses questions such as "to what extent are observed variables actually measuring the hypothesized latent variables?" (Schumacker & Lomax, 2010, p. 201), which is similar to the type of research questions being asked in this study.

According to Diamantopoulos, Siguaw, and Siguaw (2000) there are seven steps in conducting SEM (1) model conceptualization, (2) path diagram construction, (3) model specification, (4) model identification, (5) parameter (model) estimation, (6) assessment of model fit (model testing), and (7) model modification. The first two steps of SEM analysis do not involve any type of calculations or analytical tests. Model conceptualization involves an extensive literature review of the topic of interest, which is used to support a theoretical framework. The theoretical framework for this study is a mathematics identity framework and is based on a synthesis of literature referenced in the literature review. It is at this point that the two models which are integral in SEM can be clearly defined. These two models are the measurement model (describes how observed variables measure or operationalize each latent variable) and the structural model

(describes the relationships between latent variables). The details of variables being used

in the measurement model are presented in Table 3.2.

Table 3.2: Variables in Measurement Model

Latent Variable	Observed Variable	Survey Item
Interest		*Do you agree or disagree with the following statements?* *(Agree or Disagree)*
	Q44dislike	I wish I did not have to take math.
	Q44enjoy	I enjoy learning math.
	Q44interest	Math is interesting.
	Q44lookforward	I look forward to taking math.
Recognition		*Do the following people see you as a mathematics person?* *(No, not at all 1 – 6 Yes, very much)*
	Q45mathpersonp	Parents/Relatives/Friends
	Q45mathpersont	Mathematics teacher
Competence		*Do you agree or disagree with the following statements?* *(Agree or Disagree)*
	Q44understand	I understand the math I have studied.
	Q44nervous	Math makes me nervous.
	Q44persist	Setbacks do not discourage me.
Performance		*Do you agree or disagree with the following statements?* *(Agree or Disagree)*
	Q44exam	I can do well on math exams.

	Observed Variable	Survey Item
Sex	Q46gender	*Are you male or female?*
Career Choice		*Which of the following best describes your current career goal?* *(Career choices: medical professional, health professional, life scientist, earth/environmental scientist, physical scientist, engineer, computer scientist, mathematician, science/math teacher, other teacher, social scientist, business person, lawyer, English/language arts specialist, and other non-science related career)*
	Q43mathcareer	Mathematician, science/math teacher, engineer, and physical scientist

As can be seen in Table 3.2, interest, competence, and performance include variables that

are dichotomous, gender and career choice are categorical variables, and recognition

includes ordinal variables. In addition to including the variables from the FICS-Math

survey, the table indicates which variables correspond with the latent variables. The

initial structural model for this study is shown in Figure 3.2. This model includes

observed and latent variables as well as the hypothesized interactions between them

based on the theoretical framework.
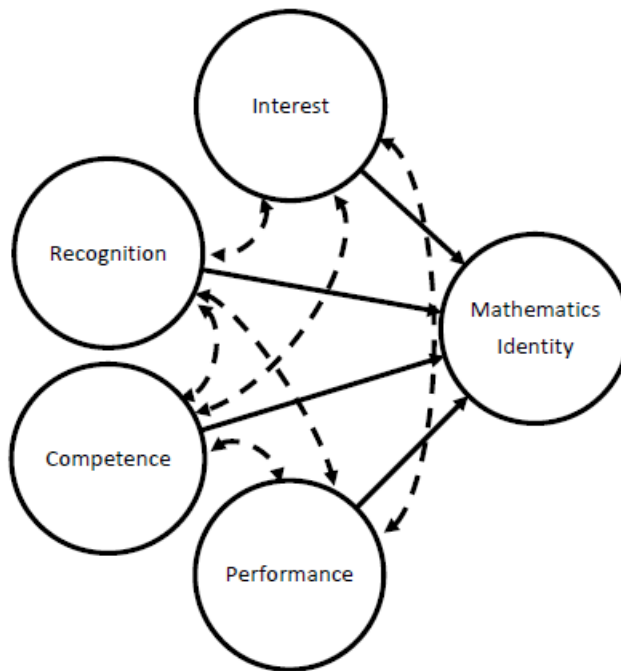
Figure 3.2: Initial Hypothesized Structural Model

Figure 3.2 highlights the inter-related nature of the sub-constructs (interest, recognition,

competence, and performance) as well as the hypothesized direct effect that these sub-

constructs have on mathematics identity. The direct effects can be seen with the solid

arrows and the covariance relationship with dotted arrows. It is also hypothesized that

this effect is positive for each of the sub-constructs.

The next step involved in SEM, model specification, was stated by Schumacker

and Lomax (2010) to be the most difficult. Model specification involves detailing the

number and characteristics of the parameters that need to be estimated. It is at this point

that the pathways are specified with a series of regression equations. Model

identification is a process of ensuring the model is determined by taking the condition

rank into account. This entails determining the number of fixed, free, or constrained

parameters that are in a model (Schumacker & Lomax, 2010). The model estimation step

is the creation of a variance-covariance (or correlation) matrix using observed variables

of interest. Because several of the observed variables being used in this study are either

dichotomous or categorical, this matrix must be calculated with methods appropriate for

these types of variables. The bootstrap method is one of several methods that have been

used in SEM research to do this (Kupek, 2006).

The sixth step entails testing the model to see if it is a good fit or if modifications

need to be made. There are many measures of fit that can be used to assess the model that

has been constructed including chi-square, goodness of fit (GFI), adjusted goodness of fit

(AGFI), root-mean-square error of approximation (RMSEA), Tucker-Lewis (TLI),

normed fit index (NFI), as well as other fit indices not listed here (Schumacker & Lomax,

2010). Due to the complexity of SEM, it is recommended that some combination of these

fit indices be reported in research results. Though there is some agreement to which fit

indices need to be included, there is variation between researchers and publication

outlets. It is recommended that at least one fit index from the different types of fit indices

be reported. These different types of fit indices are absolute fit (determines how close the

model is to a perfect fit), relative fit (compares a chi-square for the hypothesized model to

the null model), parsimonious fit (relative fit that considers adjustments made due to

model complexity), and noncentrality-based fit (based on chi-square fit which tests the

null hypotheses of $\chi^2 = 0$). Based on recommendations made through various literature

(Schumacker & Lomax, 2010; Kline, 2009) the fit indices that are reported for this study

along with their interpretation are included in Table 3.3.

Table 3.3: Fit Indices

Fit Index	Criteria for a Good Fit
Chi-square	p > 0.05, value obtained from tables using df
Goodness of fit (GFI)	> 0.90, where 0 is no fit and 1 is perfect fit
Adjusted GFI (AGFI)	> 0.90, value adjusted for df
Standardized RMR (SRMR)	< 0.10
Root-mean-square error of approximation (RMSEA)	< 0.08
Comparative fit index (CFI)	> 0.95
Tucker-Lewis index (TLI or NNFI)	> 0.90
Incremental fit index (IFI)	The higher the value, the better the model

It is important to note that though the chi-squared value is being reported for the models, it was anticipated that this value would be significant due to the large sample size (Schumacker & Lomax, 2010). It is reported because it is commonly reported in literature and the chi-squared value for the model can still provide information when compared with the chi-squared value in the null model.

Model modification is the final step in SEM and was done based on the fit indices and testing of other models. This means that pathways and variables are added or removed in an effort to improve the model. During this process all modifications are made based on the theory being tested, so that no arbitrary changes are made. This modification process allows a better data-to-model fit to be attained (Schumacker & Lomax, 2010).

Though the same SEM analysis methods were used to address Research Question 2b, a preliminary test was conducted to investigate gender differences. This preliminary test entailed conducting a Welch's t-test to determine if there was evidence of gender differences for the sub-constructs in the mathematics identity framework. Welch's t-test was selected to account for unequal variability between males and females, since the Variability Hypothesis has been a topic of discussion in research and theory investigating gender differences (Shields, 1982). Effect sizes were also calculated for each of the sub-constructs (interest, recognition, competence, and performance). A model was then created incorporating gender with the mathematics identity framework. This hypothesized structural model can be seen in Figure 3.3.
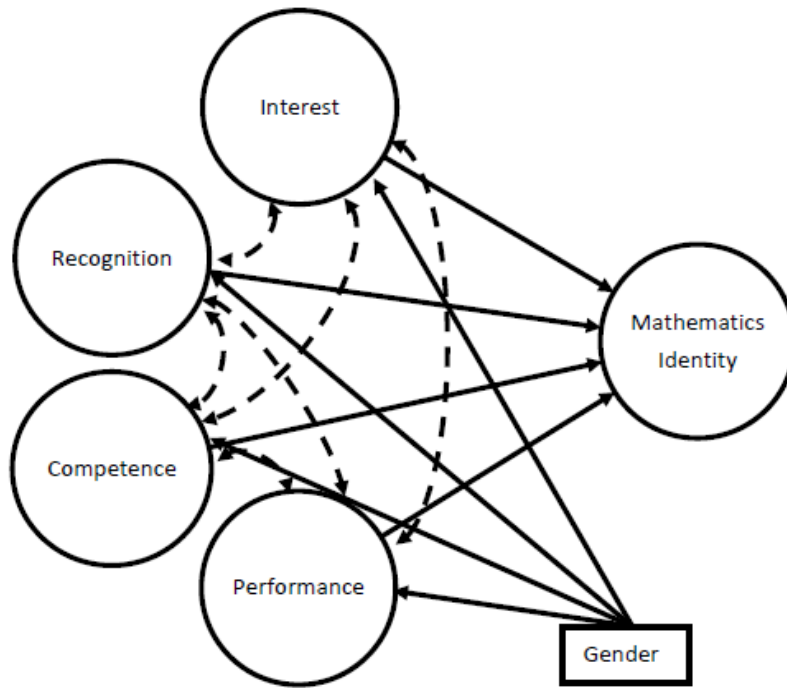
Figure 3.3: Initial Hypothesized Structural Model with Gender

The model tested the relationship between gender and the sub-constructs (interest, recognition, competence, and performance). It was hypothesized that some gender differences would be found.

Analysis for Research Question 3

To answer Research Question 3, logistic regression was used in conjunction with the results attained from SEM.

a) How strongly does mathematics identity predict career choice as a mathematician?

b) How strongly does mathematics identity predict career choice as a

mathematics or science teacher?

c) How strongly does mathematics identity predict career choice in a

STEM field?

Logistic regression was used because the outcome variables being considered were

dichotomous. Regression models were created to determine how mathematics identity

predicts students' career choice. The outcome variables of interest in this study were

students' career choice as a mathematician, as a science/math teacher, and in STEM

fields. A proxy for mathematics identity was calculated using the results from SEM

(structural coefficients). This proxy for mathematics identity acted as the independent

variable, while students' career choice acted as the dependent variable. Odds ratios were

also calculated to determine the magnitude of effects found with logistic regression.

Summary

This chapter discussed the FICS-Math study and analysis methods used in this

study. Details were provided for the development of the survey used to collect data,

validity and reliability, and sample. This chapter also provided information about the

analysis methods being used for each of the three research questions including a

discussion of why the analysis methods were appropriate and details about the variables

being used in analysis. In addition, an initial hypothesized structural model was detailed

based on the theoretical framework being used in this study. The results of the analysis

conducted in this study are reported and discussed in Chapter Four.

CHAPTER FOUR

RESULTS

This chapter details the results and how they relate to the research questions

presented in Chapter One. Each research question is addressed separately, but the chapter

begins by reporting descriptive statistics of data used for this study. The chapter is

organized in the following way: (1) summary of descriptive statistics, (2) results related

to Research Question 1, (3) results related to Research Question 2, and (4) results related

to Research Question 3.

Descriptive Statistics

All quantitative analyses were conducted using R statistical software (version

2.14.0) and are based on the data from the FICS-Math survey as previously discussed.

The percent of missing values for each of the 12 observed variables used in this study can

be seen in Table 4.1.

Table 4.1: Percent of Values Missing for FICS-Math Survey Items

Observed Variable	% Missing
Q44dislike	7.03
Q44enjoy	4.01
Q44interest	4.11
Q44lookforward	6.81
Q45mathpersons	4.11
Q45mathpersonp	4.33
Q45mathpersont	5.62
Q44exam	6.61
Q44nervous	4.47
Q44persist	4.94
Q44understand	7.08
Q43mathcareer	9.35
Q46gender	5.7

All the variables in Table 4.1 had less than 10% of values missing with Q43mathcareer having the highest percentage of missing values at 9.35%. Six of the variables (Q44enjoy, Q44interest, Q45mathpersons, Q45mathpersonp, Q44nervous, and Q44persist) had less than 5% of their values missing. Having a large sample size (N=10,492) and small percentages for missing values made listwise deletion an appropriate method for dealing with missing data values. General descriptive values for the observed variables can be seen in Table 4.2.

Table 4.2: Descriptive Statistics for Observed Variables

Observed Variable	N	Mean	Standard Deviation	Percent Agree	Percent Disagree
Q44dislike	9,754	-	-	33.31	66.69
Q44enjoy	10,071	-	-	80.47	19.53
Q44interest	10,061	-	-	83.40	16.60
Q44lookforward	9,777	-	-	58.57	41.43
Q45mathpersons	10,061	0.64	0.31	-	-
Q45mathpersont	9,902	0.63	0.29	-	-
Q45mathpersonp	10,038	0.69	0.29	-	-
Q44exam	9,798	-	-	80.87	19.13
Q44understand	9,749	-	-	86.58	13.42
Q44nervous	10,023	-	-	41.39	58.61
Q44persist	9,974	-	-	55.65	44.35

Table 4.2 details the sample size for each of the observed variables after missing values are removed. The mean and standard deviation was reported for the ordinal variables, and the frequency (reported through percent agree and disagree) is calculated for the dichotomous variables. While most of the variables were dichotomous and did not need to be rescaled, some of the variables such as Q45mathpersont and Q45mathpersonp, were rescaled to have the range of 0 to 1. This was done so that analysis could be more meaningfully interpreted because the variables were standardized before analysis was conducted.

Research Question 1

To answer the first research question (How well do the empirical data support the sub-constructs of interest, recognition, competence, and performance for composing the

construct of mathematics identity?) exploratory factor analysis (EFA) was conducted.

Two variables were reverse coded before this analysis was conducted (Q44dislike and

Q44nervous). Q45mathpersons was removed from the factor analysis because it was used

later as a scaling variable in further analysis. Preliminary results of EFA found that

Q44dislike loaded separately from other variables. This variable was removed and EFA

was conducted with the remaining nine items from the FICS-Math survey that

corresponded to the sub-constructs of mathematics identity. Factor analysis also

determined that there are three rather than four sub-constructs for mathematics identity,

which are Interest, Recognition, and Competence/Performance. The results of this

analysis including which variables loaded under each of the sub-constructs are detailed in

Table 4.3.

Table 4.3: Exploratory Factor Analysis for mathematics identity sub-constructs

Factor 1: Interest (% of cumulative variance explained = 19)		
Survey Item	**Statement**	**Loading**
Q44enjoy	I enjoy learning math	0.90
Q44interest	Math is interesting	0.76
Q44lookforw	I look forward to taking math	0.53
Factor 2: Competence and Performance (% of cumulative variance explained = 33)		
Survey Item	**Statement**	**Loading**
Q44exam	I can do well on the exams	0.70
Q44understand	I understand the math I have studied	0.55
Q44nervous	Math makes me nervous	0.46
Q44persist	Setbacks do not discourage me	0.45
Factor 3: Recognition (% of cumulative variance explained = 43)		
Survey Item	**Statement**	**Loading**
Q45mathpersonp	Degree to which parents/relatives/friends see you as a math person	0.79
Q45mathpersont	Degree to which math teachers see you as a math person	0.51

Table 4.3 indicates that all nine items loaded between 0.45 and 0.90, which is greater than the 0.40 recommended in literature related to social science research (Costello & Osborne, 2005). Competence and performance loaded under the same factor, which suggests that the two factors are closely related. Due to this result, these two factors were combined in continued analysis. All other FICS-Math survey items loaded as hypothesized. Interest accounted for 19% of the cumulative variance explained with the items loading between 0.53 and 0.90. Competence/performance accounted for an additional 14% of the variance (for a total of 33%) with items loading between 0.45 and

0.70. Recognition accounted for an additional 10% of the variance explained (for a total of 43%) with items loading between 0.51 and 0.79.

Research Question 2A: Measurement Model

In order to address Research Question 2a (To what extent do the data measure the sub-constructs of interest, recognition, competence, and performance and these sub-constructs measure the construct of mathematics identity?) structural equation modeling (SEM) was used. This entailed a two-step process. The first step was an analysis of the measurement model, and the second step was to construct the structural model to test the relationship between constructs. The exploratory factor analysis provided validation and guidance for the indicator variables were used in the measurement model.

Because SEM is a way to examine the relationship between observed variables, an inter-correlation matrix was calculated using polychoric, polyserial and Pearson depending on the observed variables being correlated. This matrix, which is Appendix A, was used to construct the initial measurement model. The results of the initial measurement model along with corresponding fit indices are included in Table 4.4. This table, which in essence represents a confirmatory factor analysis (CFA), includes the standardized factor loadings and item reliability for observed variables. Fit indices for the measurement model are also included in Table 4.4.

Table 4.4: CFA Factor Loadings, Item Reliability, Construct Reliability, Average

Variance Extracted, and Fit Indices

Latent Variable	Observed Variable	Unstd. Factor Loading	Std. Error	Item Reliability (R^2)
Interest	Q44enjoy	0.99***	0.009	0.98
	Q44interest	0.90***	0.043	0.81
	Q44lookforward	0.90***	0.047	0.81
Recognition	Q45mathpersont	0.68***	0.018	0.46
	Q45mathpersonp	0.67***	0.050	0.45
Competence/ Performance	Q44exam	0.77***	0.018	0.59
	Q44understand	0.82***	0.048	0.67
	Q44nervous	0.63***	0.014	0.40
	Q44persist	0.47***	0.016	0.22

Index	Measurement Model level
df	24
x^2	2675.2***
GFI	0.94
AGFI	0.89
SRMR	0.039
RMSEA	0.108
CFI	0.95
NNFI	0.92

Unstandardized factor loadings ranged from 0.47 to 0.99; because loadings are greater than 0.40, they are retained in the model. Though the item reliability (R^2) for Q44persist is low at 0.22, it is kept in the model because it is a significant pathway and improves the overall model fit. Item reliability for all other variables ranged from 0.40 to 0.98. Standard errors were calculated using the bootstrap method; these are generally larger than unadjusted standard errors since non-normal distribution is expected with

dichotomous variables. When looking at fit indices, the x^2 is significant, but this is not unexpected due to the sample size being large in this study. The other fit indices included in Table 4.4 provide a more accurate picture of the model fit. All fit indices were within the recommended level except for AGFI (which was only slightly low) and RMSEA. It is recommended that the value for AGFI should be greater than 0.90, but the CFA model indicates that AGFI is 0.89. RMSEA is a measure of non-centrality. Because many of the variables used in this analysis are dichotomous, it was anticipated that there would be some indication of this in the fit indices.

Research Question 2a: Structural Model

A structural model for mathematics identity was hypothesized and tested. This modified structural model is illustrated in Figure 4.1.
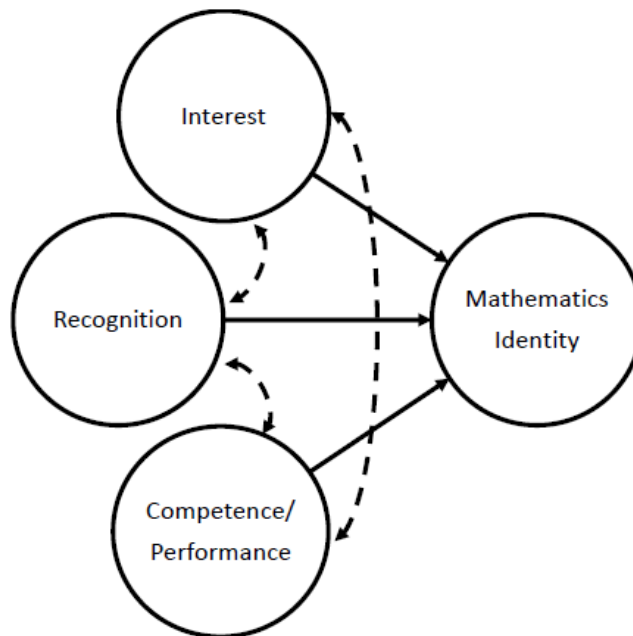
Figure 4.1: Modified Hypothesized Structural Model

Figure 4.1 has the sub-construct of competence and performance combined to form a new

sub-construct, competence/performance. All other pathways are the same as initially

hypothesized. Adjusted standard errors were also assessed and are reported in the results.

The initial (structural) model along with corresponding fit indices is shown in Figure 4.2.

Model x^2= 3204.8   Df = 30
Null Model x^2= 64765   Df = 45
GFI   0.94
AGFI   0.89
SRMR   0.039
RMSEA   0.106
CFI   0.95
NNFI   0.93
IFI   0.95
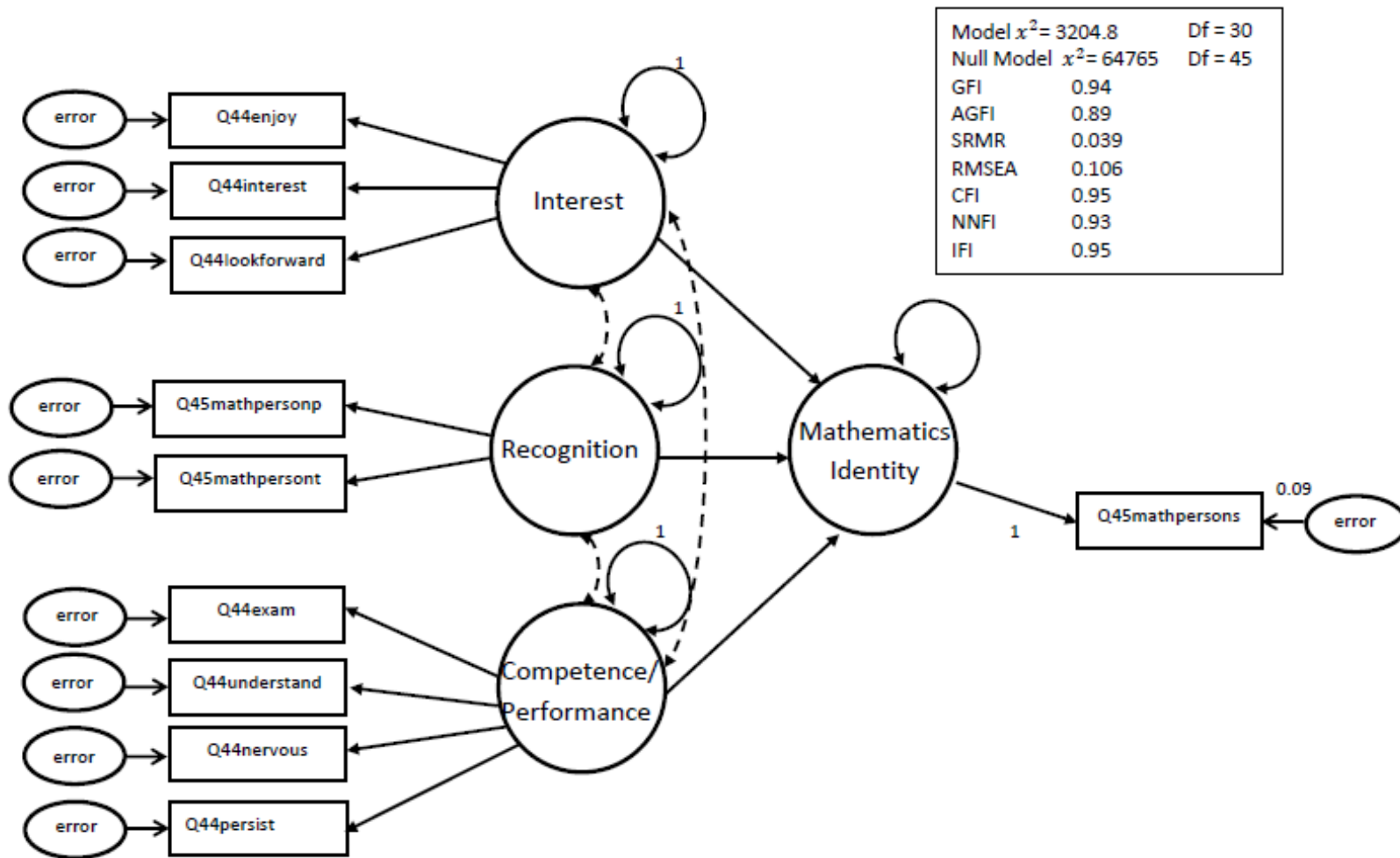
Figure 4.2: Initial Structural Model

The latent variables in Figure 4.2 are represented with circles, and the measured/observed variables are represented with rectangles. Direct effects are shown with solid lines and covariance with dotted lines. The lack of a pathway between variables represents a hypothesis that there is not a direct effect present. There are two types of error indicated in the model. The first is measurement error or residual error, which is associated with the observed variables or latent variables that are outcome (dependent) variables. This error term "represents variance unexplained by the factor that the corresponding indicator is supposed to measure" (Kline, 2009, p. 9). The second error term is a disturbance. This error term is associated with endogenous variables and accounts for "all unmeasured cases of the corresponding endogenous variable" (Kline, 2009, p. 103).

The hypothesized model includes four latent variables: interest, recognition, competence/performance, and mathematics identity. It is hypothesized that the sub-constructs of interest, recognition, and competence/performance directly predict mathematics identity. It is also hypothesized that the three sub-constructs are inter-correlated. Because latent variables are not observed directly, their unit of measurement (variance) needs to be set (Schumacker & Lomax, 2010). This can be done one of two different ways. The way it was done for the model in Figure 4.2 was by assuming that the latent variables had a standardized unit of measurement and fixing the variance of the latent variables (interest, recognition, and competence/performance) to 1. The latent variable, mathematics identity, also had to be set, but this was done the second way reported in literature by using a reference variable (Schumacker & Lomax, 2010). The reference variable for mathematics identity is Q45mathpersons and can be seen as being

fixed in the model by setting the pathway to 1. The variable that is chosen as a reference variable is typically the best indicator variable for the latent variable. In order to have an identified model, the variance error for the reference variable also had to be specified. By conducting a factor analysis, the loading and variance error for the reference variable could be established.

Results of the factor analysis indicated that q45mathpersons had the largest value at 0.91and was the best indicator variable for mathematics identity. This factor analysis was not used for any other purpose other than identifying the reference variable, so the results are not included in this section but can be seen in Appendix B. This variance error term was calculated by subtracting 1-Rxx, which is approximately one minus the variance explained for the variable (Kline, 2009). This was obtained by using the factor loading $(1 - 0.91)$, to arrive at a reasonable error variance value of 0.09. This value can be seen in Figure 4.2. The x^2 (25, N=9397) was significant at 3204.8 though this is not unexpected for a large sample (Schumacker & Lomax, 2010). GFI was greater than 0.90 at 0.94; AGFI was slightly less than 0.90 at 0.89; SRMR was less than 0.05 at 0.039; RMSEA was greater than 0.08 at 0.106; CFI was greater than 0.90 at 0.95; NNFI was greater than 0.90 at 0.93; and IFI was greater than 0.90 at 0.95. All pathways in Figure 4.2 were highly significant (p < 0.001). The goal in SEM is to achieve the best model fit based on fit indices that do not compromise the theory being represented. Two fit indices (AGFI and RMSEA) exceeded recommended levels for the initial structural model indicating that modifications could provide a better fit model. Using the mod.indices function in R, a list of the five modifications that could be made that would have the

70

greatest effect on the fit indices was given. Figure 4.3 illustrates the final structural model

along with the corresponding fit indices based on the recommended modifications and

theory being tested.
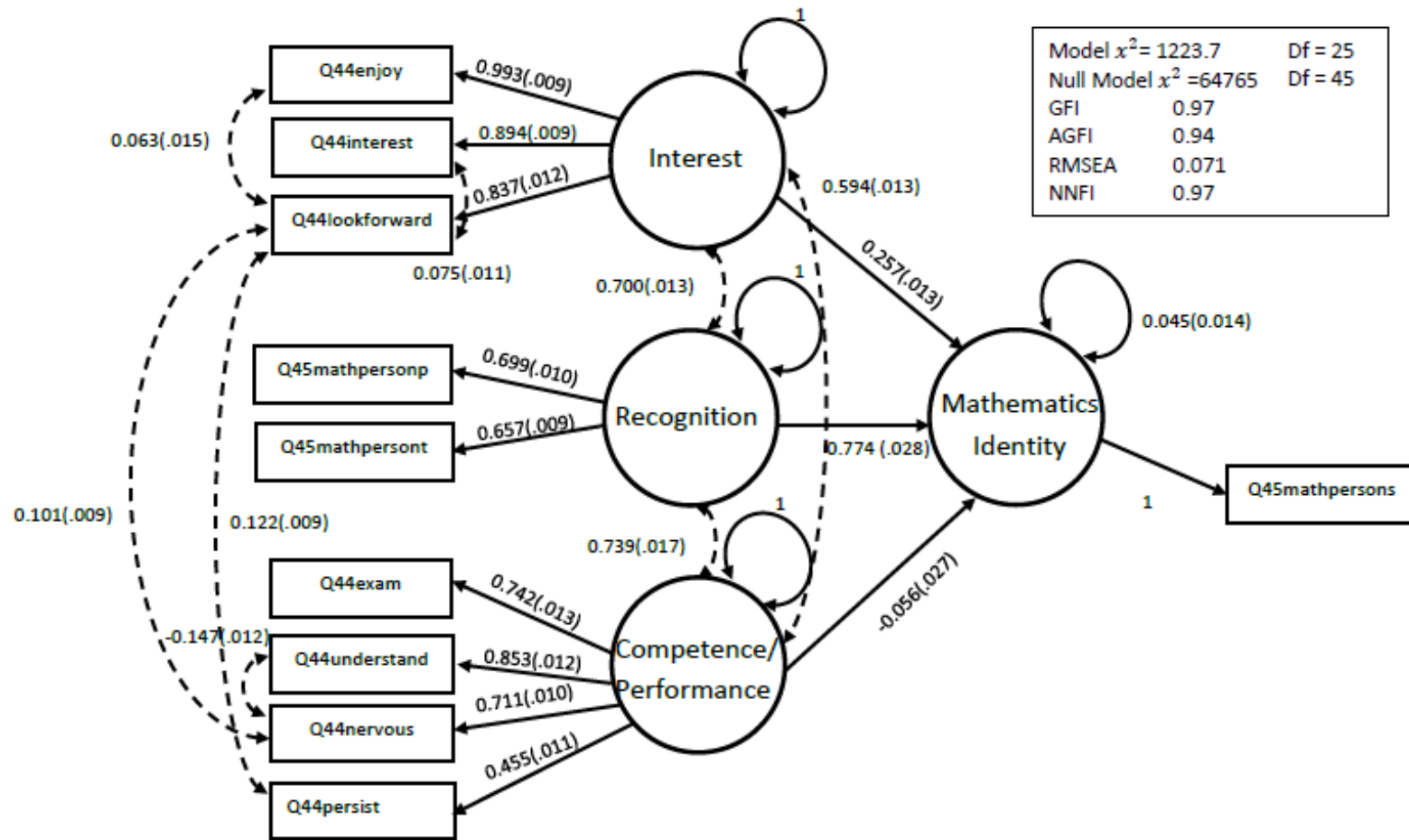
Figure 4.3: Final Structural Model

The pathways added to the final structural model were all related to correlating indicator

error terms. Five additional pathways were added correlating the measurement error of

Q44enjoy with Q44lookforward, Q44interest with Q44lookforward, Q44lookforward

with Q44nervous, Q44lookforward with Q44persist, and Q44understand with

Q44nervous. This indicates that these variables are correlated with each other, which is in

line with the theoretical framework that hypothesizes the sub-constructs as being highly

correlated. By looking at the fit indices between the initial and final model, it can be seen

that the addition of pathways made for a better fit model with fit indices, excluding x^2,

for the final model all falling within recommended levels. The x^2 (25, N=9397) was

significant at 1223.7 though this is not unexpected for a large sample (Schumacker &

Lomax, 2010). GFI was greater than 0.90 at 0.97; AGFI was greater than 0.90 at 0.94;

SRMR was less than 0.05 at 0.030; RMSEA was less than 0.08 at 0.071; CFI was greater

than 0.90 at 0.98; NNFI was greater than 0.90 at 0.97; and IFI was greater than 0.90 at

0.98. All pathways in Figure 4.2 were highly significant (p < 0.001). Table 4.4 details the

parameter estimates for the final structural model presented in Figure 4.3 including the

unstandardized estimates, adjusted standard error, and standardized estimates. Adjusted

standard errors were calculated using the bootstrap method discussed previously.

Table 4.5: Results of SEM Analysis

Parameter	Unstandardized	Adjusted Standard Error	Standardized
Structural Coefficients			
Interest → Mathematics Identity	0.257	0.013	0.269
Recognition → Mathematics Identity	0.774	0.028	0.811
Competence/Performance → Mathematics Identity	-0.056	0.027	-0.059
Factor Loadings			
Mathematics Identity			
Q45mathpersons	1.000	-	0.954
Interest			
Q44enjoy	0.993	0.009	0.993
Q44interest	0.894	0.009	0.894
Q44lookforward	0.837	0.012	0.831
Recognition			
Q44mathpersonp	0.699	0.010	0.699
Q44mathpersont	0.657	0.009	0.657
Recognition			
Q44exam	0.742	0.013	0.742
Q44understand	0.853	0.012	0.853
Q44nervous	0.711	0.010	0.710
Q44persist	0.455	0.011	0.457
Measurement error variances			
Q45mathpersons	0.090	-	0.090
Q44enjoy	0.014	0.015	0.014
Q44interest	0.201	0.014	0.201
Q44lookforward	0.314	0.020	0.309
Q45mathpersonp	0.511	0.014	0.511
Q45mathpersont	0.569	0.012	0.569
Q44exam	0.450	0.014	0.450
Q44understand	0.272	0.016	0.272
Q44nervous	0.496	0.012	0.496
Q44persist	0.785	0.008	0.791

Table 4.5 Continued

	Factor variances		
Mathematics Identity	0.045	0.014	0.049
Interest	1.000	-	1.000
Recognition	1.000	-	1.000
Competence/Performance	1.000	-	1.000
	Error covariance		
Q44enjoy ↔ Q44lookforward	0.063	0.015	0.063
Q44interest ↔ Q44lookforward	0.075	0.011	0.074
Q44lookforward ↔ Q44nervous	0.101	0.009	0.100
Q44lookforward ↔ Q44persist	0.122	0.009	0.122
Q44understand ↔ Q44nervous	-0.147	0.012	-0.147
	Factor covariance		
Interest ↔ Recogntion	0.700	0.013	0.700
Interest ↔ Competence/Performance	0.594	0.013	0.594
Recognition ↔ Competence/Performance	0.739	0.017	0.739

Note: All pathways were statistically significant at $p < 0.001$.

Factor loadings are slightly different in the structural model than the measurement model due to addition of structural pathways. The three factors (interest, recognition, and competence/performance) all have positive covariance values. The direct effects of the structural model are of particular interest for understanding the explanatory model of mathematics identity. Mathematics identity was predicted by interest (standardized coefficient = 0.269, adjusted standard error = 0.013), recognition (standardized coefficient = 0.811, adjusted standard error = 0.028), and competence/performance (standardized coefficient = -0.059, adjusted standard error = 0.027). The effect of recognition is much larger than either the interest or competence/performance factors.

Competence/performance is a negative predictor for mathematics identity although it has a very small effect on mathematics identity. There are several hypotheses that might provide insight into why this result was obtained, but these are discussed further in the next chapter.

Research Question 2b

In order to address Research Question 2b (What is the relationship between the sub-constructs of mathematics identity and gender?) independent t-tests were performed. The t-tests addressed the hypothesis of whether the mean of each mathematics identity sub-construct was significantly different at the level of 0.05 when comparing females and males. In order to do this analysis, new variables for interest, recognition, and competence/performance had to be calculated. This was done by summing the observed variables-based loadings from the EFA analysis and dividing by the number of observed variables used. For example, interest is composed of the variables Q44enjoy, Q44interest, and Q44lookforward. An interest variable was calculated in the following way.

Interest = (Q44enjoy + Q44interest + Q44lookforw)/3

The results of the Welch's t-test using their three new variables for interest, recognition, and competence/performance are shown in Table 4.6.

Table 4.6: Results of Welch's t-test

Mathematics Identity Sub-construct	Mean		t-statistic	p-value	Effect Size*
	Females	Males			
Interest	0.72	0.75	-16.95	<0.001	0.07
Recognition	0.64	0.67	-4.11	<0.001	0.10
Competence/Performance	0.67	0.72	-11.61	<0.001	0.16

* Effect size was calculated using Cohen's d

The results indicate that there is a highly significant difference between the means for all three sub-constructs (interest, recognition, and competence/performance) when comparing females and males. The effect sizes for these differences are small, but indicate that adding gender to the SEM model could provide more insight about the interaction between the sub-constructs and gender.

Three paths were added to the final SEM model that had been previously tested and found to be a good fit model in order to test gender interactions. This entailed calculating a new matrix including the variable Q46gender and modifying the structural model by adding three regression paths. This modified structural model with a gender variable included can be seen in Figure 4.4 and the resulting SEM analysis can be seen in Figure 4.4.
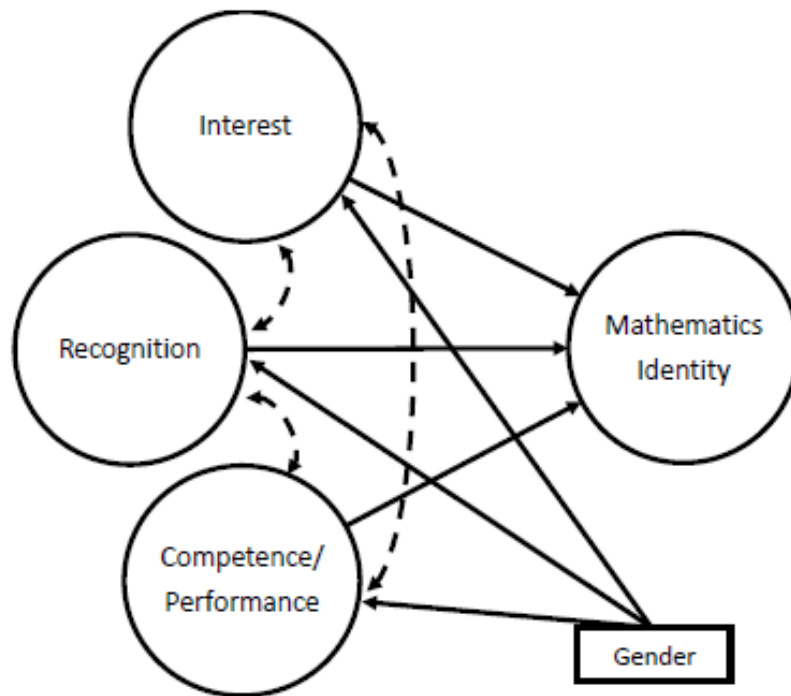
Figure 4.4: Modified Structural Model with Gender
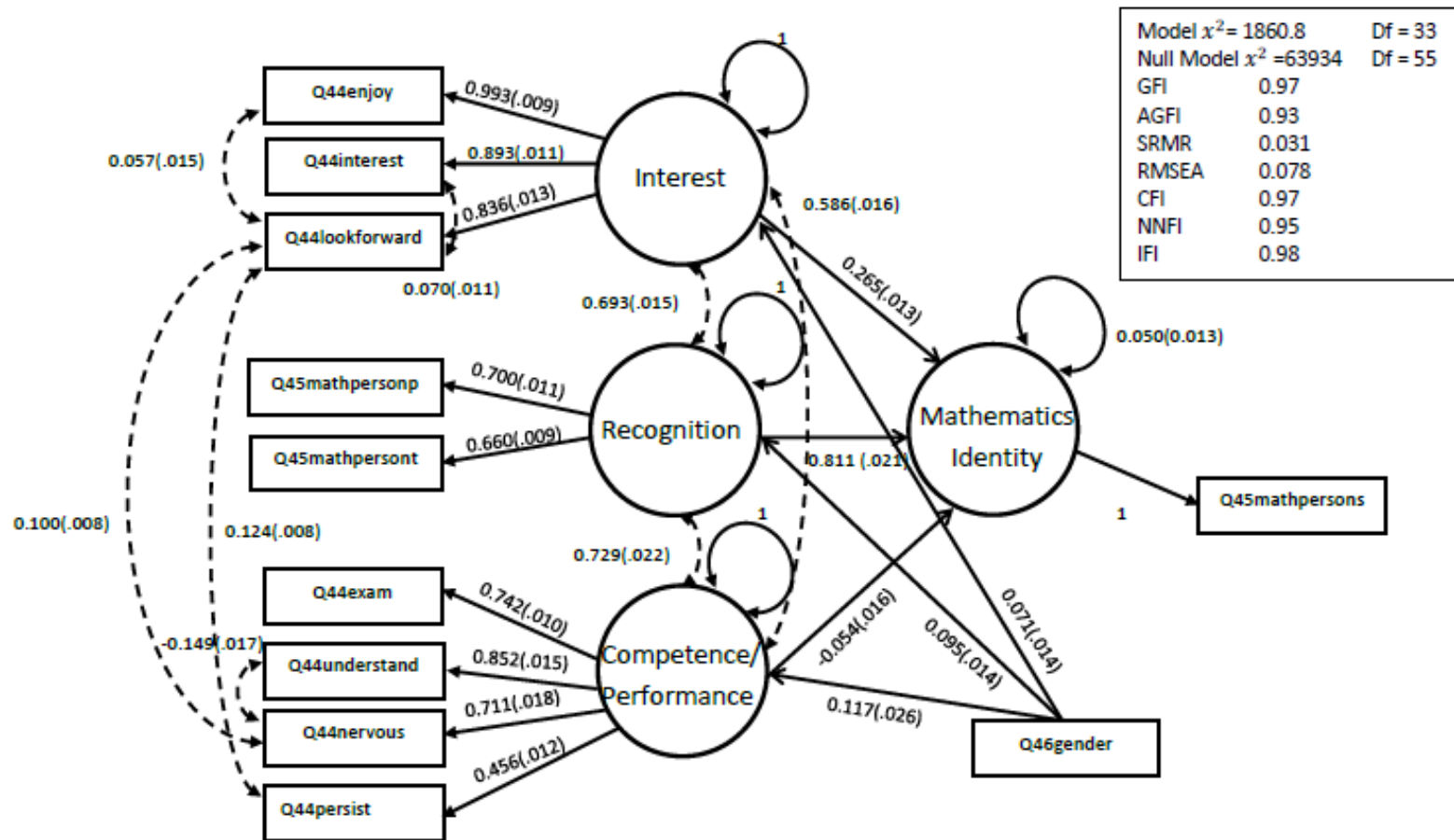
Figure 4.5: Final Structural Model with Gender

As with the previous SEM model without gender, some of the parameters were fixed in order to set the measurement variance. These fixed pathways are indicated on the figure with a 1 and include the error variance for interest, recognition, and competence/performance as well as a reference variable for mathematics identity (Q45mathpersons). For identification purposes, the error variance term for Q46gender was also set to 1. By looking at the fit indices of the final model, it can be seen that all fit indices, excluding x^2 , are within recommended levels. No other pathways were added due to the good fit of the model and the lack of viable suggestions for modifications provided by the mod.indices function in R. The x^2 (33, N=9181) is significant at 1860.8 though this is not unexpected for a large sample (Schumacker & Lomax, 2010). GFI is greater than 0.90 at 0.97; AGFI is greater than 0.90 at 0.93; SRMR is less than 0.05 at 0.031; RMSEA is less than 0.08 at 0.078; CFI is greater than 0.90 at 0.97; and NNFI is greater than 0.90 at 0.95. All pathways in Figure 4.3 are highly significant (p < 0.001) except for the pathway Competence/Performance predicting Mathematics identity, which is moderately significant (p < 0.01). The change in this significance level when gender pathways are added may indicate that gender effects accounted for some of this effect. Table 4.7 details the parameter estimates for the final structural model presented in Figure 4.5 including the unstandardized estimates, adjusted standard errors, and standardized estimates. Adjusted standard errors were calculated using the bootstrap method as previously discussed.

Table 4.7: Results of SEM Analysis with Gender

Parameter	Unstandardized	Adjusted Standard Error	Standardized
	Structural Coefficients		
Interest → Mathematics Identity	0.252	0.013	0.265
Recognition → Mathematics Identity	0.771	0.024	0.812
Competence/Performance → Mathematics Identity	-0.052	0.022	-0.054
	Gender Effects		
Gender → Interest	0.071	0.014	0.071
Gender → Recognition	0.096	0.014	0.095
Gender → Competence/Performance	0.118	0.026	0.117
	Factor Loadings		
Mathematics Identity			
Q45mathpersons	1.000	-	0.954
Interest			
Q44enjoy	0.990	0.010	0.993
Q44interest	0.891	0.009	0.893
Q44lookforward	0.840	0.013	0.836
Recognition			
Q44mathpersonp	0.697	0.010	0.700
Q44mathpersont	0.657	0.008	0.660
Recognition			
Q44exam	0.737	0.012	0.742
Q44understand	0.846	0.016	0.852
Q44nervous	0.707	0.017	0.711
Q44persist	0.451	0.011	0.456
	Measurement error variances		
Q45mathpersons	0.090	-	0.090
Q44enjoy	0.014	0.016	0.014
Q44interest	0.202	0.014	0.202
Q44lookforward	0.307	0.020	0.302
Q45mathpersonp	0.509	0.014	0.509
Q45mathpersont	0.564	0.011	0.564
Q44exam	0.449	0.003	0.449

Table 4.7 Continued

Q44understand	0.274	0.021	0.274
Q44nervous	0.496	0.018	0.495
Q44persist	0.786	0.008	0.792
Q46gender	1.000	-	1.000
	Factor variances		
Mathematics Identity	0.046	0.013	0.050
Interest	1.000	-	0.995
Recognition	1.000	-	0.991
Competence/Performance	1.000	-	0.986
	Error covariance		
Q44enjoy ↔ Q44lookforward	0.058	0.016	0.057
Q44interest ↔ Q44lookforward	0.070	0.012	0.070
Q44lookforward ↔ Q44nervous	0.101	0.009	0.100
Q44lookforward ↔ Q44persist	0.125	0.009	0.124
Q44understand ↔ Q44nervous	-0.149	0.016	-0.149
	Factor covariance		
Interest ↔ Recogntion	0.697	0.014	0.693
Interest ↔ Competence/Performance	0.591	0.015	0.586
Recognition ↔ Competence/Performance	0.738	0.024	0.729

Note: All pathways were statistically significant at $p < 0.001$.

The SEM gender model had structural coefficients, factor loadings, measurement error variance, factor variance, error covariance, and factor covariance that are almost identical to the values for the final SEM model not including a gender variable. This is expected since additional pathways or variables were not added other than those related to the gender variable. The effects of gender on the sub-constructs (interest, recognition, and competence/performance) are of particular interest for understanding gender differences; however, they do not modify the explanatory model for mathematics identity. The gender variable had the largest effect on competence/performance (standardized coefficient =

0.118, adjusted standard error = 0.026), the second largest on recognition (standardized coefficient = 0.096, adjusted standard error = 0.014), and smallest on (standardized coefficient = 0.071, adjusted standard error = 0.014). The positive standardized coefficients for each of the gender pathways indicate that males rate themselves higher than females for each of the sub-constructs.

Research Question 3

In order to address Research Question 3 (how strongly does the mathematics identity proxy predict career choice) logistic regression was performed. There are three parts to Research Question 3: career choice as a mathematician, science/math teacher, and in a STEM field. The following career choices were considered to be in a STEM field: life scientist (e.g. biologist, medical researcher), earth/environmental scientist (e.g., geologist, meteorologist), physical scientist (e.g., chemist, physicist, astronomer), engineer, computer scientist (IT), mathematician, and science/math teacher. The total number of students selecting the three career choices is given in Table 4.8.

Table 4.8: Number of Students Selecting Career Choice

Career choice	Number	Percent of Sample
Mathematician	152	1.60
Science/math Teacher	582	6.12
Non-STEM (e.g. lawyer)	3,916	41.17
STEM		
Life Scientist	329	3.46
Earth/Environmental Scientist	236	2.48
Physical Scientist	344	3.62
Engineer	3,287	34.56
Computer Scientist	665	6.99
Mathematician	152	1.60
Science/math Teacher	582	6.12
STEM Total	5,595	58.83

As can be seen in Table 4.8, there were 152 students who have selected the mathematician career choice, 582 students who selected the science/math teacher career, and 5,595 students who selected a STEM career choice. A mathematics identity proxy was calculated based on the results of the SEM analysis in order to conduct logistic regression. This proxy was then used to predict student career choice. Each of the sub-constructs of mathematics identity was weighted based on the path coefficients from the final SEM model and added to create a mathematics identity proxy (MIP).

$$MIP = [(0.269 \times Interest) + (0.811 \times Recognition) + (-0.059 \times Competence/Performance)]$$

The mathematics identity proxy was also standardized, with a mean equal to 0 and standard deviation equal to 1. This standardization was done so that results could be interpreted more readily. The independent variable is the mathematics identity proxy, and the dependent variable is student career choice in the regression model. Control variables were intentionally left out of the analysis in order to tell a clear picture of the relationship between the mathematics identity proxy and career choice.

Research Question 3a

The results for logistic regression testing whether the mathematics identity proxy predicts a career choice as a mathematician is shown below in Table 4.9.

Table 4.9: Final Logistics Regression Results for Mathematician Career Choice

	Estimate	SE	Sig	Odds Ratio
Intercept	-7.10	0.76	***	
Mathematics Identity	1.00	0.16	***	2.73

*p<0.05   **p<0.01   ***p<0.001

Results indicate that the mathematics identity proxy is highly significant (p<0.001) and is a positive predictor for career choice as a mathematician. The odds ratio indicate a shift in the mathematics identity proxy of one standard deviation corresponds to a 2.73 higher odds of choosing a career as a mathematician.

Research Question 3b

The results for logistic regression testing whether the mathematics identity proxy predicts career choice as a science/math teacher is shown in Table 4.10.

Table 4.10: Final Logistics Regression Results for Science/math Teacher Career Choice

	Estimate	SE	Sig	Odds Ratio
Intercept	-0.97	0.30	**	0.38
Mathematics Identity	0.85	0.07	***	2.33

*p<0.05   **p<0.01   ***p<0.001

Results indicate that mathematics identity is highly significant ($p < 0.001$) and is a positive predictor for career choice as a science/math teacher. The odds ratio indicates a shift in the mathematics identity proxy of one standard deviation corresponds to a 2.33 higher odds of choosing a career as a science/math.

Research Question 3c

The results for logistic regression testing whether mathematics identity predicts career choice in a STEM field is shown in Table 4.11.

Table 4.11: Final Logistics Regression Results for STEM Career Choice

	Estimate	SE	Sig	Odds Ratio
Intercept	0.52	0.05	***	1.68
Mathematics Identity	0.48	0.02	***	1.62

*p<0.05   **p<0.01   ***p<0.001

Results indicated that mathematics identity was highly significant ($p<0.001$) and was a positive predictor for career choice in a STEM field. The odds ratio indicated a shift in the mathematics identity proxy of one standard deviation corresponds to a 1.62 higher odds of choosing a career in a STEM field.

Summary

This chapter detailed the results of this study based on the three research questions presented in the first chapter. The first research question was addressed through the use of EFA. Results validated the theoretical framework hypothesized with slight modifications, which entailed combining two of the sub-constructs (competence and performance). The second research question was addressed through SEM, which entailed constructing a model to determine the effect that the sub-constructs (interest, recognition, and competence/performance) had on predicting students' mathematics identity. A second model was constructed with a gender variable added so that gender effects could be analyzed. The results indicate a good fit model was constructed that helps to establish an explanatory framework for mathematics identity. They also indicate that males rated themselves higher than females for each of the sub-constructs. The third research question was addressed through logistic regression where mathematics identity predicted career choice as a mathematician, as a science/math teacher, and in a STEM field. Results indicate that the mathematics identity proxy is a highly significant predictor for each of the career choices. The significance of these results is discussed further in the next chapter.

CHAPTER FIVE

DISCUSSION

This chapter discusses the overall findings of this study, implications for mathematics educators, and future research. There are three major outcomes of this research. The first is the development of an explanatory structural equation model for mathematics identity. This model provides a lens for educators and researchers to view mathematics identity in order to better understand students' self-perceptions about mathematics. The second outcome is a model for how gender influences students' mathematics identity. As other research has shown, males reported higher scores for their self-perceptions in relation to mathematics than females. Specifically, males rate themselves higher for each of the sub-constructs of interest, recognition, and competence/performance. Gender differences in student perceptions about mathematics can provide researchers and educators a better understanding of why gender gaps continue to persist in mathematics. The third outcome found is that mathematics identity strongly predicts students' career choice in mathematics, as science/math teaching, or in a STEM-related field. This result highlights the importance of students' self-perceptions about mathematics and the influence of these views on their career choices. These findings can provide guidance to educators and researchers in their efforts to understand how to influence students' mathematics identity as well as establish a foundation for future research. Each of these outcomes is discussed in more detail in the following sections.

Establishing an Explanatory Model

One of the purposes of this study was to develop an explanatory structural equation model to better understand what factors influence students' mathematics identity. The theoretical framework hypothesized is founded on previous empirical and theoretical literature (Carlone & Johnson, 2007; Gee, 2001; Hazari, Sonnert, Sadler, Shanahan, 2010). In order for educators and researchers to have a better understanding of what it means for students to know and learn mathematics, students' perceptions and beliefs about mathematics need to be considered. This entails taking into account the interconnected nature of identity, which is influenced by various factors. The sub-constructs of interest, recognition, and competence/performance are considered viable in the framework because they take into consideration the perceptions of students that relate to many aspects of their experiences with mathematics. For example, interest is connected to students' personal identity as well as their experiences both inside and outside of school. Recognition also focuses on multiple aspects of their identity, looking at how students perceive others including family, peers, and teachers view them. This takes into consideration students' sense of membership in a mathematics community such as a mathematics classroom. Competence and performance, discussed together since they were not quantitatively different in the analysis, relate to students' self-perceptions with respect to their prior experiences and achievement in mathematics, particularly experiences they have had with using mathematics, accomplishing mathematics related tasks, and performance in math courses. While the establishment of this explanatory model is just a picture of the possible complex interactions of these sub-constructs and

the influence on students' mathematics identity, it can also provide a lens for how students view themselves in relation to the mathematics community and what it means for them to be knowers and doers of mathematics.

Research Question 1: How well do the empirical data support the sub-constructs of interest, recognition, competence, and performance for composing the construct of mathematics identity?

The results of the first research question in this study validates the framework for mathematics identity. This validation entailed conducting exploratory factor analysis (EFA) to see if the items being used from the FICS-Math survey aligned with the hypothesized framework. Results validated the inclusion of the sub-constructs interest and recognition in the framework but indicated that the sub-constructs of competence and performance should be combined into one sub-construct. This result implies that students in the sample were not able to distinguish between what it means to understand mathematics and what it means to perform in mathematics. In Carlone and Johnson's (2007) study investigating women of color who were scientists, competence and performance were two of the emerging themes. It is possible that students who are enrolled in a single-variable college calculus course have not had significant experiences where they are able to discern that understanding and performing in mathematics are two separate concepts. This result is also evidence of the highly correlated nature of the sub-constructs, which supports the creation of a mathematics identity proxy. In Hazari, Sonnert, Sadler, and Shanahan's (2010) study using the same sub-constructs to

investigate physics identity, the same result was attained in that the competence and performance items loaded together during factor analysis. Though their study found that recognition and interest loaded separately, two other factors were present in their analysis, science interest and science activity (Hazari, Sonnert, Sadler, & Shanahan, 2010). The results of the EFA analysis in this study do not show evidence of other factors, which means that the framework is particularly applicable when exploring mathematics identity. This could be due to how mathematics is viewed by students as one unit, where science might be viewed as many different units (such as physics, biology, chemistry, etc.). Evidence from this analysis supports the continuation of analysis using the framework hypothesized with the inclusion of three rather than four sub-constructs. The importance of this model for educators and researchers is discussed later in this chapter.

Research Question 2 was divided into two parts. The first part was to establish an explanatory model for mathematics identity. The second part was to determine gender differences in the model created. These questions are addressed in order.

Research Question 2a: To what extent do the data measure the sub-constructs of interest, recognition, competence, and performance and these sub-constructs measure the construct of mathematics identity?

In order to construct an explanatory model, analysis was done through structural equation modeling (SEM). This entailed using the sub-constructs from the EFA analysis

to establish a good fit measurement model. This measurement model was tested using confirmatory factor analysis (CFA). Because this step is foundational for establishing a good fit structural model, measures were taken to insure that appropriate statistical procedures were used as detailed in previous chapters. The next step was to create a structural model so that relationships between latent variables could be explored. The results of this analysis provide insight into students' self-perceptions with regard to the sub-constructs.

Mathematics identity was predicted by the interest variable with a standardized coefficient of 0.269. This means that for a one point increase in the interest sub-construct, mathematics identity increased by 0.269 standard deviations. This is considered a statistically medium effect because it is close to 0.30 (Cohen, 1992). This is an indication of the important role that interest plays in students' mathematics identity. Thus, students who have a higher level of interest toward mathematics are more likely to have a higher mathematics identity. The vital role that interest plays has been supported by previous research in mathematics (Koller, Baumert, & Schnabel, 2001; Krapp, 1999; Renninger, Hidi, & Krapp, 1992). In a study conducted with 602 students who were tested at the end of grades 7, 10, and 12, Koller, Baumert, and Schnabel (2001) found that while interest is does not have a significant effect on achievement, it does predict students' choice of advanced mathematics courses. They also found this correlation between student interest in mathematics and achievement was mediated through instructional environment (Koller, Baumert, & Schnabel, 2001). That study supports the role that students' experiences have in their interest related to mathematics as well as how students'

academic interest influences students' future choices. Koller, Baumert, and Schnabel's (2001) study also provides support for the role that teachers play in encouraging student interest and future engagement in mathematics. Implications for mathematics educators are further discussed later in this chapter. Results from the SEM analysis indicated that students' interest in mathematics influences their mathematics identity, which could in part explain the findings from previous research on the role that interest has on students' choices and establish the mediating role that identity development may have when connecting interest to career choice.

While the Koller, Baumert, and Schnabel (2001) study allowed for interest to be considered, it did not explore the interconnected nature of students' perceptions concerning mathematics as seen through the mathematics identity framework. In a review of the literature on achievement values, goal orientations, and interest to achievement outcomes, Wigfield and Cambria (2010) stated that there was a need for studies that would build on prior research to look at "the combined influences of the values, goal orientation, and interest variables on these and other outcomes at different age levels, to provide us with a richer and more complete understanding of how motivation and major outcome variables relate" (p. 27-28). This study endeavors to provide a better understanding of how interest, as well as other student perceptions related to mathematics, influence students mathematics identity. This was done by examining how career choice was related to students' mathematics identity to further understand the complex relationship between students' perceptions and their choices. Evidence from this analysis also indicates that even at the freshman college level interest is still a predictor

of students' mathematics identity and potentially their career choices. However, though interest is a predictor of students' mathematics identity, it is not the strongest predictor in the model.

SEM analysis also indicates that mathematics identity is predicted by recognition with a standardized coefficient of 0.811. Recognition has the largest effect on students' mathematics identity, where for an increase of one point in recognition, mathematics identity increased by 0.811 standard deviations. This is a statistically large effect because it is greater than 0.50 (Cohen, 1992). This result means that being recognized by others as a "mathematics person" has a greater influence on students' mathematics identity than student interest or students' perceptions of their ability to understand or perform in relation to mathematics. This result also emphasizes the importance of considering how social aspects of students' experiences and perceptions influence their development of mathematics identity and potentially their long-term goals and choices.

Recognition, as defined in this study, takes into account students' perceptions of how their parents, relatives, and peers see them as well as how their mathematics teachers see them in relation to mathematics. The above finding is an indication of how important it is for students to be recognized by others as a "mathematics person" not only in the classroom but also in their home and community. Social learning theories and research from this perspective support the idea that learning is a social process where students negotiate meaning and are active participants (Boaler, 1998; Boaler & Greeno, 2000). In a study conducted by Solberg, Kimmel, and Miller (2012) the level of explicit math-science encouragement that was given by parents to their children had a stronger

influence for students in science, technology, engineering, mathematics, and medicine (STEMM) fields than in STEMM support occupations. This finding was evident by the percent of students who eventually became STEMM professionals with 53% reporting that parents strongly encouraged them to study mathematics and science in high school in contrast to 30% for STEMM science and technology support workers and 25% for those entering a STEMM health support occupation. That study supports the evidence found in this study for the influence that being recognized has on students' mathematics identity. While Bleeker and Jacob's (2004) study did not report on specific careers such as a career choice as a mathematician in a longitudinal study investigating the influence of parents' perceptions on students' career choice, it did explore how these perceptions influences students' career choice in mathematics and science related fields. This study expands on that research by exploring how students' perceived their parents viewed them in relation to mathematics and how these perceptions influenced their mathematics identity.

Other research supports this finding, indicating that not only is parent encouragement important for students' development of a sense of efficacy in mathematics, but teachers' support is also integral (Friedel, Cortina, Turner, Midgley, 2007). NCTM (2000) acknowledged the important role that teachers play in students' experiences with mathematics. They state that "effective teaching conveys a belief that each student can and is expected to understand mathematics and that each will be supported in his or her efforts to accomplish this goal" (NCTM, 2000, p. 18). Evidence from this study supports this in that students' perceptions that their teacher views them as

95

a "mathematics person" are important for their sense of recognition in mathematics and ultimately the development of their mathematics identity. This strong influence that being recognized as a "mathematics person" has on students' mathematics identity is also an indication of how students value external acknowledgement. Students value how others view them and this perception influences how they see themselves. This finding is important to consider because students' perceptions have the potential to influence their behavior and choices, such as the choice to take advanced mathematics courses or pursue a mathematics related career.

In addition, mathematics identity was predicted by competence/performance with a standardized coefficient of -0.059. Competence/performance had the smallest effect on students' mathematics identity, where for an increase of one point in competence/performance, mathematics identity decreased by -0.059 standard deviation. This finding means that student perceptions about their ability to perform or understand mathematics had a negligible effect on their mathematics identity for this population. This effect was significant but small because it was less than 0.10 (Cohen, 1992). This result was not what was initially hypothesized, but further reflection could provide some insight into this finding.

It is first important to consider that the effect size for the competence/performance variable was so small that it was almost a negligible effect. This result might be a consequence of the nature of the sample in that there might be less variability between students who are enrolled in college calculus classes. Recall that interest was not an emergent theme in Carlone and Jonhson's (2007) study as the participants in her study

were practicing female scientists. This indicates that interest did not add insight into their identity at that stage in their careers. Similarly, students enrolled in college calculus may be at a stage in their mathematics careers where perceptions about their ability to understand and perform in mathematics are no longer adding to their mathematics identity. In essence, students taking college calculus have similar perceptions regarding their ability to understand or perform in mathematics. It is also important to note that the survey was given to students at the beginning of the semester before the college calculus class had time to influence student's mathematics identity either positively or negatively. A different result might have been attained if students were surveyed at the end of the semester.

Though evidence about the competence/performance sub-construct indicated that this sub-construct may not be viable for the mathematics identity framework, it was retained in the framework for several reasons. First, a follow up study is being conducted using the same methodology as the FICS-Math study with a different population of students. By surveying students who are enrolled in introductory college English classes, the framework can be further tested with a population of students who have a higher degree of variability in regards to their perceptions of their mathematics abilities. Another reason why this sub-construct was retained was to explore gender influences later in the study. If further analysis indicates that the sub-construct is not significant for students' development of mathematics identity, it will be removed and the theoretical framework modified.

Research Question 2b: What is the relationship between the sub-constructs of mathematics identity and gender?

The first step in exploring the influence of gender on students' perceptions was to do Welch's t-tests to see if there were any differences between males and females for the sub-constructs of interest, recognition, and competence/performance. Results from this t-tests indicates that males rate themselves higher for all three sub-constructs (p<0.001). The largest effect size for these differences is in competence/performance, which is supported by literature indicating that males have higher competency beliefs than females (Else-Quest, Hyde, Linn, 2010; Lindberg, Hyde, & Hirsch, 2008; Watt, 2004). It is important to note that all the effect sizes were small in the gender analysis (interest with an effect size of 0.07, recognition with an effect size of 0.10, and competence/performance with an effect size of 0.16). Regardless of these small effects, these results do provide further insight into what gender differences still exist and the relationship between gender and students self-perceptions in mathematics. Since the t-tests indicated that there were gender differences for the sub-constructs, analysis continued through the construction of a model using SEM.

In order to add gender to the model created through SEM, three paths were added to investigate the influence of gender on each of the sub-constructs. Results from SEM were supported by the results from the t-tests with the three pathways being highly significant (p < 0.001). Competence/performance had the largest effect when interacting with gender with a standardized coefficient of 0.117. This provides additional support that this sub-construct should be considered viable for the mathematics identity

framework. The result means that males rate themselves higher in their ability to understand and perform in mathematics than females. Similar results were found in a meta-analysis conducted by Else-Quest, Hyde, and Linn (2010) when exploring gender differences in students' attitudes and affect in relation to mathematics. For students in the United States, the cross-national study found gender differences in students' attitudes and affect about mathematics where males scored themselves higher though these differences were small (with an effect size of approximately 0.05). It is important to consider these results even if they are small because competency beliefs have the potential to affect students' selection of activities and environments as discussed in cognitive social learning theory (Bandura, 1997; Bussey & Bandura, 1999) and, compounded with other gender differences, can ultimately result in large overall gender gaps.

The second largest effect when considering gender differences was for the sub-construct recognition, with a standardized coefficient of 0.095. This result means that males rate themselves higher in how they feel perceived by others (parents, relatives, peers, and mathematics teachers) as compared to females. This finding could provide insight into gender stereotyping as has been previously discussed in literature (Beyer, 1999; Fennema & Sherman, 1977; Lindberg, Hyde, & Peterson, 2010; Furnham, Reeves, Budhani, 2002; Frome & Eccles, 1998). While there is still evidence of a gender gap in how students believe others view them in relation to mathematics, it is encouraging to see that this effect is small.

A study conducted by Kiefer and Sekaquaptewa (2007) provides insight into the results of this study. They conducted a study with undergraduate women enrolled in

college calculus to investigate women's gender identification and gender stereotyping. When discussing gender stereotyping, the researchers made a distinction between explicit and implicit stereotyping. Implicit stereotyping is associated with unconscious qualities that are attributed to particular social groups, while explicit stereotyping is intentional or conscious. In their study, they found that explicit stereotyping did not predict students' performance or career goals even when considering gender identification. In contrast to this finding, implicit stereotyping did influence students' performance and career goals (Kiefer & Sekaquaptewa, 2007). That study suggests that there may be more to consider when investigating gender differences in the mathematics identity framework. Because this current study did find gender differences when investigating mathematics identity, considering how explicit and implicit gender stereotyping relates to mathematics identity would be of interest in future studies. Also, for the choice of STEM fields, mathematics identity is not the only consideration. For example, physics identity is important for students' physics career choice, and the gender gaps found in a study exploring physics identity are much larger than the gender gaps that were found in this study (Hazari, Sonnert, Sadler, & Shanahan, 2010). This could correspond to the large gaps that are seen when comparing the percentage of females employed as a physicist/astronomer at 13.8% with females employed as mathematical scientists at 38.9% in 2006 (NSF, 2011).

The smallest effect when considering gender differences was for the sub-construct of interest with a standardized coefficient of 0.071. This finding means that males' interest in relation to mathematics is greater than females' interest. Though previous research has reported gender differences in students' interest in mathematics

100

(Lichtenfeld, Frenzel, & Pekrun, 2007; OECD, 2004), the small effect found in this study indicates that this difference is not substantial though still significant. Su, Rounds, and Armstrong (2009) conducted a meta-analysis investigating sex differences in interests for different age groups ranging from a mean age of 12.50 to 42.55. They found that the effect size for differences in interest in mathematics and sciences was small, even though this effect size was in favor of men. This result was in contrast to the effect size for differences in interest related to engineering, which was found to be very large (Su, Rounds, & Armstrong, 2009). Evidence from that study supports the small effect that was found in this study. Because interest is ultimately related to students' career goals (Wigfield & Cambria, 2010), the large effect in gender differences in engineering makes sense considering that the underrepresentation of females in engineering is larger than in mathematics (NSF, 2011). Though females' and males' mathematics identity as defined in this study is similar, the small effects in gender differences cannot completely account for the gender gap that continues to persist in some STEM fields.

Research Question 3 was divided into three parts looking at how mathematics identity predicted student's career choice. These three questions are addressed separately.

Research Question 3: How strongly does mathematics identity predict career choice as a mathematician? How strongly does mathematics identity predict career choice as a mathematics or science teacher? How strongly does mathematics identity predict career choice in a STEM field?

In order to address the third research question, a proxy for mathematics identity was created. This analysis entailed using the coefficients from the SEM analysis to calculate a new variable, which was used as a mathematics identity proxy. Once this proxy was created, it was used to predict students' career choice. Because each of the career variables is a dichotomous variable, logistic regression was used.

The first part of Research Question 3 investigated a career choice as a mathematician. Results indicate that the mathematics identity proxy is a strong predictor for students' career choice with a p-value less than 0.001. A shift in the mathematics identity proxy of one standard deviation corresponds to a 2.73 higher odds of choosing a career as a mathematician. This finding means that compared to the baseline of a student who is neutral with regards to their mathematics identity (baseline of 0 where the student does not identify with mathematics either positively or negatively), a student who has a mathematics identity that is one standard deviation greater than the baseline is nearly three times more likely to choose a career as a mathematician. Figure 5.1 demonstrates the magnitude of the influence that mathematics identity has on a student's career choice as a mathematician.
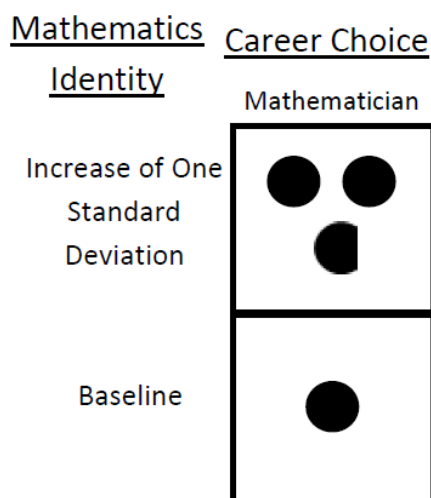
Figure 5.1: Effect of Students' Mathematics Identity on Choice of Career as a

Mathematician

This result highlights the significance that students' mathematics identity has on their career choice as a mathematician and supports the construct as a way of investigating students' career choices in mathematics related fields.

The results from Research Questions 3b and 3c also support the previous statement in that the mathematics identity proxy is a positive predictor for a student's career choice as a science/math teacher and generally for STEM fields. A shift in the mathematics identity proxy of one standard deviation corresponds to a 2.33 higher odds of choosing a career as a science/math teacher. This result means that compared to the baseline of a student who is neutral in regards to their mathematics identity, a student who has a mathematics identity that is one standard deviation greater than the baseline is over two times more likely to choose a career as a science/math teacher. Figure 5.2

demonstrates the magnitude of the influence that mathematics identity has on a student's career choice as a science/math teacher.
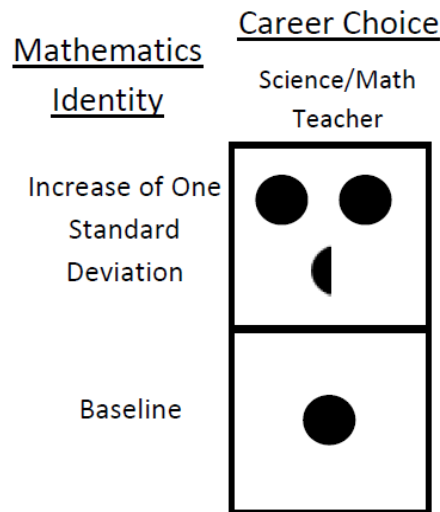
Figure 5.2: Effect of Students' Mathematics Identity on Choice of Career as a

Science/math Teacher

This result also means that while the mathematics identity proxy still has a strong influence on students' career choice as a science/math teacher it has less of an influence (0.40 less odds) than it did on students' career choice as a mathematician.

Students' career choice in a STEM field was the final regression model constructed. Findings indicated that a shift in the mathematics identity proxy of one standard deviation corresponds to a 1.62 higher odds of choosing a career in a STEM field. This result means that compared to the baseline of a student who is neutral in regards to their mathematics identity, a student who has a mathematics identity that is one standard deviation greater than the baseline is over one and a half times more likely to

choose a career in a STEM field. This result also means that while mathematics identity still had a strong influence on students' career choice of a STEM field it has 1.11 lower odds of influencing students' career choice as a mathematician and 0.71 lower odds of influencing students' career choice as a science/math teacher. When considering this result it is important to keep in mind the career choices included in a STEM field. Some careers choices such as biological science and computer science might not be considered as mathematically intense by students particularly since they have fewer mathematics course requirements than other STEM majors such as the mathematical sciences. Regardless of the differing influence that students' mathematics identity has on students' career choices, findings indicated that the construct is a good predictor of students' career goals in mathematics related fields. Figure 5.3 demonstrates the magnitude of the influence that mathematics identity has on students' career choice in STEM fields.
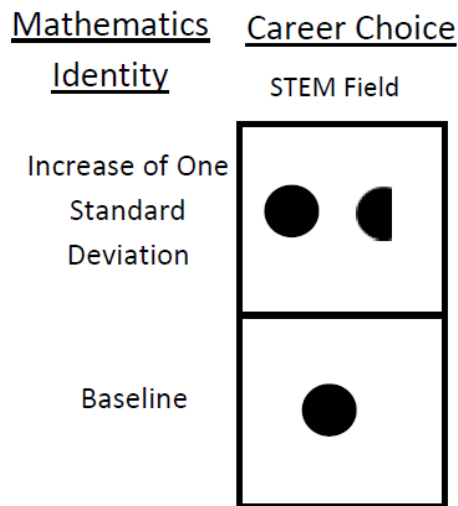
Figure 5.3: Effect of Students' Mathematics Identity on Choice of Career in STEM Fields

Wenger's (1998) discussion of communities of practice might provide insight into why the mathematics identity proxy is such a strong indicator of students' career goals. According to this theory, identity is constantly being negotiated where individuals may have an inbound trajectory with a particular community. This means that "newcomers are joining the community with the prospect of becoming full participants in its practice. Their identities are invested in their future participation, even though their present participation may be peripheral" (Wenger, 1998, p. 154). Students who have developed a sense of belonging and membership with certain communities (such as within mathematics classrooms) may be more inclined to direct their future goals and participation in relation to that community.

Summary

Evidence from this study found that students' self-perceptions related to recognition and interest are significant in their mathematics identity development, which has been a concept that NCTM (2000) has stressed as important for effective classroom instruction. Students who have an increased interest in mathematics are more inclined to develop a stronger mathematics identity. In addition, competence/performance was found to have a negative, yet negligible, effect on students' mathematics identity, though this sub-construct had the largest effect when investigating gender differences. This framework has provided a lens for students' mathematics identity to be viewed as well as highlighted gender differences in students' perceptions in relation to mathematics. These differences could provide further insight if explored since mathematics identity is a way

of understanding student persistence in terms of career choice. Previous research has linked identity to students' career choices (Carlone & Johnson, 2007; Hazari, Sonnert, Sadler & Shanahan, 2010), which has been further supported through the results in this study. While mathematics identity can be mapped to short-term classroom effects, as seen through normative and personal identity research (Cobb & Hodge, 2011), broader effects were explored through a global perspective to mathematics identity development and students' career choice. These findings also have important implications for mathematics educators and provide the groundwork for future research.

Implications

As mathematics education has increasingly been discussed as an issue of equity, it is important to understand students' beliefs about mathematics and how their experiences are influencing their mathematics identity. Cobb and Hodge (2011) proposed a definition of equity, which emphasizes the significance of exploring students' identity as it relates to mathematics. They state that equity "encompasses students' development of a sense of efficacy (empowerment) in mathematics together with the desire and capability to learn more about mathematics when the opportunity arises" (Cobb & Hodge, 2011, p. 181). Their definition of equity includes "students' motivations to continue to study mathematics and their persistence while doing so" (p. 181). The explanatory framework proposed in this study could provide a way for educators and researchers to better understand and further explore student persistence and ways that teachers, parents, schools, and community members could provide opportunities for students to develop

this sense of efficacy and motivation toward mathematics. In particular, providing opportunities both inside and outside of the classroom where students can be recognized in relation to mathematics could help students develop a positive sense of affiliation with mathematics. This could include a focus on participatory methods in the classroom or possibly students tutoring peers outside of the classroom. Research further exploring the connection between instructional practices and students' self-perceptions could provide more insight into how these practices influence students' mathematics identity. The mathematics identity framework also provides a better understanding of how students' experiences with mathematics might influence their perceptions of mathematics and, as Cobb (2004) stated, a "more enduring sense of who they are and who they want to become." (p. 336). If educators want to find ways to provide students with the experiences and opportunities with mathematics that empower them and open doors for future engagement with mathematics, understanding students' mathematics identity development is essential. This research not only provides a picture of the broader influence students' mathematics identity has in terms of career choice, but has the potential to provide insight for curriculum design and instructional practices.

When considering implications for curriculum design and instructional practices, it is important to reflect on how student interest and recognition influences students' mathematics identity and subsequent career choice. Cobb and Hodge (2011) contend that "supporting students' development of a sense of affiliation with mathematics as it is realized in their classrooms should be an explicit goal of both instructional design and teaching" (p. 186). The significance of the sub-constructs, interest and recognition, can

give insight into how teachers can help students develop this sense of affiliation with mathematics. NCTM (2000) stated that including relevant mathematics is a way of capturing student interest. Other research, such as work conducted through a hybrid space framework, has focused on including instructional practices that are culturally relevant to students (Flessner, 2009; Nasir, Hand, & Taylor, 2008; Gonzalez, 1995; Gonzalez & Amanti, 1997). While teacher practices can encourage student engagement and interest through this perspective, the sociomathematical norms that teachers construct are vital for providing a classroom that allows students to be active participants. These norms support or impede taking part in classroom discourse and help students see themselves as knowers and doers of mathematics. This concept goes beyond a focus on student interest and reveals how students come to see themselves in relation to the mathematics they do. Knowing the important role that being recognized as a "mathematics person" plays in students' mathematics identity development provides support for this focus. Teachers need to incorporate practices that allow for students to be recognized by others as contributors to mathematical knowledge and understanding. It is important for students to be recognized by others as knowers and doers of mathematics. When students are recognized, both inside and outside of the classroom, they have the potential to develop a stronger mathematics identity.

Another implication for mathematics educators concerns students not being able to distinguish between what it means to understand and perform in mathematics and that the contribution of competency/performance beliefs to students' mathematics identity is very small at the college calculus level. Though more research needs to be conducted in

order to understand why differences were not found between competency and performance beliefs, it is problematic for students to not be able to distinguish between these concepts, particularly since it has been found that scientists clearly distinguish between these ideas (Carlone & Johnson, 2007). One hypothesis for this result is the focus on high-stakes testing in K-12 education, which not only influences teachers' instructional practices but also students' perceptions of what type of mathematics is valued. This pressure and value toward performing in mathematics may diminish the value of learning mathematics for understanding. Shepard and Dougherty (1991) found standardized testing results in teachers placing a greater emphasis on basic skills instruction as well as limiting instruction on content that was not being tested. These results were based on the responses of 360 teachers from 100 different schools on a questionnaire (Shepard & Dougherty, 1991). This means that students would be engaged in "drill and skill" type of instruction that allowed for limited use of reform practices focused on discussion and meaning making with mathematics content. Another study conducted by Betts, Hahn, and Zau (2011) investigated how mandatory diagnostic testing affected students' achievement. Results found that this mandatory testing did improve students' achievement scores in mathematics, but there were several caveats made by the researchers. One was that the diagnostic testing needed to be followed by intervention to help students who were struggling and effects dissipated after a few years if this method of diagnostic testing and intervention was not maintained.

The question then becomes, what does it mean for students to achieve on this type of testing? Is testing concerned mostly with rote mathematics skills or are these types of

tests asking for students to apply critical thinking skills with mathematics? This has implications not only for practitioners but also for other educators and curriculum specialists who could provide students with opportunities to make sense of mathematics. Stevens (2000) had the same type of questions in mind when conducting his study investigating problem-based mathematics in a middle school classroom. His work highlights the difficulty of presenting reformed curriculum in an environment where the teacher and students are used to traditional methods of instruction and mathematics is considered a set of skills and algorithms. What counts as mathematics and who makes those decisions needs to be a continuing conversation in the mathematics community. Stevens made a poignant statement in the concluding remarks of his study concerning the battle between testing and learning in mathematics.

> I see raising standardized test scores as one sort of objective, but I see helping most students learn to use mathematical tools and ideas to support arguments, to work together, to make things, and to resolve problematic situations from daily life as very different sorts of objectives. More important ones, I would argue. And while I do not propose that current versions of PBM [problem-based mathematics] education will achieve these objectives, I do propose that we consider this a better starting point than the alternatives (Stevens, 2000, p. 139).

When students are presented with opportunities to make meaning of mathematical tasks, they can develop a deeper level of understanding of mathematics that goes beyond performing on standardized tests. Even more beneficial, students are presented with a

different picture in terms of what is valued as mathematics. This focus might help students discern between understanding and performing in mathematics.

It is important to keep in mind how influential teachers are in enculturating students into the mathematics community. Knowing how teachers can influence students' views of mathematics through their instructional practices and development of sociomathematical norms within the classroom is a vital component to incorporate for effective teaching practices. Results from this study indicate that a focus on student interest and recognizing each student as a "mathematics person" are two ways that teachers can influence students' mathematics identity particularly for students who may already have well-developed performance/competency beliefs. The fact that mathematics identity can be used as a way of explaining student persistence in mathematics, e.g. mathematics related career choices, only solidifies the important role that teachers might play in helping students to have meaningful experiences with mathematics. Future research might provide more insight into these relationships.

Future Research

This study provides many new directions for future research in the area of mathematics identity. The framework that was used as well as the subsequent explanatory model for students' mathematics identity development provides a foundation for other research exploring student persistence. This might go beyond students' career choice to explore other outcome measures. Further research needs to also investigate what students mean when they state that they are recognized and who is recognizing them.

Research also needs to explore how teachers' instructional practices might influence students' mathematics identity. Boaler and Greeno (2000) stated that while many educators, both with a traditional or reform-based focus, might consider the act of learning mathematics and the final product or knowledge that students attain as separate, recent theories of learning and mathematical knowledge do not agree with this idea. They claim that these theories, such as sociocultural (Rogoff, 2008) and situative theories (Greeno & MMAP, 1998; Lave, 1988; Lave & Wenger, 1991), have the view that "the *practices* of learning mathematics define the knowledge that is produced" (Boaler & Greeno, 2000, p. 172). These theories stress the importance of teachers' instructional practices as they have the potential to influence students' agency and mathematics identity. This might include practices that build student interest in mathematics as well as practices that provide opportunities for students to be recognized as knowers and doers of mathematics.

Students also need to see mathematics as important to their everyday lives and be able to incorporate what they are learning in their everyday practices outside of the classroom. It would be particularly helpful to understand students' views of these instructional practices, as it is their perceptions of these practices that are helpful in understanding how they add to students' agency and identity in relation to mathematics. Boaler and Greeno (2000) stated that "what happens in the mathematics classrooms matter less within representations of figured worlds than the teachers' and students perceptions of what happens" (p. 189). In their study, 48 students taking AP Calculus from 6 high schools were interviewed concerning their experiences in the mathematics

classroom as well as their beliefs about mathematics. Results indicated that the mathematics environment influenced students' ways of knowing and identification with mathematics. Many of the students in didactic classrooms, where they were passive participants and had a received form of knowing, were alienated. In contrast, students in discussion-oriented classrooms were engaged in other forms of knowing, which gave them agency (Boaler & Greeno, 2000). When students are not afforded opportunities for interpretation, expression, and agency in mathematics classrooms, they tend to become disengaged with the content and do not pursue mathematics. Exploring this avenue of research might provide practical guidance to practitioners as to how they could help their students persist in mathematics as well as provide opportunities for their students to be engaged in meaningful learning.

Other research needs to further develop and explore the framework proposed in this study. One way this can be done is to better understand how students describe their experiences in mathematics as they relate to the three sub-constructs (interest, recognition, and competence/performance). Identity development is complex and further research can help to delve into the complexities of the sub-constructs and other factors that could potentially influence students' mathematic identity, such as how students' other social and personal characteristics interact with mathematics identity development. This line of research might also provide some insight into whether students might be able to distinguish qualitatively between what it means to understand and perform in mathematics. Another way to investigate this concept is to conduct another study with a different population of students. In particular, students enrolled in freshman college

calculus might provide a different picture for how students' perceptions in their ability to understand and perform in mathematics influence their mathematics identity. This might be the case because individuals from different walks of life and age groups may not have similar beliefs about these perceptions as was the case in this study. In this way, the framework can be further tested and a better understanding of students' mathematics identity can be developed.

In conclusion, mathematics identity is a good lens for understanding mathematics related behaviors and choices. With the focus of mathematics education being discussed by some educators and researchers as equated with issues of equality, it is imperative to understand how students are developing a sense of identification with mathematics. This is especially the case for students who might have been traditionally marginalized. This is a topic of interest for many researchers and educators because it has the potential to consider the complex interactions that are occurring in students' lives.

The model for mathematics identity presented in this study adds to our current understanding of mathematics identity and how it influences students' career choices. This model is only the beginning of the research that needs to be conducted to better understand mathematics identity and presents some clear directions for how research can continue. Because identity research is complex, many avenues of research can be expanded as related to the model. As these areas of research are expanded, ways that educators and researchers can positively influence students' mathematics identity can be explored. In this way it might be possible to fulfill the vision of equity as discussed by NCTM (2000), where all students are presented with worthwhile opportunities in

mathematics. Perhaps then we will finally be able to challenge the "pervasive societal belief in North America that only some students are capable of learning mathematics" (NCTM, 2000, p. 12).

APPENDICES

A. Correlation Matrix Used for SEM analysis: Without Gender Interactions

Table A.1: Correlation Matrix Used for SEM analysis: Without Gender Interactions

	1	2	3	4	5	6
1. Q44enjoy	1					
2. Q44interest	.89***	1				
3. Q44nervous	.40***	.32***	1			
4. Q44persist	.23***	.18***	.37***	1		
5. Q44lookforw	.89***	.81***	.44***	.33***	1	
6. Q45mathpersont	.43***	.38***	.35***	.22***	.38***	1
7. Q45mathpersons	.76***	.69***	.50***	.26***	.64***	.59***
8. Q45mathpersonp	.50***	.47***	.33***	.17***	.38***	.46***
9. Q44understand	.53***	.49***	.48***	.38***	.48***	.47***
10.Q44exam	.42***	.38***	.50***	.38***	.35***	.44***

* p<0.05, ** p<0.01, *** p<0.001

	7	8	9	10
1. Q44enjoy				
2. Q44interest				
3. Q44nervous				
4. Q44persist				
5. Q44lookforw				
6. Q45mathpersont				
7. Q45mathpersons	1			
8. Q45mathpersonp	.65***	1		
9. Q44understand	.56***	.41***	1	
10.Q44exam	.48***	.33***	.64***	1

* p<0.05, ** p<0.01, *** p<0.001

B. Correlation Matrix Used for SEM analysis: With Gender

Table A.2: Correlation Matrix Used for SEM analysis: With Gender

	1	2	3	4	5	6
1. Q44enjoy	1					
2. Q44interest	.89***	1				
3. Q44nervous	.40***	.32***	1			
4. Q44persist	.23***	.18***	.37***	1		
5. Q44lookforw	.89***	.81***	.44***	.33***	1	
6. Q45mathpersont	.43***	.39***	.35***	.23***	.38***	1
7. Q45mathpersons	.76***	.68***	.50***	.26***	.64***	.59***
8. Q45mathpersonp	.50***	.47***	.33***	.17***	.38***	.46***
9. Q46gender	.05**	.13***	.09***	.12***	-.00	.06***
10.Q44understand	.53***	.49***	.47***	.37***	.49***	.47***
11.Q44exam	.42***	.37***	.50***	.38***	.35***	.44***

* p<0.05, ** p<0.01, *** p<0.001

	7	8	9	10	11
1. Q44enjoy					
2. Q44interest					
3. Q44nervous					
4. Q44persist					
5. Q44lookforw					
6. Q45mathpersont					
7. Q45mathpersons	1				
8. Q45mathpersonp	.65***	1			
9. Q46gender	.09***	0.05***	1		
10.Q44understand	.56***	.41***	.07**	1	
11.Q44exam	.48***	.33***	.07***	.64***	1

* p<0.05, ** p<0.01, *** p<0.001

C. Factor Analysis with Q45mathpersons to Determine Reference Variable for SEM

Table A.3: Exploratory Factor Analysis for mathematics identity sub-constructs: including Q45mathpersons

Factor 1: Interest		
Survey Item	**Statement**	**Loading**
Q44enjoy	I enjoy learning math	0.88
Q44interest	Math is interesting	0.75
Q44lookforw	I look forward to taking math	0.50
Factor 2: Competence and Performance		
Survey Item	**Statement**	**Loading**
Q44exam	I can do well on the exams	0.73
Q44understand	I understand the math I have studied	0.57
Q44nervous	Math makes me nervous	0.38
Q44persist	Setbacks do not discourage me	0.40
Factor 3: Recognition		
Survey Item	**Statement**	**Loading**
Q45mathpersons	Degree to which you see yourself as a math person	0.91
Q45mathpersonp	Degree to which parents/relatives/friends see you as a math person	0.75
Q45mathpersont	Degree to which math teachers see you as a math person	0.57

D. FICS-Math Survey

Harvard-Smithsonian Center for Astrophysics
Science Education Department

FICSMath:

Factors Influencing College Success in Mathematics

Survey of Students in College Calculus

Researchers at the Harvard-Smithsonian Center for Astrophysics and the Harvard Graduate School of Education are interested in your experiences in learning mathematics. By filling out this questionnaire you will help us find ways to improve mathematics education for future students. Make your best estimate for each item and answer as many questions as possible. Your participation is entirely voluntary. Thank you for your help.

This survey should take no longer than 15-20 minutes to complete.

Your name will NOT be included in our study. After your instructor enters your final grade on the last page, he/she will tear off this sheet before sending us the questionnaire.

Use a No. 2 pencil or blue or black ink pen only.

CORRECT MARK ●   **INCORRECT MARKS** ✓ ✗ ➖ ⦁

Student Name (Please print.)

Course Name/Number

1

PLEASE DO NOT MARK IN THIS AREA

09334

Because our survey hopes to aid in understanding high school mathematics education, having some knowledge of the type of high school you attended is important. Please answer the following questions concerning your high school education:

1. **Where did you receive a majority of your high school education?**
   ○ A school in the US   ○ American school abroad   ○ A high school in another country

2. **What type of high school did you go to?** *Mark all that apply.*
   ○ Public   ○ Private, non-Parochial   ○ Private, Parochial   ○ All-male
   ○ Public Charter   ○ Vocational   ○ Magnet School   ○ All-female
   ○ International Baccalaureate ○ Home Schooled

3. **To help us estimate the size of the community you come from, please provide your home ZIP Code (when you graduated from high school) and bubble in the corresponding numbers.**

4. **What was the size of your graduating class?**

≤25	26–75	76–200	201–400	401–600	601–800	801–1000	1001–1200	>1200
○	○	○	○	○	○	○	○	○

CONCERNING YOUR COURSES AND ACTIVITIES IN HIGH SCHOOL:

5. **What grade did you get in your last high school English course?**

A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
○	○	○	○	○	○	○	○	○	○	○	○	○

6. **Which of the following calculus courses were offered in your high school?** *Mark all that apply.*
   ○ Calculus (Non AP)   ○ AP Calculus AB   ○ AP Calculus BC   ○ Dual Credit

7. **For each (non-AP) mathematics course listed below that you took, please indicate the level of the course, in what year in school you took the course, what grade you earned in each course, and the gender of the teacher.** *(If you repeated a course, provide info only for the last time you took the course. If your high school had "Integrated Math," mark all the years in which you took such courses and fill out the Course Level, Final Grade, and Teacher Gender portions for the last course.)*

HS Course Subject	Course Level			Last Year Taken in HS	Final Grade	Teacher Gender
	Regular	Honors	IB			
Algebra I	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Geometry	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Algebra II	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Integrated Math	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Pre-Calc./Trig.	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Statistics (Non AP)	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Calculus (Non AP)	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ
Other:	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ Ⓟ	Ⓜ Ⓕ

8. **For each of the following AP courses that you took, please indicate the score you earned on the exam, in what year you took the course, what grade you earned in the course, and the gender of the teacher.**

AP Course	AP Exam Score	Year Taken in HS	Final Grade	Teacher Gender
Calculus AB	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ	Ⓜ Ⓕ
Calculus BC	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ	Ⓜ Ⓕ
Statistics	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ	Ⓜ Ⓕ
Physics (any)	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ Ⓐ A- B+ Ⓑ B- C+ Ⓒ C- D+ Ⓓ D- Ⓕ	Ⓜ Ⓕ

CONCERNING THE ORGANIZATION AND STRUCTURE OF YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

9. **How long has it been since you completed your most advanced high school mathematics course?**
   ○ 0–12 months   ○ 1–2 years   ○ 2–4 years   ○ 5–9 years   ○ 10+ years

10. **How many students were in that mathematics course?**
    ○ 5 or fewer   ○ 6–10   ○ 11–15   ○ 16–20   ○ 21–25   ○ More than 25

11. **On average how many days each week did that mathematics course meet?**
    ○ Two or less   ○ Three   ○ Four   ○ Five or more

3

PLEASE DO NOT MARK IN THIS AREA

09334

Because our survey hopes to aid in understanding high school mathematics education, having some knowledge of the type of high school you attended is important. Please answer the following questions concerning your high school education:

1. **Where did you receive a majority of your high school education?**
   ○ A school in the US   ○ American school abroad   ○ A high school in another country

2. **What type of high school did you go to?** *Mark all that apply.*
   ○ Public   ○ Private, non-Parochial   ○ Private, Parochial   ○ All-male
   ○ Public Charter   ○ Vocational   ○ Magnet School   ○ All-female
   ○ International Baccalaureate   ○ Home Schooled

3. **To help us estimate the size of the community you come from, please provide your home ZIP Code (when you graduated from high school) and bubble in the corresponding numbers.**

4. **What was the size of your graduating class?**

≤25	26–75	76–200	201–400	401–600	601–800	801–1000	1001–1200	>1200
○	○	○	○	○	○	○	○	○

CONCERNING YOUR COURSES AND ACTIVITIES IN HIGH SCHOOL:

5. **What grade did you get in your last high school English course?**

A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
○	○	○	○	○	○	○	○	○	○	○	○	○

6. **Which of the following calculus courses were offered in your high school?** *Mark all that apply.*
   ○ Calculus (Non AP)   ○ AP Calculus AB   ○ AP Calculus BC   ○ Dual Credit

7. **For each (non-AP) mathematics course listed below that you took, please indicate the level of the course, in what year in school you took the course, what grade you earned in each course, and the gender of the teacher.** *(If you repeated a course, provide info only for the last time you took the course. If your high school had "Integrated Math," mark all the years in which you took such courses and fill out the Course Level, Final Grade, and Teacher Gender portions for the last course.)*

HS Course Subject	Course Level			Last Year Taken in HS	Final Grade	Teacher Gender
	Regular	Honors	IB			
Algebra I	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Geometry	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Algebra II	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Integrated Math	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Pre-Calc./Trig.	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Statistics (Non AP)	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Calculus (Non AP)	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F
Other:	○	○	○	⑧ ⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F P	M F

8. **For each of the following AP courses that you took, please indicate the score you earned on the exam, in what year you took the course, what grade you earned in the course, and the gender of the teacher.**

AP Course	AP Exam Score	Year Taken in HS	Final Grade	Teacher Gender
Calculus AB	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F	M F
Calculus BC	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F	M F
Statistics	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F	M F
Physics (any)	① ② ③ ④ ⑤	⑨ ⑩ ⑪ ⑫	A+ A A- B+ B B- C+ C C- D+ D D- F	M F

CONCERNING THE ORGANIZATION AND STRUCTURE OF YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

9. **How long has it been since you completed your most advanced high school mathematics course?**
   ○ 0–12 months   ○ 1–2 years   ○ 2–4 years   ○ 5–9 years   ○ 10+ years

10. **How many students were in that mathematics course?**
   ○ 5 or fewer   ○ 6–10   ○ 11–15   ○ 16–20   ○ 21–25   ○ More than 25

11. **On average how many days each week did that mathematics course meet?**
   ○ Two or less   ○ Three   ○ Four   ○ Five or more

3

PLEASE DO NOT MARK IN THIS AREA

09334

12. How often did that mathematics course meet for longer than an hour?

○ Every class   ○ 3 or 4 classes per week   ○ 1 or 2 classes per week   ○ Never

13. What was the length of that mathematics course?

○ A full year   ○ One semester   ○ Less than a semester

14. In terms of learning the material, that mathematics course required:

Very little memorization of procedures |— 1 — 2 — 3 — 4 — 5 — 6 —| A lot of memorization of procedures
Very little conceptual understanding |— 1 — 2 — 3 — 4 — 5 — 6 —| A lot of conceptual understanding

15. What type of calculator did you use most often in that mathematics course?

○ A graphing calculator   ○ A non-graphing calculator   ○ No calculators were used

16. In what ways were you allowed to use calculators for your work in that course? *Mark all that apply.*

○ For simple calculations (e.g., adding/subtracting, multiplying/dividing)   ○ On exams
○ To compute numeric values of derivatives/integrals   ○ For homework
○ To plot graphs of functions   ○ Only after a technique had been practiced
○ For trigonometric calculations   heavily using paper and pencil

17. How often did the following occur throughout that high school mathematics course:

	Never	Few times a year	Monthly	Weekly	Every Class
Used graphing calculator	○	○	○	○	○
Used computers for support/instruction	○	○	○	○	○
Given online assignments or homework	○	○	○	○	○

18. How strongly were the following emphasized in that high school mathematics course:

	Not emphasized at all					Emphasized heavily
Functions	1	2	3	4	5	6
Vocabulary	1	2	3	4	5	6
Precise mathematical definitions	1	2	3	4	5	6
Hands-on activities/labs	1	2	3	4	5	6
Mathematical proofs	1	2	3	4	5	6
Memorization of formulas	1	2	3	4	5	6
Mathematical reasoning	1	2	3	4	5	6

19. Concerning class discussions, please indicate how often the following occurred:

	Never					Always
You felt comfortable asking questions	1	2	3	4	5	6
Students' questions and comments were valued	1	2	3	4	5	6
Classroom discussions were useful	1	2	3	4	5	6
Teacher's answers to questions were valuable	1	2	3	4	5	6

CONCERNING YOUR TEXTBOOK, HOMEWORK, AND IN-CLASS ASSIGNMENTS IN YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

20. How large a role did a textbook play in your high school mathematics course?

Not used much |— 1 — 2 — 3 — 4 — 5 — 6 —| Followed it closely

21. How many minutes did you spend reading the textbook both in class and for homework each day on average?

○ 0   ○ 10   ○ 20   ○ 30   ○ 40   ○ More than 40

22. On average, how many minutes did you spend studying or doing work for math outside of class each day?

○ 0   ○ 5   ○ 10   ○ 15   ○ 30   ○ 45   ○ 60   ○ More than 60

23. Indicate the number of problems of each type you had to work on in class:

	None	One/ Month	One/ Week	Two/ Week	1–2/ Class	3–4/ Class	5–6/ Class	6+/ Class
Problems with written explanations	○	○	○	○	○	○	○	○
Problems with multiple choice/true-false	○	○	○	○	○	○	○	○
Problems with fill-in the blank	○	○	○	○	○	○	○	○
Problems with multiple parts	○	○	○	○	○	○	○	○
Word problems	○	○	○	○	○	○	○	○
Problems that involved estimation	○	○	○	○	○	○	○	○
Problems requiring graphing by hand	○	○	○	○	○	○	○	○
Problems requiring graphing by calculator	○	○	○	○	○	○	○	○
Problems that involved proofs	○	○	○	○	○	○	○	○

4

24. For problems involving calculation, how often were you required to check whether your numerical answer was reasonable?

Very Rarely |—①—②—③—④—⑤—⑥—| Every Class

CONCERNING YOUR TESTS AND QUIZZES GIVEN IN YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

25. Which of the following types of questions were typically included on your tests or quizzes? *Mark all that apply.* Problems that:

- ◯ Required calculations without a calculator
- ◯ Involved data presented in tables
- ◯ Concerned material tested earlier in the course
- ◯ Required memorization of terms or facts
- ◯ Were drawn from homework
- ◯ Required essay responses
- ◯ Required sketching, drawing, or graphing by hand
- ◯ Came from standardized exams
- ◯ Required new insight and creativity

26. Which of the following were typically involved with your tests or quizzes? *Mark all that apply.*

- ◯ The teacher gave study guides or practice exams before a test or quiz
- ◯ The teacher allowed "cheat sheets" during tests or quizzes
- ◯ The teacher allowed students to retake or rework an exam for a grade
- ◯ The teacher gave bonus points or extra credit

CONCERNING THE TEACHER OF YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

27. How would you rate your high school mathematics teacher on the following characteristics?

	Low					High
Was enthusiastic about mathematics	①	②	③	④	⑤	⑥
Treated all students with respect	①	②	③	④	⑤	⑥
Used graphs, tables, and other illustrations	①	②	③	④	⑤	⑥
Highlighted more than one way of solving a problem	①	②	③	④	⑤	⑥
Made mathematical errors	①	②	③	④	⑤	⑥
Explained ideas clearly	①	②	③	④	⑤	⑥

28. How often did disruptive students interfere with your learning in your *most advanced* high school mathematics class?

◯ Rarely   ◯ Once a week   ◯ 2–3 times per week   ◯ Once a class   ◯ Several times during each class

29. What percent of class time were students focused on learning math?

◯ Less than 25%   ◯ 25–50%   ◯ 50–75%   ◯ More than 75%

CONCERNING CLASS TIME AND METHODS IN YOUR *MOST ADVANCED* HIGH SCHOOL MATHEMATICS COURSE:

30. Regarding class and teacher interaction, please indicate how often the following occurred:

	Very rarely	Once/ Month	Once/ Week	2–3 times/ Week	Every class
The teacher lectured to the class	◯	◯	◯	◯	◯
Small group discussion/work was held	◯	◯	◯	◯	◯
Whole class discussions were held	◯	◯	◯	◯	◯
Students spent time doing individual work in class	◯	◯	◯	◯	◯
Classmates taught each other	◯	◯	◯	◯	◯
You taught your classmates	◯	◯	◯	◯	◯

31. Regarding math connections, please indicate how often the following occurred:

Connected math to your everyday-life	◯	◯	◯	◯	◯
Connected math to real-life applications	◯	◯	◯	◯	◯
Connected math to other subject areas	◯	◯	◯	◯	◯
Examples from everyday world were used	◯	◯	◯	◯	◯

32. Regarding problem solving, please indicate how often the following occurred:

Teacher solved example problems after presenting new material	◯	◯	◯	◯	◯
Teacher presented various methods for solving problems	◯	◯	◯	◯	◯
Students solved problems on board	◯	◯	◯	◯	◯

33. Regarding teaching aids, please indicate how often the following occurred:

Manipulation of physical objects	◯	◯	◯	◯	◯
Use of computer simulations or applets	◯	◯	◯	◯	◯
Teacher incorporated games and prizes into lessons	◯	◯	◯	◯	◯

34. Regarding the use of class time, please indicate how often the following occurred:

Tests or quizzes were given	◯	◯	◯	◯	◯
Class time spent preparing for class-related quizzes/tests	◯	◯	◯	◯	◯
Time spent going over assigned homework	◯	◯	◯	◯	◯
Time spent reviewing past lessons	◯	◯	◯	◯	◯
Class time was spent preparing for standardized math exams	◯	◯	◯	◯	◯
Spent time correcting your own work	◯	◯	◯	◯	◯

5

35. Regarding the level of respect, please indicate how often the following occurred:

	Very rarely	Once/ Month	Once/ Week	2–3 times/ Week	Every class
Students were disrespectful to you	○	○	○	○	○
Students were disrespectful to other students	○	○	○	○	○
Students were disrespectful to the teacher	○	○	○	○	○
The teacher was disrespectful to me	○	○	○	○	○
The teacher was disrespectful to other students	○	○	○	○	○

36. How often did you tutor other students in any math topic while you took your *most advanced* high school mathematics course?

○ Never ○ Once/month ○ Once/week ○ 2–3 times/week ○ 4 or more times/week

CONCERNING THE EFFECT BACKGROUND HAS ON LEARNING MATHEMATICS *THIS SEMESTER:*

37. Did you take a calculus course in COLLEGE prior to this one?

○ Yes ○ No *(If No, go to question #40)*

38. If yes, where?

○ At this university ○ Another 4-yr university ○ Another 2-yr university

39. Why are you taking the course again?

○ It did not count towards the credits I need ○ I passed, but I need/want a higher grade (e.g. for my major)
○ I did not pass the course ○ I dropped the class

40. Did you take a pre-calculus course in COLLEGE prior to this course?

○ Yes ○ No *(If No, go to question #42)*

41. If yes, where?

○ At this university ○ Another 4-yr university ○ Another 2-yr university

42. Is any type of tutoring or outside help available for *this college calculus class?*

○ Yes, and I plan to take advantage of it ○ No, but I would take advantage of it if it were
○ Yes, but I do not plan to take advantage of it ○ No, but I probably would not take of advantage of it

CONCERNING YOUR CAREER PLANS:

43. Which of the following BEST describes your current career goal? *Mark only ONE choice.*

	Current goal		Current goal		Current goal
Medical professional (e.g., doctor, dentist, vet.)	○	Engineer	○	Social scientist (e.g., psychologist, sociologist)	○
Health professional (e.g., nurse, medical technician)	○	Computer scientist, IT	○	Business person	○
Life scientist (e.g., biologist, medical researcher)	○	Mathematician	○	Lawyer	○
Earth/Environmental scientist (e.g., geologist, meteorologist)	○	Science/Math teacher	○	English/Language Arts specialist	○
Physical Scientist (e.g., chemist, physicist, astronomer)	○	Other teacher	○	Other non-science related career	○

CONCERNING YOUR PERSONAL THOUGHTS AND FEELINGS TOWARDS MATHEMATICS:

44. Do you agree or disagree with the following statements?

	Agree	Disagree		Agree	Disagree
I enjoy learning math.	○	○	I can do well on math exams.	○	○
Math is interesting.	○	○	I look forward to taking math.	○	○
Math makes me nervous.	○	○	I wish I did not have to take math.	○	○
Math is relevant to real life.	○	○	I understand the math I have studied.	○	○
Setbacks do not discourage me.	○	○			

45. Do the following people see you as a mathematics person?

	No, not at all					Yes, very much
Yourself	①	②	③	④	⑤	⑥
Parents/Relatives/Friends	①	②	③	④	⑤	⑥
Mathematics teacher	①	②	③	④	⑤	⑥

CONCERNING YOUR FAMILY'S AND YOUR OWN BACKGROUND:

46. Are you male or female?

○ Male ○ Female

6

09334

127

47. What is your race? *(For multi-racial, mark all that apply.)*

- ◯ White
- ◯ Black
- ◯ Asian
- ◯ Pacific Islander
- ◯ American Indian or Alaskan Native
- ◯ Other: _____

48. Are you of Hispanic origin?

- ◯ Yes
- ◯ No

49. Was English the primary spoken language in your household?

- ◯ Yes
- ◯ No

50. What year are you in college?

- ◯ Freshman
- ◯ Sophomore
- ◯ Junior
- ◯ Senior
- ◯ Graduate Student
- ◯ Other

51. What is your current type of college enrollment?

- ◯ Full-time
- ◯ Part-time

52. Was your home environment supportive of math?

Not supportive at all ├─①─②─③─④─⑤─⑥─┤ Very Supportive

53. Who encouraged you to take mathematics classes? *Mark all that apply.*

- ◯ No One
- ◯ Mother/Female Guardian
- ◯ Father/Male Guardian
- ◯ Sibling(s)
- ◯ Other Relative
- ◯ School Counselor
- ◯ Math Teacher
- ◯ Other Teacher
- ◯ Coach

54. What was the highest level of education for your male parent or guardian?

- ◯ Did not finish high school
- ◯ High school
- ◯ Some college
- ◯ Four years of college
- ◯ Graduate school

55. What was the highest level of education for your female parent or guardian?

- ◯ Did not finish high school
- ◯ High school
- ◯ Some college
- ◯ Four years of college
- ◯ Graduate school

56. Which category best fits you and your parents' or guardians' background?

	Born in United States	
You	◯ Yes	◯ No
Male Parent or Guardian	◯ Yes	◯ No
Female Parent or Guardian	◯ Yes	◯ No

57. Which of the following statements best describes your family's interest in mathematics? *Mark all that apply.*

- ◯ Math is involved in both parents' jobs.
- ◯ Math is involved in one parent's job.
- ◯ Math is a way for you to have a better career.
- ◯ Math was a series of courses that I had to pass.
- ◯ Math was not an interest of my family.
- ◯ My parents were able to help me with math.
- ◯ My parents were willing and able to get me a tutor for math.

58. For each of the following standardized tests, please indicate the highest score you earned on that test by marking the oval closest to your score.

SAT Exam	SAT Score							
	200	300	400	500	600	700	800	Did not take
Math								
Critical Reading								
Writing								

ACT Exam	ACT Score											
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36											Did not take
Math												
English												
Science Reas.												
Reading												

SAT Subject Test	SAT Subject Test Score							
	200	300	400	500	600	700	800	Did not take
Math Level 1								
Math Level 2								

7

128

E. Portions of R Code Used in Analysis

```
library(car)
library(sem)
library(polycor)
library(psych)
library(semGOF)

# EFA Analysis
Factors <-
na.omit(cbind(q44enjoy,q44exam,q44interest,q45mathpersonpr,q45mathpersontr,
q44lookforw,q44understand,q44persist,q44nervousr))
fit <- factanal(Factors, 3, rotation="promax")
print(fit, digits=2, cutoff=0.3, sort=T)
coef(fit)

library(nFactors)
ev <- eigen(cor(Factors))
ap <- parallel(subject=nrow(Factors),var=ncol(Factors),rep=100,cent=.05)
nS <- nScree(ev$values, ap$eigen$qevpea)
plotnScree(nS)
---------------------------------------
# Create subset for SEM Analysis
mathmodel1 <- as.data.frame(cbind(q44enjoy, q44interest, q44nervousr, q44persist,
q44lookforw, q45mathpersontr, q45mathpersonpr,q45mathpersonsr,
q44understand, q44exam))

#classifying variables
mathmodel1$q44enjoy <- factor(mathmodel1$q44enjoy, labels= c("Disagree", "Agree"),
ordered=F)
mathmodel1$q44interest <- factor(mathmodel1$q44interest, labels= c("Disagree",
"Agree"), ordered=F)
mathmodel1$q44lookforw <- factor(mathmodel1$q44lookforw, labels= c("Disagree",
"Agree"), ordered=F)
mathmodel1$q44exam <- factor(mathmodel1$q44exam, labels= c("Disagree", "Agree"),
ordered=F)
mathmodel1$q44understand <- factor(mathmodel1$q44understand, labels= c("Disagree",
"Agree"), ordered=F)
mathmodel1$q44nervousr <- factor(mathmodel1$q44nervousr, labels= c("Disagree",
"Agree"), ordered=F)
mathmodel1$q44persist <- factor(mathmodel1$q44persist, labels= c("Disagree",
"Agree"), ordered=F)

newmathmodel1 <- as.data.frame(cbind(q44enjoy, q44interest, q44nervousr, q44persist,
```

q44lookforw, q45mathpersontr, q45mathpersonpr,q45mathpersonsr,
q44understand, q44exam))

#List-wise deletion of missing values
mathmodel1 <- na.omit(mathmodel1)

hcor <- function(data) hetcor(data, std.err=FALSE)$correlations
R.Observ <- hcor(mathmodel1)
R.Observ

nrow(mathmodel1)

#Running SEM
Model.mathmodel1 <- specifyModel( )
mathid->q45mathpersonsr, NA, 1
q45mathpersonsr<->q45mathpersonsr, NA, 0.09
mathid<-> mathid, psi1, NA
recognition->mathid, gam2, NA
competence->mathid, gam3, NA
interest->mathid, gam1, NA
interest->q44enjoy, lam4, NA
interest->q44interest, lam5, NA
interest->q44lookforw, lam6, NA
recognition->q45mathpersonpr, lam7, NA
recognition->q45mathpersontr,lam8, NA
competence->q44exam, lam10, NA
competence->q44understand, lam11, NA
competence->q44persist,lam12, NA
competence->q44nervousr,lam13, NA
interest<->interest, NA, 1
recognition<->recognition, NA, 1
competence<->competence, NA, 1
q44enjoy<->q44enjoy, thd2, NA
q44interest<->q44interest, thd3, NA
q44lookforw<->q44lookforw, thd4, NA
q45mathpersonpr<->q45mathpersonpr, thd5, NA
q45mathpersontr<->q45mathpersontr, thd6, NA
q44exam<->q44exam, thd8, NA
q44understand<-> q44understand, thd9, NA
q44persist<->q44persist,thd10, NA
q44nervousr<->q44nervousr, thd11, NA
interest<->competence, phi4, NA
interest <->recognition, phi5, NA
recognition<->competence, phi6, NA

```
q44enjoy<->q44lookforw, phi10, NA
q44interest<->q44lookforw, phi21, NA
q44nervousr<->q44understand,phi22,NA
q44lookforw<->q44persist,phi23, NA
q44lookforw<->q44nervousr,phi24, NA

sem.mathmodel1 <- sem(Model.mathmodel1, R.Observ , N=9397)
summary(sem.mathmodel1)
summaryGOF(sem.mathmodel1)

system.time(boot.mathmodel1 <- bootSem(sem.mathmodel1, R=100, cov=hcor,
data=newmathmodel1), gcFirst=TRUE)
summary(boot.mathmodel1, type="norm")
std.coef(sem.mathmodel1)

mod.indices(sem.mathmodel1)

---------------------------------------
#Create math identity sub-construct variables
interest=(q44enjoy+q44interest+q44lookforw)/3
recognition=(q45mathpersonpr+q45mathpersontr)/2
comp_perf=(q44exam+q44understand+q44nervousr+q44persist)/4

#Create mathematics career goal variable
q43mathcareer=recode(q43careergoal, '1:4=0; 5:6=1; 7=0; 8:9=1;
10:15=0',as.factor.result=FALSE)
mathmodel=cbind(mathmodel,q43mathcareer)
table(q43mathcareer)
describe(q43mathcareer)

#New variable for math identity using SEM analysis
mathid = ((0.269*interest)+(0.811*recognition)-(0.059*comp_perf))
mathidr <- rescaler(mathid, type="sd")
describe (mathidr)

##Welch's t-tests for gender analysis
t.test(q46gender, interest, na.rm=False)
t.test(q46gender, recognition, na.rm=False)
t.test(q46gender, comp_perf, na.rm=False)

#Create mathematics career goal variable
#Create mathematian career goal variable
q43math=recode(q43careergoal, '1:7=0; 8=1; 9:15=0',as.factor.result=FALSE)
mathmodel=cbind(mathmodel,q43math)
```

```
#Logistic regression
model <- glm(q43math~mathidr,family=binomial(link=logit),data=mathmodel)
summary(model)
vif(model)

# odds ratio
exp(model$coefficients)

#Create math/science teacher career goal variable
q43teach=recode(q43careergoal, '1:8=0; 9=1; 10:15=0',as.factor.result=FALSE)
mathmodel=cbind(mathmodel,q43teach)
table(q43teach)

#Logistic regression (model 3)
model3 <- glm(q43teach~mathidr,family=binomial(link=logit),data=mathmodel)
summary(model3)

# odds ratio
exp(model3$coefficients)

#Create STEM career goal variable
q43STEM=recode(q43careergoal, '1:2=0; 3:9=1; 10:15=0',as.factor.result=FALSE)
mathmodel=cbind(mathmodel,q43STEM)
table(q43STEM)

model12 <- glm(q43STEM~mathidr,family=binomial(link=logit),data=mathmodel)
summary(model12)

# odds ratio
exp(model12$coefficients)
```

REFERENCES

Armstrong, J. M. (1981). Achievement and participation of women in mathematics: Results of two national surveys. *Journal for Research in Mathematics Education, 12,* 356-372.

Ashforth, B. E., & Mael, F. (1989). Social identity theory and the organization. *Academy of Management Review,14,* 20-39.

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review, 84*(2), 191-215.

Bandura, A. (1986). *Social foundations of thought and action*. Prentice Hall, New York.

Bandura, A. (1997). *Self-efficacy: The exercise of control.* Worth Publishers.

Betts, J. R., Hahn, Y., & Zau, A. C. (2011). *Does diagnostic math testing improve student learning?* Public Policy Institute of California.

Beyer, S. (1990). Gender differences in the accuracy of self-evaluations of performance. Jour*nal of Personality and Social Psychology, 59,* 960-970.

Beyer, S. (1995). Maternal employment and children's academic achievement: Parenting style as mediating variable. *Developmental Review, 15,* 212-253.

Beyer, S. (1998). Gender differences in self-perception and negative recall biases. *Sex Roles, 38,* 103-133.

Beyer, S. (1999). Gender differences in causal attributions by college students of performance on course examinations. *Current Psychology, 17,* 346-358.

Bleeker, M. M., & Jacobs, J. E. (2004). Achievement in math and science: Do mothers' beliefs matter 12 years later?. *Journal of Educational Psychology, 96*(1), 97-109.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. Journal for Research in Mathematics Education, 29, 41-62.

Boaler, J. (2000). Introduction: Intricacies of knowledge, practice, and theory. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 1-18). Ablex Publishing Corporation.

Boaler, J. (2002) *Experiencing School Mathematics: Teaching styles, sex and setting.* Buckingham: Open University Press.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple Perspectives on Mathematics Teaching and Learning,* 171-200. Ablex Publishing.

Bouchey, H.A., Harter, S. (2005). Reflected appraisals, academic self-perceptions, and math/science performance during early adolescence. *Journal of Educational Psychology*. 673–686.

Buckley, L. (2010). Unfulfilled hopes in education for equity: Redesigning the mathematics curriculum in a US high school. *Journal of Curriculum Studies, 42*(1), 51-78.

Burton, L. (Ed.) (1999). *Learning Mathematics: from hierarchies to networks.* London: Falmer.

Bussey, K., & Bandura, A. (1999). Social cognitive theory of gender development and differentiation. *Psychological Review, 106*(4), 676-713.

Carlone, H. B., & Johnson, A. (2007). Understanding the science experiences of successful women of color: Science identity as an analytic lens. *Journal of Research in Science Teaching, 44*(8), 1187-1218.

Cobb, P. (2004). Mathematics, literacies, and identity. *Reading Research Quarterly, 39*(3), 333-337.

Cobb, P., & Hodge, L. L. (2011). Culture, identity, and equity in the mathematics classroom. *A Journey in Mathematics Education Research,* 179-195.

Cohen, J. (1992). A power primer. *Psychological Bulletin, 112*(1), 155-159.

Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation, 10*(7), 1-9.

Crosswhite, F. J. (1972). Correlates of attitudes toward mathematics (Report NLSMA Reports No. 20): Ohio State University.

DeBellis, V. A., & Goldin, G. A. (1997). The affective domain in mathematical problem-solving. *Proceedings of 21st Conference of the International Group for Psychology of Mathematics Education*, Lahti, Finland, 601-606.

Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behavior.* Springer.

Dekkers, J., De Laeter, J.R., & Malone, J.A. (1986) *Upper secondary school science and mathematics enrollment patterns in Australia, 1970-1985*. Bentley, Western Australia: Science and Mathematics Education Centre, Western Australian Institute of Technology.

Dewey, J. (1913). *Interest and effort in education.* Houghton Mifflin Co.

Diamantopoulos, A., Siguaw, J., & Siguaw, J. A. (2000). *Introducing LISREL: A guide for the uninitiated.* Sage Publications Ltd.

Eccles-Parsons, J. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivations* (pp.75–121). San Francisco, CA: Freeman.

Eccles-Parsons, J. E., Adler, T. F., & Kaczala, C. M. (1982). Socialization of achievement attitudes and beliefs: parental influences. *Child Development, 53*(2), 310–321.

Eccles, J. S. (1994). Understanding women's educational and occupational choices: Applying the Eccles et al. model of achievement-related choices. *Psychology of Women Quarterly, 18,* 585–609.

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology, 53*(1), 109-132.

Eccles, J. S., Wigfield, A., Flanagan, C., Miller, C., Reuman, D., & Yee, D. (1989). Self-concept, domain values, and self-esteem: Relations and changes at early adolescence. *Journal of Personality*, 57, 283–310.

Einarsdo´ttir, S., & Rounds, J. (2009). Gender bias and construct validity in vocational interest measurement: Differential item functioning in the Strong Interest Inventory. *Journal of Vocational Behavior, 74,* 295–307.

Else-Quest, N.M., Hyde, J.S., Linn, M.C. (2010). Cross-national patterns of gender differences in mathematics and gender equity: A meta-analysis. *Psychological Bulletin*. 103–127.

Erickson, E. H. (1968). *Identity: Youth and crisis.* W.W. Norton & Company, Inc.

Felson, R. B. (1989). Parents and the reflected appraisal process: A longitudinal analysis. *Journal of Personality and Social Psychology, 56,* 965–971.

Fennema, E., & Romberg, T. A. (1999). *Mathematics classrooms that promote understanding.* Lawrence Erlbaum Associates.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. *American Educational Research Journal, 14*(1), 51-71.

Fennema, E. H., & Sherman, J. A. (1978). Sex-related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics Education,* 189-203.

Ferla, J., Valcke, M., & Schuyten, G. (2010). Judgments of self-perceived academic competence and their differential impact on students' achievement motivation, learning approach, and academic performance. *European Journal of Psychology of Education, 25*(4), 519-536.

Ferry, T. R., Fouad, N. A., & Smith, P. L. (2000). The role of family context in a social cognitive model for career-related choice behavior: A math and science perspective* 1. *Journal of Vocational Behavior, 57*(3), 348-364.

Flessner, R. (2009). Working toward a third space in the teaching of elementary mathematics. *Educational Action Research, 17*(3), 425-446.

Fouad, N. A. (1999). Validity evidence for interest inventories. In M. L. Savickas & R. L. Spokane (Eds.), *Vocational interests: Meaning, measurement, and counseling use* (pp. 193–209). Palo Alto, CA: Davis-Black.

Fouad, N.A., Smith, P.L., & Zao, K.E. (2002). Across academic domains: Extensions of the social–cognitive career model. *Journal of Counseling Psychology*, 49(2)

Fox, J. (2006). TEACHER'S CORNER: Structural equation modeling with the sem package in R. *Structural Equation Modeling, 13*(3), 465-486.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 225-256). Reston, VA: National Council of Teachers of Mathematics.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: Growth trajectories in two male-sex-typed domains. *Developmental Psychology, 38*(4), 519.

Frenzel, A. C., Goetz, T., Pekrun, R., & Watt, H. M. G. (2010). Development of mathematics interest in adolescence: Influences of gender, family, and school context. *Journal of Research on Adolescence, 20*(2), 507-537.

Friedel, J. M., Cortina, K. S., Turner, J. C., & Midgley, C. (2007). Achievement goals, efficacy beliefs and coping strategies in mathematics: The roles of perceived parent and teacher goal emphases. *Contemporary Educational Psychology, 32*(3), 434-458.

Frome, P. M., & Eccles, J. S. (1998). Parents' influence on children's achievement-related perceptions. *Journal of Personality and Social Psychology*, 435–452.

Furnham, A., Reeves, E., Budhani, S. (2002). Parents think their sons are brighter than their daughters: Sex differences in parental self-estimations and estimations of their children's multiple intelligences. *Journal of Genetic Psychology*. 24–39.

Gee, J. P. (2001). Identity as an analytic lens for research in education. *Review of Research in Education, 25*(1), 99.

GonzáLez, N. (1995). Processual approaches to multicultural education. *The Journal of Applied Behavioral Science, 31*(2), 234-244.

González, N., & Amanti, C. (1997). Teaching anthropological methods to teachers: The transformation of knowledge. In C. Kottak, J. White, R. Furlow, & P. Rice (Eds.), *The teaching of anthropology: Problems, issues and decisions* (pp. 353–359). Mountain View, CA: Mayfield.

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (2001). Continuity of academic intrinsic motivation from childhood through late adolescence: A longitudinal study. *Journal of Educational Psychology*, 93, 3–13.

Greeno, J. G., & MMAP. (1998). The situativity of knowing, learning and research. American Psychologist, 53(1), 5-26.

Harris, M. J., & Rosenthal, R. (1985). Mediation of interpersonal expectancy effects: 31 meta-analyses. *Psychological Bulletin, 97*(3), 363-386.

Hazari, Z., Sonnert, G., Sadler, P. M., & Shanahan, M. C. (2010). Connecting high school physics experiences, outcome expectations, physics identity, and physics career choice: A gender study. *Journal of Research in Science Teaching,* 47(8), 978-1003.

Hedges, L.V., Nowell A. (1995). Sex differences in mental test scores, variability, and numbers of high-scoring individuals. *Science*, 41–45.

Helwig, R., Anderson, L., & Tindal, G. (2001). Influence of elementary student gender on teachers' perceptions of mathematics achievement. *The Journal of Educational Research, 95*(2), 93-102.

Hidi, S. (2000). An interest researcher's perspective: The effects of extrinsic and intrinsic factors on motivation. In C. Sansone & J. M. Harackiewicz (Eds.), *Intrinsic and extrinsic motivation: The search for optimal motivation and performance.* New York: Academic.

Hidi, S. (2001). Interest, reading and learning: Theoretical and practical considerations. *Educational Psychology Review*, 13, 191–210

Hidi, S., & Harackiewicz, J. (2000). Motivating the academically unmotivated: A critical issue for the 21st century. *Review of Educational Research,* 70, 151–179.

Hogan, R., & Blake, R. (1999). John Holland's vocational typology and personality theory. *Journal of Vocational Behavior*, 55, 41–56.

Holland, D., Lachicotte, W. Jr., Skinner, D., & Cain, C. (1998). Identity and agency in cultural worlds. Cambridge, Massachusetts: Harvard University Press.

Holland, D., & Lave, J. (2001). History in person: An introduction. *History in Person: Enduring Struggles, Contentious Practice, Intimate Identities,* 3-33.

Hyde, J.S., Fennema, E., Lamon, S. (1990) Gender differences in mathematics performance: A meta-analysis. *Psychological Bulletin*, 139–155.

Hyde, J.S., Fennema, E., Ryan, M., Frost, L.A., Hopp, C. (1990). Gender comparisons of mathematics attitudes and affect. *Psychology of Women Quarterly*, 299–324.

Jacobs, J. E., Davis-Kean, P., Bleeker,M.M., Eccles, J. S.,&Malanchuk,O. (2005). 'I can, but I don't want to': The impact of parents, interests, and activities on gender differences in math. A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach* (pp. 246–263). New York: Cambridge University Press.

Jacobs, J. E., & Eccles, J. S. (1992). The impact of mothers' gender-role stereotypic beliefs on mothers' and children's ability perceptions. *Journal of Personality and Social Psychology, 63*(6), 932.

Jacobs, J. E., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Changes in children's self-competence and values: Gender and domain differences across grades one through twelve. *Child Development,* 509-527.

Kiefer, A. K., & Sekaquaptewa, D. (2007). Implicit stereotypes, gender identification, and math-related outcomes. *Psychological Science, 18*(1), 13-18.

Kline, R. B. (2009). *Principles and practice of structural equation modeling.* The Guilford Press.

Köller, O., Baumert, J., & Schnabel, K. (2001). Does interest matter? the relationship between academic interest and achievement in mathematics. *Journal for Research in Mathematics Education, 32*(5), 448-470.

Krapp, A. (1999). Interest, motivation, and learning: An educational-psychological perspective. *European Journal of Psychology and Education*, 14, 23-40.

Kupek, E. (2006). Beyond logistic regression: Structural equations modelling for binary variables and its application to investigating unobserved confounders. *BMC Medical Research Methodology, 6*(1), 13.

Lapan, R. T., Hinkelman, J. M., Adams, A., & Turner, S. (1999). Understanding rural adolescents' interests, values, and efficacy expectations. *Journal of Career Development, 26*(2), 107-124.

Lave, J. (1988). Cognition in practice. Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge university press.

Laverty, S. M. (2008). Hermeneutic phenomenology and phenomenology: A comparison of historical and methodological considerations. *International Journal of Qualitative Methods, 2*(3), 21-35.

Lester, F. K. (2007). *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics.* Information Age Publishing.

Li, Q. (1999). Teachers' beliefs and gender differences in mathematics: A review. *Educational Research, 41*(1), 63-76.

Lichtenfeld, S., Frenzel, A. C., & Pekrun, R. (2007). Gender differences in students' emotions in elementary school. *Paper presented at the 12th Biennial Conference of the European Association for Research on Learning and Instruction*, Budapest, Hungary.

Lichtwarck-Aschoff, A., van Geert, P., Bosma, H., & Kunnen, S. (2008). Time and identity: A framework for research and theory formation. *Developmental Review, 28*(3), 370-400.

Liu, S., & Meng, L. (2010). Re-examining factor structure of the attitudinal items from TIMSS 2003 in cross-cultural study of mathematics self-concept. *Educational Psychology, 30*(6), 699-712.

Lindberg, S.M., Hyde, J.S., Hirsch, L.M. (2008). Gender and mother-child interactions during mathematics homework. *Merrill-Palmer Quarterly*, 232–255.

Lindberg, S. M., Hyde, J. S., Petersen, J. L., & Linn, M. C. (2010). New trends in gender and mathematics performance: A meta-analysis. *Psychological Bulletin, 136*(6), 1123-1135.

Marcia, J. E. (1966). Development and validation of ego-identity status. *Journal of Personality and Social Psychology, 3*(5), 551-558.

Marsh, H. W., & Yeung, A. S. (1998). Longitudinal structural equation models in academic self-concept and achievement: Gender differences in the development of math and English constructs. *American Educational Research Journal*, 35, 705–738.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics (pp. 575-596). New York: Macmillan.

Meece, J. L., Parsons, J. E., Kaczala, C. M., Goff, S. B., & Futterman, R. (1982). Sex differences in math achievement: Toward a model of academic choice. *Psychological Bulletin, 91,* 324–348.

Meeus, W., Iedema, J., Helsen, M., & Vollebergh, W. (1999). Patterns of adolescent identity development: Review of literature and longitudinal analysis* 1. *Developmental Review, 19*(4), 419-461.

Meyer, M.R. (1989). Gender differences in mathematics. In M.M. Lindquist (Ed.), *Results from the fourth mathematics assessment of the national assessment of educational programs* (pp. 149-159). Reston, VA: National Council of Teachers of Mathematics.

Michaelides, M. (2008). Emerging themes from early research on self-efficacy beliefs in school mathematics. *Electronic Journal of Research in Educational Psychology, 6*(1), 219-234.

Miserandino, M. (1996). Children who do well in school: Individual differences in perceived competence and autonomy in above-average children. *Journal of Educational Psychology, 88*(2), 203-214.

Mullis, I. V. S., Martin, M. O., Foy, P., Olson, J. F., & International Association for the Evaluation of Educational Achievement. (2008). *TIMSS 2007: International mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades* IEA TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. *Review of Research in Education, 32*(1), 187-240.

National Science Foundation (NSF), Division of Science Resources Statistics (2011). *Women, Minorities, and Persons with Disabilities in Science and Engineering: 2011*. Special Report NSF 11-309. Arlington, VA. Available at http://www.nsf.gov/statistics/wmpd/.

National Science and Technology Council (2006). Review and appraisal of the federal investment in STEM education research. Retrieved August 28, 2011, from http://www.whitehouse.gov/files/documents/ostp/NSTC%20Reports/STEMEducationResearchOctober06.pdf

National Center for Education Statistics (2010). The Nation's Report Card: Grade 12 Reading and Mathematics 2009 National and Pilot State Results(NCES 2011–455). Institute of Education Sciences, U.S. Department of Education, Washington, D.C.

NCTM. (2000). Principles and standards for school mathematics. *Reston, VA.*

OECD. (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris, France: OECD Publications.

Osborne, J. W. (1994). Some similarities and differences among phenomenological and other methods of psychological qualitative research. *Canadian Psychology/Psychologie Canadienne, 35*(2), 167-189.

Osborne, J. W., & Costello, A. B. (2009). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Pan-Pacific Management Review, 12*(2), 131-146.

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students* 1,* 2. *Contemporary Educational Psychology, 24*(2), 124-139.

Parsad, B., Lewis, L., & Greene, B. (2003). *Remedial education at degree-granting postsecondary institutions in Fall 2000* (NCES 2004-010). Washington, DC: National Center for Education Statistics.

Parsons, J. E., Adler, T., & Meece, J. L. (1984). Sex differences in achievement: A test of alternative theories. *Journal of Personality and Social Psychology, 46,* 26–43.

Philipp, R. A. (2007). Mathematics teachers' beliefs and Affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematic* (257-315). Information Age Publishing, Charlotte, NC.

Peterson, K.D. (1990). Assistance and assessment for beginning teachers. In J. Millman & L. Darling-Hammond (Eds.), *The new handbook of teacher evaluation: Assessing elementary and secondary school teachers* (pp. 104-115). Newbury Park, CA: Corwin.

Pietsch, J., Walker, R., & Chapman, E. (2003). The relationship among self-concept, self-efficacy, and performance in mathematics during secondary school. *Journal of Educational Psychology, 95*(3), 589-603.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated components of classroom academic performance. *Journal of Education & Psychology, 82,* 33–40.

Putnam, R. T., Heaton, R. M., Prawat, R. S., & Remillard, J. (1992). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. *The Elementary School Journal,* 213-228.

R Development Core Team (2011). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria.

Renninger, K. A., Hidi, S., & Krapp, A. (1992). *The role of interest in learning and development.* Lawrence Erlbaum.

Reyes, L. H. (1984). Affective variables and mathematics education. *The Elementary School Journal,* 558-581.

Rogoff, B. (2008). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. *Pedagogy and Practice: Culture and Identities.* 58–74.

Rounds, J. (1995). Vocational interests: Evaluation of structural hypotheses. In D. Lubinski & R. V. Dawis (Eds.), *Assessing individual differences in human behavior: New concepts, methods, and findings* (pp. 177–232). Palo Alto, CA: Consulting Psychologists Press.

Schiefele, U. (1991). Interest, learning, and motivation. *Educational Psychologist, 26*(3), 299-323.

Schraw, G., Flowerday, T., & Lehman, S. (2001). Increasing situational interest in the classroom. *Educational Psychology Review, 13*(3), 211-224.

Schumacker, R. E., & Lomax, R. G. (2010). *A beginner's guide to structural equation modeling* Lawrence Erlbaum.

Seegers, G., & Boekaerts, M. (1996). Gender-related differences in self-referenced cognitions in relation to mathematics. *Journal for Research in Mathematics Education,* 215-240.

Sfard, A., & Prusak, A. (2005). Identity that makes a difference: Substantial learning as closing the gap between actual and designated identities. Paper presented at the *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education,* 37-52.

Shepard, L. A., Dougherty, K. C., & Educational Resources Information Center (US). (1991). *Effects of high-stakes testing on instruction.* Spencer Foundation.

Shields, S. A. (1982). The variability hypothesis: The history of a biological model of sex differences in intelligence. *Signs, 7*(4), 769-797.

Silvia, P. J. (2006). *Exploring the psychology of interest.* Oxford University Press, USA.

Simpkins, S. D., & Davis-Kean, P. E. (2005). The intersection between self-concepts and values: Links between beliefs and choices in high school. *New Directions for Child and Adolescent Development, 2005*(110), 31-47.

Simpkins, S. D., Davis-Kean, P. E., & Eccles, J. S. (2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology, 42*(1), 70-83.

Solberg, V. S., Kimmel, L. G., & Miller, J. D. (2012). Pathways to STEMM support occupations. *Peabody Journal of Education, 87*(1), 77-91.

Solomon, Y. (2007). Not belonging? what makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education, 32*(1), 79-96.

Steffe, L. P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25*(6), 711-733.

Stevens, R. (2000). Project-based classroom. *Multiple Perspectives on Mathematics Teaching and Learning, 1*, 105.

Stevens, T., & Olivarez Jr, A. (2005). Development and evaluation of the mathematics interest inventory. *Measurement and Evaluation in Counseling and Development, 38*(3), 141-152.

Strong American Schools. (2008).*Diploma to nowhere*. Washington, DC. Retrieved from www.deltacostproject.org/resources/pdf/DiplomaToNowhere.pdf

Su, R., Rounds, J., & Armstrong, P. I. (2009). Men and things, women and people: A meta-analysis of sex differences in interests. *Psychological Bulletin, 135*(6), 859-884.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Thorndike, R.M. (1997). Measurement and evaluation in psychology and education (6th ed., pp. 116–117). Upper Saddler River, NJ: Merrill.

Turner, S. L., Steward, J. C., & Lapan, R. T. (2004). Family factors associated with sixth-grade adolescents' math and science career interests. *Career Development Quarterly, 53*, 41-52.

Watt, H. M. G. (2004). Development of adolescents' self-perceptions, values, and task perceptions according to gender and domain in 7th through 11th grade Australian students. *Child Development*, 75, 1556–1574.

Wenger, E. (1998). Communities of practice: Learning, meaning, and identity. Cambridge University Press.

Wigfield, A., Eccles, J. S., & Pintrich, P. R. (1996). Development between the ages of 11 and 25. *Handbook of Educational Psychology,* 148-185.

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30*(1), 1-35. doi:10.1016/j.dr.2009.12.001

Windschuttle, K. (1988). *Education in the U.S.A. Statistical comparisons with Australia Canberra*. Austrialian Publishing Service.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113-121.