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SECONDARY PREPARATION FOR SINGLE VARIABLE COLLEGE CALCULUS: SIGNIFICANT PEDAGOGIES USED TO REVISE THE FOUR COMPONENT INSTRUCTIONAL DESIGN MODEL

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SECONDARY PREPARATION FOR SINGLE VARIABLE COLLEGE CALCULUS: SIGNIFICANT PEDAGOGIES USED TO REVISE THE FOUR COMPONENT INSTRUCTIONAL DESIGN MODEL

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ABSTRACT

There is abundant evidence that many students in the United States are not adequately prepared for college calculus. How to design and implement instruction to adequately prepare secondary students for college calculus is a concern to both college mathematics professors and secondary mathematics teachers. While both groups agree that rigorous instruction promotes mathematical understanding, they hold different opinions about how to optimally prepare high school students for single variable college calculus. This is important because readiness for success in college calculus is a known gatekeeper for success in STEM majors. The data used in this study was drawn from the Factors Influencing College Success in Mathematics (FICSMath) project, which focuses on finding evidence for effective strategies that prepare students for college calculus success. Funded by the National Science Foundation (NSF award #0813702), FICSMath is a large-scale study from the Science Education Department at the Harvard-Smithsonian Center for Astrophysics, which surveyed a nationally representative sample of college students who were enrolled in single variable college calculus courses in the fall semester of 2009. The purpose of the FICSMath study was to gain insight into what high school teachers did that had a significant effect on single variable college calculus performance. The development of the FICSMath survey was informed by several components in order to establish content validity. One particularly informative source was the open-ended responses gathered from mathematics professors and secondary mathematics teachers across the nation, via an online survey. The mathematics professors were asked, "What do high school teachers need to be doing to prepare their students for college calculus success?" and the mathematics teachers were asked, "What are you doing that you think prepares students for college calculus success?" An unequal status concurrent mixed-method design was used to analyze the data. Phenomenographical analysis compared the variation between the mathematics professors' and secondary mathematics teachers' responses. The quantitative data came from students who were in two and four-year large, medium, and small colleges and universities across the nation who completed the FICSMath survey. Participating schools of higher-ed administered the 61-item FICSMath survey in the beginning of the fall semester of 2009. The professors held the surveys until the end of the semester, at which time they recorded the grades earned, and then returned the surveys to Harvard University. The surveys included questions on students' demographics, academics, and pedagogical practices from their last high school mathematics course. The sample included in the analysis were pre-calculus, non-AP Calculus, AP Calculus AB, and AP Calculus BC students who moved directly from secondary mathematics to single variable college calculus where the FICSMath survey was administered. The dependent variable was performance in college calculus and the independent variables were pedagogical variables that aligned with components of the Four Component Instructional Design (4C/ID) model. This model was designed from cognitive load theory and has four distinct components. The support, procedure, learning task, and part-task components were placed together by van Merriënboer and other cognitive load researchers in order to assist with the instruction of complex tasks and to enhance transfer of learning. Using multiple-regression analysis, two models were created that are predictive of college calculus performance, one for pre-calculus students,

and one by combining all three levels of secondary calculus students together. The pre-calculus model (n=964) had four significant pedagogical variables, one with positive effect on performance, and three with negative effects. The predicted difference for those experiencing positive verses negative predictors was a total predicted difference of 19.9 points earned in students' final college calculus grade. Because there was no significant difference between the mean high school performance across the three levels of secondary calculus, or in the mean performance in college calculus, all three levels of secondary calculus were grouped together to create the calculus model. The calculus model (n=1,999) revealed 11 significant pedagogical variables. Four variables had a positive effect on performance and seven had negative effects. The predicted difference for those experiencing positive verses negative predictors was 25.29 points in college calculus performance. Seven of the eleven categories from the phenomenography aligned with significant variables from the two multiple regression models. The triangulation of findings led to functions being classified as a learning task variable, meaning working with functions is a specific complex task for students. Triangulation also revealed that even though professors and teachers believed that connecting mathematics to the real world is important, such pedagogy was not predictive of future performance in college calculus. Chapter 6 addresses the change in variability in the data that is explained when student affect variables are added to the models, and also provides an implications section for teachers. The 4C/ID model was modified to the 3C/ID Pre-Calculus Model for College Calculus Performance and the 3C/ID Secondary Calculus Model for College Calculus Performance.

DEDICATION

This dissertation is dedicated to my husband, Tim Wade, the professional engineer, who taught me to love mathematics; To my Mother, Bonnie Henderson, the science teacher who taught me value education; to my Father, Alvin Henderson, a veteran of the 101st Airborne Screaming Eagles 501 Parachute Infantry Regiment and POW survivor, who taught me to be tough; and to Jesus Christ who made it all possible.

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CHAPTER 1

INTRODUCTION

Secondary Preparation for College Calculus

There is abundant evidence that many students in the United States are not adequately prepared for their first college level mathematics course (Harwell et al., 2009). Most often single variable calculus is the first college level mathematics course that counts toward degree credits (Smith, 1998). There is agreement among college mathematics professors and secondary mathematics teachers that rigorous instruction promotes mathematical understanding, but there is less agreement on how to implement instruction in order to better prepare secondary students for college calculus (Harwell et al., 2009).

Empirical findings from science, technology, engineering, and mathematics (STEM) research reveal that factors such as environment, demographics, and pre-college preparation impact college performance (Crisp, Nora, & Taggart, 2009). How to prepare secondary students for college level mathematics so the first calculus course is not a "gate keeper" which discourages students from pursuing STEM degrees has been a concern since the New Math era in the 1960s. At first curriculum changes were considered to be the correct way to add rigor to the learning of mathematics (Harwell et al., 2009). The defenders of the Commercially Developed (CD) mathematics curricula and the National Science Foundation-Funded (NSFF) curricula had opposing views, which resulted in the "math wars" of the 1980s (Harwell et al., 2009). For example, the CD curricula focused mainly on algorithms and procedures while the NSFF curricula aligned with problem solving and conceptual understanding as advocated in the *Curriculum and Evaluation Standards for School Mathematics* (Harwell et al., 2009). Research revealed that both the CD and NSFF curriculum prepared students to enroll in a college course that should have been completed in high school (Harwell et al., 2009). Therefore it was shown that neither curriculum was predictive of college mathematics preparation for the average student. As a result, the research focus shifted to teacher instructional practices and student reasoning in the mathematics classroom (NCTM, 1989, 2008). Attempts in mathematics education research to understand the complexity of variables that influence teachers' instructional practices, and how these variables impact student achievement, have been largely inconclusive (Mewborn, 2007). What is known is that each student's mathematical understanding and problem solving ability is primarily shaped by the teaching experiences they encounter in school (Mewborn, 2007).

A better understanding of what prepares students for college level mathematics, specifically college calculus, is important since calculus is the foundation for many STEM degrees. More schools are adopting the goal of preparing all students for college, yet American College Testing (ACT) research revealed that too few high school graduates leave high school prepared for college level work in mathematics and science (Camacho & Cook, 2007). Some researchers claim that students are underprepared for college calculus because teachers tend to focus on procedural instruction instead of conceptual understanding (Tall, 1992). Furthermore, research has shown that when secondary mathematics teachers incorporate opportunities for students to develop conceptual understanding they are more likely to be able to problem solve (Haskell, 2001). The term problem solve has multiple meanings but it is most often associated with solving nonstandard problems (Darken, Wynegar, & Kuhn, 2000). Schoenfeld (1992) stated that when problems are placed in the back of a section as examples of how the mathematics can be useful students come to believe that mathematics should have a ready method for problem solving, and the method was just covered in that chapter. Such problems are considered to be standard problems. Non-standard problems require students to "grapple with new and unfamiliar tasks when the relevant solution methods are not known" (Schoenfeld, 1992, p. 56).

AP Calculus Trends

In 1965 the Mathematical Association of America (MAA) expressed concern over the small number of students who took the Advanced Placement (AP) calculus exam (Bressoud, 2009). At that time there were 1.4 million students who began college in the fall, but only 9,000 of those students had taken the AP calculus exam the previous spring. Comparatively, in 2008 there were 21 million students who began college in the fall, and 300,000 of them had taken the AP Calculus exam the previous spring. The percent increase of students who had taken the AP Calculus exam with respect to the overall college entrance population more than doubled from 0.6 percent to 1.4 percent across 43 years. Additionally, the MAA reported there were at least 200,000 more students who completed some other form of secondary calculus (Bressoud, 2009). These numbers indicate that more students are taking AP or another type of calculus course at the secondary level than ever before (Bressoud, 2009). It is conceivable, then, that colleges and universities should be seeing an increase in the number of students that are prepared for college level calculus. However, the last longitudinal study by The National Education Longitudinal Study of the high school class of 1992 reported that one-third of all students who studied calculus in high school had to take pre-calculus in college (Bressoud, 2009).

The US Department of Education stated that of the 430,000 students who took calculus in high school in 2004, only 52 percent of them took the AP calculus exam (Bressoud, 2009). However, in 2009 it was predicted that 575,000 students would be in secondary calculus and the College Board estimated that 75 percent of them would take the AP calculus exam (Bressoud, 2009). From the college professor perspective, it is not well known how AP Calculus benefits students in college calculus, and most certainly not known how non-AP calculus benefits students in college calculus (Bressoud, June 14, 2010, personal communication). The students who take AP Calculus may receive credit for college level work and exempt single variable or multiple variable college calculus. The College Board claims that high performance on the AP Calculus AB exam is equivalent to material learned in single variable college calculus, and high performance on the AP Calculus BC exam is equivalent to multi-variable college calculus (Pieronek, 2007). While receiving college credit is based on policy, typically if a student earns a 3 (qualified), 4 (well qualified), or 5 (extremely well qualified) on the AP Calculus exam, they may receive college credit and opt to exempt college calculus courses (Pieronek, 2007). There is limited research on student performance for those who choose to exempt calculus because of AP Calculus credits earned. A detailed review of these studies is presented in chapter two.

College Calculus Trends

A trend concerning first semester college calculus since the 1980s is one that should receive serious consideration (Friedler, 2004). Between 1980 and 2000 there was a 99 percent increase in the number of students enrolled in post-secondary institutions for higher learning, but a 3.4 percent decrease in the number of students who took first semester college calculus. However, in terms of raw numbers, there are more students than ever before taking both secondary and college level calculus, making this an increasing important area to examine. Research about success in college calculus indicates there has been a consistent trend of 65 percent of students earning a C or better in college calculus since 1985 (Maggelakis & Lutzer, 2007). This means that 35 percent of students who enter college calculus either fail the course or pass with a D. This should not be viewed just as students who were unsuccessful in college calculus, but as students who may be transferring out of STEM areas of study.

Powerful Learning Environments

How to help students understand mathematics in a meaningful way to elicit conceptual understanding has been a concern for many years in the mathematics education community. In the 1960s and 1970s, instructional design theories were used in education research to gain an understanding of how instructional strategies helped students develop sophisticated mathematical reasoning and problem solving abilities (Gravemeijer, 2004). During this time instructional designs were considered to be in vogue within the research community, but eventually interest faded as instructional designs were perceived as being in conflict with constructivism (Gravemeijer, 2004). From the constructivist point of view, learners of mathematics must be active participants in constructing understanding and not just passive recipients of instruction (Von Glassersfeld, 1987b). Some viewed instructional design models as following the task analysis approach, which infers that the models were based on procedures that experts used to solve problems (Gravemeijer, 2004). Basing mathematics instruction on proceduralized problem solving steps was seen to be in conflict with how students construct their own understanding. However, some contrasting research over the past decade has focused on instructional designs for "powerful learning environments" (van Merriënboer & Paas, 2008).

Powerful learning environments, as described by van Merriënboer and Paas (2008), should enhance the learning of complex material, aid in the transfer of learning from one environment to another, encourage collaboration, and enable students to construct their own understanding. Transferring mathematical knowledge from secondary mathematics to single variable college calculus is one example of transferring learning to different environments. Secondary mathematical preparation for single variable college calculus involves learning complex mathematical information. For example, calculus is often the first time that students are confronted with limits, continuity, differentiation, integration, and the real life applications of displacement, optimization, related rates, area, and volume problems. All of these topics require understanding of many complex mathematical ideas and require vertical transfer from previous mathematics courses. Vertical transfer occurs when prior learning is transferred to a new learning environment that is higher in a knowledge hierarchy (Haskell, 2001).

The Four Component Instructional Design Model

Van Merriënboer, a cognitive load researcher, developed an instructional model based on powerful learning environments and the human cognitive architecture in the early 1990s, called the Four Component Instructional Design (4C/ID) Model. This model was designed for environments where complex learning occurs, but it was not specifically designed for mathematics instruction. The 4C/ID model was created by cognitive load theorists to enhance the learning of complex material, to aid in the transfer of learning from one environment to another, to encourage collaboration, and to enable students to construct their own understanding (Van Merrienboer et al., 2008). Cognitive load theory is based upon the idea that working memory, which was previously referred to as short-term memory, is limited in space and duration, while long term memory is unlimited in both (Sweller, van Merriënboer, & Paas, 1998; Paas & van Merriënboer, 1994; Paas, Renkl, & Sweller, 2003; Sweller & Chandler, 1991). The definition of learning, from a cognitive load perspective, is defined as a permanent change in long-term memory (Sweller et al., 1998; Sweller et al., 1991; Sweller & Candler, 1994). This has specific implications since the goal of instruction is to create schemas in long-term memory (Sweller et al., 1998). According to schema theory, knowledge is stored in long-term memory in the form of schemas, which categorize elements of information based on the manner in which the information will be used (Sweller et al., 1998). More information about cognitive load theory will be discussed in chapter two.

The 4C/ID model is based upon the idea that learning environments for complex tasks can be described using four components. These components will be discussed in detail in chapter two, but briefly, the model is designed from: (1) support component; (2) procedure component; (3) learning task component; and (4) the part-task component (van Merriënboer & Paas, 2008). According to van Merriënboer et al., (2006) complex learning tasks have "many different solutions, are ecologically valid, cannot be mastered in a single session, and pose a very high load on the learners cognitive system" (p. 343). From the learning for transfer perspective, learning tasks must be considered as a whole first and foremost, and then the parts needed to accomplish the task are considered. This is based on the whole-part method of decreasing cognitive load, which increases the potential of learning for transfer. Supportive information is based upon conceptual understanding and reasoning of new information, problem solving, collaboration, and cognitive assessment (van Merriënboer, Clark, de Croock, 2002). Procedural information is based upon examples and previous learning, or schemas in long-term memory, which help process complex tasks. Part-task practice promotes the compilation of procedures or rules, such as teaching for automaticity (van Merriënboer et al., 2008; van Merriënboer, Kirschner, & Kester, 2003; van Merriënboer & Sluijsmans, 2008; van Merriënboer et al., 2006; Feldon, 2007; Schankman, 2006).

Learning Mathematics is a Complex Task

Cognitive load theorists would refer to secondary preparation for single variable college calculus as a complex task. Each of the aforementioned descriptions of a complex task may be mapped to the learning of mathematics. For example, mathematics problems can have different solutions depending upon algebraic, graphic, or symbolic representation. Cognitive load theory states that problems based in real life help to create complex learning environments. The National Council for Teachers of Mathematics (NCTM) states that good mathematical tasks relate to familiar everyday worlds of students, but not always (NCTM, 1991). Some tasks are theory based or are “fanciful” tasks, which challenge students intellectually such as number theory problems or formal proofs for mathematical ideas (NCTM, 1991). Thus, all mathematics can be connected to real-life applications, but not all complex problems in mathematics are connected to real life. Some problems are mathematical in nature and the complexity resides in seeking patterns and solutions, which may eventually be used to solve real-world problems. In reference to the inability to master complex tasks in one session, mathematical reasoning and sense making takes time. The NCTM has defined reasoning and sense making in high school mathematics. By definition, reasoning is observing generalizations, making connections between numbers and ideas, and drawing conclusions on the basis of evidence or stated assumptions (NCTM, 2009). Sense making refers to developing mathematical understanding of a situation, context, or concept by connecting learning to previous knowledge (NCTM, 2009). The NCTM (2009) states that, “Reasoning and sense making are intertwined across the continuum from informal observations to formal deductions" (p. 9). Relative to sense making, cognitive load theory describes this processing as encoding new information in working memory for storage into long-term memory (Sweller & Chandler, 1991). This process changes the learners' schema structure, or conceptual understanding of mathematics. Lastly, learning a complex task is stated to cause a high working memory load on the learners' cognitive system. If working memory load is too high, mathematical information cannot be processed or encoded for storage into long-term memory. A high working memory load is considered to be that which limits processing and encoding of new mathematical information for storage into long term memory. Thus, a high working memory load hinders mathematical sense making. The connection between studying how students learn information and mathematics began years ago when it was evident that learning mathematics is difficult. The development of a meaningful curriculum in mathematics was slow to be developed, and in fact, the first widely accepted curriculum was only established a little more than 30 years ago.

Historical Background

The MAA is the professional organization for college mathematics professors, and the NCTM is the professional organization for K-12 mathematics educators. Mathematics professors and educators have worked together to establish mathematics content and curriculum since the creation of the MAA in 1915 (Straley, 2010) and of the NCTM in 1920 (Gates, 2003). Historically, both groups have addressed challenges such as limited course requirements, the need for students to be exposed to meaningful problem solving, and the need for rigor in mathematics courses (Bressoud, 2009; NCTM, 2008). The Joint Commission of the MAA and the NCTM in 1935 was established to unify attempts of mathematics professors and teachers to establish new objectives for the study of secondary school mathematics (Garrett & Davis, 2003). The consensus was that the high school curriculum needed to include less arithmetic, more meaningful mathematics, and more problem solving, yet little direction was offered from the commission regarding how to develop such programs for all students. The creation of a meaningful secondary mathematics curriculum was essentially left to colleges of education and secondary education systems.

A decade later, mathematics professors complained about their students' lack of college readiness and blamed "colleges of education and the administrative circles in the secondary school system" for low expectations and limited standards in high schools (Kempner, 1948, p. 415). Some professors did acknowledge that they had been "willing to let the professional educators do all the hard and dirty work" of creating a credible curriculum (Garret & Davis, 2003; Kempner, 1948, p. 415). In the 1980s there was, for the first time, considerable data from the National Science Foundation (NSF) Priorities in School Mathematics (PRISM) studies that could offer evidence and new directions in the creation of a secondary mathematics curriculum. These results laid the foundation for the NCTM's A*genda for Action: More Math Study.* The *Agenda* included three recommendations relevant to secondary preparation for college calculus: (1) School districts should increase the amount of time students spend in the study of secondary mathematics; (2) mathematics educators and college mathematicians should reevaluate the role of calculus in high school mathematics programs; and (3) a curriculum that stresses problem solving must focus on the problem solving process, not just content (NCTM, 1980).

In 1989 the NCTM *Principles and Standards for School Mathematics* provided the first specific guidelines for a secondary mathematics curriculum. Initially the 1989 NCTM Standards caused controversy between the MAA and the NCTM for reasons most easily described as a conflict between content and pedagogy (Klein, 2001). If instructional decisions are based on content considerations then the choices of pedagogy may be limited, and conversely, the choices of pedagogy can also limit the amount and type of content that can be covered (Klein, 2001). Mathematicians have typically focused on mathematical content while mathematics educators have been trained to focus on content and pedagogical strategies for heterogeneous groups of students who may or may not be college bound. The concerns over the 1989 Standards led the MAA to appoint *The President's Task Force on the NCTM Standards*, which served as a review group and provided advice concerning how to resolve the disagreements between the NCTM and the MAA (Ross, 2000). The MAA stated, "The members of the Task Force applaud the NCTM for its courage in formulating a set of standards for school mathematics" (MAA, 1997). They also presented nine specific concerns that they believed needed to be addressed. The NCTM addressed the concerns of the MAA and revised the 1989 Standards. The new version of the Principles and Standards for School Mathematics was published in April of 2000.

The fundamental idea of the 2000 Standards has been that problem solving, reasoning and proof, mathematical connections, communication and collaboration, and representation are all involved in the process of learning mathematics (NCTM, 2008). The 2000 Standards document provided the first widely accepted secondary mathematics curriculum in the United States (US). However, each state in the US determines independently what standards are followed, which has caused inconsistencies across the nation. Currently, a new paradigm shift is moving the curriculum away from the NCTM Standards to the *Common Core State Standards* initiative in order to align mathematics standards and curriculum across all states.

The Common Core State Standards (CCSS) initiative has been an effort coordinated by the National Governors Association Center for Best Practices to create common mathematics standards across the US (Common Core State Standards, 2010). As of November 29, 2010, 42 states have officially adopted the new CCSS. The CCSS align with college expectations, are clear and understandable, include rigorous content and application knowledge, are informed by top performing countries, and have been built upon the strengths and lessons of the NCTM 2000 Standards (Common Core 2010). Just as it is important to have well-established standards that are used across the nation, it is also important that the autonomy of the teacher in the classroom be respected. Mewborn (2007) stated that each student's mathematical understanding and problem solving ability is primarily shaped by the teaching experiences they encounter in school. Therefore research in what teachers to do help prepare students for college calculus success is important.

The Focus of this Research

The purpose of this research is to gain a better understanding of what instructional practices secondary mathematics teachers employ that prepare pre-calculus and calculus students for success in single variable college calculus. There is currently no instructional model that is based on pedagogies that prepare pre-calculus and calculus students for single variable college calculus. The 4C/ID model is appropriate to use as a theoretical lens through which to view secondary preparation for college calculus because three of the four components of the model explicitly consider instruction for transfer of learning (van Merriënboer, Kester, Paas, 2006). The qualitative data from mathematics professors and secondary mathematics teachers, and the quantitative data from students in single variable college calculus courses, were analyzed concurrently in an un-equal status concurrent mixed methods design. The results were triangulated to modify the 4C/ID model. Two models based upon pedagogical practices that best prepare students for single variable college calculus success have been created.

Research Questions

Three research questions are addressed in this study:

Research Question 1: What categories from the phenomenography of college mathematics professors and high school mathematics teachers align with the 4C/ID model?

Research Question 2: How well do the components in the 4C/ID model represent pedagogies that predict pre-calculus and calculus students' success in single variable college calculus?

Research Question 3: How can the 4C/ID model be modified to reflect pedagogies that are predictive of pre-calculus and calculus students' success in single variable college calculus?

Theoretical Perspective

Van Merriënboer and other cognitive load theorists developed the 4C/ID model in the early 1990s. The basic premise of the model is that learning tasks should always be combined with methods that have been shown to enhance learning for transfer. Complex tasks are often considered to be that which connects learning to 'real life' tasks since real life connections provide opportunities to present overarching concepts to learners (Merrill, 2002; van Merriënboer et al., 2008). The assumption is that the interacting elements in complex learning tasks can be limited in order to enhance memory and thus augment transfer of learning (Merrill, 2002; van Merriënboer et al., 2008). The model was not designed specifically for mathematics instruction but for learning environments where transfer of learning is the goal, and complex problems are the basis of instruction. The components of the 4C/ID model are designed to guide instruction to increase the likelihood of transfer of learning by creating a way of scaffolding the whole-part instructional method (van Merriënboer et al., 2006). The 4C/ID model shares the same assumptions about the human cognitive architecture as cognitive load theory. The assumptions are that working memory is limited in space and duration while there appears to be no limit to either the space or duration of long-term memories. It is also assumed that there are three sources of working memory load. The first is extraneous cognitive load, which comes from how the material is presented during instruction. The second is intrinsic cognitive load, which comes from the element interactivity of the mathematics to be processed in working memory (J. Sweller et al., 1994; van Merriënboer et al., 2006). Element interactivity occurs because of the interacting parts of the mathematics that must be addressed in solving complex tasks. Element interactivity is inherent in secondary preparation for college calculus because of the many interacting mathematical concepts involved in pre-calculus and calculus problem solving. The third is germane cognitive load, which is the only load in working memory that is necessary for learning (Van Merriënboer, Jeroen, & Sweller, 2005) Germane cognitive load hooks new information that has been processed and encoded for storage into long term memory to existing schemas. Thus the new information becomes part of the learners' schema and can be brought back into working memory as a chunk of information to help process more new mathematics. Both intrinsic cognitive load and germane cognitive load will be discussed in detail in chapter two.

Sources of Data

The research team for Factors that Influence College Success in Mathematics (FICSMath) conducted the first large-scale national study seeking to better understand secondary preparation for college calculus. The purpose of the FICSMath study was to gain insight into what high school teachers do that best prepare students for single variable college calculus success. “Sadler’s conundrum” is the name given for the disconnect between the claim from high school teachers that they prepare their students for college mathematics and mathematics professors who lament that their students are not prepared for college level mathematics (Sadler, June 14, 2010). The qualitative data came from mathematics professors and high school mathematics teachers’ open-response on-line surveys. The mathematics professors were asked, “What do high school teachers need to be doing to prepare their students for college calculus success?” and the mathematics teachers were asked, “What are you doing that you think prepares students for college calculus success?”

The quantitative data comes from students’ responses to items on the FICSMath survey. A random sample of schools across the nation were contacted by a team of recruiters, one of whom was the researcher for this study, asking college and university mathematics department chairs if their calculus students could participate in the FICSMath study. A random sampling of schools was conducted to ensure a nationally representative sample of students attending college calculus courses. Participation required students in single variable college calculus courses to complete a 61-item survey. At the end of the 2009 fall semester, the professors recorded the students’ final grades on the survey and 10,492 surveys were sent back to Harvard with no student identifiers. The students’ grade is the dependent variable, and the independent variables are the items from the FICSMath survey that align with the components of the 4C/ID model.

Significance of the Study

This study is significant because: (1) it utilizes data from the first large-scale nation-wide study on what factors from secondary mathematics instruction prepare students for success in single variable college calculus; (2) there is currently no instructional model designed from the correlation between secondary pre-calculus and calculus pedagogical practices and college calculus performance; and (3) there is currently no model that is predictive of future performance in college calculus that has been designed from the 4C/ID model. Controls were put in place to only include data from students who transferred directly from secondary pre-calculus or calculus to single variable college calculus in an effort to better understand what teachers do that helps students vertically transfer mathematical knowledge to college calculus. There were 1,287 pre-calculus students and 2,160 calculus students across the nation who moved directly from secondary pre-calculus or calculus to a single variable college calculus course where the FICSMath survey was completed. The large sample size of students in the study may allow small differences in responses about teacher instructional practices to be statistically significant. These findings may indicate variables worthy of future experimental research.

Limitations of the Study

The open ended nature of the survey for the mathematics professors and secondary mathematics teachers was beneficial in obtaining rich data, however, the discrepancy between the number of professors and teachers who responded to the survey may be a limitation of this study. In an effort to limit the effect from the unequal group size, all data was converted to percentage of statements from professors for each category and for the clustered statements. Likewise, the same was done for the teachers' statements, which allowed for comparison of the descriptions for each category across groups.

The students are in different calculus courses at different colleges and universities, which means hierarchical linear modeling could be used because of the different levels represented in the quantitative data. However, only seven percent of the variability of the data came from the course level, four percent came from the school level, while 89 percent came from the student level. Therefore linear modeling will be used instead of hierarchical linear modeling since the course and school levels captured so little variability.

There are many variables that can influence performance in first semester single variable college calculus. Variables such as roommate situation, college or university activities, course load, instructional practices of the mathematics professor, and student commitment to attend class and study can all effect performance. Such questions are not within the scope of this study. This study is seeking to measure what pedagogies from students' senior year in pre-calculus or calculus effected single variable college calculus performance. Even though the questions on the FICSMath survey were carefully constructed to have high content validity, concerns remain because what students read and what they understand may be different than the intent of the question.

The 4C/ID model has been proposed as a theoretical model for teaching complex tasks to increase the likelihood of transfer of learning. However, this model has not been used as a framework to structure pedagogical practices that are predictive of college calculus success in the past. Therefore the components of the model may not capture the essence of what teachers do to prepare students for college calculus success.

Definitions and Key Terms

A *complex task* is defined in cognitive load literature as having many different solutions, as being ecologically valid, as being content that cannot be mastered in a single session, and as content that places a high cognitive load on the learner's cognitive system (Van Merriënboer et al., 2006; Van Merriënboer, Jeroen J. G. & Sweller, 2005a).

Extraneous cognitive load originates from poorly designed instruction and can be a cause of high working memory load (F. Paas, Renkl, & Sweller, 2003).

Four Component Instructional Design (4C/ID) model is an instructional model based on cognitive load theory that focuses on complex learning environments and teaching for transfer of learning.

Germane cognitive load encodes working memory and sends the encoded information to be stored in long-term memory (Van Merriënboer et al., 2006).

Intrinsic cognitive load comes from element interactivity, or the multiple elements that must be considered in a complex problem-solving task. It is determined by the interaction between the nature of the learning tasks that must be learned and the expertise of the learner (Van Merriënboer, Jeroen J. G. & Sweller, 2005a).

Learning task component of the 4C/ID model is the complex task that requires the support of the other components in the model. Such tasks are often real life, whole task problems (Van Merriënboer et al., 2006).

Long-term memory is a major aspect of human cognitive architecture and is considered to be quantitative due to its seemingly limitless size (J. Sweller & Chandler, 1991).

The *Outcome space* from a phenomenography, as described by Marton (1994) is the "ordered and related set of categories of description" of the concept being studied.

A *schema* is a cognitive construct that allows multiple elements of information to be treated as a single element categorized according to its use (Birney, Fogarty, & Plank, 2005).

Part task component of the 4C/ID model aligns with part-whole scaffolding and is based on sufficient practice for automaticity.

Phenomenography research has the goal of capturing the variability, both the similarities and differences, in the ways a phenomenon is perceived by different groups and expressed in qualitative data (Dahlin, 1999). For this research the two different groups are mathematics professors and secondary mathematics teachers.

Procedures, when learning complex upper level mathematics content, is defined as knowledge of the order of steps, goals and subgoals of steps, knowledge of the situation where the procedure is used, and consideration of constraints and heuristics inherent in the situation (Star, 2000).

Procedural information component of the 4C/ID model is supported by conceptual understanding and reasoning, which aligns with the mathematics education perspective that procedures do not stand alone.

Powerful learning environments aim at the development of complex learning, deep conceptual understanding, and for learners to accept responsibility and regulate their learning.

Real-life learning experiences allow for the simultaneous use of non-recurrent (new information to be learned) and recurrent information (information learned earlier and recalled back into working memory from long term memory) and confront learners with all of the multiple parts that combine to create a complex learning task.

Supportive information component of the 4C/ID model supports the learning of complex tasks and focus on learning new content by elaborating on conceptual understanding, reasoning, problem solving, and cognitive assessment (Van Merriënboer et al., 2002; Van Merriënboer et al., 2003).

Working memory is the seat of consciousness, and has been referred to in the past as short-term memory (F. Paas et al., 2003; J. Sweller & Chandler, 1994).

CHAPTER 2

THE LITERATURE REVIEW

A better understanding of what prepares students for single variable college calculus is important since this course is often the foundation for many STEM degrees. In the past there has been a lack of consensus on what secondary preparation for college calculus should be, especially since each state independently established mathematics requirements for high school graduation (Reys, Dingman, Nevels, Teuschner, 2007). It is not yet known if the alignment of mathematics standards across the nation with the recently adopted Common Core State Standards (CCSS) will create a greater consensus of what secondary practices align with college calculus success. What is known is that since 1985 there has been an upward trend of two factors that have served as indicators of college calculus success (Ferrini-Mundy & Gaudard, 1992). One factor is an increase in the number of mathematics courses required at the secondary level across the nation (Ferrini-Mundy & Gaudard, 1992) while the other factor is the growth in the number of students who take the Advanced Placement (AP) calculus exam (Bressoud, 2009). These two factors indicate that more students are pursuing mathematics beyond the minimum requirements in the high school curriculum than ever before (Bressoud, 2009). Before the 2010 adoption of the CCSS, five states required four mathematics credits for graduation, 26 states required three, and 15 required two, while nine states' requirements were not provided on the 2010 *Education Commission of the States* website (ecs.org/html/IssueSection.asp). It is not known how the CCSS will impact mathematics courses needed for graduation, but it is expected that course requirements will become more standardized across the states because of the common state standards. Rigorous preparation for college mathematics was defined seven years ago as passing three high school mathematics courses (Greene & Forster, 2003). Typically these three courses would be Algebra I, Algebra II, and Geometry (Reys et al., 2007). Research indicates there is a gap between requirements for high school graduation and what colleges and universities require for admittance (Greene & Forster, 2003). The CCSS may help to reduce this gap since one of their claims is that these standards are aligned with college and work expectations. In 2007, it was found that College Board AP exam participation and performance were two of the strongest predictors of college mathematics preparation (Byrd, 2007). However Bressoud (2009), a college mathematics professor and the 2009 president of MAA, stated that the benefits of secondary calculus are not well known.

AP Calculus and College

The research on student performance in college mathematics for those who choose to exempt college calculus courses because of AP Calculus exam scores is limited. However, what follows is a summary provided by various researchers in the field. Education Testing Service (ETS) research indicates that 24 percent of students who earned a three on the AP Calculus AB exam took no additional calculus and 17 percent took a remedial course (Klopfenstein & Thomas, 2005). The need for remedial mathematics courses is determined by additional placement tests required by individual college mathematics departments (Klopfenstein et al., 2005). The National Research Council (NRC) also determined that secondary calculus prepares students for the rigors of college when teachers are not pressured to sacrifice depth for breadth (Klopfenstein & Thomas, 2005). For example, the NRC stated that the inclusion of too much content in the College Board calculus curriculum might prevent students from achieving a deep understanding of calculus concepts (Klopfenstein et al., 2005). Furthermore, Klopfenstein and Thomas (2005) researched the effect of AP Calculus on early college success using the Texas Schools Micro-data Panel and determined that AP Calculus had no effect in early college success for the average college student (Klopfenstein et al., 2005).

A study in 2002 investigated what happened to students that received a 3 or higher on an AP Calculus exam (Bressoud, 2009). Research on 435 randomly selected students was carried out to investigate what percent of students take advantage of earning a score of three or higher and exempt single variable college calculus. For AP Calculus AB, 84 percent of the students who made a score of 5, 82 percent of the students who earned a score of 4, and 60 percent of those who received a score of 3 chose to receive college credit (Bressoud, 2009). If credit was not received, about half of the students said it was because the college did not give credit and the other half stated they chose to enroll in single variable college calculus even though they could have exempted the course (Bressoud, 2009). For AP Calculus BC, 79 percent of students who scored a 3, 4, or 5 exempted single variable college calculus. This study did not provide information of how students who exempted single variable calculus performed in multi-variable calculus (Bressoud, 2009).

Bressoud (2009) provided results of a large-scale study conducted in the fall of 1994 at 22 colleges and universities that received the greatest number of AP Calculus scores. The advantage of this study is that these researchers not only provided average calculus grades for the different levels of AP exam scores, and the average grade for the students who earned single and multi-variable calculus the traditional way (in college), but they also adjusted the SAT grades of the students who exempted single and multi-variable calculus (Bressoud, 2009). These comparisons are presented in Table 2.1, which indicates that students who chose to exempt calculus are the students who are predicted to be successful in college, as indicated by their SAT scores. Colleges have accepted the SAT as an indicator of students' ability for academic success in college; however, Sadler and Tai (2007a) stated that high school grades are considered to be the best predictor of college performance. What Table 2.1 may really show is that characteristics of students may be the true indicator of college success instead of their AP scores (Dougherty, Mellor, & Jian, 2005). Students who take AP courses typically have better academic preparation, stronger motivation, and more family advantages than non-AP

Table 2.1

AB and BC Calculus Grades and Adjusted SAT Grades For Comparison of Calculus II Performance For Students Who Did and Did Not Take AP Calculus AB

Placed via	Average Calculus II Grade	SAT Adjusted Grade	Placed via	Average Calculus III Grade	SAT Adjusted Grade
Calculus I	2.43	-------	Calculus II	2.50	-------
3 on AB exam	2.69	2.64	3 on BC exam	3.00	2.92
4 on AB exam	2.90	2.78	4 on BC exam	3.45	3.35
5 on AB exam	3.34	3.15	5 on BC exam	3.46	3.27

students (Dougherty et al., 2005). It is reasonable to expect that these characteristics would also be an advantage when taking the SAT test as well.

Researchers at Notre Dame examined the performance of AP Calculus students who earned at least a four on the AP Calculus BC exam and compared their performance in differential equations, or the third level of calculus, to the students who entered the course because of earning the previous two calculus credits the traditional way in college (Pieronek, 2007). In 2005 and 2006, 45 percent of the students in differential equations exempted single and multi-variable calculus because of AP credits (Pieronek, 2007). The analysis of the performance of students in differential equations revealed that students with AP credits earned higher average final grades and had a higher proportion of top grades than students who entered into differential equations the traditional way (Pieronek, 2007). However, there is more to this story. The students who earned AP credits and scored high in differential equations also entered Notre Dame with 10 or more AP credits (Pieronek, 2007). Again, it appears that the personal characteristics of students may be the true indicator of success instead of AP scores.

Other research finds no conclusive evidence that the AP experience provides superior college preparation when compared to a non-AP curriculum that is rich in mathematics and science (Klopfenstein et al., 2005). Sadler and Tai (2005a) found that students with low grades in honors and AP courses perform worse in college courses than students who had courses from the standard curriculum and earned high grades. Bressoud (2009) stated the most glaring observations from the few studies available about AP calculus students' performance in college indicate that little is known about the effects of secondary calculus instruction.

Problem Solving

The 1989 NCTM Standards focused on problem solving and called for mathematics instruction to abandon curricula that promoted thinking about mathematics as a rigid system of rules (Battista, 1994). A mathematics curricula based on algorithms without meaning is limited because mathematics is dynamic, it reveals patterns and relationships, and learners can use it to seek solutions, formulate conjectures, and solve meaningful problems (Schoenfeld, 1992). Curricula standards for high school mathematics provide learning expectations that should be the focus of mathematics instruction, yet since the 1989 NCTM Standards there has been inconsistency across states concerning what standards are used and how they are used (Reys et al., 2007). Most states have referred to the NCTM Standards and have organized high school mathematics curricula using the 9^{th} through 12^{th} grade band based on the traditional subjects of Algebra 1, Geometry, Algebra II, Pre-calculus, Trigonometry, Probability and Statistics, and Calculus (Reys et al., 2007). The new paradigm shift in mathematics education from the NCTM Standards to the CCSS is expected to align standards across all states for more consistency in mathematics education across the nation.

Traditional Problem Solving

Historically textbooks have included problem-solving sets that are contrived to illustrate the mathematical techniques provided by the instructor (Schoenfeld, 1992). Such problems have typically been added at the end of chapters as if they justify why students are to learn the mathematical material (Schoenfeld, 1992). A consequence of this type of problem solving in the traditional mathematics curricula is that students get the impression that there is only one right way to solve the problems, and that way was just demonstrated by the instructor (Schoenfeld, 1992). Traditional problem solving has been perceived as being superficial because it typically introduces a technique, illustrates the technique, and then provides similar problems for students to practice (Schoenfeld, 1992). This type of problem solving leads students to believe there is a fixed algorithm for every problem, and solving problems should require little time and effort (Schoenfeld, 1992). Traditional problem solving requires students to cognitively process less information since their thinking is limited to the examples just provided by the instructor (Schoenfeld, 1992).

Real Problem Solving

The NCTM call for mathematics educators to change their pedagogical practices from procedural instruction to "real" problem solving provided an opportunity for transformative changes for both curricular content and pedagogy (Schoenfeld, 1992). The types of problem solving advocated by the NCTM Standards were to enable students to apply mathematics with flexibility and resourcefulness (Schoenfeld, 1992). Hence forth in this document, problem solving is defined as that which requires students to grapple with new and unfamiliar tasks when the method to solve the problem and the solution is not readily known. Students who engage in such problem solving must learn to work with complex problems of significant difficulty (Schoenfeld, 1992). The 4C/ID model was designed under the assumption that powerful learning environments exist where learners grapple with whole-task, complex problems (van Merriënboer et al., 2006).

The Complexity of Learning Mathematics

Real life applications help students understand the connections between content, mathematical reasoning, and sense making (NCTM, 2009) but real-life tasks are not what makes learning mathematics a complex task. Learning mathematics is a complex task because mathematics is abstract, and reasoning is required to understand abstract information (Russell, 1999). A focus on reasoning and sense making implies that "covering" mathematics in the curriculum is insufficient and that the goal of instruction should be mathematical reasoning and sense making (NCTM, 2009). Sense making is developing an understanding of a situation, context, or concept by connecting mathematics with existing knowledge (NCTM, 2009). Thus the goal of instruction should be that students both understand and can use what they have been taught (NCTM, 2009).

The complexity of learning mathematics can be better understood by considering the problem solving process. When presented with a mathematics problem, students must: (1) analyze the problem; (2) consider a strategy to solve the problem; (3) make connections to prior mathematical knowledge; and (4) reflect on the solution (NCTM, 2009). The first step, analyzing a mathematics problem, involves students being able to identify relevant mathematical concepts, procedures, or representations that reveal information about the problem; define relevant variables and conditions given; seek patterns and relationships; look for hidden structure; consider special cases or simpler analogs; make connections across various mathematical domains, contexts, and representations; make preliminary deductions and conjectures; and decide if a statistical approach is appropriate (NCTM, 2009). For the second step, implementing a strategy, the students must make a purposeful use of procedures; organize calculations, algebraic manipulations, and data displays; make logical deductions based on current progress by verifying conjectures and initial findings; and they must monitor progress toward a solution (NCTM, 2009). The third step, making connections to prior mathematical knowledge, requires students to recall previously learned mathematics in problem solving (NCTM, 2009). For example, algebra is fundamental to solving calculus problems and if students cannot recall essential algebraic elements, they cannot solve calculus problems. Lastly, students need to revisit initial assumptions while being mindful of special cases and extraneous solutions; reconcile different problem solving approaches; effectively provide the solution; and then in order to make significant mathematical connections, they need to generalize the solution to a broader class of problems while looking for connections with other problems (NCTM, 2009). Also, if the problem is statistical in nature, students must recognize and consider the scope of inference (NCTM, 2009).

Conceptual Understanding and Procedural Knowledge

There has been criticism, based on students inappropriate use of algorithms, that mathematics instruction focuses on rote memorization of procedures while neglecting conceptual understanding required for meaningful problem solving (Bosse & Bahr, 2008). The criticism stems from the fact that students tend to learn algorithms by rote without developing any understanding of what the algorithm is for or how to use it (Hiebert & Lefevre, 1986; Star, 2000). Problem solving in mathematics requires both conceptual understanding and procedural knowledge, and the NCTM *Standards* states there should be a balance between them (NCTM, 2008; Bosse et al., 2008).

Research in this area has often addressed which occurs first, learning concepts or procedures, but Star (2000) claims this is less important than understanding what each means when learning advanced mathematics. The current assumption in mathematics education research is that the end goal for conceptual understanding is knowledge that can be used to "recognize, identify, explain, evaluate, judge, create, invent, compare, and choose; in other words, when such knowledge is understood" (Star, 2000, p. 82). By contrast, the end goal for the acquisition of procedures is when "skills become routine and can be executed with fluency; in other words, when such knowledge has become automatized" (Star, 2000, p. 82). Such distinction about knowledge of concepts and procedures has its origins in a philosophical framework, which relates conceptual understanding as "knowing how" and procedural knowledge as "knowing that" with the latter being perceived as straightforward and rather uninteresting (Star, 2000, p. 82). However, considering procedures is more complex when examining abstract algebraic and geometric procedures required in learning pre-calculus and calculus content. Superficially, procedures may be represented simply as a chronological list of actions or steps (Star, 2000), such as finding common denominators in order to add fractions. On a more abstract level, procedures in complex mathematical tasks and in the mathematics classroom may include:

> Planning knowledge -- knowledge of such things as the order of steps, the goals and subgoals of steps, the environment or type of situation in which the procedure is used, constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge which are inherent in the environment or situation. This knowledge is abstract (and deep), but not necessarily conceptual (Star, 2000, p. 85).

The calculus curriculum has been at the center of the procedural skills verses conceptual understanding debate (Chappell & Kilpatrick, 2003). One common example is the criticism from mathematics professors that the College Board AP calculus curriculum is so broad that the students move through the course by learning procedures instead of concepts, which will not be beneficial in college calculus (Bressoud, 2009). A specific example is the research by Chappell and Kilpatrick (2003) based on the charge that the calculus reform movement watered down secondary and post secondary calculus courses by teaching only a superficial use of skills. Star's definition of procedural knowledge may be different than that of Chappell & Kilpatrick's "superficial use of skills" (p. 18). However, Chappell & Kilpatrick (2003) claim that the reform movement has significantly impacted the beliefs of secondary and post-secondary teachers; therefore their study should be presented despite the possible difference in the definition of procedural knowledge.

Chappell & Kilpatrick's (2003) research at a large state university involved one class with a focus on conceptual understanding and seven classes with a focus on procedure-based instruction. The focus of the conceptual understanding course was to link students' entry knowledge to more formal concepts; to use multiple methods of representation, such as numeric, graphic, and algebraic; and for students to explain the variety of methods employed as they problem solved (Chappell et al., 2003). The focus for the procedures courses were the teaching of procedures, algorithms, and skills where algebraic solutions were emphasized over non-algebraic solutions, and students were not required to explain their problem solving methods (Chappell et al., 2003). Other qualities of the courses were kept constant, except for the number of students in each treatment group. Table 2.2 show the results of the first study, which indicate that the concepts based group performed significantly better on their midterm than the procedural groups. A replication study ensured that the difference did not come from the initial mathematics abilities of the students in the concepts based group (Chappell & Kilpatrick 2003). In the replication study students in the concepts-based group performed better not only on their midterm, but also on their final exam. In the discussion the authors state, "Results from a variety of assessment measures demonstrate that it is possible to devote significant class time to the development of conceptual understanding without sacrificing skill proficiency" (p. 32). The 4C/ID model, introduced in chapter one, has a support component, based on reasoning and conceptual understanding, and a procedural component. The discussion provided here should help in understanding and defining the procedural component relative to learning complex mathematics.

Table 2.2

Descriptive Statistics and Univariate Results for Conceptual/Procedural Study One (n=1,164)

	Concept Based Course		Procedure Based Course			
Exam	No.	Means (Std. Dev)	No.	Means (Std. Dev)	t-value	Effect
Skills 1	72	85.63 (15.52)	231	87.34 (14.09)	0.879	
Skills 2	70	79.36 (17.34)	213	82.32 (17.73)	1.221	
Midterm	72	119.38 (19.95)	232	104.61 (23.96)	4.743*	0.64
Final	69	129.80 (27.36)	205	125.85 (29.89)	0.969	

*p<0.05

Theoretical Framework

Cognitive Load Theory and the 4C/ID model provide guidelines for lowering working memory load associated with learning rich complex tasks (van Merriënboer et al., 2008; van Merriënboer et al., 2006). Both theories share the same assumptions about the human cognitive architecture and both distinguish three types of working memory load. First, a brief description of the assumptions will be discussed followed by a discussion of the three different types of cognitive load.

There are five assumptions concerning the human cognitive processing system (Sweller et al., 1998). The first assumption is that long-term memory holds cognitive schemata, which vary in complexity and provide organization to knowledge (van Merriënboer et al., 2005a; Sweller, 2009). It is assumed that a huge amount of information is accumulated and stored in long-term memory (Leahy & Sweller, 2008), however, humans are not directly conscious of that information (Sweller et al., 1998). Awareness of long-term memory content is only known when schemas are filtered back into working, or conscience, memory as chunks of knowledge (Sweller et al., 1998). When a person reads a mathematics problem, a number of nodes in that person's schematic network begin to fire, which spreads and activates other nearby semantically related nodes (Mestre, 2005). Soon there is a chunk of memory activated that can assist in the encoding of new mathematics that is recalled back into working memory. Schemas are referred to in mathematics education as a "web" of mathematical memory (Russell, 1999). The web means that mathematical ideas are connected and work together to create a large memory base of previously learned mathematics.

The second assumption is that the bulk of stored information in long term memory is borrowed from other people through reading what they write, listening to what they say, or by imitating what they do (Leahy et al., 2008; Sweller, 2009). "Borrowing" does not imply that meaningful information becomes part of ones own repertoire of knowledge simply because one paid attention to what others know. Rather, it is understood that learners are exposed to information, and they must make it their own if there is to be any application or transfer of learning, either in or outside of the knowledge domain. Schoenfeld (1992) also addressed learning mathematics in this way by stating that students learn mathematics by observing the process of mathematics, listening to explanations of mathematics, and by practicing mathematics.

The third assumption is that all human learning is critically dependent on a "random generation and test of effectiveness process during problem solving" (Sweller, 2009, p. 13 & 14). When faced with problem solving, the choices of how to proceed may be so many that they seem like random trials, so humans have no choice but to make a move, test the move for its effectiveness, discard ineffective moves, and then proceed to the next step (Sweller, 2009). In mathematics education, this is similar to the idea of seeking strategies for problem solving and flawed reasoning (NCTM, 1999). During mathematical reasoning and problem solving, students must try a strategy from their repertoire of mathematical memory, and sometimes problem solving strategies seem to work, when in fact they do not. Even though the incorrect process did not further the solution, the student can learn from the flawed reasoning process. Thus the strategy that led to "flawed reasoning" is "discarded" and a different strategy must be used.

The fourth assumption is that the structure of working memory limits the "explosive growth" in the potential number of possible combinations that can be processed in working memory (Sweller, 2009). According to Sweller (2009), "Working memory acts as an intermediary between long term memory and the environment" (p. 14). Therefore there is a massive amount of information that can be brought into working memory from the environment through our senses. The structure of working memory limits the processing capacity to about seven plus or minus two items of information at a time (Miller, 1956). Knowledge of the limits of working memory suggests that humans are particularly poor at complex reasoning unless most of the elements with which we reason have previously been stored in long term memory (Sweller et al., 1998). Working memory is simply not capable of processing complex interactions between elements that have not been previous stored in long-term memory (Sweller et al., 1998). Thus, sense making in mathematics aligns directly with this assumption. Sense making states that mathematical understanding of concepts and content should be connected to existing knowledge, which is in long term memory, because working memory is not capable of processing complex mathematical ideas independent of mathematical memory (NCTM, 2009; Sweller et al., 1998).

The last assumption is that there are no limitations in working memory when dealing with chunks of previously organized schemas from long-term memory (Sweller, 2009). In fact, the capacity and duration of working memory, when dealing with chunks of information from schemas in long-term memory, are considered to be limitless (Sweller, 2009). Thus, a leading premise of cognitive load theory is that the seat of human intellectual ability resides in long-term rather than in working memory (Sweller et al., 1998). However, the process of moving items from working memory into long-term memory can be hindered because of working memory load.

Cognitive load theory has described working memory load as coming from two distinct sources that limit learning, extraneous and intrinsic load, and one source that is necessary for learning, germane cognitive load. Figure 2.1 presents an image, modified from cognitive information processing theory, of the three loads on working memory and the learning process. Van Merriënboer et al., (2006) claim that teaching complex tasks requires the learning process. Van Merriënboer et al., (2006) claims that teaching complex tasks requires intrinsic load to be balanced, or limited, in order to enhance germane cognitive load. When this occurs then students can transfer complex learning to a new and different environment (van Merriënboer et al., 2006).

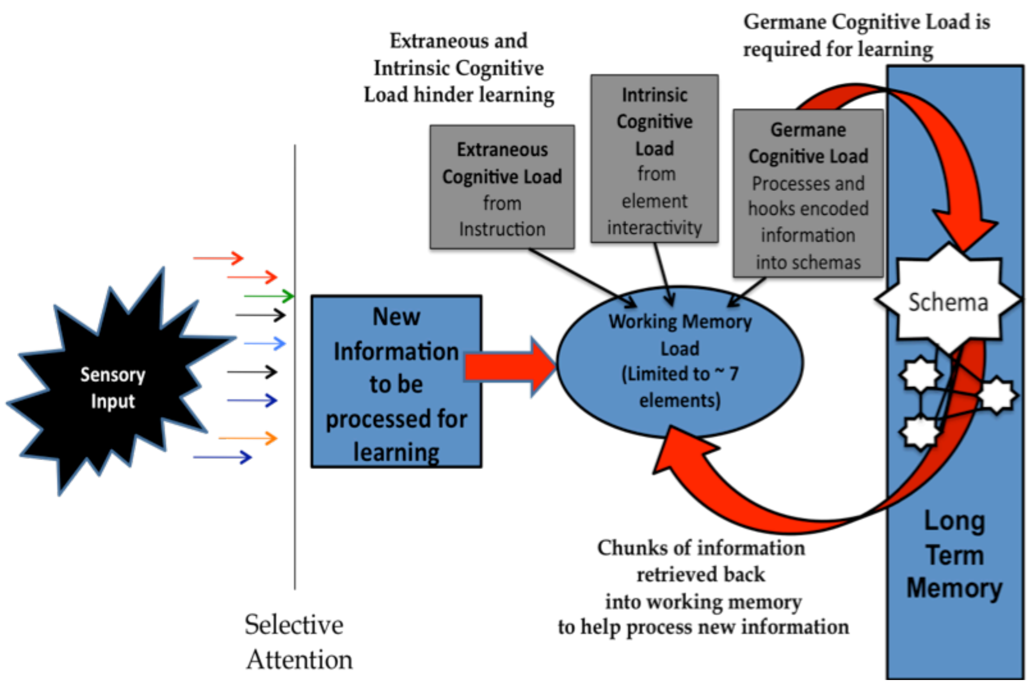

Figure 2.1. Cognitive Load Theory, Image modified from Driscoll (2005).

Extraneous Cognitive Load

Extraneous cognitive load may occur from instructional practices, which has the potential to limit the processing ability in working memory (van Merriënboer et al., 2005a). Considering a graph with multiple sources of information that are not integrated provides an example of how high extraneous cognitive load can occur. If a graph of simultaneous equations is presented with both graph and text, but the text is captioned at the bottom of the graph, instead of with the appropriate function, this may cause the split-attention effect (van Merriënboer et al., 2005a). Theoretically, this means the learner must use two of the seven available processing capabilities in working memory in order to understand the graph (van Merriënboer et al., 2005a). If the text is integrated into the graph next to the appropriate function, then extraneous cognitive load is limited because there is not a need to integrate the two disparate information sources (van Merriënboer et al., 2005a). Even if extraneous cognitive load is low, working memory can still be hindered because of the complexity of the material to be learned since extraneous and intrinsic cognitive load are considered to be additive (Sweller et al., 1998; van Merriënboer et al., 2002; van Merriënboer et al., 2005a).

Intrinsic Cognitive Load

Intrinsic cognitive load exists from the complexity of the content to be learned (Ayres & Gog, 2009). Element interactivity refers to how individual elements of a task interact with other tasks in a specific learning activity and is considered to be the main generator of intrinsic cognitive load (Ayres, 2006). High element interactivity imposes a high working memory load (Ayres, 2006) and is inherent in mathematics due to the complexity of learning mathematics. The descriptions of analyzing a problem, implementing a strategy, seeking connections across mathematical domains, and reflecting on the solution to a problem presented earlier is an example of high element interactivity. If each one of the processes presented were considered individually, and not as a chunk of information from schema, then the seven available processing capabilities in working memory would be used just by analyzing the problem. This means there would be no processing capability left for reasoning, sense making, or solving the problem. Even if there is no extraneous cognitive load, intrinsic cognitive load can be so high that learning is hindered. Learning occurs only if there are some of the "seven plus or minus two" processing capabilities in working memory available for germane cognitive load (Sweller et al., 1998; van Merriënboer et al., 2002).

Germane Cognitive Load

Germane cognitive load is required for schema formation, and is the only working memory load that is necessary for learning (Ayres et al., 2009). Germane cognitive load processes and encodes information for storage into long-term memory, and then hooks the information to existing schemas (van Merriënboer et al., 2005a). After new information is stored in long term memory, as part of a schema, or part of the mathematical web of knowledge, it can then be brought back into working memory later as a chunk of knowledge (Sweller et al., 1998). Cognitive load theorist believe infinite chunks can be sent back into working memory to help process more new information (Sweller et al., 1998; van Merriënboer et al., 2002). Therefore, sense making in mathematics means that learners are processing mathematical information for storage into long-term memory as part of the mathematical memory. If this process occurs, then students are learning mathematics for understanding and later use.

The 4C/ID Model

Since the late 1990s, authentic learning tasks have been considered to help learners integrate knowledge and abilities needed for understanding and performance (van Merriënboer et al., 2003). The philosophy is that such tasks allow learners to transfer what is learned in their current environment to a new and different environment (van Merriënboer et al., 2003). The risk of such tasks is that element interactivity can be high and learning can be hindered. As a result, cognitive load theory states that whole-part practice should be used to scaffold complex learning tasks (van Merriënboer et al., 2003; van Merriënboer et al., 2006). Scaffolding is useful in that it allows learners to achieve a goal or action not achievable without that support (van Merriënboer et al., 2003).

Whole-part instruction means a complex problem is presented in its full complexity right from the beginning, but teachers focus the learners' attention on subsets of the interacting elements (van Merriënboer et al., 2003; van Merriënboer et al., 2006). For example, a typical calculus related rates problem may read, "Water flows into a cylindrical tank at a steady rate of 15 cubic feet per minute. The tank is 16 feet wide and 30 feet deep. Find the rate the height of the water is changing when there is 10 feet of water in the tank." One way to emphasize the interacting elements of this problem is to require learners to focus attention on the parts of the problem. These could be described as: (1) the set up; (2) similar triangles from *geometry* to solve for the radius in term of the height; (3) *algebraic* manipulation to restate the volume equation in terms of height;

(4) *calculus* to take the derivative; (5) *algebraic* substitution of what is known into the derivative; and (6) *algebraic* manipulation to simplify and solve the problem. However, in mathematics education, it is perceived as unbeneficial to parse the problem into geometry, algebra, and calculus as if mathematics is disconnected. This method is similar to the part-whole, or simple-to-complex method, which has been shown to make complex problem solving too “piece meal and fragmented to allow for transfer to new problem situations” (van Merriënboer et al., 2003, p. 6). Instead, when learning mathematics, it is perceived as more beneficial to focus on the concepts of the problem, using reasoning and sense making to choose the strategies needed to solve the problem (NCTM, 2009). The 4C/ID model can be helpful in this process. The four distinct parts of the model are the learning task component, support component, procedure component, and part-task component (van Merriënboer et al., 2002; van Merriënboer et al., 2003; van Merriënboer et al., 2006). Figure 2.2 shows van Merriënboer & Paas (2008) conceptual framework of the 4C/ID model. The observer should notice that the largest component for complex learning tasks is the supportive information component, which in mathematics education is related to conceptual understanding and reasoning. The foundation for procedural information is also the support component, and together the concepts and procedures support learning complex authentic tasks. The part task practice component is relative to instruction when enough practice is available for the information to become automatized.

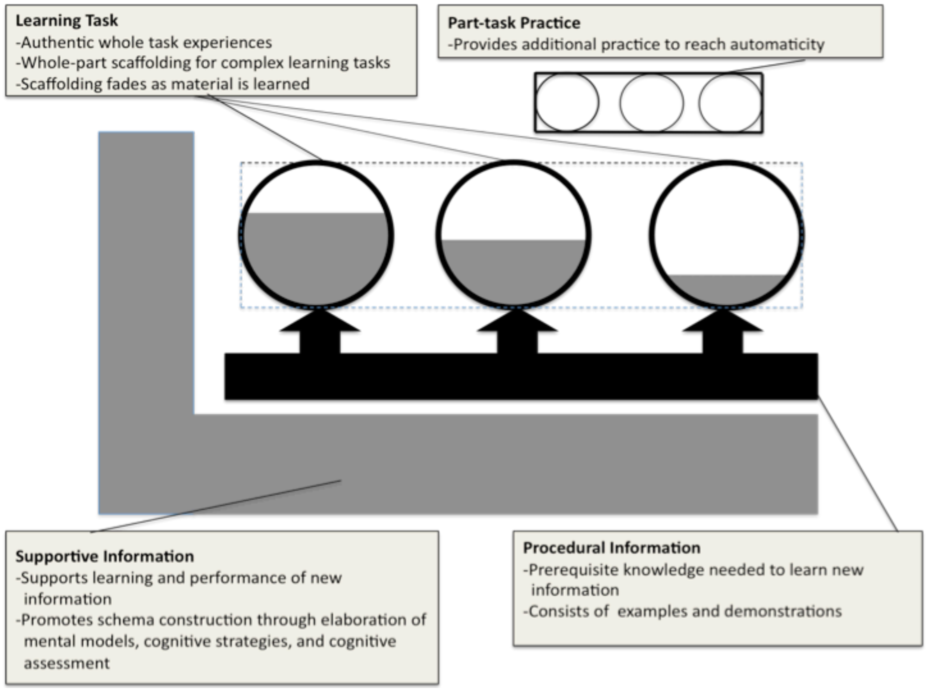

Figure 2.2. The 4C/ID Model Components (van Merriënboer & Paas, 2008)

The Learning Task Component

The learning task component should engage the learner in meaningful problem solving that requires mental processes to move from the initial state of the problem to an acceptable solution (van Merriënboer et al., 2003). Preferably learning tasks are authentic, real life problems that are presented with the whole part scaffolding method (van Merriënboer et al., 2002; van Merriënboer et al., 2003; van Merriënboer et al., 2006). The NCTM (2009) states, “High school mathematics prepares students for possible post-secondary work and study in three broad areas: (1) mathematics for life; (2) mathematics for the workplace; and (3) mathematics for the scientific community” (p. 3). Van Merriënboer and colleagues describe the 4C/ID model as connecting the world of work and education through the use of authentic real-life problems.

The Support Component

Supportive information is best presented before learners begin the complex learning task (van Merriënboer et al., 2003). Cognitive load theorists believe this will allow schema to be constructed in long-term memory, which can ultimately be sent back to working memory in chunks of information and assist with the learning task (van Merriënboer et al., 2003). Conceptual understanding, reasoning, problem solving, and cognitive assessment are all parts of this component that aid in creating a supportive learning environment (van Merriënboer et al., 2002; van Merriënboer et al., 2003; van Merriënboer et al., 2006). Each will be discussed relative to how these fit in the support component and in the learning of mathematics.

Supportive information promotes schema construction through elaboration by helping students establish non-arbitrary relationships (van Merriënboer et al., 2002). Supportive information aids in conceptual understanding, and provides knowledge of structures and causal relationships in complex learning tasks (van Merriënboer et al., 2002). Conceptual models focus on how elements are interrelated, structural models describe how elements are organized, and causal models help to interpret processes, give explanations for events, and make predictions (van Merriënboer et al., 2002). The integration of such cognitive models helps students understand nonarbitrary relationships in complex learning tasks. For example, relative to the aforementioned related rate problem, understanding conceptually that as the water flows into the tank, the volume, height, and radius of the water are all changing with respect to time as the water rises, which should help the learner understand why implicit differentiation is needed to solve related rates problems. Structural organization of the problem reveals that there are three unknowns, the volume, the radius, and the height, but only information for two of these have been provided. Thus, there must be a method of solving for one unknown in terms of another. Finally, the units in the problem assist in interpreting the solution since the unknown is the rate that the water is rising in the tank, meaning a final solution should be in feet per minute.

During problem solving students must analyze a problem, implement a strategy, and reflect on a solution, which aligns with what van Merriënboer et al., 2003 stated as the problem solving process where students use heuristics unique to interacting elements. Literally supporting the problem solving process, the support component connects complex elements to theories, contains concrete, abstract and general knowledge, and provides reasoning opportunities (van Merriënboer et al., 2002).

Lastly, cognitive assessment and feedback are also part of this component. The idea is that cognitive assessment promotes schema construction and stimulates learners to reflect on the quality of their problem solving processes (van Merriënboer et al., 2002). The goal is for cognitive feedback to encourage learners to seek more effective mental models and problem solving strategies. This is contingent upon the teachers' feedback being valuable and providing opportunities for reflection.

Support Component Summary

The classroom environment where mathematics is learned should be one that is focused on conceptual understanding, mathematical reasoning, problem solving, and cognitive assessment. When this exists there is a supportive environment for learning mathematical information and solving complex problems. Learning mathematics is complex; therefore, instead of the support component existing only when real-life authentic learning tasks are integrated into the curriculum, the support component in mathematics education should be considered as the supportive *environment* in which meaningful mathematics is learned.

Procedure Component

The relationship between learners' knowledge of concepts and their ability to execute procedural skills has been a concern in mathematics education research for many years (Star, 2000). The concern is that learners of mathematics tend to learn algorithms by rote without developing an understanding of what the procedure is for, why it is important, and how and when to use it (Star, 2000). In cognitive psychology both the understanding of concepts and the acquisition of procedures has been researched, but the relationship between them is not well understood (Star, 2000). Procedural information is presented to learners because it helps them perform routine aspects of complex learning tasks (van Merriënboer et al., 2006). The recall and manipulation of algebra, when problem solving with new pre-calculus and calculus concepts, may be anything but routine. A plausible example of a routine aspect of learning in a secondary mathematics class may be the level of complexity in which graphing calculators are used. The NCTM states, "Technology can relieve students of burdensome computations giving them the freedom and the need to think strategically" (NCTM, 2009, p. 14). A traditional approach to procedural information has been memorization, however neither the NCTM nor cognitive load theorists advocate learning by memorization (van Merriënboer et al., 2002; van Merriënboer et al., 2003, NCTM, 1999).

Van Merriënboer et al., (2002) stated that supportive information pertains to learning new information, while procedural information pertains to knowledge previously learned that is stored in long-term memory (van Merriënboer et al., 2002). Considering which occurs first, conceptual or procedural knowledge, has not been established as important in mathematics education. In mathematics there is agreement that knowledge of concepts and procedures are positively correlated and the two are learned in tandem rather than independently (Star, 2000).

One of the goals of the 4C/ID model is to connect rules, or procedures that combine rules, to knowledge elements such as concepts through examples or demonstrations when the learners need the information (van Merriënboer et al., 2002). Presenting procedural information when it is needed helps to prevent the split attention effect (van Merriënboer et al., 2002; van Merriënboer et al., 2003). Van Merrienboer et al., (2003) states if procedural information is presented at the time that the learner needs it then integration with concepts is more likely. The split attention effect may cause a misapplication of procedures that results in errors, and van Merriënboer et al., (2002) stated, "It is important that learners learn to recognize their own errors and how to recover from them" (p. 53). When learning a complex task it is practically impossible to prevent errors, therefore it is important to give meaningful corrective feedback as soon as possible after the misapplication of a procedure or rule occurs (van Merriënboer et al., 2002).

Worked examples may help students learn the connection between procedures and concepts because examples can focus the learners' attention on particulars of the complex learning task (van Merriënboer et al., 2006). When whole part scaffolding is used the complex task is presented in its entirety, and worked examples are one way to focus the learners attention on specific parts of the problem without making the task be too piecemeal as in part-whole scaffolding. Schoenfeld (1992) stated that worked examples are often contrived to illustrate why specific mathematical information is needed when such problems in real life would rarely be encountered. This view aligns with Gravemeijer & Doorman (1999) perspective that contextual problems used to be limited to applications addressed at the end of a learning sequence "as a kind of add on" (p. 111). Currently worked examples of contextual problems have a more central role in learning the connection between concepts and procedures because of the emphasis on student understanding that mathematics is useful and also because of the presumed motivational power (Gravemeijer & Doorman, 1999).

Authentic learning tasks is the terminology used by cognitive load theorists to describe problems placed in context, and are described as problems that have "many different solutions, are ecologically valid, cannot be mastered in a single session, and pose a very high load on the learners cognitive system" (van Merriënboer et. al, 2006, p. 343). The related rate problem presented earlier is an example of an authentic real life complex problem where whole-part instruction could scaffold understanding with a worked example. When such a complex problem is presented in its entirety the complexity may be overwhelming, but an example can focus attention on the dimensions of the tank and the changing dimensions of the water as it fills the tank over time. Assuming students understand implicit differentiation, observing a knowledgeable teacher provide an example of related rates has the potential of helping students understand why they need to know and be able to correctly work through the mathematical process of implicit differentiation. After students have observed how related rates problems are set up and solved, then they can begin to use what they have observed to help them solve problems themselves. However, in mathematics education there is a concern that worked examples give students the impression that there is one way to solve the given set of problems and that method was just provided by the instructor (Schoenfeld, 1992). When dealing with complex mathematical topics such as related rates, each problem may appear completely different, which requires students to consider the worked example but to think about each problem individually. For example, the sequential related rates problem may read, "A car is traveling north toward an intersection at a rate of 60 mph while a truck is traveling east away from the intersection at a rate of 50 mph. Find the rate of change of the distance between the car and the truck when the car is 3 miles south of the intersection and the truck is 4 miles east of the intersection." This problem is very different from the tank problem, but both problems are dealing with changing phenomenon over time. Thus students would need to: (1) consider the worked example; (2) consider the similarities of the changing dimensions of water rising in a tank and movement relative to an intersection; (3) find a mathematical way to express the changing rates for the intersection by considering what is given in the problem; and (4) apply implicit differentiation to solve and ultimately answer the problem. Even though a worked example has been provided, the learning task is still complex and requires students to grapple with the problem in order to understand and solve it. Teachers should be aware that a disadvantage of worked examples relative to learning mathematics is that learners may not study them carefully (Renkl, Stark, Gruber, & Mandl, 1998). Learners may only briefly refer to examples when they have difficulties performing task (Renkl et al., 1998). In this case, worked examples are not beneficial for learning.

Procedure Component Summary

The procedural information component has been placed in the 4C/ID model because it promotes schema automation by embedding new information in situation specific rules that connect particular conditions to particular actions, and this process is called "proceduralization" (van Merriënboer et al., 2008, P. 11). Relative to learning complex mathematics in pre-calculus and calculus that will transfer to single variable college calculus, it is the researchers perspective that Star's (2000) definition of procedures being:

> Knowledge of such things as the order of steps, the goals and subgoals of steps, the environment or type of situation in which the procedure is used, constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge which are inherent in the environment or situation.

This knowledge is abstract (and deep), but not necessarily conceptual" (p. 85). What is most important to consider, relative to the 4C/ID model, is that procedures do not stand alone. Procedures must be supported by conceptual understanding and reasoning within the supportive information component (see Figure 2.2).

Part Task Component

When learning has occurred to levels of automaticity this means some specific task can occur with little effort, requires little conscious monitoring, can occur rapidly, and utilizes few cognitive resources (Feldon, 2007). The part-task component is part of the 4C/ID model because there are times that instruction allows repeated practice of information to the point of automaticity. This can both benefit and hinder meaningful learning of mathematics. For example, finding solutions to quadratic equations is a common task in pre-calculus and calculus, and if students can efficiently work through the steps of completing the square then they can focus on complex algebraic manipulations in order to find the solutions. However, if completing the square is difficult students may become overwhelmed with both the process of completing a square and complex mathematical manipulations. This is an example of automaticity being beneficial. A counter example is when students know the unit circle and can draw it with correct quadrants, angles, degrees, and polar and rectangular coordinates, yet they do not know how to use the information. This type of automaticity, or memorization in mathematics, is not beneficial for learning complex pre-calculus and calculus concepts.

Part Task Summary

Van Merriënboer et al., (2003) stated that the part task practice component had not yet been substantiated in the whole-task theoretical framework. As stated previously, the whole-task scaffolding method is more appropriate for considering complex tasks, such as solving related rate problems, than the part task scaffolding method. The part task method made the complex task too "piece meal" for conceptually understanding concepts such as related rates. In general, overreliance of instruction for automatization, or for part task practice, is not helpful in complex learning (Van Merriënboer et al., 2003).

CHAPTER 3

METHOD

The Factors that Influence College Success in Math (FICSMath) is a cross-sectional study that gathered data from calculus students in single variable calculus across the nation from two and four year small, medium, and large colleges and universities. Data from this study was used to determine what pedagogies teachers employ that best prepare students for college calculus. Dr. Phil Sadler, the principal investigator of the FICSMath study, is located at the Science Education Department within the Harvard Smithsonian Center for Astrophysics. FICSMath is Sadler's fourth nation-wide study of freshman at randomly selected colleges and universities. The first study was a pilot program, followed by the Factors Influencing College Science Success (FICSS) study, which began in 2002. The third study was the Persistence Research in Science and Engineering (PRiSE) study, which began in 2006. The FICSS study revealed important information about secondary mathematical preparation for college science courses. Collectively, these studies add to the validity of the FICSMath survey. The FICSMath study is funded through the National Science Foundation (NSF award # F15226-105), and is the first nationwide study seeking to identify secondary mathematics teachers' pedagogy, assessment practices, along with other techniques that lead to success in single variable college calculus. An epidemiological research method was used in the design of the FICSMath study.

Epidemiological Research Method

The epidemiological method of research relies on the natural variation within the students' experiences, backgrounds, and personal decisions rather than on explicit comparison of treatment and control groups (Hazari, 2006). When natural variation exists in a large sample from a heterogeneous population, it is advantageous to use a research method that allows for capturing this variation. For example, the FICSMath randomized sample from two and four year small, medium, and large colleges and universities has variability in three distinct areas: the college or university level, the course level, and the student level. Relative to the college or university level, there was four percent variability in college calculus performance (Sonnert, 2010). The course level, whether designed for engineers, mathematics, science, or for non-STEM majors, captured seven percent variability in college calculus performance (Sonnert, 2010). At the student level, 89 percent of the variability in college calculus performance was captured from the wide range of experiences from students' last high school mathematics courses (Sonnert, 2010). The epidemiological research method has the power to simultaneously test many variables across all levels, but with the variability from the school and course level being small, in comparison to the student level, the research focused only on the student level. Epidemiological studies are based on correlations, and such information can reveal relationships that exist, or fail to exist.

Development of FICSMath Survey

The development of the FICSMath survey was guided by five major components. The first was an extensive literature review of mathematics education journals from the past ten years with special attention given to demographic and academic variables that affected performance in high school or college mathematics (Sadler, 2010). The second component was the transcribed report of the first FICSMath advisory board meeting, comprised of board members from the field of mathematics and mathematics education (Sadler, 2010). The third component examined what was learned from the FICSS and PRiSE studies (Sadler, 2010). The fourth was open response information from students in college calculus who were asked what their high school mathematics teacher did that helped to prepare them for college calculus (Sadler, 2010). The fifth component was an online survey sent to mathematics professors and mathematics teachers across the nation. The mathematics professors were asked, "What can high school teachers do to prepare students for success in college calculus courses?" Secondary mathematics educators were asked, "What do you do, as a mathematics teacher, that you think make a positive difference in helping your students succeed in college calculus?" This information was obtained through an online survey and collectively there were 185 mathematicians and 84 mathematics teachers who responded to surveys (Watson, 2010). The information from these five components was synthesized and used to create the FICSMath survey.

FICSMath Survey

The FICSMath survey had a total of 61 questions divided into 9 different sections that questioned students regarding content, teacher instructional practices, and assessment methods used in their last high school mathematics class. There was a demographic section along with an area for the students to report all of the secondary mathematics courses taken, along with their grades. The survey included a section where the student's calculus professor recorded their final grade in the course. For this study, this grade is the dependent variable, and the independent variables are the pedagogical practices that align with the components of the 4C/ID model.

Validity of FICSMath Survey

The content validity for the FICSMath survey was established through the five components that were used to create the survey. Each component addressed specific levels of concern in the area of mathematics education. The literature review provided what researchers are currently addressing, and the FICSMath advisory board meeting addressed legitimate concerns from both the secondary classroom and the college calculus classroom. The FICSS and PRiSE survey items that had high correlation between college calculus students and secondary mathematics educator responses were used on the FICSMath survey (Sadler, 2010). The responses from the college calculus students about their high school mathematics teacher was a way to ensure that the information gained from other sources aligned with what is currently happening in the secondary mathematics classroom (Sadler, 2010). Lastly, the online surveys provided views unique to mathematics professors who teach college calculus, and to secondary mathematics teachers who seek to prepare students for college level mathematics. All of these sources of information were used to ensure that the content on the FICSMath survey addressed what is "currently on the mind" of the various groups of people who have an invested interest in secondary preparation for college calculus (Sadler, 2010).

Reliability of the FICSMath Survey

The reliability of the survey was established by the test-retest method. Students in calculus classes at four different universities were given the same FICSMath survey two weeks apart. The professor for the calculus classes would leave twenty minutes early and the researchers would enter with the surveys for the students to take. There were 148 students in the fall of 2009 that took both the test and retest surveys. The comparison of the test-retest responses revealed symmetry where some responses were one or two higher or lower than the first, but for all types of questions there was an identical response spike with an overall shape of symmetry (Tai, 2010). Even though all responses were not the same, the identical response spike allows the researchers to have a high confidence in the reliability of the FICSMath survey (Tai, 2010).

The Study Population and Sample

There were 633 small, medium, and large two-year colleges, and 1,591 small, medium, and large four-year colleges and universities across the nation that were randomly chosen to participate in the FICSMath study. The researcher was part of a team

Figure 3.1. FICSMath Sample (FICSMath Advisory Board Meeting, June 14, 2010)
Legend: Red=2-year small schools, Blue=2-year medium schools, Purple=2-year large schools. Green=4-year small schools, Yellow=4-year medium schools, Orange=4-year large schools

of recruiters who began to contact mathematics departments early in the fall semester of 2009. Figure 3.1 shows the location of the colleges and universities from the population of schools across the nation that participated in the FICSMath study. Schools that did not offer mathematics, or mathematics departments that did not offer single variable calculus, were eliminated from the list. Mathematics departments that offered single variable calculus were asked for course numbers, which were used to classify the different type of calculus courses represented in the study, such as courses for engineering, mathematics, science or non-STEM majors. There were 485 two-year schools that were contacted by either email, voicemail, or by person-to-person phone contact, and 94 of these schools agreed to participate. Likewise, 625 four-year schools were contacted in the same manner and 89 agreed to participate in the study. The surveys were sent to mathematics departments that had indicated they would participate in the study in the fall of 2009. Of the 94 two-year colleges that agreed to participate 73 returned the surveys, and of the 89 four-year colleges and universities that agreed to participate 62 returned the surveys.

Table 3.1 below provides the corresponding colors with the name of the state in which the college or university is located, the number of calculus courses that are offered at that institution for the fall semester of 2009, and the average student grade. In the spring of 2010, a database of 10,492 survey responses with almost 500 variables was created. This represents 135 colleges and universities, 224 single variable calculus courses, and 336 calculus professors.

Table 3.1

Color Coded Sample of Colleges and Universities Correlated to Map in Figure 3.1

College or University*	State Where College or University Located	Number Calculus Courses	Number Students in Class	Percent of Sample	Average Grade	Standard Deviation
2-Year Small Colleges or Universities						
1	Colorado	1	11	0.10%	75.54	7.06
2	North Carolina	1	14	0.13%	72.73	22.68
3	Montana	1	24	0.23%	70.05	12.87
4	Georgia	1	15	0.14%	81.53	14.75
5	Minnesota	1	9	0.09%	87.83	7.07
6	Kansas	1	20	0.19%	84.50	8.58
7	Minnesota	1	9	0.09%	87.00	7.07
8	Pennsylvania	1	17	0.16%	88.03	9.96
9	Alabama	2	37	0.35%	83.44	12.88
Totals and Grand Mean 2-Year Small		**10**	**156**	**1.49%**	**81.18**	**11.44**
2-Year Medium Colleges or Universities						
10	New Jersey	3	75	0.71%	77.56	13.37
11	California	1	31	0.30%	72.73	22.68
12	California	1	11	0.10%	87.50	10.59
13	California	2	46	0.44%	80.45	12.05
14	California	2	47	0.45%	85.48	12.03
15	New York	1	8	0.08%	70.94	20.84

16	Wisconsin	1	16	0.15%	86.47	8.42
17	New Jersey	1	23	0.22%	77.89	15.54
18	Maryland	2	66	0.63%	85.22	8.75
19	Pennsylvania	2	32	0.30%	81.93	13.48
20	Iowa	1	15	0.14%	85.00	7.90
21	New York	3	64	0.61%	68.48	19.07
22	Minnesota	1	22	0.21%	83.88	9.21
23	Indiana	2	34	0.32%	84.55	13.07
24	Michigan	1	37	0.35%	86.23	12.13
25	Illinois	1	19	0.18%	80.69	18.94
26	California	1	12	0.11%	78.63	16.13
27	California	1	7	0.07%	81.43	7.80
28	Massachusetts	1	18	0.17%	83.36	12.97
29	New Jersey	1	144	1.37%	79.33	11.97
30	Minnesota	1	30	0.29%	79.33	11.97
31	Minnesota	1	134	1.28%	83.26	10.17
32	Texas	2	42	0.40%	78.96	14.42
33	Mississippi	1	38	0.36%	73.95	17.18
34	Texas	4	84	0.80%	81.88	12.41
35	Kentucky	1	10	0.10%	73.50	21.12
36	Missouri	1	14	0.13%	81.65	9.95
37	Arizona	3	63	0.60%	72.44	14.75
38	Massachusetts	1	23	0.22%	76.66	16.54
39	New Jersey	2	75	0.71%	83.76	13.62
40	Tennessee	1	3	0.03%	95.67	2.02
41	Minnesota	1	9	0.09%	83.56	12.15
42	Illinois	1	35	0.33%	85.13	10.68
43	New York	2	76	0.72%	74.17	20.35
44	New Mexico	1	19	0.18%	79.00	13.34
45	California	1	21	0.20%	72.62	22.44
46	Tennessee	1	21	0.20%	60.48	21.12
47	Washington	1	34	0.32%	76.05	21.32
48	Arizona	1	34	0.32%	79.18	12.01
Totals and Grand Mean: 2-Year Medium		**56**	**1492**	**14.22%**	**79.72**	**13.96**
2-Year Large Colleges or Universities						
49	California	1	21	0.20%	72.21	14.21
50	California	3	88	0.84%	80.44	15.43
51	Illinois	2	88	0.84%	75.3	16.2
52	Maryland	3	90	0.86%	77.06	15.55
53	Iowa	2	45	0.43%	75.45	17.77

54	Texas	1	68	0.65%	81.99	12.62
55	California	1	26	0.25%	76.55	15.16
56	Illinois	1	184	1.75%	75.49	14.94
57	Florida	1	24	0.23%	86.26	9.46
58	Kansas	3	124	1.18%	78.83	14.7
59	Texas	3	73	0.70%	80.81	16.94
60	Michigan	3	172	1.64%	81.71	12.46
61	New York	2	95	0.91%	77.77	16.58
62	Maryland	1	50	0.48%	85.63	11.1
63	Illinois	3	72	0.69%	80.09	12.58
64	California	1	28	0.27%	80.48	12.42
65	Florida	2	61	0.58%	75.13	15.52
66	California	1	32	0.30%	73.46	17.09
67	Arizona	1	59	0.56%	60.58	20.6
68	Utah	1	31	0.30%	75.59	22.11
69	Texas	1	21	0.20%	68.42	15.27
70	California	1	32	0.30%	79.54	11.02
71	Florida	1	132	1.26%	78.06	14.87
72	California	2	42	0.40%	78.89	11.17
73	New Jersey	3	131	1.25%	70.88	18.44
Totals and Grand Mean: 2-Year Large		**44**	**1789**	**17.05%**	**77.06**	**14.97**
4-Year Small Colleges or Universities						
74	Minnesota	1	24	0.23%	82.35	10.04
75	South Dakota	1	29	0.28%	82.97	11.2
76	Maine	1	19	0.18%	81.02	13.73
77	Indiana	6	242	2.31%	85.71	9.63
78	Maryland	1	33	0.31%	85.27	11.19
79	Arizona	1	23	0.22%	74.8	16.72
80	New York	1	22	0.21%	82.14	16.64
81	Texas	1	16	0.15%	76.44	17.36
82	South Dakota	1	10	0.10%	87.15	8.61
83	Pennsylvania	1	100	0.95%	86.91	11.39
84	Nebraska	1	43	0.41%	82	13.42
85	Texas	1	28	0.27%	87.29	8.57
86	New York	1	25	0.24%	86.94	6.46
87	Virginia	3	41	0.39%	76.61	13.5
88	Florida	1	43	0.41%	82.67	11.6
89	North Carolina	1	21	0.20%	83	12.09
90	New York	1	53	0.51%	85.11	9.21
91	Texas	3	55	0.52%	75.42	14.31

92	California	1	68	0.65%	78.13	19.16
93	Illinois	1	10	0.10%	89.5	7.07
94	Oklahoma	1	8	0.08%	77.71	20.11
95	Montana	1	35	0.33%	83.9	7.87
96	Tennessee	1	22	0.21%	84.86	10.94
Totals and Grand Mean: 4-Year Small		**32**	**970**	**9.25%**	**82.52**	**12.21**
4-Year Medium Colleges or Universities						
97	Ohio	1	42	0.40%	84.69	8.64
98	California	1	9	0.09%	73.33	20.1
99	Illinois	2	88	0.84%	77.81	11.53
100	Idaho	3	104	0.99%	80.09	16.45
101	Virginia	2	252	2.40%	82.37	12.51
102	Pennsylvania	1	45	0.43%	78.26	15.39
103	Tennessee	4	178	1.70%	85.33	11.98
104	Florida	2	89	0.85%	81.31	11.96
105	New York	1	65	0.62%	82.92	9.62
106	Louisiana	2	152	1.45%	79.17	11.9
107	Minnesota	1	25	0.24%	81.88	12.83
108	North Dakota	1	255	2.43%	77.45	15.34
109	Oklahoma	1	37	0.35%	80.51	13.18
110	Pennsylvania	1	38	0.36%	75.88	10.69
111	Virginia	2	88	0.84%	79.58	14.05
112	Connecticut	3	49	0.47%	73.54	14.93
113	Massachusetts	1	18	0.17%	82.86	10.78
114	New York	3	198	1.89%	76.81	14.59
115	New York	1	12	0.11%	85.32	7.99
116	Texas	1	43	0.41%	80.98	15.32
117	New Jersey	1	100	0.95%	84.94	9.17
118	Michigan	1	38	0.36%	85.39	11.05
119	Massachusetts	1	49	0.47%	76.72	15.36
120	Minnesota	1	320	3.05%	75.22	18.63
121	Alabama	1	36	0.34%	75.22	18.63
122	North Dakota	2	113	1.08%	75.09	15.21
123	Colorado	3	65	0.62%	76.64	19.58
124	South Carolina	2	60	0.57%	77.67	13.26
Totals and Grand Mean: 4-Year Medium		**46**	**2568**	**24.48%**	**79.54**	**13.60**
4-Year Large Colleges or Universities						
125	Alabama	7	465	4.43%	83.33	12.35
126	Indiana	2	65	0.62%	75.05	17.13

127	Iowa	12	1162	11.08%	80.67	13.56
128	Oklahoma	1	209	1.99%	78.87	14.47
129	Oregon	2	83	0.79%	82.43	11.13
130	Minnesota	1	111	1.06%	66.94	19.59
131	Texas	1	176	1.68%	73.07	16.53
132	Texas	2	292	2.78%	77.34	16.38
133	Kentucky	5	373	3.56%	82.53	11.27
134	North Carolina	2	312	2.97%	78.63	13.38
135	South Carolina	1	269	2.56%	78.56	15.55
Totals or Total Average: 4-Year Large		**36**	**3517**	**33.52%**	**77.95**	**14.67**
Totals and Grand Mean: All Participating Colleges or Universities		**224**	**10492**	**100%**	**79.66**	**13.48**

* Names of Colleges and Universities not provided

Analysis

The unequal status-concurrent mixed-method design was used to analyze the data. A phenomenography was used to answer the research question, “What categories from the phenomenography of college mathematics professors and high school mathematics teachers align with the 4C/ID model?” A phenomenography seeks to represent the relationship between some phenomena that is experienced by different groups (Johnson & Onwuegbuzie, 2004). The phenomenon of preparing students for college calculus experienced by the secondary mathematics teachers is different from the phenomenon of teaching single variable college calculus. The phenomenographic method can help identify the commonalties and variances experienced at both levels.

Mapping the questions from the FICSMath survey to the components of the 4C/ID model, and multivariate linear regression was used to answer the research question, “How well do the instructional components in the 4C/ID model represent pedagogies that are predictive of pre-calculus and calculus students performance in single variable college calculus?" Data from students who had either pre-calculus or calculus (non-AP, APAB, or APBC) in the twelfth grade was used to build the model. The 4C/ID instructional model was designed to enhance transfer of learning of complex tasks. Therefore the sample was comprised of students who moved directly from their last high school mathematics class to the single variable college calculus course where the FICSMath survey was taken. Controls were used in an effort to assure the significant variables were from secondary pedagogical practices and not other variables such as gender and ability. Stepwise regression was used to find the significant variables for each component of the 4C/ID model beginning with the supportive information component. The model was built in the following order: controls and foundation knowledge component, support component, procedure component, learning task component, and finally the part-task component. Each variable was entered into the model progressively as the model was built in order to assure that variables in the model remained significant.

Results from the phenomenography and multiple regression were triangulated to answer the research question, "How can the 4C/ID model be modified to better reflect pedagogies that best prepare pre-calculus and calculus students for single variable college calculus?" Triangulation of data was used to discover similarities, paradoxes, and contradictions (Johnson & Onwuegbuzie, 2004). More focus was placed on the quantitative results than qualitative results during triangulation.

CHAPTER 4

QUALITATIVE RESULTS

A PHENOMENOGRAPHY OF MATHEMATICS PROFESSORS' AND MATHEMATICS TEACHERS' PERSPECTIVES OF SECONDARY PREPARATION FOR COLLEGE CALCULUS

The data used for this phenomenography was drawn from the Factors Influencing College Success in Mathematics (FICSMath) project, which focused on finding evidence for effective strategies that prepare students for college calculus success. FICSMath is a large-scale study conducted within the Science Education Department at the Harvard-Smithsonian Center for Astrophysics, which surveyed a nationally representative sample of college students who were enrolled in single variable college calculus courses in the fall semester of 2009. The survey included questions on students' demographics, academics, interest in mathematics, and high school mathematics experiences. The development of the FICSMath survey was led by several components in order to establish content validity. One particularly informative source was open-ended responses gathered from mathematics professors and secondary mathematics teachers across the nation, via an online survey. The survey gathered open response data concerning what professors believe teachers should be doing, and what teachers state they are doing, to optimally prepare students for single variable college calculus. These responses have been analyzed using a phenomenographic approach and the results are presented in this chapter.

The mathematics professors responded to the question, “What can high school teachers do to prepare students for success in college calculus courses?” The mathematics teachers responded to the question, “What do you do, as a high school mathematics teacher, that you think prepares students for college calculus success?” Success is considered to be that which prepares students to move through the first college calculus course, or single variable college calculus, and be prepared for the subsequent multi-variable calculus course required in a STEM major. The respondents were asked to describe up to three interventions they considered to be beneficial to students' preparation for single variable college calculus. Responses were collected from 185 mathematics professors (62 percent male) and 84 mathematics teachers (52 percent male) responded to the survey. Because more professors than teachers responded to the survey, the numbers of statements in all categories were converted to percent of statements from the professors and teachers groups, which allowed for equitable comparison of statements across groups. Relative to teaching experience, 30 percent of professors had less than 10 years experience teaching single variable college calculus while 67 percent had 10 or more years experience, with three percent not responding; 91percent of the teachers had 10 or more years experience teaching secondary mathematics.

Phenomenographic Approach

The objective of phenomenographic research is to capture the variability, both the similarities and differences, in the ways a phenomenon is perceived by different groups and expressed in qualitative data (Dahlin, 1999). The phenomenographic research method assumes the different categories that emerge from coding describe ways of experiencing a phenomenon, and the categories are logically related to one another (Akerlind, 2005). High school mathematics teachers and college mathematics professors are two different groups with distinct beliefs based on their education and teaching experiences. Secondary mathematics teachers form beliefs about student readiness for college calculus, and professors' perceptions about student preparation are developed when these same students are in their college calculus courses. Consequently, professors and teachers have similar yet different perspectives concerning student preparation for college calculus. Thus a phenomenography is an appropriate method for seeking to understand the ways professors and teachers experience, conceptualize, and understand the aspects of preparing students for success in college calculus.

Data Analysis

In phenomenographical analysis, the meaning of each statement is considered holistically within groups, not as individual responses. Statements within groups are organized together to form categories, and the categories create an outcome space (Dahlin, 1999; Akerlind, 2005). An outcome space, as described by Marton (1994) is the "ordered and related set of categories of description" of the concept being studied. Open coding was used to find categories of description, meaning all responses were considered to be important and were carefully read. For each group, responses were organized into categories with similar statements, which captured holistic beliefs for the professors and teachers. Figure 4.1 displays the outcome space of this research created from the comparison of categories across groups. Figure 4.1 reveals the percentage of statements in each category and indicates how strongly the professors and teachers believe the phenomenon addressed some aspect of college calculus success. However, it should be noted that the percentage of statements does not reveal the commonalities or disparities between their beliefs, but only reveals the percent of statements made by each group. Figure 4.1 show that professors had significantly more statements addressing algebra and pre-calculus content than teachers. Likewise, teachers made significantly more statements concerning classroom environment and real world problems than

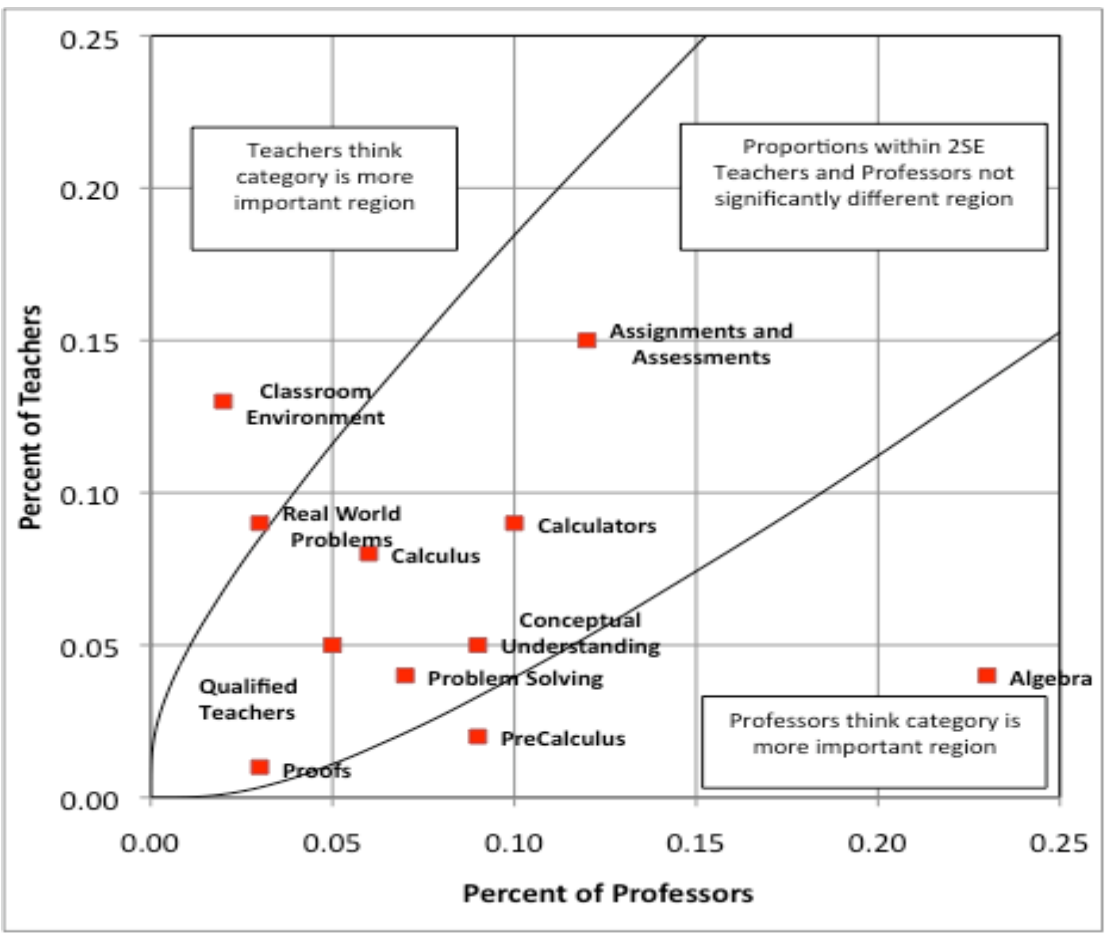

Figure 4.1. Categories of Beliefs Compared Across Professors and Teachers

professors. This aligns with Akerlind (2005), who stated that categories in a phenomenography create an outcome space that provides a way of looking at a collective experience holistically from more than one perspective. Typically, mathematics professors tend to focus on content while secondary teachers focus on pedagogical interventions in an attempt to make mathematics understandable to students who may or may not be college bound.

The categories in the center region of Figure 4.1 do not have a significant difference in the percent of statements between groups, but there is a significant difference between the percent occurrence of the statements and zero, therefore these categories will be discussed. There were seven categories with very few statements from professors and teachers, causing the percentage of statements to not be significantly different from zero. The categories of geometry, vocabulary, additional support for learning, student motivation, memorization, multiple representations, and textbooks are not included in the comparative analysis or the discussion. In order to compare the categories presented in Figure 4.1 across groups, the statements within the categories were clustered together into similar topics, and descriptions were provided of the comparative qualities for the categories. This step was critical because it provided a way to discuss the categories using the voice of the respondents.

Validity and Reliability

The validity of phenomenographic research is based on three factors (Dahlin, 1999). The first factor is the logic of the system of categories that emerged from the analysis; the categories must be logically separate and exclusive. The second factor is the correspondence between results and what is known from previous studies of mathematics professors and secondary mathematics teachers. The last factor is the plausibility that the categories represent actual or possible human experiences.

The reliability in the coding was established through inter-rater reliability, which is the extent that two raters agreed on the coding of statements that created the categories. The first rater created precise definitions of the categories (available in the appendix) and the second rater used these definitions in order to code 10 random responses from each group. Each response had multiple components so the total number of statements coded was 55. A contingency table was created from responses and Cohen's Kappa was computed. The measure of agreement between the two raters was calculated as 0.74, which is considered good agreement between the raters (Landis & Koch, 1977).

Phenomenography Categories and the 4C/ID Components

Research Question 1 states, "What categories from the phenomenography of college mathematics professors and high school mathematics teachers align with the 4C/ID model?" To answer Research Question 1 the categories from Figure 4.1 were related to the components of the 4C/ID model that were discussed in detail in Chapter 2. The categories from Figure 4.1 were placed on the appropriate component as seen in Figure 4.2.

The Support Component was summarized in Chapter 2 as being a classroom environment where the focus of mathematics instruction is conceptual understanding, mathematical reasoning, problem solving, and cognitive assessment. Because learning

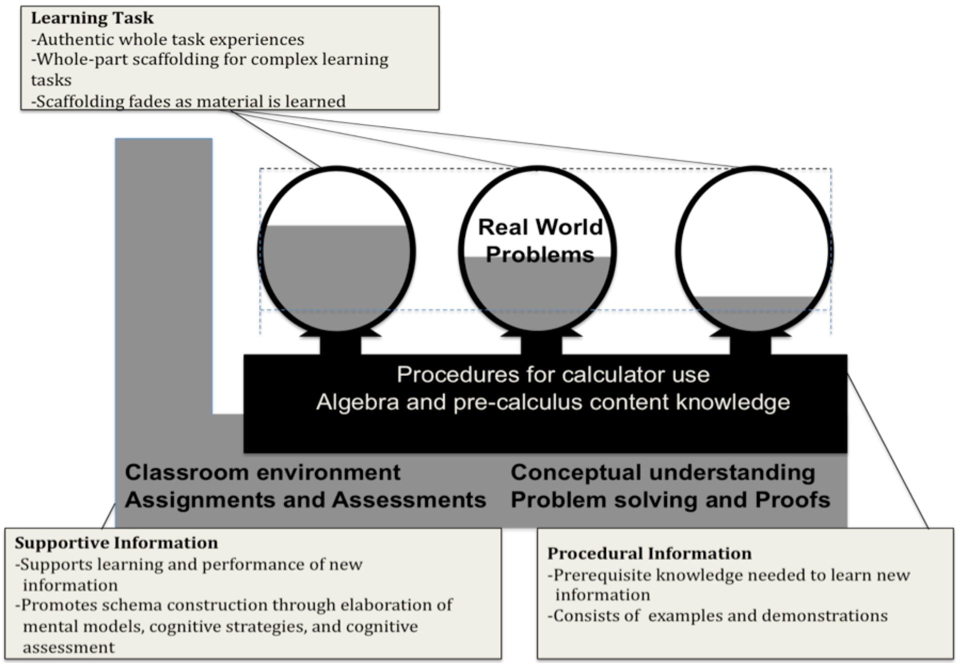

Figure 4.2. The 4C/ID Model With Phenomenography Categories Placed Into The Corresponding Components

mathematics is complex, and the 4C/ID model is being connected specifically to the learning of mathematics, the support component does not only exist when real-life authentic learning tasks are integrated into the curriculum. Instead the support component is considered as a supportive environment in which meaningful mathematics is learned. The model places the procedures and learning task component on the support component because, based on the model, without the support component learning for transfer cannot occur. Van Merriënboer (2002) stated that the support component promotes schema construction through elaboration by helping students establish non-arbitrary relationships in complex learning tasks. The NCTM (2010) stated that reasoning in mathematics is often understood to encompass formal proofs that are logically deduced from assumptions and definitions, thus proofs are one way that teachers may present mathematical content, which requires students to use reasoning for schema construction and to establish non-arbitrary mathematical relationships. When students work toward understanding complex mathematical ideas during problem solving they must analyze a problem, understand the concepts, implement a strategy, and reflect on a solution. The solution reveals how well the concepts were understood and cognitive assessment provides opportunities for learners to reflect on the quality of the problem solving process (van Merriënboer et al., 2003). Van Merriënboer et al., (2002) stated that students use heuristics unique to interacting elements in the problem solving process, and high element interactivity hinders the process of understanding concepts. For mathematical understanding to transfer from one topic to another, and to subsequent mathematics or otherwise courses, concepts must be stored in long term memory and accessed during recall to help process more new mathematical material. A supportive classroom environment literally supports conceptual understanding and mathematical reasoning for transfer of learning. Students must be able to make sense of mathematical ideas presented formally by understanding a situation, context, or concept by connecting it to existing knowledge (NCTM, 2010).

The procedural component was placed in the 4C/ID model because it promotes schema automation by embedding new information in situation specific rules that connect particular conditions to particular actions, and this process is called “proceduralization” (van Merriënboer et al., 2008, P. 11). The cognitive load definition of proceduralization for the use of calculators or for recalling previously learned algebra or pre-calculus content is not as appropriate as Star's definition. Star (2000) stated that procedures are

> Planning knowledge -- knowledge of such things as the order of steps, the goals and subgoals of steps, the environment or type of situation in which the procedure is used, constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge which are inherent in the environment or situation. This knowledge is abstract (and deep), but not necessarily conceptual (Star, 2000, p. 85).

What is most important to consider, relative to the 4C/ID model and the learning of mathematics, is that procedures do not stand alone. Procedures must be supported by conceptual understanding and reasoning within the supportive information component (see Figure 4.2).

The learning task component in the 4C/ID model represents learning tasks that are preferably authentic, real life problems that are presented using the whole part scaffolding method (van Merriënboer et al., 2002; van Merriënboer et al., 2003; van Merriënboer et al., 2006). Van Merriënboer and colleagues (2003) describe the 4C/ID model as connecting the world of work and education through the use of authentic real-life problems. Cognitive load theorists describe a complex, authentic learning task as real life problems that have "many different solutions, are ecologically valid, cannot be mastered in a single session, and pose a very high load on the learners cognitive system" (van Merriënboer et. al, 2006, p. 343). There are no categories from Figure 4.1 that connect to the part task component of the 4C/ID model, therefore this component is not addressed.

Results of Phenomenography

The discussion of categories reveals the commonalities and differences between professors' and teachers' beliefs. The broad categories from Figure 4.1 with the largest variability are presented with graphs that compare the percent of professors' and teachers' beliefs. The graphs also have a line of equal emphasis that indicates where there is agreement, if any, and allows for comparison of the differences of beliefs for each group. Any emphases within the quotes are from the respondents.

The frequency with which professors and teachers advocated specific aspects of students' algebra background (e.g. symbolic manipulation, focus on functions) is summarized in Figure 4.3. Although both groups had concerns about students' algebra knowledge, professors mentioned the importance of focusing on algebra significantly more than teachers. Both groups shared concerns that "students should begin algebraic reasoning and symbolic notation earlier" and agreed that it is important to focus on functions. One professor stated:

> Students need an operational understanding of functions by focusing on proper function notation, function concepts, function composition, and function families while emphasizing basic algebra rules so students know how to rearrange formulas, factor complex statements, and solve equations.

A few professors de-emphasized the importance of a high school calculus background. One stated, “A low level understanding of calculus benefits students much less than a high level of algebraic understanding when they get to a college course.” The professors

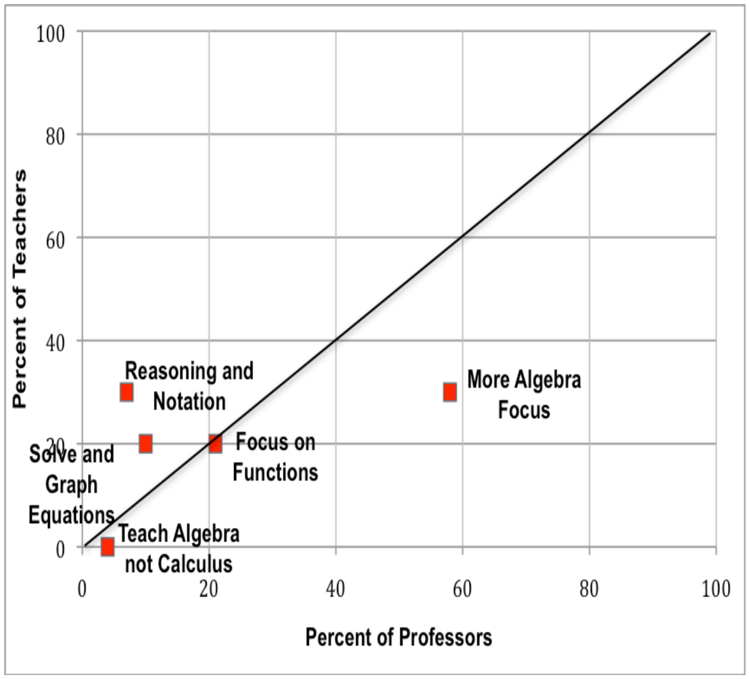

Figure 4.3. Percent of Teachers’ versus Percent of Professors’ Beliefs for Algebra Category

had concerns that “students with the weakest algebra skills are the students to most likely drop out of the calculus sequence” and were convinced that “they [professors] could teach the upper level content if teachers would just teach the basics of algebra.” Teachers also stressed that the “mastery of algebra is essential because the concepts of calculus are relatively easy but they produce very difficult algebra problems.” However, teachers stressed the importance of solving and graphing equations. One advocated focusing on “all types of functions from a graphical point of view with emphasis on their general behavior and, as much as possible, without the use of a graphing calculator.” Several teachers encouraged students to develop algebra skills. For example, one stated, they "encourage students to develop rigor and to be proud when they solve a long difficult algebra problem."

Professors and teachers shared concerns that students have weak algebra II and geometry background knowledge needed to learn pre-calculus content (see Figure 4.4). One professor stated, "the students have learned to solve pre-calculus problems by rote."

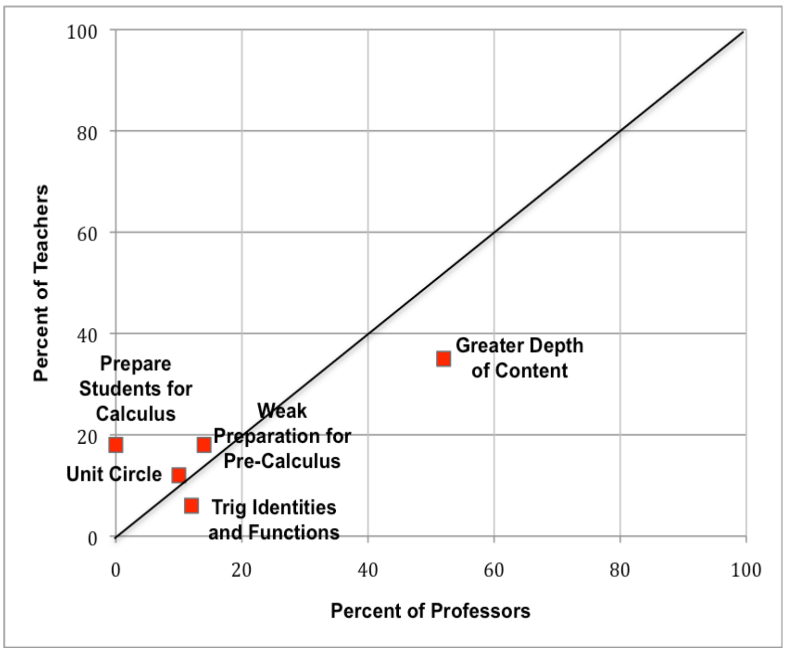

Figure 4.4. Percent of Teachers' versus Percent of Professors' Beliefs for Pre-Calculus Category

There was agreement that there should be more focus on the unit circle, trig functions and the trig identities. The following statement by one professor typified professors' beliefs, "teachers should provide a greater depth in their coverage of trigonometry, dig deeper." In support, one teacher stated, "I choose to teach a few topics well, rather than trying to skim over a broad range of topics." Often the same teachers that taught calculus also taught pre-calculus and they stated that their "choices of what to teach in pre-calculus are influenced by their experiences as a calculus teacher," which did not align with beliefs of the professors.

Professors and teachers agreed that a classroom environment should: (1) be a positive atmosphere where students can "learn mathematics without being intimidated;" (2) be supportive of "students working together so they can discuss their problem solving strategies;" (3) build the confidence level of students (see Figure 4.5).

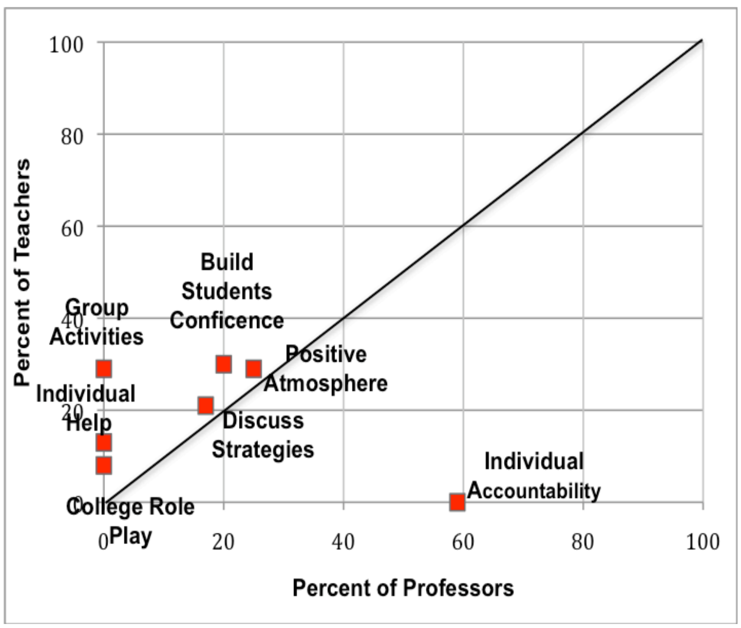

Figure 4.5. Percent of Teachers' versus Percent of Professors' Beliefs for Classroom Environment Category

More than 50 percent of the statements from professors in this category stated that teachers should hold students individually responsible for learning. One professor stated, "Teachers need to help students realize that learning mathematics requires work—it isn't enough to understand when the teacher explains a solution. They must work thorough it themselves." Teachers use different types of group work activities, for example, one teacher stated, “I use guided practice that bleeds into independent practice and incorporate a lot of group-oriented activities using homogeneous grouping.” Several teachers thought it was important to prepare students for the quick pace that material will be covered in college. One teacher stated, “Sometimes I do a unit called College Role Play where the teacher role plays a college professor, moving through material much faster than normal. We cover an entire chapter in 3 lecture days.”

Both professors and teachers believed that other courses such as physics and chemistry connect mathematics to real life problems; however, about 30 percent more of professors than teachers believed that mathematics should be connected to real life (See Figure 4.6). Professors conveyed, “One reason real world problems are beneficial is that they place mathematics in context and allow out of the box problems.” Professors also believed that real world problems provide reasons for students to check their work. One

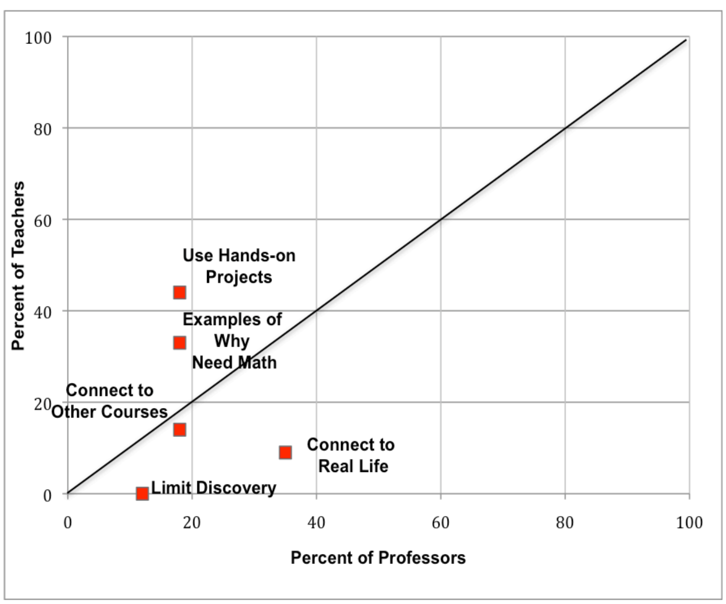

Figure 4.6. Percent of Teachers’ versus Percent of Professors’ Beliefs for Real World Problems Category

stated, “Teachers should ask their students if it would make sense for a satellite to orbit the earth 10 feet above the surface.” A few professors stressed concern that real life problems may distract from learning mathematics because, “teachers sacrifice instruction for student discovery of real world problems when time would be better spent with teacher directed development of concepts.” Teachers provide examples of why students need to learn mathematics and assign hands on projects that require students to think problems through.

There is more disagreement than agreement between professors and teachers in the category of assignments and assessments (See Figure 4.7). Professors and teachers agreed that students should justify their solutions, but a greater percentage of professors stated that teachers should require homework and assess students’ performance in some

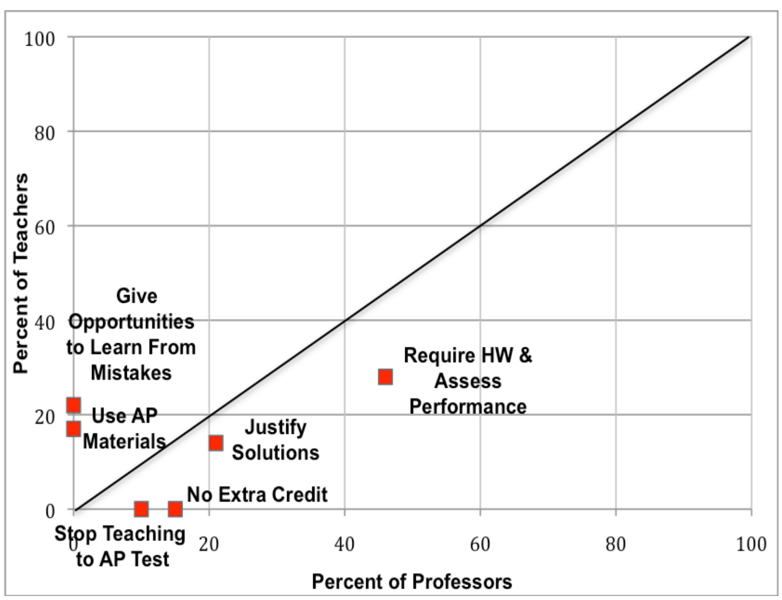

Figure 4.7. Percent of Teachers’ versus Percent of Professors’ Beliefs for Assignments and Assessments Category

meaningful way. Professors believed teachers should "stop teaching to the AP Test and base their assignments more on important foundations." Professors also believed that teachers should "stop giving extra credit in lieu of learning material and help students take responsibility for their own learning." One professor stated:

> High school teachers need to ask more of their students-- no more extra credit, re-taking exams, or open book exams because students come out of high school thinking that seat time should equal passing.

On the other hand, teachers seek to align assessments with AP Calculus content and with the types of questions they will see on college exams. One teacher stated, "I use AP test items as test and quiz questions" and another stated they create tests that have, "conventional questions similar to what a college professor might give and an AP part consisting of multiple choice and free response questions similar to questions on past AP exams." Teachers "do not want students to become discouraged from taking hard classes in high school," which can lower their GPA, therefore they give students "opportunities to learn from their mistakes by correcting errors, which helps them understand better what they need to know."

Professors and teachers shared concerns that students are too dependent on calculators. There is no figure for this category since there was less variance in the responses between groups. The groups agreed that: (1) calculators can enhance the understanding of concepts; (2) calculators should be used for complicated computations; and (3) students should not be allowed to use graphing calculators on all sections of formal assessments. Professors believe teachers should help students learn that graphing calculators provide “approximation techniques” which should not be used as “replacement for mathematical thought.” Many professors believed teachers should not allow students to use calculators in mathematics class. One stated, “teachers should have higher expectations of their students than to allow them to use calculators” while several professors specifically stated that “teachers should make students throw away the calculators” because students think, “if I don't know how to use my calculator to solve this problem, then I don't know how to solve this problem.” Aligning with the idea of limiting calculator use, teachers stated they: (1) “wean students away from calculators while working on logs, rational expressions, and radicals;” (2) “incorporate meaningful graphing calculator activities, using the calculator as a tool to bring a concrete picture to the mathematics;” (3) discourage students from using a calculator just as a “number cruncher” and instead encouraged the use of calculators to “connect mathematical concepts.”

There is agreement between professors and teachers that the focus of instruction should be conceptual understanding (see Figure 4.8). Both groups agreed that the focus of instruction should be on understanding concepts rather than test preparation. One teacher stated they encouraged their students to get past “the academic bulimia of learning material just to regurgitate it on an examination.” More teachers than professors stated that there should be a balance between concepts and procedures during instruction; however, many professors stated that they believe that teachers tended to focus on procedures much more than on concepts. For example, one professor stated:

If teachers teach calculus, the focus should be conceptual understanding, not a 'plug-and-chug' approach that the students will have to (resentfully) abandon in favor of a deeper conceptual understanding when they arrive in college calculus.

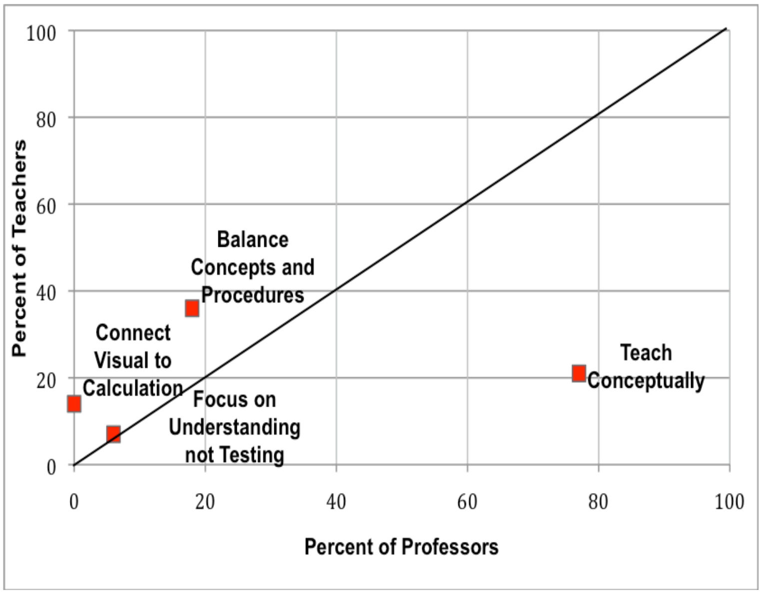

Figure 4.8. Percent of Teachers' versus Percent of Professors' Beliefs for Conceptual Understanding Category

Teachers help students make connections and focus on concepts by, "making mathematics visual at all levels, and by connecting the calculation part to the picture." One teacher stated they "work very hard to help students understand the *why* of mathematics and not just the *how*."

Professors and teachers agreed that it is important to provide students with problem solving opportunities. There is no figure for this category due to limited variability between professors and teachers beliefs. They agreed that: (1) students should be given challenging problems that take more than a few minutes to solve; (2) students should be required to think instead of just manipulate an algorithm; and (3) students should solve problems that have “various representations and multiple problem solving paths.” Professors stated they wanted teachers to “help students realize that simple calculation problems are not reflective of the problems they will encounter in college courses.” Professors believe teachers should allow the students to struggle with problems before providing guidance. One professor stated teachers should require students to:

> Identify what is given, identify the theoretical basis of the problem, draw and label a picture of the problem, identify the variables, determine what is requested as an answer, apply the rules of the theory, and isolate the solution.

Teachers stated they provided lots of problem solving opportunities and provided “problems that required students to critically think and work with paper and pencil.”

Professors and teachers both believe that secondary calculus should be a rigorous course (see Figure 4.9), but professors also believe teachers should focus more on

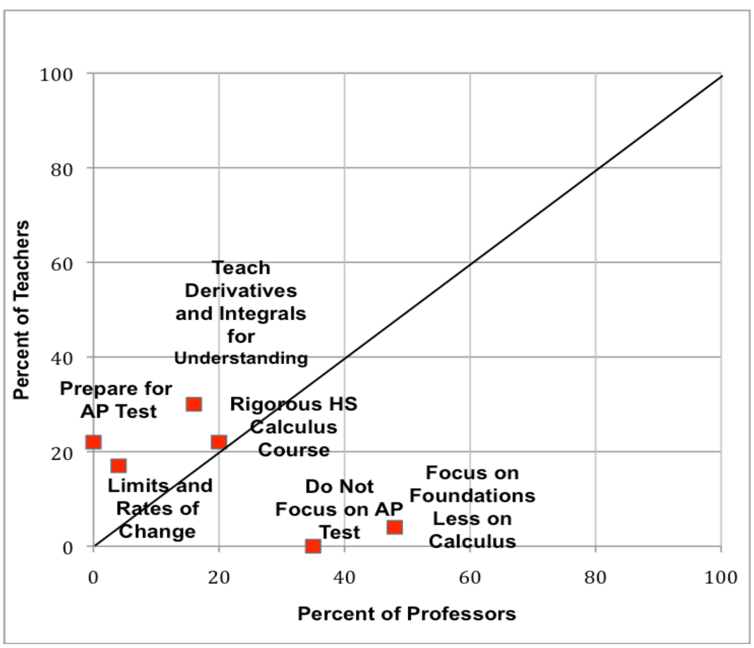

Figure 4.9. Percent of Teachers’ versus Percent of Professors’ Beliefs for Calculus Category

foundations and less on teaching calculus. One professor stated:

> Teachers should stop teaching bad calculus courses because knowing algebra would be better for students than having the misunderstanding that the only thing in calculus that matters is knowing how to take a derivative.

Professors believe there is too much focus on the AP Calculus curriculum and not enough on concepts in secondary calculus. One professor stated:

> AP calculus is beneficial for some students but not most because MOST high school students planning on going to college would be better served by focusing more on the PRE-Calculus mathematics and getting more repetition, linkage and depth in their study of mathematics, rather than our current method of making the high school mathematics curriculum a mile wide, and a *half*-inch deep - if you know what I mean....It's NOT a race to see who can get there FIRST.

Teachers stated they focused on students understanding the concepts of derivatives and integrals. For example, one teacher stated, "I stress to the students THE DERIVATIVE GIVES THE SLOPE OF A CURVE AT EVERY POINT(!), using no rules (shortcuts) for derivatives, just concepts." Many teachers addressed AP Calculus exam preparation and believe the AP curriculum is rigorous and prepares students for success in college calculus. One teacher stated:

> I begin a formal review for the AP test in late February. Each student is given a notebook with every free response question since 1990, and every released

> multiple-choice test since 1985. They are assigned 3 free response and 10 multiples choice every day and are all scored based on the standards.

Even though teachers seek to prepare students to pass the AP Calculus exam, they also stressed that "students should also take single variable calculus in college for a multitude of reasons."

There is no agreement between groups in the category of proofs. While there is a figure for this category (see Figure 4.10) it should be noted by the reader that only a small percent of professors and teachers mentioned this category (see Figure 4.1). One professor stated that more teachers should emphasize proofs, which could "help students avoid proof by erasure." Professors also believe teachers need to do a better job of explaining theories to students. One professor stated:

> I'm not sure how much formal reasoning is done in HS anymore, but it seems that students are less familiar with theory and are more schooled in doing computational problems, practicing problem solving methods, and using their calculators. As a result they have a very difficult time understanding theories and justifying their approach.

Another professor stated teachers needed to help students to reason mathematically by:

> Teaching mathematics the old-fashioned way, with proofs of fundamental statements so students get a feeling that mathematics is powerful, universal, logically united and politically independent.

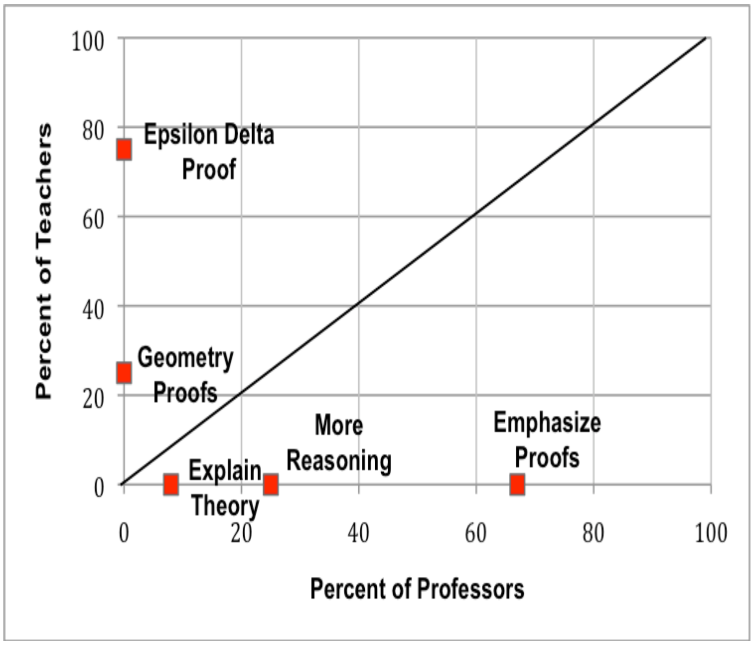

Figure 4.10. Percent of Teachers' versus Percent of Professors' Beliefs for Proof Category

Teachers, on the other hand only referred to formal proofs in terms of geometry and the epsilon-delta definition of limits. One teacher stated they, "help students think and reason through elementary proofs in geometry" while another stated they, "Teach the epsilon-delta proofs the first couple of weeks of school to prepare students for college--not AP since these proofs aren't required for AP."

The category "Qualified Teachers" did not specifically the answer the questions, "What can high school teachers do to prepare students for success in college calculus courses?" or "What do you do, as a high school mathematics teacher, that you think prepares students for college calculus success?" However, the Qualified Teachers category addresses these questions from the perspective of curriculum policies, teacher preparation, or professional development. Because there was a significant difference between the percent of statements and zero, the graph showing the similarities and disparities of beliefs are presented in Figure 4.11 with a discussion following. Both groups believe that mathematics teachers needed stronger mathematics preparation, but more professors made statements concerning this than teachers. One professor stated:

> Too many high school teachers lack the academic background to teach mathematics and high schools should only employ math teachers with a BS degree in mathematics or a closely related discipline (i.e., physics or engineering). A *math ed* degree should not be recognized as an equivalent or substitute.

Relative to this, one teacher stated, "What we need are more teachers with strong mathematics backgrounds (maybe more pay for math and science teachers?)" Professors believe the certification process of teachers should be re-examined. One professor stated, "certifying teachers based on the number of math credits without mandating specific courses creates a loophole in the certification system." Another professor stated "a

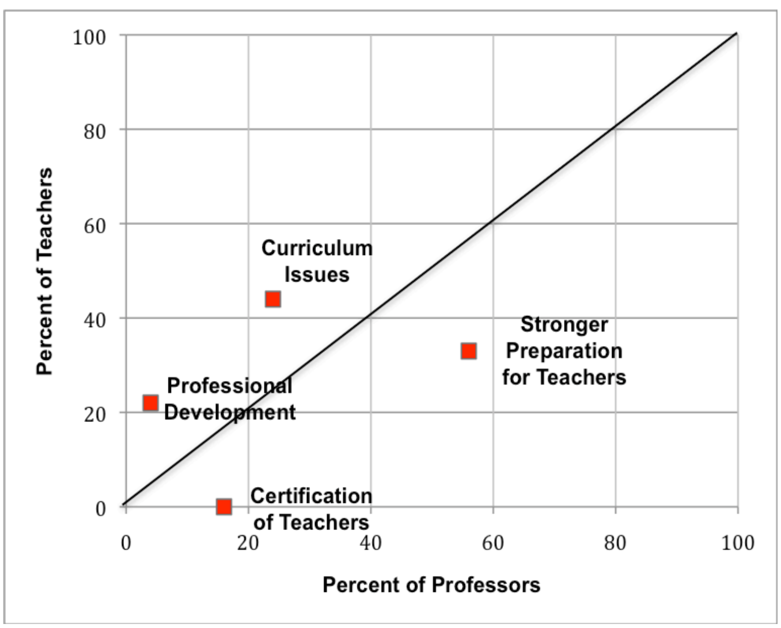

Figure 4.11. Percent of Teachers' versus Percent of Professors' Beliefs for Qualified_Teachers Category

bachelor degree to teach secondary mathematics is not enough, one should have at least a Masters Degree to be certified to teach." More teachers than professors mentioned curriculum issues. One professor stated schools should "mandate ALL high school seniors to take a math course in their last year, preferably Pre-Calculus or higher." Teachers relayed their frustration with policies that dictated curriculum that inhibits instruction of rigorous content. One teacher stated:

> Our district adopted Contemporary Math in Context, therefore we now teach math appreciation and NONE of our students will ever be adequately prepared for anything let alone Calculus. Teachers need to be able to teach what is needed for college preparation- and- "No child left behind = No child gets ahead."

More teachers than professors mentioned professional development. Professors stated that teachers should use professional development to "brush up on their own skills and understanding of not only the material they are teaching but also the subsequent material so teachers can show students what is to come." One teacher stated, "The most effective reform in public education today is high quality professional development."

Discussion

Figure 4.1 identified categories that had a significant difference between the percent of statements and zero on a 95 percent confidence interval. The categories outside of the lower and upper bounds had a significant difference of statements between the professors and teachers. The professors made significantly more statements concerning algebra and pre-calculus content than teachers, and teachers made significantly more statements addressing classroom environment and real world problems than professors. All of the categories in Figure 4.1 represent unique findings and plausible phenomena within the context of secondary preparation for college calculus success. This aligns with two of the factors that establish content validity for a phenomenography. Another factor that establishes content validity is the connection of the results to previous studies. The 1997 Presidents Task Force Report (PTFR) and the MAA outlined concerns from mathematics professors relative to the 1989 NCTM Standards, which provided guidelines for secondary mathematics instruction. Comparing the results of this phenomenography to the PTFR allows for a comparison between what mathematics professors considered important a little more than a decade ago and what they consider important now.

There were originally nine concerns presented in the PTFR to the NCTM concerning 1989 *Standards* for School Mathematics. The concerns from the PTFR that relate to this phenomenography are: (1) conceptual understanding; (2) proofs; (3) too many standards to teach; and (4) technology. First the concerns from the PTFR with comments from the MAA will be presented, and then findings from the phenomenography will be discussed, which will allow a comparison to determine if and how the concerns have changed.

CONCERN 2 presented in the PTFR stated that the mastery of basic skills was not sufficiently addressed in the 1989 NCTM Standards. The authors agreed that drills of important algorithms enabled students to master topics and learn mathematical reasoning. The MAA's comment was that teachers must maintain a balance between helping

students develop conceptual understanding and procedural facility (Ross, 2000). This phenomenography revealed that more than 75 percent of the descriptors from professors in the conceptual understanding category addressed the need for teachers to teach conceptually (see Figure 4.8). Professors' concerns about procedural instruction were often connected to teachers preparing students for the AP Calculus exam. Bressoud, the 2009-2010 President of MAA, stated that many colleges now teach a more theoretical differential calculus and postpone integration techniques and applications until the second semester of calculus (Bressoud, 2010). This is different from the AP Calculus curriculum, which covers application material from both derivatives and integrals. The AP Calculus curriculum is considered by many mathematics professors as a "breadth of material to be mastered" and professors believe students often earn a satisfactory grade by focusing on algorithms and procedures instead of understanding (Bressoud, 2010, p. 3 of 4). The statement from the PTFR that "drills of important algorithms enabled students to master topics and learn mathematical reasoning" is not reflective of the professors concerns revealed in this phenomenography. For example, professors stated:
(1) "Teachers should provide more exploration of concepts rather than rote algorithms" and (2) "Many students believe that math is a bunch of algorithms to be memorized, rather than a cohesive system of thought." These statements indicate that mathematics professors' do not currently believe that teachers should stress the use of algorithms.

CONCERN 3 presented in the PTFR stated, "the [1989] *Standards* did not sufficiently address issues of mathematical reasoning, the need for precision in mathematical discourse, and the role of proof in the curriculum" (Ross, 2000, p. 3).

Figure 4.1 reveals less than five percent of both groups addressed the category of proofs, yet the statements from professors indicate strong beliefs about the importance of proofs. One professor's statement typified beliefs that teachers should, "Include ideas crucial for proof, such as making logical, sequential mathematical arguments that rely on these skills." The findings in this phenomenography align to some extent with the previous concern that teachers should address reasoning and formal proofs.

CONCERN 6 of the PTFR stated "the 1989 Standards recommended the inclusion of more topics," which led the mathematicians in the MAA to have concerns that the secondary curriculum would become "superficial" and be a "mile wide and an inch deep" (Ross, 2000). Over 50 percent of professors stated there should be greater depth of content in pre-calculus (Figure 4.4), and almost 50 percent of statements within the category of calculus (Figure 4.9) described that professors want teachers to provide a greater focus on foundations and less on calculus. One professor stated teachers covered so much material that mathematics instruction is "a mile wide and a *half*-inch deep" (see calculus discussion).

CONCERN 9 in the PTFR was that "technology should not be used as a replacement for basic understanding and intuitions; rather it can and should be used to foster those understandings and intuitions" (Ross, 2000). This phenomenography revealed that both groups shared the view that calculators should be used appropriately. One professor expressed concerns about calculator usage with the same tone as the PTFR by stating, "teachers should not allow students to use calculators to replace mathematical thought." There was limited variability in the comparison of beliefs across groups for this category.

The discussion of the remaining categories is not connected to the PTFR since the findings were not listed as concerns that the MAA had relative to the 1989 NCTM Standards. Teachers made more statements addressing assignments and assessments than any other category, and the percent of statements from professors was second only to algebra (see Figure 4.1). Professors believe that high school grades should be valid and representative of students' mathematical knowledge and ability. One professor stated:

> Give realistic grades!!!!! If students don't know what they are doing then they shouldn't get "A"s (or even "B"s or "C"s). The students are too used to being coddled. It's a real shock to these students when they get to college and fail the courses when they don't know how to do the problems.

While this is a valid concern, the teachers stated that such grade adjustments "allowed more students to take the class [calculus] that otherwise would never have taken a senior course because of the *I don't want to lower my GPA* argument." Therefore teachers considered formal assessment as an opportunity for students to learn from their mistakes. Teachers may also be more aware of the motivational aspects of assessment than professors. Poor performance on assessments may depress student's motivation toward learning mathematics.

The "Real World Problems" category is on the outside border of the 95 percent confidence interval (shown in Figure 4.1), which indicates there was more than two standard errors difference in the percent of statements from teachers and professors.

Within this category more teachers specifically stated *how* they connected mathematics to real life while professors made more general statements that teachers should connect mathematics to the real world. Discovery learning is one type of inquiry learning methods that is often related with teaching real world problems. Some professors believed such methods were a waste of class time. One professor stated, "Recognize that the traditional methods of instruction are the best - they do not need to be *improved on* least of all by the calculus reform movement or other such woolly-minded groups."

Limitations of the Phenomenography and Future Research

The open-ended nature of the survey was beneficial in obtaining the rich data that was presented in this phenomenography, however, the discrepancy between the number of professors and teachers who responded to the survey may be a limitation of the study. Also, the possibility of obtaining responses from professors and teachers that are not representative of the population of college mathematics professors and secondary mathematics teachers are a limitation as well. The open-ended nature of the survey may have allowed respondents to express strong opinions and did not compel fence sitters to respond. In an effort to limit the effect from the unequal group size, all data was converted to percentage of statements from professors for each category and for the clustered statements. Likewise, the same was done for the teachers' statements. This allowed for comparison of the descriptions for each category across groups even though the group sizes were different.

Conclusion

Overall professors believe high school mathematics teachers should not push students into calculus but should instead provide a greater depth of understanding of algebra and pre-calculus concepts. If students do take secondary calculus, professors want them to take AP Calculus and not a "watered down version of calculus" however, teachers should not be teaching to the AP Calculus exam. Professors have concerns that the secondary mathematics curriculum and the AP Calculus syllabus covers too much content therefore teachers and students focus on procedures instead of concepts. Many professors stated that teachers should focus on student understanding of concepts, and one way they believe this can be done is by connecting mathematics to real life problems so students can consider the reasonableness of their answer. Professors also believe teachers should base their instruction on theory, hold students individually accountable for reasoning, and not allow students to earn extra points on formal assessments.

Teachers, on the other hand, encourage students to take secondary calculus and believed this course helps to deepen student understanding of algebra concepts. One teacher stated, "the concepts of calculus are easy but they produce very difficult algebra problems." Teachers use AP Calculus materials for covering calculus concepts and for preparing students for the AP Calculus exam. Teachers allow students in secondary calculus to earn points back on formal assessments and believe this helps them to learn from their mistakes. Teachers also believe students should plan to take single variable college calculus independent of AP calculus test scores.

The variance across groups reveals professors and teachers are their own professional communities of practice, yet not all professors and teachers fall into the profiles described in this phenomenography. Often what professors believe teachers should be doing teachers state they are doing, as evidenced in the calculator and problem solving category. Klein (2001) stated that the conflict between mathematics professors and mathematics teachers could be described as a conflict between pedagogy and content. Pedagogy, in mathematics education, is considered to be that which teachers do to help students understand and be able to do and use mathematics (Brown & Smith, 1997). Figure 4.1 reveals that teachers think the classroom environment and integrating real world problems into instruction are significantly more important than professors, and professors believe that algebra and pre-calculus content knowledge is more important than teachers. However, Figure 4.1 only represents how often statements were made for each category, not the level of agreement or disagreement within categories between groups.

The responses to the FICSMath survey from students in college calculus courses across the nation can provide insight into what secondary pre-calculus and calculus teachers did that helped to prepare students for college calculus success. The quantitative findings can reveal if what the professors believed the teachers should be doing and what the teachers did helped students transfer learning from secondary mathematics to college calculus. The quantitative results are reported in Chapter 5.

CHAPTER 5

QUANTITATIVE RESULTS

MULTIPLE REGRESSION MODELS DESIGNED FROM THE 4C/ID MODEL THAT ARE PREDICTIVE OF SECONDARY PRE-CALCULUS AND CALCULUS STUDENTS' PERFORMANCE IN SINGLE VARIABLE COLLEGE CALCULUS

The 4C/ID model was designed by cognitive load theorists for the instruction of complex tasks to enhance transfer of learning (van Merriënboer, 2002). This model is an appropriate framework for modeling pedagogical practices that enhance transfer of learning from high school mathematics to college calculus. The definition of learning, based on cognitive load theory, is that learning only occurs when there is a permanent change in long-term memory. Only mathematical information that is stored in long-term memory will transfer from one environment to another. Vertical transfer occurs when prior learning is transferred to a new learning environment that is higher in a knowledge hierarchy (Haskell, 2001). Transferring knowledge from high school mathematics to college calculus is an example of vertical transfer. Henceforth in this document, secondary calculus refers to AP Calculus AB, AP Calculus BC, and non-AP calculus, unless otherwise noted. Likewise, college calculus refers to single variable calculus, unless specified differently. Typically single variable college calculus is the first calculus course at the college level. The models have been created based on performance in single variable college calculus only and are not predictive of performance in multivariable calculus for students who opt to exempt courses because of AP Calculus credit.

FICSMath Data Included in Analysis

The FICSMath survey had a total of 61 questions divided into 9 different sections to which 10,492 students in college calculus courses across the nation responded to. Students from two and four year small, medium, and large colleges and universities completed surveys that addressed content, pedagogy, and assessment methods used in their last high school mathematics course. The survey included a broad demographic section, which included what secondary mathematics courses were taken with corresponding final grades earned. At the end of the 2009 fall semester the FICSMath surveys were returned to Harvard University with the students' final grades reported by the calculus professors of record for each course. This grade is the dependent variable and the independent variables are the survey questions that align with the components of the 4C/ID model.

Data from respondents who had taken either pre-calculus or calculus courses their senior year in high school were chosen for analysis. Students from these courses were most likely to be prepared to move directly from high school mathematics to college calculus. The goal was to analyze what teachers did that helped students be successful in college calculus therefore controls were put in place to only include students in the analysis who moved directly from high school mathematics to college calculus. There were 2,483 students who completed the FICSMath survey that were in a secondary pre-calculus course their senior year, and 1,287 of them moved directly from high school mathematics to their first college calculus course where the FICSMath survey was administered. For this later group, no college level pre-calculus course was taken in between high school mathematics and college calculus, and no college level calculus course was taken previous to the calculus course where the FICSMath survey was completed. Likewise, there were 4,229 students from secondary calculus and 4,159 of them moved directly to college calculus course where the FICSMath survey was taken. The sample sizes are reported in the tables that present the regression results.

Analysis Method

The respondents who completed the FICSMath surveys were nested in a hierarchy where effects from high school preparation for college, college calculus content and pedagogy, and college or university requirements could affect performance. Under such conditions hierarchical linear modeling is appropriate to use as the analysis tool, however, only four percent of variability in the data came from college or university entrance requirements (Sonnert, 2010). Likewise, only seven percent of variability in the data came from the design of the calculus course for either STEM or non-STEM majors (Sonnert, 2010). At the student level, 89 percent of the variability was captured from a wide range of experiences including the respondents' last high school mathematics course, as well as a wide range of control variables from students' demographics and foundation knowledge variables (Sonnert, 2010). Therefore multiple-regression was used for analysis instead of hierarchical linear modeling because the other two levels captured only a small percent of the total variability.

Stepwise multiple-regression was used to find the significant variables for the controls and for each of the components of the 4C/ID model. This type of multiple-regression is particularly useful when there are a large number of possible predictors, the sample size is large, and the analysis goal is prediction (Keith, 2006). First, all variables for one component were entered using stepwise regression to find the significant variables. Then the model was built, one component at a time, by entering the significant variables individually and progressively. This method assured the variables in the model remained significant as additional variables and components were added.

Reporting of Effect Size

The effect size is reported for each component and for both models in the order of controls, the support component, the procedure component, and the learning task component. All variables reported were statistically significant, and the reporting of the effect size addressed the practical importance of the results. The findings from this research address what teachers do in secondary pre-calculus and calculus courses that enhance transfer of learning from high school mathematics to college calculus. Such information has practical importance, and will be reported using the adjusted R^2 value. The adjusted R^2 value is considered to be more stable with larger samples (Keith, 2006). The effect sizes are small conservative estimates, but this is reasonable since it is unlikely that a large amount of variance from college calculus performance is explained only by secondary mathematics teachers' pedagogical practices. There are many secondary and post-secondary variables that may predict college calculus performance, however this study is only focused on secondary mathematics teachers pedagogical practices that predicted performance. Therefore the small effect sizes are reasonable. The impact that small effect sizes have on performance is demonstrated in examples at the end of each component and for the entire model. These examples demonstrate the effect that teachers' pedagogical practices have on the subsequent learning of mathematics. Learning, as defined by cognitive load theorists, is a permanent change in long-term memory. Only what students store in long-term memory can be transferred to a different learning environment.

Alignment of FICSMath Items With 4C/ID Model Components

In order to answer Research Question 2, "How well do the components in the 4C/ID model represent pedagogies that predict pre-calculus and calculus students' success in single variable college calculus?" the questions from the FICSMath survey were first aligned with the components in the 4C/ID model. Pedagogy, in mathematics education is described as, "The ways in which mathematics teachers help their students come to understand and be able to do and use mathematics" (Brown & Smith, 1997, p. 138). The independent variables address many different strategies and classroom procedures used during instruction. The pedagogical variables from the FICSMath survey that aligned with each component of the 4C/ID model are presented in the order that the models were built. The models for pre-calculus and calculus were built successively and the effect size is reported for each component as well as cumulatively for the composite model.

All variables from the FICSMath survey that aligned with the 4C/ID model components are presented, both significant and non-significant variables. Observing all variables from the FICSMath survey that aligned with the 4C/ID components provides information about what teachers did that had a significantly positive or negative effect, or no effect, on preparation for college calculus success. There were no FICSMath variables that aligned with the part-task component for either set of data. This does not mean that automaticity is not important in learning and processing new mathematical information, but only implies that the independent variables were believed to not measure this component. The variables that addressed memorization of formulas or procedures aligned better with the procedure component based on the mathematics education literature. The variables that end with the letter 'l' are variables that have been linearized. This means the original variable was recoded so that the scale was linear. For example, the responses for connecting mathematics to everyday life (Q31everydaylifel) were: (1) very rarely; (2) once a month; (3) once a week; (4) two to three times a week; and (5) every class. These responses are different than responses that were on a linear scale, such as responding to how strongly mathematical reasoning was emphasized (Q18reason), with a range of responses from one to six for not emphasized at all to emphasized heavily. All linear scales from one to six were rescaled so that zero aligned with the response choice of not at all. Often linearized variables have different max or min values than non-linearized variables.

Support Component Variables

Conceptual understanding, reasoning, problem solving, and cognitive assessment are all parts of the support component (van Merriënboer et al., 2002; van

Merriënboer et al., 2003; van Merriënboer et al., 2006). Supportive information: (1) promotes schema construction through elaboration by helping students establish non-arbitrary relationships (van Merriënboer et al., 2002); (2) aids in conceptual understanding; (3) provides knowledge of structures and causal relationships in complex learning tasks (van Merriënboer et al., 2002); (4) provides feedback through cognitive assessment by providing opportunities for students to reflect on the quality of their problem solving processes (van Merriënboer et al., 2002). Literally supporting the problem solving process, the support component connects complex elements to theories, contains concrete, abstract and general knowledge, and provides reasoning opportunities (van Merriënboer et al., 2002). The questions that align with the support component are listed in Table 5.1.

Procedure Component Variables

The procedure component promotes schema automation by embedding new information in situation specific rules that connect particular conditions to particular actions, and this process is called "proceduralization" (van Merriënboer et al., 2008, P. 11). Procedural information is presented to learners because it helps them perform routine aspects of complex learning tasks (van Merriënboer et al., 2006). Van Merriënboer et al., (2002) stated that supportive information pertains to learning new information, while procedural information pertains to knowledge previously learned that is stored in long-term memory (van Merriënboer et al., 2002). One of the goals of the

Table 5.1

Variable Names and FICSMath Items that Align With the Support Component of the 4C/ID Model

Variable Name	Category and Description of Variable
	Variables that require conceptual understanding or mathematical reasoning
Q14concept	Extent of conceptual understanding required in most advanced HS math course
Q18funct	Emphasis on functions in most advanced HS math course
Q18vocab	Emphasis on vocabulary in most advanced HS math course
Q18def	Emphasis on precise mathematical definitions in most advanced HS math course
Q18proof	Emphasis on mathematical proofs in most advanced HS math course
Q18reason	Emphasis on mathematical reasoning in most advanced HS math course
	Variables that may support the problem solving process
Q19ask	Frequency of feeling comfortable asking questions in class discussions in most advanced HS math course
Q19value	Frequency of students' questions and comments being valued in class discussions in most advanced HS math course
Q19useful	Frequency of class discussions being useful in most advanced HS math course
Q19teachval	Frequency of teacher's answers being valuable in class discussions in most advanced HS math course
Q30smallgroupl	Regarding class and teacher interaction small group discussion/work was held
Q30alldiscl	Regarding class and teacher interaction whole class discussions were held
Q30indivl	Regarding class and teacher interaction students spent time doing individual work in class
Q30peerteach	Regarding class and teacher interaction classmates taught each other
Q30youteach	Regarding class and teacher interaction you taught your classmates
Q27alternat	Teacher highlighted more than one way of solving a problem
Q32variousmethl	Teacher presented various methods for solving problems
Q32board	Students solved problems on board
	Frequency of types of problems solved in class
Q23tfl	Frequency of problems with multiple choice/true-false
Q23blankl	Frequency of problems with fill-in the blank
Q23multl	Frequency of problems with multiple parts
Q23wordl	Frequency of word problems
Q23estiml	Frequency of problems with estimation
Q23graphhl	Frequency of problems with graphing by hand
Q23graphcl	Frequency of problems with graphing by calculator
Q23proofl	Frequency of problems with proofs
Q24checkl	Frequency of checking whether numerical answer calculated was reasonable
	Variables that align with cognitive assessment
Q25nocalc	Tests and quizzes required calculation without calculator
Q25table	Tests and quizzes involved data presented in tables
Q25prevtest	Tests and quizzes concerned material tested earlier in course
Q25homework	Tests and quizzes included questions that were drawn from homework
Q25essay	Tests and quizzes required essay responses
Q25sketch	Tests and quizzes required sketching, drawing, or graphing by hand
Q25standard	Tests and quizzes included questions from standardized exams
Q25insight	Tests and quizzes required new insight and creativity

4C/ID model is to connect rules, or procedures that combine rules, to knowledge elements such as concepts through examples or demonstrations (van Merriënboer et al., 2002). Worked examples may help students learn the connection between procedures and concepts because examples may focus the learners' attention on particulars of the complex learning task (van Merriënboer et al., 2006). Research has also shown that examples designed to prepare students for standardized test have led to a decrease in students' higher order thinking skills (Shepard & Dougherty, 1991). It has been shown that teachers that placed a high emphasis on standardized tests preparation often led students to memorize procedures and to focus on the surface features of problems (Cankoy & Tut, 2005). Cognitive load theory states when teachers provide blocked practice of similar type problems for test preparation they are creating a low contextual interference learning environment (Van Merriënboer, Kester, & Paas, 2006).

The procedural information component is part of the 4C/ID model because it was designed to promote schema automation by embedding new information in situation specific rules that connect particular conditions to particular actions (van Merriënboer et al., 2008, P. 11). Star's (2000) definition of procedures is particularly helpful in understanding procedures in the learning of mathematics:

> Knowledge of such things as the order of steps, the goals and subgoals of steps, the environment or type of situation in which the procedure is used, constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge which are inherent in the environment or situation. This knowledge is abstract (and deep), but not necessarily conceptual" (p. 85).

The questions that align with the procedure component are listed in Table 5.2.

Table 5.2

Variable Names and FICSMath Items that Align With the Procedure Component of the 4C/ID Model

Variable Name	Category and Description of Variable
	Proceduralization of content in most advanced HS math class
Q14mem	Extent of memorization of procedures
Q18memor	Emphasis on memorization of formulas
Q25mem	Required memorization of terms and facts
	Environment or type of situation procedure is used in most advanced HS math course
Q16simple	Allowed to use calculators for simple calculations
Q16derivint	Allowed to use calculators to computer numeric values of derivatives/integrals
Q16graph	Allowed to use calculators to plot graphs of functions
Q16trig	Allowed to use calculators for trigonometric functions
Q16exam	Allowed to use calculators on exams
Q16home	Allowed to use calculators for homework
Q16after	Allowed to use calculators only after a technique had been practiced with paper and pencil
Q17graphcalcl	Frequency of using graphing calculator
Q17compl	Frequency of using computer
	Reviewing knowledge previously learned
Q34prepl	Class time spent preparing for class-related quizzes/tests
Q34homel	Class time spent going over assigned homework
Q34reviewl	Class time spent reviewing past lessons
Q34standardl	Class time spent preparing for standardized math exams
Q34correctl	Class time spent correcting your own work
	Aids for accessing knowledge from long-term memory
Q26prep	Teacher gave study guides or practice exams before tests or quizzes
Q26cheatsheet	Teacher allowed cheat sheets on tests or quizzes
Q26retake	Teacher allowed students to retake or rework an exam for a grade
Q26bonus	Teacher allowed additional bonus points or extra credit on tests or quizzes
	Examples or demonstrations
Q27illust	Teacher used graph, tables, and other illustrations
Q27clear	Teacher explained ideas clearly
Q30lecturel	Teacher lectured to the class
Q32exampleprobl	Teacher solved example problems after presenting new material

What is most important concerning the procedure component for the learning of mathematics is that procedures do not stand alone but must be supported by conceptual understanding and reasoning, or the support component.

Learning Task Component Variables

Van Merriënboer et al., (2006) stated that the learning task component should represent complex tasks that have "many different solutions, are ecologically valid, cannot be mastered in a single session and pose a very high load on the learners cognitive system" (p. 343). Complex problems in secondary pre-calculus and calculus courses that are not based in real life may still have: (1) many different solutions such as an algebraic, graphical, or analytical solutions; (2) provide application problems for trig ratios, derivatives, and integrals; (3) typically cannot be mastered in a single session; and (3) pose a high working memory load on the learners cognitive system (Van Merriënboer et al., 2006). The difficulty of assigning variables from the FICSMath survey to the learning task component existed because secondary preparation for college calculus *is* a complex task. Van Merrienboer et al., (2006) define a learning task as being "preferably based on real-life tasks" (p. 349). Therefore the variables from the FICSMath survey that aligned with real life tasks were assigned to this component. The questions that align with the learning task component are listed in Table 5.3. Also, variables that addressed specific complex mathematical tasks, such as working with functions, mathematical reasoning, and mathematical proofs, which are complex tasks that can connect concepts for understanding, are also placed in this component. Conceptual understanding, the essence of the support component, and learning tasks are not mutually exclusive.

Table 5.3

Variable Names and FICSMath Items That Align With The Learning Task Component of the 4C/ID Model

Variable Name	Category and Description of Variable Description of Variable
	Connecting mathematics to real world problems
Q31everydaylifel	Connected math to your everyday life
Q31realappl	Connected math to real-life applications
Q31othersubl	Connected math to other subject areas
Q31examplesl	Examples from everyday world were used
	Specific mathematical complex tasks (also listed in support component above because these tasks can also connect mathematical concepts together for understanding)
Q18funct	Emphasis on functions in most advanced HS math course
Q18proof	Emphasis on mathematical proofs in most advanced HS course
Q18reason	Emphasis on mathematical reasoning in most advanced HS course
	Scaffolding real world problems
Q33objectl	Regarding teaching aids manipulation of physical objects was used
Q33compl	Regarding teaching aids teacher used computer simulations or applets
Q18handson	Emphasis on hands-on activities or labs

Model for Pre-Calculus

There were 2,483 (62 percent male) students who took pre-calculus their senior year in high school. There were 1287 (60 percent male) that moved directly from secondary pre-calculus to the college calculus course where the FICSMath survey was taken. The first part of creating the model to predict performance in college calculus, based on pedagogical practices in secondary pre-calculus, was to identify the significant variables that create the control and the foundation knowledge component. The control and foundation knowledge component was generated from significant variables that measured gender, SES, and previous performance in secondary mathematics courses. The phenomenography findings in Chapter 4 informed that algebra, pre-calculus, and secondary calculus is foundational knowledge for learning single variable college calculus. Controls were variables such as gender, size of graduating class, education of parents or guardians, support for learning mathematics at home, just to name a few. Performance on the SAT and/or ACT mathematics section, and performance in secondary mathematics courses were considered as foundational knowledge needed for vertical transfer of knowledge to single variable college calculus. The control and foundation knowledge component captured 15 percent of the variability in the pre-calculus model. The significant variables are presented in Table 5.4. Females were coded as zero and males were coded as one, meaning females' final average in college calculus was about three points higher than males. Only algebra and pre-calculus grades have the same scale while the others have different scales, so the standardized coefficients must be observed to determine the strongest effect for the component. By observing the standardized coefficient it is confirmed that performance in pre-calculus is the strongest predictor of performance from this component, which would be expected based on the idea of transfer of learning.

The intercept for the pre-calculus model is 43.75. An example is provided to demonstrate how the significant parameter estimates predict performance. In the calculus model, presented in the next section, the degree of home environment supportive of learning mathematics was not a significant variable. In order to compare to the calculus model, the variable for support for learning mathematics in the home is set to zero. Assume a female student scored an above average SAT/ACT math concordance score (600); made a B in algebra 2 (3); and a B in pre-calculus (3). The equation that predicts performance would be: 43.750 + $\{0.016 \times 600.000 + 2.334 \times 3.000 + 5.022 \times 3.000\}$ which predicts performance in college calculus to be 75.418. The R^2 for this component is 0.15, meaning the effect size is 15 percent, or that 85 percent of performance in college calculus is explained by variables other than gender, support for learning mathematics in the home, and teachers' pedagogical practices.

Table 5.4

Significant Controls and Foundational Knowledge for Pre-Calculus Model (n=1007)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Gender	-2.839**	0.915	-0.096	F=0.000	M=1.000	--------
Degree of home environment supportive of learning mathematics	1.179**	0.365	0.098	0.000	5.000	3.930
SAT/ACT Math Concordance Score	0.016**	0.005	0.101	240.000	800.000	597.870
HS Algebra 2 Grade	2.334**	0.806	0.102	0.000	4.330	3.600
HS Pre-Calculus Grade	5.022***	0.713	0.250	0.000	4.330	3.500

p<0.01,**; p<0.001,***

Significant Support Component Variables

The significant support variables from Table 5.1 were added to the control variables in Table 5.5. This captured an additional 2.2 percent of the variability in the pre-calculus data. The total variability explained with the controls and the support component is 17.2 percent. The significant support component variables are presented in

Table 5.5. Mewborn (2007) stated that each student's mathematical understanding and problem solving ability is primarily shaped by the teaching experiences they encounter in school. Thus, it is reasonable to believe that more support variables, other than the ones listed in Table 5.5, may significantly impact learning pre-calculus in high school but were not predictive of college calculus performance. It is also reasonable to believe that vertical transfer from secondary pre-calculus to college calculus is so great that how teachers made pre-calculus content understandable is less predictive of performance than for secondary calculus students; secondary calculus covers more material that aligns with college calculus than pre-calculus. The strongest predictor of performance for the support component was that tests and quizzes required new insight and creativity (Q25insight). This was a dichotomous variable (0=no, 1=yes) and 21 percent of the respondents stated that their tests from pre-calculus required new insight and creativity. This variable may be a positive predictor of college calculus performance because tests in college calculus may be perceived as challenging, which may align with "new insight and creativity." Several comments from professors from the phenomenography (Chapter 4) revealed their belief that teachers created exams that were too easy. One professor stated:

> University mathematicians almost always devise questions that have the purpose of trying to distinguish between students who really understand and those who don't. The HS teachers were asking questions that were, somehow, entirely predictable and very similar to those of the examples worked in the text. Our research show that university exams typically had at least 20 - 30% questions that were a bit more difficult.

Table 5.5

Significant Support Variables for Pre-Calculus Model (n=1005)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Tests and quizzes required new insight and creativity (Q25insight)	4.128***	1.053	0.118	0.000	1.000	0.210
Teacher highlighted more than one way to solve a problem (Q27alternat)	-1.084**	0.331	-0.101	0.000	5.000	3.630

p<0.01,**; p<0.001***

Highlighting more than one way to solve a problem (Q27alternat) was a negative predictor of college calculus performance. Van Merriënboer et al., (2006) stated that complex learning tasks have more than one solution, and typically different solutions are presented to students by using various methods of problem solving. Such methods may be based on solving problems algebraically, graphically, and analytically. Van Merrienboer et al., (2006); Van Merrienboer et al., (2002); stated that teaching complex tasks requires scaffolding for processing and storing complex mathematical knowledge in long-term memory. Cognitive load research has shown that the split-attention effect occurs when multiple sources of information are not integrated well (van Merriënboer & Sweller, 2005). The NCTM (2009) stated that "covering mathematical topics is not enough, students need to experience and develop mathematical reasoning themselves" (p. 9). This indicates that the importance is on students working to make sense of multiple ways to solve problems, not the teacher presenting multiple ways to students. Also, highlighting multiple ways of solving a problem may be beneficial for learning pre-calculus content, but this method of instruction was not predictive of future performance in college calculus.

Assume the same female student from the previous example had a pre-calculus teacher who required new insight and creativity and often highlighted more than one way to solve a problem. The second bracketed computation shows how this may affect college calculus performance: $43.750+\{0.016\times 600.000+2.334\times 3.000+5.022\times 3.000\}+\{4.128\times 1.000-1.084\times 3.000\}$. With the intercept, the controls and foundational knowledge component, and the support component, the student is predicted to score 76.294 in college calculus. This predicted performance score is almost a point more than the previous example. Thus insight and creativity on assessments boosted college calculus performance more than average use of alternative solutions hurt.

Significant Procedure Component Variables

The 1997 MAA President's Task Force report, discussed in Chapter 4, stated that teachers needed to balance instruction in order to help students develop conceptual understanding and to use procedures in an effective manner. Bosse & Bahr (2008) stated that pedagogy that is based on procedures has received criticism because students tend to use procedures in an inappropriate way. Pedagogy that proceduralizes instruction has been shown to lead to memorization and to neglecting conceptual understanding (Bosse & Bahr, 2008). Such research reveals that there have been concerns about procedures and the learning of mathematics. The NCTM (2008) stated that there should be a balance between conceptual understanding and procedural knowledge.

There was only one variable from the procedure component that was a significant predictor of future performance in college calculus, and this variable is shown in Table 5.6. As will be seen in Table 5.10, there are more significant variables in the procedure component for the calculus model, which may point to vertical transfer of content from secondary pre-calculus to college calculus. The procedure component only added 0.6 percent to the explained variability in the pre-calculus data, for a cumulative total of 17.8 percent of the variance explained with both components and the controls. Research has

Table 5.6

Significant Procedure Variables for Pre-Calculus Model (n=985)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Time spent preparing for standardized math exams (linearized) (Q34standardl)	-1.016**	0.338	-0.089	0.100	5.000	0.918

p<0.01,**

shown that when teachers' focus on preparing students for standardized tests that their worked examples parallel closely to test questions (Shepard & Dougherty, 1991). The negative parameter indicates that such instruction in pre-calculus does not enhance transfer of learning to college calculus. Assume the female student had a pre-calculus teacher that placed a less than average emphasis on standardized test preparation (0.5). The model predicts that her performance in college calculus would be 75.786, which is almost one-half of a point less than with just the support component.

Significant Learning Task Component Variables

There was also only one significant variable for the learning task component, which is shown in Table 5.7. Manipulation of physical objects during instruction may be a negative predictor for college calculus performance because such instructional strategies may not be used in college calculus. Secondary pre-calculus instruction presents information to students with high element interactivity. Many connections are needed in order to compress mathematical knowledge, or create schema, that can be recalled in a different environment that requires vertical transfer. Manipulating objects,

Table 5.7

Significant Learning task Variables for Pre-Calculus Model (n=964)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Manipulation of physical objects (Q33objectl)	-1.068*	0.431	-0.075	0.100	5.000	0.669

$p<0.05$,*

as an aid to instruction, may not assist student understanding unless the students themselves manipulate the object and construct their own understanding. It takes work to process *why* and not just *how* when learning mathematics, and the goal of manipulating objects during instruction should be to emphasize both. Assume the same female student had a pre-calculus teacher that manipulated objects as teaching aids about once a week (1). The equation that predicts college calculus performance from all components is

$$43.750 + \{-2.839 \times 0 + 1.179 \times 0.000 + 0.016 \times 600.000 + 2.334 \times 3.000 + 5.022 \times 3.000\} +$$

$$\{4.128 \times 1.000 - 1.084 \times 3.000\} + \{-1.016 \times 0.500\} + \{-1.068 \times 1.000\}$$ which renders a predicted score of 74.718.

Conclusion of Pre-Calculus Model

The pre-calculus model explains a total of 18.100 percent of the variance in college calculus performance for pre-calculus students that transferred knowledge directly from high school to college calculus. This means that 81.900 percent of the variability in college calculus performance came from other variables. Only 3.100 percent of the variability is explained specifically by pedagogical practices from pre-calculus teachers. If only the pedagogical practices with negative parameter estimates are applied at the highest level, while the positive practices are applied at the lowest level, the predicted college calculus grade may decrease by 15.800 points. Conversely if only the positive pedagogical practices are applied at the highest levels, and the negative practices at the lowest level then college calculus performance may increase by 4.100 points. The point increase is less than the point decrease because there were more pedagogical practices that negatively effected college calculus performance. According to the model an overall difference of 19.900 points in college calculus performance can be explained by teachers' pedagogical practices. The significant variables have been placed on the 4C/ID model in Figure 5.1.

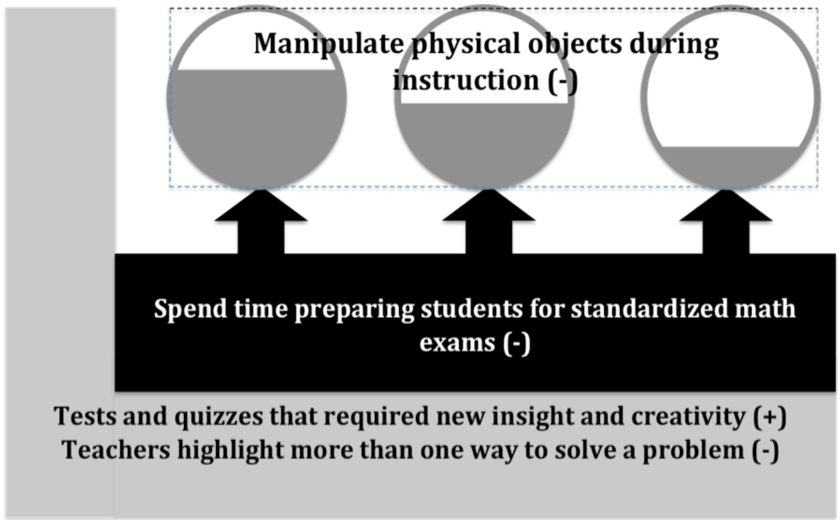

Figure 5.1. Significant Pre-Calculus Pedagogical Practices Placed on the 4C/ID Model

Model for Secondary Calculus

There were 4,021 students that completed either non-AP Calculus, AP Calculus AB, or AP Calculus BC their senior year in high school mathematics, and 2160 of them transferred knowledge directly to a single variable college calculus course where the FICSMath survey was completed. Figure 5.2 displays the comparison of these two groups for the three different levels of secondary calculus. There is a significant difference between the total number of students that took college calculus and the number of students that moved directly from high school mathematics to the course where the FICSMath Survey was taken. The red group is one that has been analyzed to discover how pedagogical practices from high school mathematics instruction effected college calculus performance. All respondents in the red group were placed together for analysis. Initially there were concerns about grouping students from non-AP Calculus, AP Calculus AB, and AP Calculus BC together because the three courses may be different in the amount of content they cover and the speed at which the content is covered. For

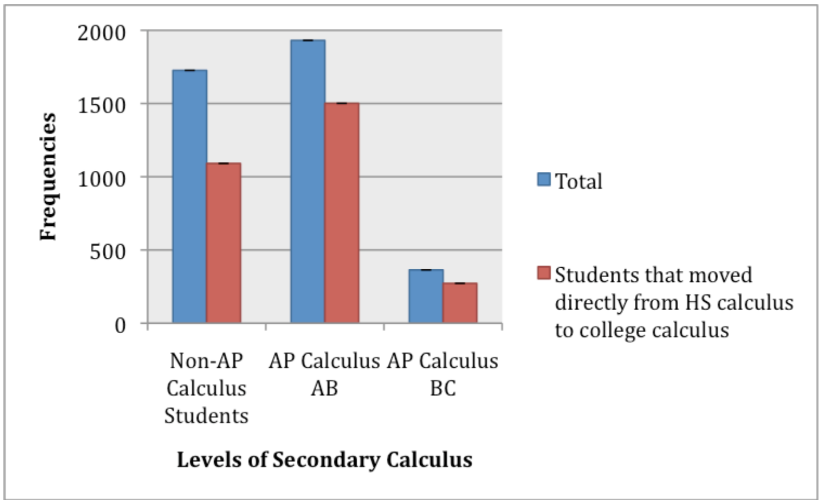

Figure 5.2. Comparison of Students in Three Levels of Secondary Calculus and Number of Students that Moved Directly to the College Calculus Course Where the FICSMath Survey was Taken

example, some AP Calculus teachers try to finish teaching course content early in order to prepare students for the AP Calculus Exam. Also, AP Calculus BC covers sequences and series. These concerns were addressed by comparing the performance in high school calculus and in college calculus across the three levels. Figure 5.3 show that the high school grade point average for the three levels of secondary calculus were all between 3.42 and 3.57, or in the B range. The error bars indicate there is not a significant difference between the mean grades across the three levels. Figure 5.3 also shows the average performance for all three levels in college calculus was 3.90. The error bars indicate more variance in the performance of AP Calculus BC students, but there is not a significant difference in college calculus performance across the three levels. AP Calculus BC covers more content at a faster pace yet the mean performance of this group was the same as the group that took non-AP Calculus. Because of the similarity of performance in secondary calculus and college calculus for the three groups, all groups were combined for analysis.

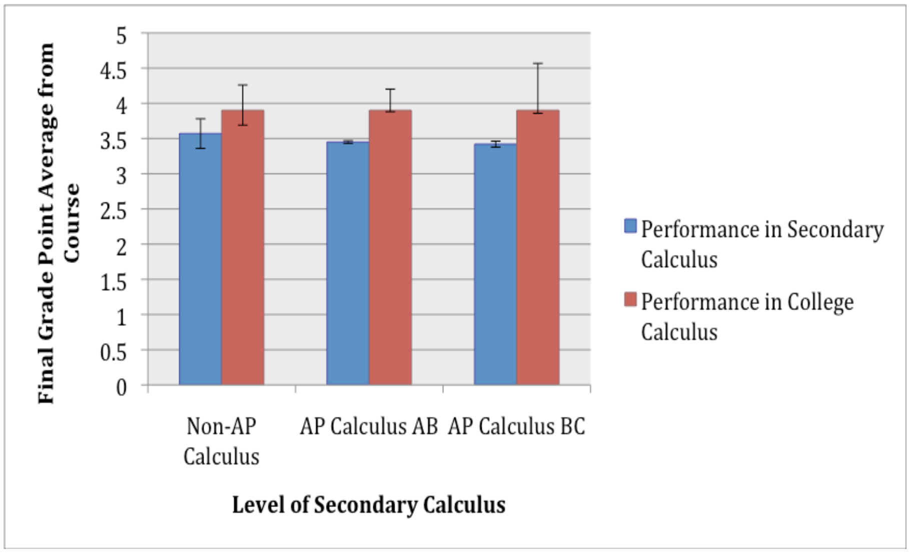

Figure 5.3. Three Levels of Secondary Calculus and Average High School Grades

The control and foundational knowledge component explained 15.7 percent of the variability in college calculus performance. This component is presented in Table 5.8. The constant in the model, or the y-intercept, is 47.34. As in the pre-calculus model, the females' predicted performance is higher than males. The standardized coefficients reveal that the largest coefficient is the secondary calculus grade, which is expected since students are transferring knowledge from secondary calculus to college calculus. Because there are three different scales presented in the model, the standardized coefficients are needed to determine the parameter with the strongest effect in the component. There is no AP Calculus exam score included in the control and foundational knowledge component because inclusion of this variable would exclude the group of students in non-AP Calculus.

Table 5.8

Significant Control and Foundational Knowledge Variables for Secondary Calculus Model (n=2151)

Variable Description and Name	Parameter Estimate/Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Gender	-2.040***	0.494	-0.087	F=0.000	M=1.000	--------
SAT/ACT Math Concordance Score	0.022***	0.003	0.156	200.000	800.000	625.760
HS Algebra 2 grades	2.133***	0.462	0.102	0.000	4.330	3.650
Secondary Calculus grades	3.858***	0.356	0.240	0.000	4.330	3.490

p<0.001,***

Assume that a female college calculus student had scored about average on the SAT/ACT math test (600); had earned a B in Algebra-2 (3) and an average in secondary calculus (3). The model predicting his performance would be calculated by 47.320+ $\{0.022 \times 600.000 + 2.133 \times 3.000 + 3.858 \times 3.000\}$. This renders a predicted performance of 78.493 in college calculus. Compared to the pre-calculus predicted performance of 75.418, with the same levels placed into the significant variables, this is three points higher.

Significant Support Component Variables

The significant support variables were found using the same method as described in the pre-calculus model and are displayed in Table 5.9. Students who took secondary calculus have more content knowledge that may transfer to college calculus; therefore the teachers' pedagogical practices may be more predictive of performance for calculus students than for pre-calculus students. This may explain why there are more variables in the support component for this model than for the pre-calculus model.

Table 5.9

Significant Support Variables for Secondary Calculus Model (n=2076)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Extent of conceptual understanding (Q14concept)	0.772***	0.214	0.078	0.000	5.000	3.680
Emphasis on vocabulary (Q18vocab)	0.575**	0.182	0.069	0.000	5.000	2.370
Frequency of checking whether numerical answer was reasonable (Q24check)	-0.470**	0.179	-0.055	0.000	5.000	3.350

p<0.01,**; p<0.001***

Conceptual understanding is the largest positive coefficient for the support component. Tall (1991) stated that mathematics instructors typically make the error of breaking up complex mathematical information into small pieces when teaching calculus. This may provide an ordered sequence from the experts view, but the student may perceive instruction as in separate pieces and not perceive the overall concepts (Tall, 1991). Students may perceive instruction broken into pieces "like separate pieces of jigsaw puzzles for which no total picture is available (Tall, 1991, p.17). This aligns with what van Merriënboer & Kester (2008) describes as part-whole instruction, which is claimed to cause instruction of complex tasks to be too “piecemeal” (p. 444.) Tall (1991) also argued for more emphasis on visualizing overarching mathematical concepts, which aligns with what van Merriënboer et al., (2002) and van Merriënboer et al., (2006) describe as whole-part instruction. The 4C/ID model was designed as an instructional tool to help teachers present complex tasks using the whole-part scaffolding method. The support component is representative of overarching concepts scaffolding instruction for a complex learning task.

The support component explains an additional 2.2 percent of the variability in the calculus data, meaning that 17.9 percent of the overall variance in college calculus performance is explained by both the foundation knowledge and the support components. The variability captured by the amount of emphasis placed on functions (variable Q18funct) is reported in the learning task component instead of the support component even though it is listed in both Table 5.1 (because functions are foundational to understanding concepts in mathematics) and Table 5.3 (because functions are a specific complex task for students). The variable Q18funct was a significant variable predictive of future performance in college calculus, but it was discussed in the Learning Task Component (Table 5.11) because: (1) there were 34 statements, made by professors across eight of the 11 categories presented in the phenomenography, that revealed that students struggle with functions at the college level, indicating this is a specific complex task for students; and (2) the learning task component is supported by the understanding of concepts in the support component. The Pearson Correlation coefficient between conceptual understanding and the emphasis on functions was r=0.27, which is a positive but weak relationship between conceptual understanding and the emphasis on functions.

The emphasis on vocabulary (Q18vocab) is the second and last positive parameter estimate for the support component. Vocabulary was not discussed in the phenomenography (Chapter 4) because the percent of statements from teachers and professors for the vocabulary category was not significantly different than zero. This indicates that very few statements were made that addressed vocabulary from both groups. On the FICSMath survey, the mean response reported by students that their last high school mathematics teacher placed an emphasis on vocabulary was 2.370 out of a scale of not emphasized at all (0) to emphasized heavily (5), indicating that teachers somewhat emphasized vocabulary. Tall (1993) stated that calculus is the first time students are exposed to vocabulary such as, "limit as x approaches some value, towards infinity the limit tends to, as small as we please, a variable getting arbitrarily large, N approaches infinity" (p. 2). He also stated that such terms are used colloquially but have specific unique meanings in calculus.

Teachers requiring students to check whether their numerical answer was reasonable was a negative predictor of performance. This question from the FICSMath survey was placed in the support component because checking the reasonableness of an answer is linked to understanding the concepts in a problem. In the phenomenography, professors stated that teachers should provide real world problems because it gave students a reason to check if their answer was reasonable. However, students being required to check the reasonableness of their answer was not a positive predictor of performance in college calculus. One explanation of the variable Q24check being a negative predictor of future performance in college calculus may be explained by considering the validity of the term "reasonable". Yakel and Hanna (2007) stated "reasoning in mathematics is complicated by the term *reasoning*," which is used widely with the implicit assumption that there is agreement on its meaning (p. 228). This may explain why Q18reason was not a significant predictor, either negative or positive, of college calculus performance. The NCTM (2009) stated that reasoning is often understood to encompass formal reasoning, or proof, but instead can take many forms ranging from informal explanation and justification of formal deduction or inductive observations. The NCTM (2010) stated, "As students develop a repertoire of increasingly sophisticated methods of reasoning and proof during their time in high school, "standards for accepting explanation should become more stringent" " (p. 4; NCTM 2000a, p. 342). Requiring students to check the reasonableness of their answer may be one way to increase the rigor of student explanation, however, the NCTM (2010) may be addressing the *process* more than the final answer. The practice of requiring students to check their final answer may be beneficial for secondary calculus instruction, but it was not a positive predictor of future performance in college calculus.

Assume the same student discussed in the control and foundation knowledge component had a secondary calculus teacher who: focused on conceptual understanding a lot (5); emphasized vocabulary very little (2); and required students to check whether their numerical answer was reasonable in every class (5). This student's predicted college calculus performance from the model with control and support components would be: 47.34+ $\{0.022 \times 600 + 2.133 \times 3.000 + 3.858 \times 3.000\} + \{0.772 \times 5.000 + 0.575 \times 2.000 - 0.470 \times 5.000\}$. This renders a predicted score of 81.153 for college calculus performance. Again, this model explains 17.9 percent of the variability in the college calculus performance. The remaining variability would most likely be captured by other variables, such as those from the actual calculus course and the effort of the student.

Significant Procedure Component Variables

The 1997 MAA President's Task Force report, discussed in Chapter 4, stated that teachers needed to balance instruction in order to help students develop conceptual understanding and to use procedures in an effective manner. Bosse & Bahr (2008) stated that pedagogy that is based on procedures has received criticism because students tend to use procedures in an inappropriate way. Pedagogy that proceduralizes instruction has been shown to lead to memorization and to neglecting conceptual understanding (Bosse & Bahr, 2008). The NCTM (2008) stated that there should be a balance between conceptual understanding and procedural knowledge. Van Merriënboer et al., (2002) stated the procedure component (1) provides support of the concepts presented in the support component through examples; (2) provides directions during practice; (3) describes rules for procedures and knowledge elements; and (4) that support should fade as learners gain more expertise. Also, Star (2000) defined procedures in the learning of mathematics as knowledge of the order of steps, or the environment or type of situation in which the procedure is used. Such research reveals that there have been concerns about procedures, about what procedures are, and how they are beneficial in the learning of mathematics. The 4C/ID model was designed as an instructional model to emphasize the whole-part scaffolding method, indicating that the procedure component does not stand alone but is supported by conceptual understanding, or the whole overarching concepts, in the support component.

The procedure component explained an additional 1.1 percent of the variability in the calculus data for an overall explained variance of 19.0 percent of the variability explained for college calculus performance. The significant procedure variables are listed in Table 5.10. Again, as for the support component, there are more significant variables for the procedure component than there were for the pre-calculus model. The first three variables are dichotomous variables where the students answered no (0) or yes (1) by marking all that applied. There are no averages provided for these variables in Table 5.10 but the percent that stated yes (1) will be presented in the discussion. Out of six significant variables in the procedure component, only one is a positive predictor of performance in college calculus.

Using the calculator in class to plot graphs of functions (Q16graph) had the second largest negative parameter estimate. There were 81 percent of students that reported their teachers allowed them to use calculators to plot graphs of functions. Many professors stated in the phenomenography that students should be able to work mathematics problems and graph functions without calculators. Many teachers stated they taught content first by hand and then added the calculator, however, the variable, "Calculator was allowed for use only after a technique had been practiced with paper and pencil" was not a significant variable. The sample has two different levels of AP Calculus included in it, and the AP Calculus exam has a multiple choice calculator section. Therefore it is reasonable that AP students were expected to learn how to use graphing calculators to help them answer the multiple choice sections of the AP exam, however, independent of the exam, allowing students to use their calculators to plot graphs of functions was a negative predictor of college calculus performance. The negative parameter may also indicate that students could not graph by hand without their calculators, which hurt their performance in college calculus.

Table 5.10

Significant Procedure Variables for Secondary Calculus Model (n=2032)

Variable Description and Name	Parameter Estimate/ Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Allowed to use calculator to plot graphs of functions (Q16graph)	-1.907**	0.614	-0.063	0.000	1.000	------
Teacher allowed cheat sheets (Q26cheatsheet)	-2.125**	0.677	-0.064	0.000	1.000	------
Teacher allowed additional bonus points or extra credit on tests or quizzes (Q26bonus)	-1.260**	0.459	-0.056	0.000	1.000	------
Class time spent preparing for class-related quizzes/test, linearized (Q34prepl)	-0.372*	0.188	-0.042	0.100	5.000	1.370
Time spent going over assigned homework, linearized (Q34homel)	0.326*	0.135	0.052	0.100	5.000	3.220
Time spent reviewing past lessons, linearized (Q34reviewl)	-0.475**	0.182	-0.058	0.100	5.000	1.340

$p<0.05$,*; $p<0.01$,**

The negative parameter estimates for teacher allowing bonus points or extra credit and the use of cheat sheets were all discussed by professors and teachers in the phenomenography for the assignments and assessments category (Figure 4.7). Such pedagogy was perceived by professors as: (1) enabling students to believe that seat time equaled passing; and (2) reporting performance scores from secondary calculus that were not predictive of what students actually knew and could do. Teachers, on the other hand, stressed that they used such pedagogical practices because it encouraged seniors to take a rigorous mathematics class without using the "I do not want to lower my GPA" argument. There were 13 percent that reported their teacher allowed the use of cheat sheets, and 48 percent that reported their teachers gave bonus points or extra credit on tests or quizzes.

The variables Q34 prepl and Q34 reviewl may be negative parameter estimates because such pedagogy is typically not part of a college calculus class. Most professors expect students to prepare for formal assessments on their own time. College calculus courses typically have a syllabus that details the topics that will be covered each day, and students that need extra help reviewing past lessons are expected to obtain extra help on their own time. Time spent going over homework was a positive predictor (Q34homel), yet some college calculus courses do not spend class time discussing students' problems with assignments. Again, if students have a difficult time with homework they are most often expected to see the professor for individual help. The positive parameter estimate for spending time going over homework may be explained with cognitive load theory. When teachers address students' questions about problems they have attempted, they may be breaking down the interacting elements of the complex task, which allows germane cognitive load in working memory to process and store the information in long term memory for future recall. Van Merriënboer et al., (2002) stated that worked examples may be one way teachers answer questions, provide an understanding of rules, procedures that combine rules, and unite other knowledge elements together that support learning the overarching concept.

Assume the same female student had a secondary calculus teacher that regularly allowed the class to use calculators to plot graphs of functions (1); did not allow students to use cheat-sheets on tests and quizzes (0); regularly gave bonus points on tests and quizzes (1); spent time in class preparing for quizzes or tests about once a month (0.25); spent time going over assigned homework daily (5); and typically spent time reviewing past lessons once a week (1). The predicted performance in college calculus with the control and foundation knowledge, support, and procedure component is 79.048. This is a little more than two points less than the predicted performance with just controls and the support component.

Significant Learning Task Component Variables

The learning task component explains an additional 0.4 percent of the variability in college calculus performance from the calculus data, for a total of 19.3 percent of the total variability explained from the components in the calculus model. Without the variable Q18funct there was no additional variability explained in college calculus performance even though there is one addition variable in the learning task component. The variables in the learning task component are listed in Table 5.11.

Table 5.11

Significant Learning Task Variables for Secondary Calculus Model (n=1999)

Variable Description and Name	Parameter Estimate/Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Manipulation of physical objects (Q33objectl)	-0.791***	0.210	-0.078	0.100	5.000	0.836
Emphasis on functions (Q18funct)	0.724**	0.272	0.059	0.000	5.000	3.860

p<0.01, **; p<0.001, ***

The emphasis on functions was placed in the learning task component because the findings from the phenomenography (Chapter 4) revealed that functions is a specific complex task for college calculus students, and it is the professors' belief that more focus on functions may better prepare secondary mathematics students for college calculus success. The placement of the variable Q18funct in the learning task component is appropriate because the learning task component is supported by the overarching (whole) concepts in the support component. Van Merriënboer et al., (2002); Van Merriënboer et al., (2003); Van Merriënboer et al., (2006) state that such instruction, along with the algorithms (parts) in the procedure component, increases the likelihood of transfer of learning. The mean emphasis on functions (Q18funct) was 3.86 out of a scale of not emphasized at all (0) to heavily emphasized (5). This indicates that, on the average, secondary calculus teachers emphasized functions, but not heavily. Tall (1997) stated one purpose of the function is to represent how things change, and calculus is often referred to as the mathematics of change. Functions are used in calculus to “do and undo” visual-spatial, numeric, symbolic, and graphic, representations of change in mathematics (Tall, 1997, p. 7).

The only other significant variable in the learning task component was Q33objectl, which was also a significant variable in the pre-calculus model. The mean response of the linearized variable indicates that calculus teachers manipulated objects during instruction about once a week. From a cognitive load perspective, if the manipulation of objects was not integrated well with the learning task then instruction may have cause a split attention effect. This means the connection between the teachers' manipulation of objects and what the student needed to do to problem solve was not well connected. Students often have a difficult time visualizing what an integral is representing. For example, understanding a volume by slicing or rotation problem can be difficult if the students cannot visualize the problem. Students observing their teacher manipulate physical objects are most likely not what would help students understand such complex calculus concepts.

What is interesting about this component is that no variables that measured the connection between mathematics and real life applications were significant predictors of college calculus performance. Problems such as displacement problems, related rates, optimization, area and volume problems, are all problems that connect calculus to changing phenomenon related to “real life”. Such problems may be considered as real life problems, but they may also be considered “as problems included in the mathematics curriculum because they provide justification for teaching mathematics at all (Schoenfeld, 1992, p. 13). It is reasonable to believe that students perceived such problems not as connected to real life but as complex word problems. Van Merriënboer et al., (2006) stated the benefit of real life problems is they present opportunities to present context as a whole from the start. However, secondary preparation for college calculus is a complex task even without real life application problems, and complex mathematical concepts can be presented from the whole conceptually from the start without real life applications.

Assume the same student from the previous calculus model examples had a calculus teacher that placed a lot of emphasis on the functions (4) and manipulated physical objects during instruction about once a week (1). The predicted college calculus performance is 81.153. This is a little more than five points higher than the predicted performance from the control and foundational knowledge component.

Conclusion of Calculus Model

Mewborn (2007) stated that what is known about learning mathematics is that each student's mathematical understanding and problem solving ability is primarily shaped by the teaching experiences they encounter in school. By observing only the pedagogical practices from the support, procedure, and learning task components with the highest possible positive parameter estimates and the lowest possible negative parameter estimates the college calculus performance may increase by 11.82 points. Conversely, if the negative parameter estimates are quantified at the highest amount of pedagogical practices and the positive parameter estimates at the lowest amount then the college calculus performance is predicted to decrease by 15.79 points. This indicates there is a total difference of possible performance in college calculus of 27.61 points. This could be the difference between passing and failing college calculus. All of the significant variables have been placed on van Merrienboer's et al., (2002) model and are displayed in Figure 5.4.

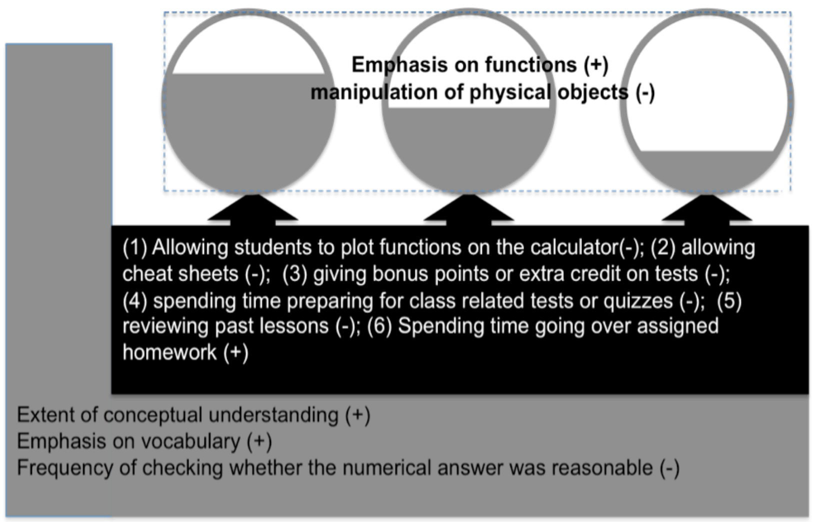

Figure 5.4. The Significant Pedagogies from Secondary Calculus Sample Placed on the 4C/ID Model

Summary of Models

Research Question 2 asked, "How well do the components in the 4C/ID model represent pedagogies that predict pre-calculus and calculus students' success in single variable college calculus?" If the pre-calculus and calculus models have positive predictors of performance in the support and procedure components then the components represent pedagogies that may be predictive of future success in single variable college calculus. The learning task component represents a complex task that must be supported by concepts first and foremost and then with procedures, or algorithms, required to move from a problem to an accepted solution. Concerning the learning task component, van Merriënboer et al., (2006) stated that a complex task "has many different solutions, are ecologically valid, cannot be mastered in a single session, and pose a high load on the learners cognitive system" (p. 343). Each of these descriptors of a complex task was related to learning mathematics in Chapter 2. Figure 5.5 displays the number of significant variables for each component for both models.

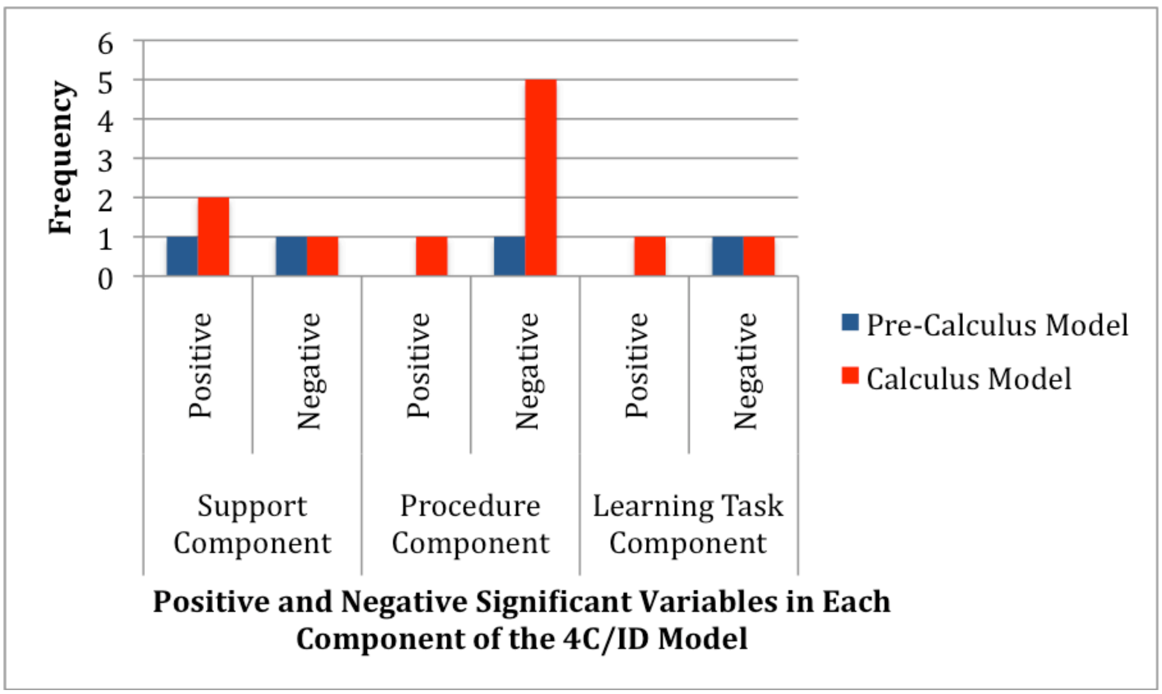

Figure 5.5. Comparison of Significant FICSMath Variables in the 4C/ID Components for Both Models

The pre-calculus model had only one positive predictor of performance and one negative predictor for each component. There are concerns that there were no other positive predictors of future performance in college calculus, which could indicate that: (1) the components do not model pedagogies that predict success in single variable college calculus well; (2) the content gap between secondary pre-calculus and college calculus might be so great that *how* teachers presented pre-calculus content may have a limited effect; (3) secondary pre-calculus content did not adequately prepare students to learn college calculus content; or (4) as expressed by the professors and teachers in the phenomenography, so much content was covered, "a mile wide and a half-inch deep" which resulted in shallow preparation for the deep understanding required to learn college calculus. There may be more reasons why the model for pre-calculus had so few significant variables, which indicates the need for more understanding concerning secondary pre-calculus preparation for college calculus success.

Figure 5.5 shows the 4C/ID instructional model components are more likely to be predictive of secondary calculus success than the pre-calculus model. The calculus model had positive predictors of performance in all three components. Also, the significant positive predictors were "focus on conceptual understanding" and "focus on vocabulary," which can be used to structure whole concepts. The positive predictor of performance in the procedure component, "teacher goes over assigned homework" provides opportunities for "just in time" information (van Merriënboer et al., 2002, p. 51), which provides learners with knowledge they need in order to move from a problem situation to an acceptable solution. Figure 5.5 also shows the component that is most predictive of lower performance in college calculus is the procedure component. The reader should note that the effect size and the strength of the variables, as determined by the standardized coefficient and the parameter estimates, are not considered in the comparisons in Figure 5.5. Only the number of significant variables in each component was considered.

CHAPTER 6

4C/ID MODEL MODIFIED AS TWO DIFFERENT MATH MODELS FOR COLLEGE CALCULUS PERFORMANCE

The FICSMath survey provided valuable information from college calculus students' perspectives concerning what their last high school mathematics teacher did to prepare them for college calculus success. Cognitive load theory provided a theoretical framework for learning, and the 4C/ID model provided an instructional model for teaching complex tasks for transfer of learning. These three have been merged in an unequal status-concurrent mixed-method design to study what pedagogical practices are predictive of college calculus performance. A phenomenography analyzed what professors believed secondary mathematics teachers should be doing, and what secondary mathematics teachers are doing to prepare students for college calculus success. Two multiple regression models were built, one from the sample of pre-calculus students, and the other from the sample of calculus students that moved directly from secondary mathematics to college calculus where the FICSMath survey was administered. The models are based on the correlations between what the respondents reported their secondary mathematics teacher did to prepare them for college calculus and their college calculus performance. The alignment of the results from the phenomenography and the findings from pre-calculus and calculus models are shown in Table 6.1.

Table 6.1

Alignment of Categories from Phenomenography with Findings from Secondary Pre-Calculus and Calculus Models with Positive (P) or Negative (N) Effect on Performance in College Calculus

4C/ID Component	Pre-Calculus Model Significant Variables	Calculus Model Significant Variables	Phenomenography Categories
Support-Supports learning and performance of new information. Promotes schema construction through elaboration of mental models, cognitive strategies, and cognitive assessment.	Tests and Quizzes required new insight and creativity (P) Teacher highlighted more than one way of solving problems (N)	Extent of conceptual understanding (P) Focus on vocabulary (P) Require students to check whether their numerical answer was reasonable (N)	-Conceptual understanding, -problem solving, -classroom environment, -assignments and assessments
Procedure-Prerequisite knowledge needed to learn new information, examples and demonstrations, knowledge of order of steps, environment or situation in which procedure is used	Time spent preparing for standardized math exams (N)	Go over assigned homework (P) -Teacher gave bonus points or extra credit on tests or quizzes (N) -Teacher allowed cheat sheets on tests or quizzes (N) -Class time spent preparing for tests or quizzes (N) -Time spent reviewing past lessons in class (N)	-Assignments and assessments, -classroom environment
		-Allowed to plot graphs of functions on calculator (N)	Calculator use
	Significant Foundational knowledge-Algebra 2, and pre-calculus (P)	-Significant Foundational knowledge-Algebra 2 and calculus (P)	Algebra, pre-calculus, and calculus
Learning Task Authentic whole task experiences. Best if use whole-part scaffolding for learning complex tasks. Scaffolding fades as material is learned	Teacher manipulated objects during instruction (N)	Focus on functions (P) Teacher manipulated objects during instruction (N)	Algebra, pre-calculus, calculus content (functions) and problem solving
Categories from the phenomenography that did not align with models			Real world problems, proofs and qualified teachers

There were 35 statements from the professors across seven of the categories presented in Figure 4.1 that addressed the fact that students struggle with functions, and that teachers need to focus more on a broad range of algebraic, trigonometric, and transcendental functions, as well as families of functions, operations on functions, and generating functions from given information or from patterns. For sure, functions are foundational to understanding concepts in mathematics, but they were revealed in the phenomenography as a unique complex task with which students struggle. Therefore the variable, "focus on functions" was moved from the support component to the learning task component, as discussed in Chapter 5.

Creation of New Models from Findings

The 4C/ID model was created to help teachers with the instruction of complex tasks, but not specifically for the instruction of mathematics. Even though the learning task component had the fewest significant variables from the FICSMath survey, and the part-task component was dropped because there were no items that aligned with this component from the FICSMath survey, the 4C/ID model is appropriate to use as a framework. This is because: (1) secondary preparation for college calculus *is* a complex task (as stated in Chapter 2); and (2) Van Merriënboer et al., (2003) stated that the part-task component had not yet been substantiated in the whole-part theoretical framework. The 4C/ID model was created to help teachers with the difficulty of structuring complex learning tasks with a focus on the "whole" over arching concepts first, and then incorporating procedures needed to move from an initial problem to an acceptable solution. Such instruction is perceived to help students create schemata in long term memory by decreasing intrinsic cognitive load, or element interactivity, so chunks of knowledge can be transferred back into working memory in order to process more new (mathematical) information. Thus whole-part instruction is claimed to help students transfer learning to a new and different environment. Tall (1991) stated that once mathematical concepts are understood there is often a tremendous mental compression and mathematical ideas can be filed away, recalled quickly, and used when needed in just one step in some other metal process (p. 4). Both professors and teachers made statements in the phenomenography that secondary pre-calculus students had "weak preparation for pre-calculus" (see Figure 4.4). It is possible that students were taught the correct content to prepare pre-calculus students for college calculus, but the content did not transfer to a different environment. Haskell (2001) stated: "Part of the problem of transfer is that our learning tends to be welded to a place" (p. 10). The struggle to recall information from secondary pre-calculus to help with the process of learning college calculus may be an example of learning being "welded to a place."

The 4C/ID model was used in this study to determine if pedagogical practices that align with the components are predictive of college calculus performance. The significant findings from the pre-calculus and calculus models, and the findings from the phenomenography, were used to modify the 4C/ID model and answer Research Question 3, "How can the 4C/ID model be modified to reflect pedagogies that are predictive of pre-calculus and calculus students' success in single variable college calculus?"

Van Merriënboer et al., (2002); van Merriënboer et al., (2003); van Merriënboer et al., (2008) image of the 4C/ID model is a complex network of components represented by various shapes and circles with partial shading representing decreased scaffolding over time as students learn the content. The goal of the modifying the 4C/ID model was to present the findings from the FICSMath study in a clear and concise way that can be easily understood by those who have an invested interest in secondary preparation for college calculus. The findings from the pre-calculus model are presented in Figure 6.1.

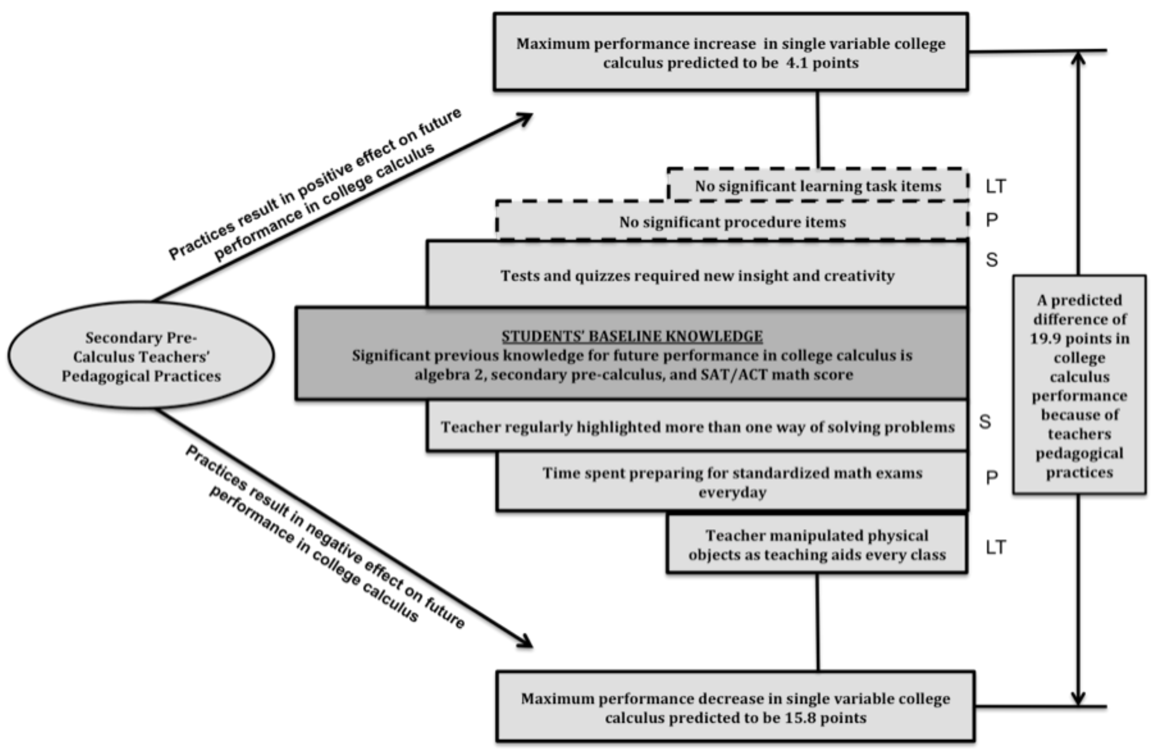

Figure 6.1. The 3C/ID Pre-Calculus Model for College Calculus Performance

The maximum performance increase was computed by multiplying the positive parameter estimates with the highest possible numeric response (e.g. 6=emphasized heavily) and the negative parameter estimates with the lowest possible response (e.g. 0=none, or 0.1=very rarely, linearized). The maximum performance decrease was computed by multiplying the negative parameter estimates with the highest possible numeric response (e.g. 6=emphasized heavily) and the positive parameter estimates with the lowest possible response (e.g. 0=none, or 0.1=very rarely, linearized). These extremes were used to reveal the large predicted change in final college calculus grade based solely upon teachers' pedagogical practices. The only parameter estimates used in the maximum performance increase and decrease computations were significant pedagogical variables from the support, procedure, and learning task components. In Figure 6.1, the students' baseline knowledge is the foundation upon which the model is built. The phenomenography revealed that algebra and pre-calculus content are foundational to learning college calculus. The SAT/ACT concordance score for mathematics was a positive predictor of college calculus performance, as revealed in Chapter 5, and this variable was also included as part of the students baseline knowledge. It should be observed that the small amount of variability captured from teachers' pedagogical practices (3.1 percent) reveal a large predicted difference in points earned in final college calculus grade. The positive and negative sloped lines in the model represent the effect on future performance and align with the positive and negative parameter estimates from the models presented in Chapter 5.

Bressoud (2010) stated that it is not known what effect AP Calculus or non-AP calculus has on college calculus performance. The mean performance of the pre-calculus group is the only group that performed lower in college calculus than in secondary mathematics. The error bars indicate a larger variance in the performance of the pre-calculus group in college calculus. Based on the mean performance across groups it appears that taking secondary calculus was beneficial for college calculus performance. Some professors stated that high schools should stop teaching calculus and let calculus be a college level course (Chapter 4). Many of the professors believed that high school mathematics should focus on foundational knowledge needed to learn calculus, such as functions, algebra, and pre-calculus content. However, these views do not align with the mean performance of pre-calculus and calculus students in college calculus that are presented in Figure 6.2.

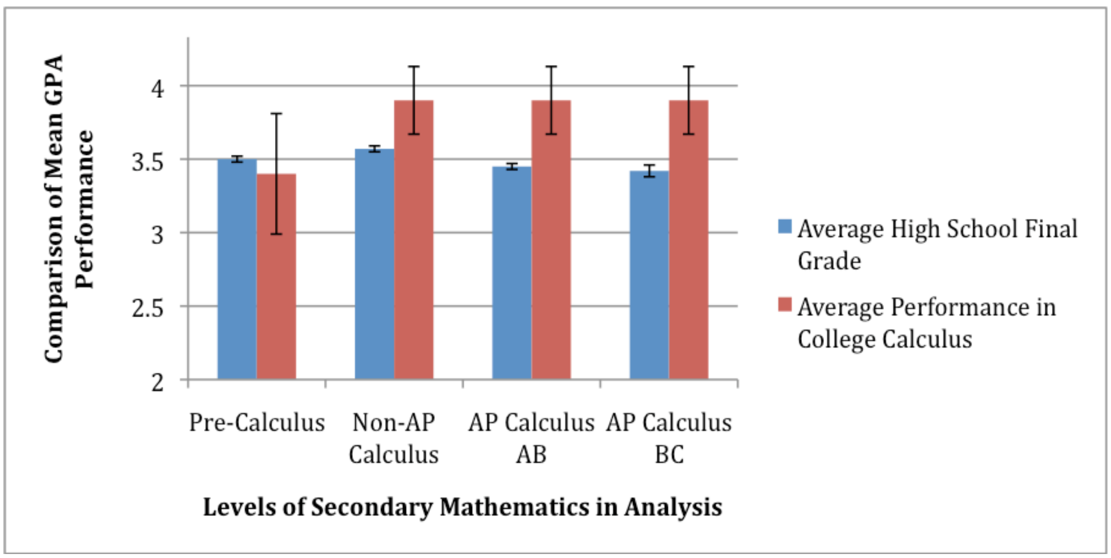

Figure 6.2. Comparison of Pre-Calculus and Three Levels of Secondary Calculus Performance and College Calculus Performance

The results of the secondary calculus model are displayed in Figure 6.3. There are more variables predictive of performance in this model, possibly because of less content knowledge difference between secondary calculus and college calculus. Similar to the pre-calculus model, there are more pedagogical practices that predicted future performance in college calculus negatively than practices that had a positive effect on future performance. The maximum performance increase and decrease was computed using the same method described for the pre-calculus model. Also, the only parameter estimates used to compute the performance increase or decrease were from the support, procedure, and learning task components. The students' baseline knowledge is the same for the pre-calculus model except that performance in secondary pre-calculus was not a significant predictor for secondary calculus students' performance in college calculus. Thus performance in secondary calculus is a part of the students' baseline knowledge,

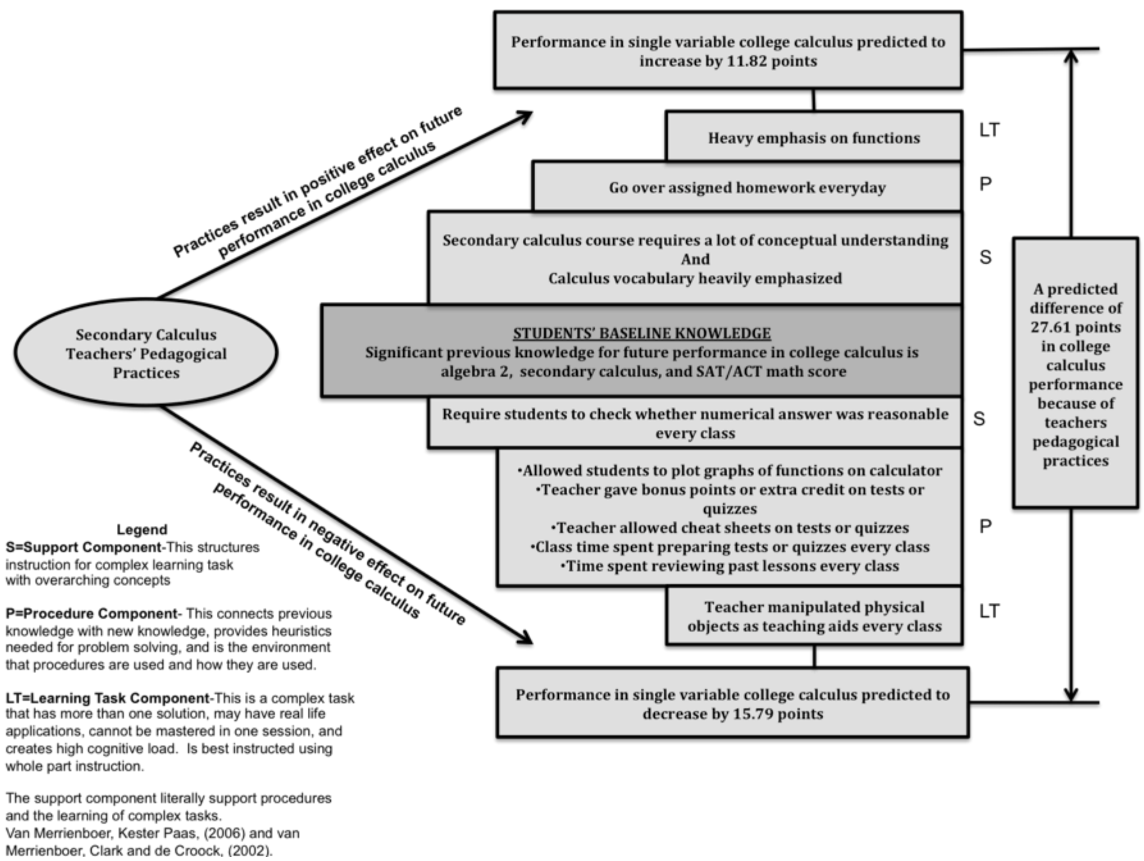

Figure 6.3. The 3C/ID Secondary Calculus Model for College Calculus Performance

since this was a positive predictor of future success in college calculus performance. The small amount of variability captured from teachers' pedagogical practices (3.6 percent) indicates a large predicted difference in points earned in the final college calculus grade.

Effect Size Discussion

The pre-calculus model explained 18.1 percent of the variability in the data from students' that had pre-calculus their senior year, while the calculus model explained 19.3 percent of the variability in the data from students' that had calculus their senior year. The intent was to study teachers' pedagogical practices that transferred from one course to the next course hierarchically, and how these practices were predictive of college calculus success. The 4C/ID model provided a framework for this study. The variability captured explained how gender, student experiences, foundational knowledge, and pedagogical practices predicted future performance in college calculus. When considering exclusively teachers' pedagogical practices, only 3.1 percent of the variability in the pre-calculus data, and 3.6 percent of the variability from the calculus data explained future performance in college calculus. In order to better understand where more of the variability in the data may be captured, 12 variables that were designed to measure student affect were included into both models.

Student Affective Variables and the Pre-Calculus Model

Variables were placed on the FICSMath survey with the intent of measuring student affect concerning their beliefs about learning mathematics. These variables were not originally included into the models because student beliefs about learning mathematics are different than pedagogical practices used to communicate the complex ideas of mathematics in a way that is understandable to others. These variables were added to the pre-calculus model and the significant variables are presented in Table 6.2. Also presented in Table 6.2 are the variables from the pre-calculus model that remained significant predictors of future performance in college calculus. The model presented in

Table 6.2

Pre-Calculus With Student Affective Variables (n=1098)

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Control and Foundational Knowledge Component; n=1,181; Adjusted R^2=0.11						
Gender	-4.02***	0.77	-0.14	0.00	1.00	-----
Grade in pre-calculus	4.10***	0.52	0.21	0.00	4.33	3.86
With Support Component Added; n=1,163; Adjusted R^2=0.12						
Tests or quizzes required new insight and creativity	3.46***	0.92	0.10	0.00	1.00	-----
Teacher highlighted more than one way of solving a problem	-1.06***	0.29	-0.10	0.00	5.00	3.63
With Procedure Component Added; n=1,152; Adjusted R^2=0.13						
Time spent preparing for standardized math exams (linearized)	-0.89**	0.30	-0.08	0.10	5.00	0.92
With Student Affective Variables Added; n=1,098; Adjusted R^2=0.31						
I can do well on exams	8.271***	0.998	0.230	0.00	1.00	-----
I wish I did not have to take math	-2.245**	0.83	-0.07	0.00	1.00	-----
I understand the math I have studied	7.45***	1.12	0.19	0.00	1.00	-----
Math teacher sees you as a math person	1.49***	0.29	0.15	0.00	5.00	3.21

$p<0.01$, **; $p<0.001$, ***

Chapter 5 (and Figure 6.1) changed because some of the pedagogical variables were no longer significant after including the student affect variables. The variables that were no longer significant were: (1) degree to which home environment was supportive of learning math; (2) SAT/ACT math concordant score; (3) grade in algebra 2; and (4) teacher manipulated physical objects during instruction. The significant student affect variables in Table 6.2 may measure student motivation to learn mathematics, anxiety about leaning mathematics, or student efficacy or identity. The model shows that teachers' pedagogical practices, along with gender and grade in pre-calculus, captured 13 percent of the variability in the data. This is 5.1 percent less than the model presented in Chapter 5 because of the variables that dropped out of the model. What should be noticed is the large jump in the adjusted R^2 value when student affect variables were added to the model. The significant student affect variables explained an additional 18 percent of the variability in the pre-calculus data, for a total of 31 percent of the variability explained in the model with all of the components together. The standardized coefficients should be observed since they reveal the variables with the strongest effect on the model since the scales are not the same. Next to grade in pre-calculus, the variables "I can do well on exams" and "I understand the math I have studied" were the strongest predictors of performance in college calculus. This information is only provided to show where additional variability of the data may be captured. The instructional model presented by van Merriënboer et al., (2002); van Merriënboer et al., (2003); and van Merriënboer et al., (2008) does not include components for student motivation. More research should be done relative to the significant student affect variables in order to determine what motivational constructs are represented in the significant affect variables presented in Table 6.2. The intent is not to create a new model that has both, teacher pedagogical practices and student affective variables. The researcher is not a motivation or identity expert, so more research is needed in order to determine how to best design a model that includes teachers pedagogical practices and student affective variables that will be predictive of secondary pre-calculus students' performance in college calculus.

Calculus Model with Affective Variables

The same process described above for the pre-calculus model was also applied to the calculus model from Chapter 5. There was one variable, "time spent going over assigned homework" that was no longer significant after the affective variables were added into the calculus model. Table 6.3 displays the calculus model with the additional variables. The total variability explained in the calculus model increased 10.7 percent because of the significant student affect variables included in the model, which increased the total explained variability to 30 percent. Table 6.3 is presented to display in a concise way the large change in the explained variability that occurs because of adding the significant student affect variables to the model. The parameter estimates are close to being the same as reported in Chapter 5, but different because this research is based on correlations. Removing one variable from the procedure component and adding four significant student affect variables changed the number of respondents that survived the model slightly, as well as the parameter estimates and other reported values. It is interesting that the significant pedagogical practices predictive of college calculus performance for the pre-calculus and calculus models are all different except for the

Table 6.3

Calculus Model Including Control and Foundation Knowledge, Calculus Teachers' Pedagogical Practices, With Student Affective Variables

Variable Description and Name	Parameter Estimate and Significance	Standard Error	Standardized Coefficients	Min	Max	Mean
Control and Foundational Knowledge Component; n=2,125; Adjusted R^2=0.16						
Gender	-2.58***	0.47	-0.11	0.00	1.00	-----
SAT/ACT math score	0.01***	0.00	0.09	200.00	800.00	625.76
Grade in algebra 2	1.69***	0.44	0.08	0.00	4.33	3.65
Grade in secondary calculus course	2.67***	0.35	0.17	0.00	4.33	3.49
With Support Component Added; n=2,081; Adjusted R^2=0.18						
Extent of conceptual understanding	0.40*	0.20	0.04	0.00	5.00	3.68
Emphasis on vocabulary	0.48**	0.17	0.06	0.00	5.00	2.37
Frequency of checking whether numerical answer was reasonable	-0.58**	0.17	-0.07	0.00	5.00	3.35
With Procedure Component Added; n=2,039; Adjusted R^2=0.19						
Allowed to plot graphs of functions	-1.57**	0.58	-0.05	0.00	1.00	-----
Cheat sheets allowed on tests or quizzes	-1.61*	0.64	-0.05	0.00	1.00	-----
Bonus points or extra credit allowed on tests or quizzes	-1.06*	0.43	-0.05	0.00	1.00	-----
Class time spent preparing for class related quizzes or tests (linearized)	-0.44*	0.18	-0.05	0.10	5.00	1.37
Time spent reviewing past lessons (linearized)	-0.33*	0.17	-0.04	0.10	5.00	1.34
With Learning Task Component Added; n=2014; R^2=0.193						
Manipulation of physical objects	-0.79***	0.197	-0.08	0.100	5.000	0.73
Emphasis on functions	0.52*	0.26	0.04	0.000	5.000	3.86
With Student Affective Variables Added; n=1,959; Adjusted R^2=0.30						
I can do well on exams	6.65***	0.73	0.19	0.00	1.00	-----
I wish I did not have to take math	-1.41**	0.52	-0.06	0.00	1.00	-----
I understand the math I have studied	3.59***	0.92	0.08	0.00	1.00	-----
Math teacher sees you as a math person	1.44***	0.19	0.16	0.00	5.00	3.21

common variable “manipulation of physical objects.” However, the same four out of 12 affect variables were significant for both the pre-calculus and the calculus model.

Implications for Instruction

The 4C/ID model has also been used as a framework to consider teachers’ characteristics because teaching secondary mathematics is a complex task. Feldon (2007) stated that when teachers must “meet the needs and behaviors of an entire classroom while also trying to remember and implement a lesson plan” they might experience cognitive overload; such concerns especially exist for novice teachers (p. 123). When teachers gain expertise in the classroom they develop elaborate schemas to process information, which requires less mental effort (Feldon, 2007). Teaching is complex, and it is important that research in mathematics education inform practice in practical and meaningful ways that will help secondary pre-calculus and calculus teachers with the task of making mathematical ideas understandable to students. Therefore considerations of some of the “take away points” for practioners from this study are important. A discussion of some of the negative predictors of future performance in college calculus that seemed contradictory to what may be expected follows.

The variable “teacher highlighted more than one way to solve a problem” was a negative predictor for secondary pre-calculus students’ future performance in college calculus. The NCTM (2010) stated that teachers implement reasoning and sense making in the classroom is by “monitoring student progress toward a solution including reviewing a chosen strategy and other possible strategies generated by *oneself* (the teacher) or others" (italics and parenthesis added) (p. 10). Therefore if the teacher presents various strategies to solve a problem, it is important that the connections are provided for the students concerning why the various strategies work. Otherwise, as cognitive load theory describes, the split attention effect may occur (van Merrienboer & Sweller, 2005). When split attention effect occurs multiple sources of information are not integrated, which causes disjointed understandings to occur instead of one method increasing understanding of the other. Therefore, more consideration from the teacher concerning *why* various methods solve one problem may be beneficial for teachers as they seek to help students make better connections between mathematical ideas.

The variable "Regarding teaching aids, how often did the teacher manipulate physical objects" was a negative predictor of both pre-calculus and calculus students' future performance in college calculus. The NCTM (2010) stated that in order to develop reasoning habits in the classroom teachers should "require students to figure things out for themselves" and "allow students to explore problems further by using models" which indicates that what is important is that *students* manipulate the objects for understanding, not the teachers. If teachers do provide such instruction, it should be as van Merrienboer et al., (2002) stated as "just in time" guidance where the students' need-to-know has been established. It is best if teachers are patient and allow students to struggle, and then when students ask questions teachers can provide "just in time" scaffolding to advance students ability to use models for problem solving (p. 51).

The variable "for problems involving calculation, how often were you required to check whether your numerical answer was reasonable" was a negative predictor of secondary calculus students' future performance in college calculus. Concerning this, the professors made many statements in the phenomenography that addressed the difference between the "process of solving a problem" and the "final answer for a problem." For example: (1) "Make students show their work. Math is about the process as well as the answer. If you cannot see the process, how do you know where the answer came from?" (2) Students need to move away from "getting the right answer" to "learning the correct process" and (3) "Take away their calculators. Students loose the ability to move through the process of solving a problem and when asked what is a reasonable answer they do not know." Therefore, it seems the focus should be on the process of problem solving instead of the final answer. Maybe a better question would be, "How often were you required to check if your problem solving process was reasonable?"

Future Research

The epidemiological research method was described in Chapter 3 as having the power to simultaneously test many independent variables at one time and identify important variables for future research. This method is different from quasi-experimental research designs where the researcher seeks to hold other classroom variables constant, which is a difficult task in the messiness of classroom research. Regardless of the research design, the 16 significant pedagogical variables identified from the large sample of pre-calculus and calculus students provide important information about variables worthy of future research in secondary mathematics education relative to pedagogical practices that are predictive of college calculus performance.

The significant affect variables shown in Table 6.2 and Table 6.3 also reveal variables concerning student beliefs about learning mathematics that should be studied further. The respondents completing the FICSMath survey were in college calculus and answered questions concerning their last high school mathematics class. Future research should investigate if such affective characteristics as "I can do well on exams" and "Math teacher sees you as a math person" transfer from one mathematics class to another. If students believe that they perform well on exams in high school, but then move to college calculus and do poorly on exams, do they have the tenacity to continue, or do they transfer out of a STEM major because of performing poorly on exams in college calculus? Also, what framework should be used that combines teachers' pedagogical practices and students' beliefs about mathematics, especially in light of the additional variability that the affective variables explained in the pre-calculus and calculus models.

One variable from the affect group of questions on the FICSMath survey, that was not a significant predictor of future performance in college calculus, was "Math is relevant to real life." This aligns with the fact that none of the learning task variables presented in Table 5.3 under the heading, "connecting math to real world problems" were significant pedagogical practices predictive of future performance in college calculus. It is possible that the models presented in Chapter 5 may have captured more variability if students perceived mathematics as being connected to real world problems. The constructivist perspective is that no one context can offer real world applications that are meaningful for all students (Boaler, 1993), however, none of the variables that addressed how teachers connect instruction to real world problems were significant predictors of future performance in college calculus. One factor from previous research, which identified how to help prepare students to learn college level STEM content, was to increase the relevance of the course content to real world problems (McKenna et al., 2001). Single variable college calculus is often the first mathematics course required for STEM majors. The fact that real world problems have been the focus in inquiry methods of instruction, yet such pedagogical practices were not predictive of pre-calculus or calculus students future performance in college calculus reveal that further research is needed concerning: (1) what teachers consider as real world problems; (2) what students consider as real world problems; (3) how to write survey questions so students are not interpreting "real world problems" as "just hard word problems" and (4) how to effectively implement real-world problems in the classroom.

The professors and teachers expressed concerns about the broad range of standards in secondary mathematics in the phenomenography. Professors stated that secondary mathematics teachers needed to focus on foundational topics in mathematics, and some stated that secondary mathematics teachers did not need to teach secondary calculus. The NCTM (2010) stated that the new Common Core State Standards (CCSS) provide "fewer and more rigorous standards" with the "goal of increased clarity" that "aligns with college and career expectations" (Slide 6, CSSM_HighSchool_120210v.2(1) ppt). The new high school CCSS provide a "common core-domain" that focuses on "Overarching "big ideas" that connect topics across the grades" (slide 17, CSSM_ HighSchool_120210v.2(1) ppt). The CCSS, like the NCTM standards, stress the importance of a balance between concepts and procedures. These concepts align with the support and procedure components of the 3C/ID Math Models for College Calculus Performance. The procedure component does not stand alone but is supported by the schema formation of concepts in the support component. The paradigm shift in mathematics education from the NCTM mathematics standards to the CCSS provides an opportunity to study if the change in standards effect secondary preparation for college calculus. The structure of the 3C/ID Math Models for College Calculus Performance should be studied further, and the significant variables identified by the two models should be studied further because they correlated with secondary pre-calculus and calculus students' increased future performance in college calculus.

APPENDICES

A. Hard copy FICSMath Survey available only

B. Inter-Rater Reliability for Coding of Statements for Phenomenography

Directions for Coder Reliability

For each response below place a number (1-18, or 19 if you create your own category) AFTER statements within the response indicating the appropriate category for the statement. If you believe the response only addresses one category then place the appropriate number after the entire response.

Number	Category	Description of Category
1	Support for learning mathematics	Spending time outside of regular class to help students learn course content, encouraging students to study in groups outside of class.
2	Assignments and assessments	Type of assignments (e.g. from informal to formal assignments) and what professors believe teachers should expect from students, including AP Calculus content, standardized tests, and how assignments are assessed (or should be assessed) by teachers.
3	Calculators	How teachers allow calculators to be used in class, how teachers use other technology in class
4	Classroom Environment	Whole class, small group, and individualized instruction in class; student reasoning and communication about mathematics in class
5	Conceptual Understanding	Teaching for the understanding of concepts and emphasizing mathematical reasoning during instruction, focusing on the process instead of the just the right answer.
6	Real World Problems	Hands on activities, real-world applications, teachers using models of motion, area, and volume, discovery learning
7	Memorization	Rote instruction or using methods to enforce memorization of formulas, focus on memorization instead of conceptual understanding
8	Multiple Representations	Demonstrating multiple methods of instruction such as teaching analytically, algebraically, and graphically
9	Problem Solving	Types of problems provided to students, and how problem solving is presented to students
10	Review	What teachers do to help students remember mathematics previously covered, or previously learned
11	Student Motivation	What teachers do to motivate the learning of mathematics

12	Textbooks	How textbooks and supplementary materials are used in the class
13	Vocabulary	The correct use of mathematical terms during instruction
14	Algebra	Statements relative to algebra content, students performance with algebra content, or what algebraic content should be covered
15	Calculus	Statements relative to AP or non-AP calculus content, students performance with calculus, or AP Calculus exams and AP Calculus expectations
16	Geometry	Statements relative to Geometry content, students performance with Geometry, or lack of geometric focus
17	Pre-Calculus	Statements relative to pre-calculus content, students performance with pre-calculus, or lack of focus on correct instruction of content
18	Proofs	Statements relative to Proofs, students performance with proofs, or proofs/logic that teachers need to teach
19	Other	Please specify

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