5-2013

Pair Production By Primordial Black Hole Evaporation

Shaun Hampton
Clemson University, sdhampt@g.clemson.edu

Pair Production by Primordial Black Hole Evaporation

A Masters Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Masters of Science
Physics

---

by
Shaun Hampton
May 2013

Accepted by:
Dr. Dieter Hartmann, Committee Chair
Dr. Mark Leising
Dr. Catalina Marinescu

Abstract

Here we investigate the evaporation of a 8.2×10^{10} kg primordial black hole. This mass is shown to satifisfy the compactness parameter constraint $L/R \geq 10^{31} ergscm^{-1}s^{-1}$. We assume only photon-photon production according to a Planck distribution. We calculate the distance from the black hole at which the optical depth due to photon-photon collisions to produce positron/electron pairs becomes unity in which a pair plasma is produced within a volume of inner radius of $90R_S$ and outer radius $123000R_S$ respectively. We then calculate corresponding positron/electron production rates, production rate densities, and optical depth rates due to subsequent Compton scattering by photons. We quantitatively investigate annihilation rate densities and number of annihilations for number densities generated by an initially static fireball that after a time $\triangle t_L = 5 \times 10^{-20}s$ is allowed to propagate radially at the speed of light. We show that the annihilation rate per particle is given by $< \sigma v >= 1.2 \times 10^{-39} m^3 s^{-1}$ where we approximate $\cos\theta \approx 1$ between positron and electron collisions and derive a probability distribution function by Monte Carlo methods for positron and electron velocities sourced by field and target photons emitted by the black hole given by the approximation $\gamma_+ = \gamma - \approx \frac{E+\epsilon}{2m_e c^2}$ whose corresponding velocities are directed radially. We show that no annihilations occur within the expanding fireball and that electrons and positrons freely stream from the proximity of the BH. We investigate the spectra produced by different mass (thermal energy) BH's that are currently evaporating. We analyze their limiting behavior and compare with blackbody emission from stars. We then discuss detection methods and limitations from possible gamma ray and positron sources including the 511 keV line emission from the galactic center, high energy cosmic ray positron production, and direct gamma ray burst events.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Primordial black hole evaporation is a process of interest to astrophysicists, particle physicists, astonomers, and cosmologist alike. They are of great importance because the different stages of evaporation closely resemble the different epochs after the big bang such as the planck epoch, quark epoch, among others and discovering their existence would provide a theoretical laboratory for which to study the early universe. There is also a direct correspondance between black hole physics and classical thermodynamics and confirmation of their existance would corroborate the theoretical prediction that BH's have a characteristic temperature and can indeed be described thermodynamically. From their evaporation process BH's emit successively higher energy radiation as well as various particle species due to the blueshift of their blackbody spectrum. We will discuss a particular model that will enable us to predict an emission spectrum. The model is constrained however. We hypothesize that the evaporating black hole, at a certain mass, will produce an optically thick positron/electron fireball surrounding the black hole. Once produced we shoe that the fireball expands freely at approximately the speed of light with freely streaming positrons and electrons.

There will be a brief background discussion regarding the theory of black holes, black hole evaporation and optical depth. We then present our fireball model to provide a description of the emission process of an evaporating black hole. We will then present three black holes that are currently 1 MeV, 500 MeV, and 1000 MeV sources and discuss the emission spectrum that would emerge by only considering black hole evaporation and no secondary processes. Lastly we will discuss possible methods by which to detect the presence of these objects such as through the 511 keV line observed from the galactic center originating from positron annihilation, the detection of high energy cosmic ray particles which include positrons, and the detection of gamma ray transient events directly emitted from the black hole.

Chapter 2

Background

2.1 Black Hole Physics and Black Hole Evaporation

In 1975 Stephen Hawking published a paper in Communications of Mathematical Physics describing a theoretical framework in which black holes were found to behave as blackbody emitters, possessing a characteristic temperature as well as emitting particles from the event horizon. Therefore black holes were shown to evaporate away and the emerging radiation was labeled Hawking radiation. This evaporation mechanism is a direct consequence of the strong gravitational field at the event horizon "pulling" virtual particle pairs out of the vacuum where the negative mass particle would fall in, decreasing the size of the BH, and the positive mass particle would escape to infinity becoming real. From this theoretical construct Hawking was able to show that the temperature of a Schwarzschild black hole (Hawking 1975) is given by

$$T = \frac{\hbar c^3}{8\pi G M k_B} \tag{2.1a}$$

$$= 6.2 \times 10^{-8} \left(\frac{M_{\odot}}{M}\right) K \tag{2.1b}$$

where a Schwarzschild geometry was used. This geometry can be described by the second fundamental form which is given by

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right) \tag{2.2}$$

where the covariant form of the metric tensor, $g_{\mu\nu}$, is expressed as a 4x4 matrix given as

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{2GM}{c^2}\right) & 0 & 0 & 0 \\ 0 & -\left(1 - \frac{2GM}{c^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 sin^2\theta \end{pmatrix}. \tag{2.3}$$

Therefore to calculate the power emitted by the black hole we use the Stefan-Boltzmann power law equation

$$P = \sigma_B A T^4. \tag{2.4}$$

Inserting equation (2.1) into equation (2.4), taking $R = \frac{2GM}{c^2}$, and $\sigma_B = \frac{2\pi^5 k_B^4}{15h^3c^2}$ (Stefan-Boltzmann constant), we obtain the following equation for the power emitted by an evaporating black hole,

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}. \tag{2.5}$$

Alternatively the power can also be written as $-c^2\frac{dM}{dt}$ indicating that the power emitted is equal to the rate of energy loss by the black hole. Setting this equation equal to equation (2.5) we integrate both sides and obtain the equation describing the mass of the black hole as a function of time given by

$$M(t) = \left(M_0^3 - \frac{3\hbar c^4 t}{15360\pi G^2}\right)^{1/3} \tag{2.6a}$$

$$= \left(M_0^3 - \frac{3\omega\hbar c^4 t}{G^2}\right)^{1/3} \tag{2.6b}$$

where $\omega = 2.07 \times 10^{-5}$. (Page 1976) showed that for a BH of initial mass $5 \times 10^{14}g << M << 10^{17}g$ that $\alpha = 3.6 \times 10^{-4}$ (Hawking 1975) where alpha is defined in the modified mass equation

$$M(t) = \left(M_0^3 - \frac{3\alpha\hbar c^4 t}{G^2}\right)^{1/3}. \tag{2.7}$$

originating from the modified mass loss equation, $-c^2\frac{dM}{dt} \approx \alpha\frac{\hbar c^6}{G^2M^2}$ This alteration is made because it accounts for the emission of mostly massless particles and ultrarelativistic positrons and electrons consistent with the actual species that is emitted within the mass range mentioned above. From this modification of equation (2.6) we obtain that the initial mass of a black hole that would be evaporating today is given by

$$M_0 = \left(\frac{3\alpha\hbar c^4 t_H}{G^2}\right)^{1/3} \tag{2.8a}$$

$$\approx 5\times 10^{11} kg \tag{2.8b}$$

where $t_H \approx 13.7Gyr$ (Spergel et. al. 2003). While we only utilize a Schwarzschild geometry for our analysis it should be noted that black hole evaporation can extend to rotating (Kerr) and charged-rotating (Kerr-Newmann) BH's where the general formula for temperature is given by

$$T = \frac{\hbar\kappa}{2\pi k_B c} \tag{2.9}$$

where

$$\kappa = \frac{4\pi c^2\left(r_+ - GM\right)}{A} \tag{2.10}$$

$$r_+ = \frac{4\pi\left(GM + \sqrt{G^2M^2 - \frac{J^2c^2}{M^2} - \frac{GQ^2}{4\pi\epsilon_0}}\right)}{c^2} \tag{2.11}$$

and

$$A = \frac{4\pi G\left(2GM^2 - \frac{Q^2}{4\pi\varepsilon_0} + 2\sqrt{M^4G^2 - J^2c^2 - \frac{GQ^2M^2}{4\pi\varepsilon_0}}\right)}{c^4}. \tag{2.12}$$

A represents the horizon area, Q, the charge of the BH, and J the angular momentum of the BH. κ represents the surface gravity at the event horizon and is closely related to the corresponding temperature. The term r_+ represents the outer radius of a rotating charged black hole which is the radius that encapsulates the ergosphere as well as the event horizon which is given by r_-. The region between the inner and outer radius is called the ergosphere. There are several interesting properties defined within this region that I will only mention such as frame dragging and the possibility of energy extraction from the black hole. If we take $Q, J \to 0$ then the temperature expression given in equation (2.1) is recovered.

2.2 Optical Depth

Pertinent to our dicussion is an understanding of optical depth which plays a critical role in determining the effective transport of photons through a given medium. Optical depth is a dimensionless number that measures the opacity of that medium. More specifically, it measures the likliehood that a photon moving through that medium will be absorbed, scattered, or a combination of the two. Let us consider the transfer equation for absorption. Optical depth is measured along the line of sight of a traveling photon. It is given by

$$\tau_\nu = \int \alpha_\nu ds \tag{2.13}$$

where ds represents the infinitesimal length element along the line of sight and $\alpha_\nu = n\sigma_\nu$ the absorption coefficient defined by multiplying the number density of particles within that medium by the corresponding characteristic interaction cross-section between the medium and the photon. Dimensional arguments suggest that $[n] * [\sigma_\nu] = \frac{1}{m^3} * m^2 = \frac{1}{m}$ indicating that α_ν behaves inversely proportional to length and therefore it's inverse representing the mean free path. Thus a large optical depth would produce a small mean free path, a small distance that a photon could propagate freely without absorption. By summing up the contribution of α_ν along the line of sight we obtain equation (2.13), the optical depth. Therefore the probability of propagation of a photon through a medium of given length is given by $e^{-\tau_\nu}$. This can be shown by taking the radiative transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu \tag{2.14}$$

and integrating both sides to obtain

$$I_\nu = I_0 e^{-\tau_\nu}. \tag{2.15}$$

Thus an optical depth of zero will leave the intensity of the beam unchanged as it passes through the given medium. As the optical depth increases within the medium the intensity decreases rapidly. The equation becomes increasingly complicated as we include emissivity and scattering.

Chapter 3

Fireball Model

3.1 Model Development

Due to the complexity of the black hole evaporation process and the numerous particle species produced exact photon emission is not easily elucidated. Because of the range of particles of which evaporating BH's can produce many interactions must be considered. In order to fully study and understand this phenomena it would be necessary to incorporate many scattering processes for which QCD and QED are necessary. Furthermore quantum gravitational effects must be considered at late times within the lifetime of the black hole evaporation process due to the planck scale size of which the black hole reaches.

For a realistic yet simplified model of photon emission we seek to calculate the optical depth within the vicinity of the black hole. We first present the simplest scenario. We assume that the black hole is a blackbody emitter. Secondly we assume an isotropic and homogeneous radiation field produced by the black hole. Thirdly we assume only photon production.

It is well known that the total photon number density of an isotropic and homogeneous radiation field is given by

$$n_\gamma = \left(\frac{16\varsigma(3)\pi k_B^3}{c^3 h^3} \right) T^3 \tag{3.1}$$

where $\varsigma(3) = 1.202$. We normalize the photon number density to that of the CMB which is 411 $\frac{\gamma}{cm^3}$ giving us the following expression

$$n_\gamma = 411 * \left(\frac{T(M)}{T_{CMB}} \right)^3 \tag{3.2}$$

where $T_{CMB} = 2.7K$. Assuming a Thompson scattering cross-section for photon-photon interaction we can calculate the optical depth as follows

$$\tau = R_s \int_1^x \sigma_T n_\gamma dx^{'} \tag{3.3}$$

where we have normalized the distance r, to the Schwarzchild radius defined by variable $x = \frac{r}{R_s}$. Integrating and solving for distance we obtain

$$x = \left(1 + \tau \left(\frac{1}{411 \times \left(\frac{R_s(M)\sigma_T}{10^{-6}m^3}\right)}\right)\left(\frac{T_{CMB}}{T(M)}\right)^3\right). \tag{3.4}$$

In this model we are interested in the distance from the black hole at which the optical depth becomes order unity. For a primordial black of mass $M_H = 5 \times 10^{11} kg$ where we define M_H as the Hubble mass, the mass of a black hole that requires the passage of a hubble time for complete evaporation, $T_H = 2.5 \times 10^{11} K$, and $R_H = 7.4 \times 10^{-16} m$, we obtain a distance approximately $1R_S$ at which the positron electron plasma reaches an optical depth of one. Therefore the environment immediately becomes opaque. The initial temperature of a our evaporating black hole is much too high to allow for propagation of created photons to infinity due to the high opacity very near the event horizon as well as possible creation of more massive particles. As the temperature increases the mass decreases further increasing the temperature which further decreases the mass producing a runaway effect. Near the end of its lifetime the temperatures and opacity conditions would be extreme and the physics complicated. An understanding of quantum gravity would be necessary to accurately describe the photon emission we would see.

Now we formulate a still simple, but more realistic model of the optical depth calculation. We still assume, of course, that our black hole is a blackbody emitter and that the BH only produces photons but consider an isotropic radiation field generated by a spherical surface where the photon number density is now position dependent which is more characteristic of the system in consideration. The intensity produced by a blackbody is given by Planck's Law (Lightman et. al. 1972)

$$B_\nu(\nu, T(M)) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\beta(M)h\nu} - 1} \tag{3.5}$$

where $T = \frac{\hbar c^3}{8\pi G M k_B}$ is the temperature of a Schwarzschild black hole and $\beta_{T,bh}(M) = \frac{1}{k_B T(M)}$. From Planck's Law we calculate the luminosity as follows. We first calculate the flux of a radiating spherical surface where θ_c is the critical angle at which photons emitted beyond this angle from the surface are no longer able to propagate to the point at which the flux is measured described by distance r. The flux per unit frequency

(Lightman et. al. 1972) is given by

$$F_\nu(M,\nu) = \int_0^{2\pi}\int_0^{\theta_c} B_\nu(M,\nu) cos\theta d\Omega \tag{3.6a}$$

$$= B_\nu(M,\nu)\pi\frac{R^2}{r^2} \tag{3.6b}$$

where r defines the radial distance from the surface of the emitting object to some point. This enables us to calculate the luminosity per unit frequency as

$$L_\nu(M,\nu) = \int F_\nu(M,\nu) dA \tag{3.7}$$

which can be rewritten as

$$L_\nu(M,\nu) = \int \pi R^2 B_\nu(M,\nu) d\Omega \tag{3.8a}$$

$$= 4\pi^2 B_\nu(M,\nu) R^2. \tag{3.8b}$$

We rewrite the luminosity equation per unit energy as

$$L_\epsilon(\epsilon, M) = \frac{4\pi^2 R^2 B_\nu(\epsilon, M)}{h} = \frac{32\pi^2 G^2 M^2}{h^3 c^6}\frac{\epsilon^3}{e^{\beta_{bh}\epsilon}-1}. \tag{3.9a}$$

When considering photon emission from an evaporating black hole we must consider the opacity near the horizon. At a certain compactness (Goodman 1986), given by

$$\frac{L}{R} \geq 10^{31}\frac{ergs}{cm \times s} \tag{3.10}$$

where $L = \int_{>1MeV}^{\infty} L_\epsilon(\epsilon, M) d\epsilon$ which is the total luminosity above 1 MeV and $R = \frac{2GM}{c^2}$ is the Schwarzschild radius of the black hole, the emitted photons begin producing positron-electron pairs through photon-photon collisions. In order to acquire the mass of the black hole at which this opacity condition is satisfied we must calculate the total luminosity, L, above 1 MeV divided by the Schwarzschild radius. This quantity is known as the compactness parameter. Inserting equation (3.9a) into equation (3.10) we obtain the following expression

$$\frac{L}{R} = \int_{1MeV}^{\infty} \frac{16\pi^2\epsilon^3 GM}{h^3c^4}\frac{1}{e^{\epsilon\beta_{bh}(M)}-1} d\epsilon. \tag{3.11}$$

We define

$$x = \epsilon\beta_{bh}(M) \quad y = \frac{1}{\beta_{bh}(M) m_e c^2} \tag{3.12}$$

and upon substitution and conversion to $(\frac{ergs}{cm \times s})$we obtain the following integral.

$$\frac{L}{R} = \frac{1.6 \times 10^4 \pi^2 \left(m_e c^2\right)^4 GMy^4}{c^4 h^3} \int\limits_{x=2/y}^{\infty} \frac{x'^3}{e^{x'}-1} dx' \tag{3.13a}$$

$$= 2.02 \times 10^8 My^4 \int\limits_{x_{1MeV}=2/y}^{\infty} \frac{x'^3}{e^{x'}-1} dx' \approx 10^{31} \left(\frac{ergs}{cm \times s}\right) \tag{3.13b}$$

Upon numerical integration the mass, M, at which the following relation is satisfied is $M \approx 1.3 \times 10^{11} kg$ $\left(9.5 \times 10^{11} K, 82 MeV\right)$. This condition is condsidered the threshold at which photons produced by the black hole begin forming positron/electron pairs and is called the compactness parameter. Further investigating the properties of this fireball we seek to calculate its optical depth. As mentioned we assume an isotropic radiation field created by the evaporating black hole in which a target photon will interact. The energy density per unit energy of the radiation field (Lightman et. al. 1972) is given by

$$u_{\gamma_\epsilon} = \frac{4F_\nu}{hc}. \tag{3.14}$$

Then inserting equation (3.6b) into equation (3.14) we obtain

$$u_{\gamma_\epsilon}(\epsilon, M, r) = \frac{32\pi G^2 M^2}{c^7 r^2} \frac{\epsilon^3}{h^3} \frac{1}{e^{\beta_{bh}(M) h\nu} - 1}. \tag{3.15}$$

From the isotropic energy density we calculate the number density of photons per unit energy by the following relation

$$\int u_{\gamma_\epsilon}(\epsilon, M, r) d\epsilon = \int n_{\gamma_\epsilon}(\epsilon, M, r) \epsilon d\epsilon. \tag{3.16}$$

By equating the integrands and inserting equation (3.15) we obtain the expression for the number density of photons per unit energy characterizing the radiation field produced by the evaporating BH

$$n_{\gamma\epsilon, bh}(\epsilon, M, r) = \frac{32\pi G^2 M^2}{c^7 r^2} \frac{\epsilon^2}{h^3} \frac{1}{e^{\beta_{bh}(M)\epsilon} - 1}. \tag{3.17}$$

We also consider the presence of the CMB radiation field as well which possesses a photon number density per unit energy given by

$$n_{\gamma\epsilon, cmb}(\epsilon, T) = \frac{8\pi}{c^3 h^3} \frac{\epsilon^2}{e^{\beta_{cmb}\epsilon} - 1}. \tag{3.18}$$

The contribution by the CMB radiation field is negligble to the total photon number density within the vicinity of the BH where the radiation field created by the black hole dominates. Only at large distances from the event horizon is the CMB radiation field relevant.

3.2 Fireball Production Using Minkowski Metric

We now describe the interaction that will take place within the radiation field descibed by both photon number densites. Let us consider a target photon with energy, E, produced by the evaporating black hole that travels within the total radiation field. The target photon interacts with a field photon, ϵ, at an angle theta. We write the four momentum of the system as

$$p^{\mu} = \left(p^0, p^x, p^y, p^z\right) = \left(\frac{E+\epsilon}{c}, \frac{E+\epsilon cos\theta}{c}, 0, \frac{\epsilon sin\theta}{c}\right). \tag{3.19}$$

Although our spacetime is described by a Schwarzschild geometry we analyze a simpler case by applying a Minkowski geometry to our collision process described by the metric $\eta_{\mu\nu}$ represented by a 4×4 matrix as

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.20}$$

To obtain the minimum energy, ϵ_1, in which a field photon must possess in order to produce a positron-electron pair we take the invariant product of the four momentum with itself

$$p_{\mu}p^{\mu} = \eta_{\mu\nu}p^{\mu}p^{\nu} = 4E_{cm}^2 \Rightarrow \epsilon_1 = \frac{2m^2c^4}{E(1-cos\theta)} \tag{3.21}$$

where E_{cm} represents the total energy of the electron (or positron) within the center of momentum frame, the frame in which the rest energy is equal to the total energy of the system and $\epsilon_1(E,\theta)$ represents the minimum photon energy needed to produce an electron (or positron). This calculation is performed in detail within the Appendix. We also present the interaction cross section for photon-photon collisions to produce positron/electron pairs given by (Stecker 1996),

$$\sigma[E,\epsilon,x] = \frac{3}{16}\sigma_T\left(1-\beta^2\right)\left[2\beta\left(\beta^2-2\right)+\left(3-\beta^4\right)ln\left(\frac{1+\beta}{1-\beta}\right)\right] = \frac{3}{16}\sigma_T C(\beta) \tag{3.22}$$

where $\beta = \left(1 - \frac{2\left(m_e c^2\right)^2}{E\epsilon(1-cos\theta)}\right)^{1/2}$ represents the electron (and positron) velocity in the c.m. frame. which is also calculated within the Appendix. The classical electron radius is defined as $r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2}$. The formula for optical depth is given by

$$\tau(E) = \int n_{\gamma\epsilon,total}(\epsilon, M, r)\sigma[E, \epsilon, \theta]d\epsilon dr d\Omega \tag{3.23}$$

where $n_{\gamma\epsilon,total}(\epsilon, M, r) = n_{\gamma\epsilon,BH}(\epsilon, M, r) + n_{\gamma\epsilon,CMB}(\epsilon)$ where we make the coordinate label substitution taking $x \to r$. Now inserting equations (3.17) , (3.18), and (3.22) into equation (3.23) we obtain the following expression

$$\tau(E) = \frac{3}{16}\sigma_T \int_{R_s}^{r} dr \int_0^{2\pi} d\phi \int_0^{\pi} sin\theta d\theta \int_{\frac{2m_e c^2}{E(1-cos\theta)}}^{\infty} \left(n_{\gamma\epsilon,BH} + n_{\gamma\epsilon,CMB}\right) C(\beta) d\epsilon. \tag{3.24}$$

Upon simplification and normalization of integration variables taking $f = \beta_{bh}(M)\epsilon$, $g = (1 - cos\theta)$, and $h = \frac{r}{\frac{2GM}{c^2}}$, and integrating out the azimuthal term due to axial symmetry we obtain two integral expressions, the first describing the contribution to the optical depth by the BH radiation field while the second describing the contribution by the CMB radiation field. These expressions are given as

$$\tau_{BH}(E) = \tau_{0,BH} \int_1^h \frac{1}{h'^2} dh' \int_0^2 dg \int_{\frac{2(m_e c^2)^2}{EkT(M)_{bh} g}}^{\infty} \frac{f^2}{e^f - 1} C(\beta) df \tag{3.25}$$

$$\tau_{CMB}(E) = \tau_{0,CMB} \int_1^h dh' \int_0^2 dg \int_{\frac{2(m_e c^2)^2}{EkT_{cmb} g}}^{\infty} \frac{f^2}{e^f - 1} C(\beta) df \tag{3.26}$$

where $\tau(M)_{0,bh} = \frac{3\pi^2 \sigma_T G^2 M_H^2}{c^7 h^3 R_H} \left(\frac{M_H}{M}\right) \left(\frac{R_H}{R_S(M)}\right) (kT_H)^3 = 7.28 \times 10^{-3} \left(\frac{M_H}{M}\right)^2$, $\tau(M)_{0,cmb} = \frac{6\sigma_T \pi^2 k^3 T_{cmb}^3 GM}{c^3 h^3} = 9.6 \times 10^{-36} \left(\frac{M}{M_H}\right)$,where $\sigma_T = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-29}\ m^2, r_e = 2.8 \times 10^{-15} m$, $kT_H \approx 21$ MeV, and $R_S = \frac{2GM}{c^2} = 1.2 \times 10^{-16} m$. It is evident that the CMB radiation field provides negligible contribution to the optical depth as was stated previously when analyzing the respective photon number densities within the system. Performing numerical integration to evaluate the integral over the photon field energies and angle for a range of target photon energies we obtain the following expression for optical depth,

$$\begin{aligned} \tau_{total} &= \tau_{CMB}(E) + \tau_{BH}(E) \\ &= v(E, M)\tau_{0,CMB}(M) \int_1^{h(M)} h' dh' + w(E, M)\tau_{0,CMB}(M) \int_1^{h(M)} \frac{1}{h'^2} dh' \end{aligned} \tag{3.27}$$

where h is the distance from the event horizon to a field point in units of Schwarzchild radii, $v(E,M) = \int_{\frac{2(m_e c^2)^2}{E k_B T_{cmb} g}}^{\infty} \frac{f^2}{e^f - 1} C(\beta) df$ and $w(E,M) = \int_{\frac{2(m_e c^2)^2}{E k T_{bh} g}}^{\infty} \frac{f^2}{e^f - 1} C(\beta) df$. We then integrate over h obtaining

$$\tau_{total} = v(E,M)\tau(M)_{0,cmb}(h-1) + w(E,M)\tau(M)_{0,bh}\left(1 - \frac{1}{h}\right). \tag{3.28}$$

Using an initial mass of $M = 1.3 \times 10^{11} kg$ failed to provide an optical depth of unity irrespective of distance from the BH or of the target photon energy emitted. We then allowed the BH to evaporate governed by equation (2.6) until the optical depth could reach a value of unity for any given target photon energy and distance from the black hole. This occurred at $M = 8.2 \times 10^{10} kg$. We then calculated h(M), the distance from the event horizon at a given target photon energy, where $\tau \geq 1$. Having found the inner and outer radial distances for which $\tau = 1$ we then calculate the rate of positron/electron production utilizing equation (3.25) where the lower and upper integration limits were calculated to be, respectively, $h_1 \approx 90$ and $h_2 \approx 123000$ which imply that each target photon of energy E "sees" a constant optical depth as it propagates through the shell and will produce a homogeneous shell of positrons and electrons which is a simplifying assumption,

$$\frac{dN_{E,\pm}}{dt}(E, M(t)) = 2 \times \frac{L_E(E,M)}{E}\left(1 - e^{-\tau(E(f))}\right) \tag{3.29a}$$

$$= 2 \times \frac{32\pi^2 G^2 M^2}{h^3 c^6} \frac{E^2}{e^{\beta_{bh}(M)E} - 1}(1 - e^{-\tau(E(f))}) \tag{3.29b}$$

$$\Rightarrow \frac{dN_{\pm}}{dt} = \int_0^{\infty} N_{E,\pm}(E, M(t)) dE \tag{3.29c}$$

$$= \left(7.9 \times 10^{20} s^{-1}\right) \int_0^{\infty} \frac{f^2}{e^f - 1}(1 - e^{-\tau(E(f))}) df \tag{3.29d}$$

$$\approx 2.3 \times 10^{16} s^{-1} \tag{3.29e}$$

where the probability of a positron/electron pair being generated is given by $(1 - e^{-\tau(E)})$ and the factor of two accounts for total particles created. This gives us the total number of positron/electron pairs that were created within the specified energy range. We are also able to calculate number density of positron/electron pairs per unit time. While calculating target photon energy ranges that produced optical depths of unity the initial fireball thickness was calculated as well. This provided a volume given by

$$V_{\pm}(t_e = 0) \approx \frac{4}{3}\pi R_S^3 \left(123000^3 - 90^3\right) \tag{3.30a}$$

$$= 1.4 \times 10^{-32} m^3. \tag{3.30b}$$

where the outer and inner shell radii corresponding to the region of $\tau > 1$ are $R_1 = h_1 R_S \approx 90 R_S$ and $R_2 \approx h_2 R_S = 123000 R_S$, respectively. Dividing the positron/electron production rate given in equation (3.29e) by our fireball volume we obtain

$$\frac{dn_{\pm,E}}{dt} = \frac{dN_{\pm,E}}{dt} V_{\pm}^{-1} \tag{3.31}$$

and therefore

$$\frac{dn_{\pm}}{dt} = \int_0^{\infty} \frac{dn_{\pm,E}}{dt} dE \tag{3.32a}$$

$$= \int_0^{\infty} \frac{dN_{E,\pm}}{dt} V_{\pm}^{-1} dE \tag{3.32b}$$

Inserting equations (3.29e) and (3.30b) into equation (3.32b) yields

$$\frac{dn_{\pm}}{dt} \approx 2.5 \times 10^{48} m^{-3} s^{-1} \tag{3.33a}$$

$$= 2.5 \times 10^{42} cm^{-3} s^{-1}. \tag{3.33b}$$

Utilizing our production rate density above given in equation (3.32b), assuming that we produce an initially static positron/electron fireball, and only accounting for Compton scattering we calculate the optical depth rate of change given by

$$\frac{d\tau}{dt} = \int_{R_1}^{R_2} dh' \int_0^{\infty} \frac{dn_{\pm,E}}{dt} \sigma_{KN}(E) dE \tag{3.34a}$$

$$= R_S(123000 - 90) \int_0^{\infty} \frac{dn_{\pm,E}}{dt} \sigma_{KN}(E) dE \tag{3.34b}$$

$$= \left(7.3 \times 10^{-12} m\right) \left(\frac{M}{M_H}\right) \int_0^{\infty} \frac{dN_{\pm,E}}{dt} V_{\pm}^{-1} \sigma_{KN}(E) dE \tag{3.34c}$$

$$= 7.9 \times 10^{29} \int_0^{\infty} \frac{1}{(123000^3 - 90^3)} \frac{f^2}{e^f - 1} (1 - e^{-\tau(f(E))}) T\left(f(E)\right) df \tag{3.34d}$$

and after numerical integration we obtain

$$\frac{d\tau}{dt} \approx 3.7 \times 10^{7} s^{-1} \tag{3.35}$$

where the relativistic Klein-Nishina cross-section given below was used (Jauch and Rohrlic 1959).

$$\sigma(E) = 2\pi r_e^2 \left\{ \frac{1+\frac{E}{m_e c^2}}{\left(\frac{E}{m_e c^2}\right)^3} \left[\frac{2\frac{E}{m_e c^2}\left(1+\frac{E}{m_e c^2}\right)}{1+2\frac{E}{m_e c^2}} - ln\left(1+2\frac{E}{m_e c^2}\right)\right] + \frac{ln\left(1+2\frac{E}{m_e c^2}\right)}{\frac{2E}{m_e c^2}} - \frac{1+3\frac{E}{m_e c^2}}{\left(1+2\frac{E}{m_e c^2}\right)^2} \right\} \tag{3.36a}$$

$$= \frac{3}{4}\sigma_T T(E). \tag{3.36b}$$

Therefore after one second has expired $\tau \approx 3.7 \times 10^7 \gg 1$.

3.3 Fireball Expansion Time to Reach $\tau = 1$

If we allow our positron/electron fireball to expand radially we can calculate the expansion time at which the optical depth reaches unity. We assume expansion at the speed of light and we modify equation (3.30) yielding a positron/electron fireball volume as a function of expansion time, t_e given by

$$V_{\pm}(t_e) = \frac{4}{3}\pi R_S^3 \left(\left(123000 + \frac{ct_e}{R_S(M)}\right)^3 - \left(90 + \frac{ct_e}{R_S(M)}\right)^3 \right). \tag{3.37}$$

We are interested in the expansion time after which the fireball reaches an optical depth of unity and thus we seek to satisfy the following equation

$$\frac{d\tau}{dt}\Delta t_L = 1 \tag{3.38}$$

where $\triangle t_L$ represents the positron/electron loading time which is the crossing time for which it takes the photons to propagate the length of shell thickness given by $R_2 - R_1 = (123000 - 90)\, R_S = 1.5 \times 10^{-11} m$. This expansion volume will be beneficial in subsequent calculations of number of positron/electron annihilations. Taking $\triangle t_L = \frac{(R_2 - R1)}{c} = 5 \times 10^{-20} s$ we note that the optical depth only achieves a value of $\tau = \frac{d\tau}{dt}\Delta t = \left(3.7 \times 10^7 s^{-1}\right) \times \left(5 \times 10^{-20} s\right) = 1.8 \times 10^{-12}$ before radial expanding at roughly the speed of light. Therefore the optical depth due to Compton scattering never exceeds unity but in fact remains much less than that allowing for subsequently emitted photons as well as those trapped briefly within the positron/electron fireball to stream freely towards infinity if we consider no other processes. If we consider a static positron/electron fireball for $\Delta t_L = 1s$ we can solve for the expansion time to reach optical depth of unity by inserting equation

(3.37) into equation (3.34c) and that expression into equation (3.38) where we obtain

$$1 = \left(7.9 \times 10^{29} \int_0^\infty \frac{1}{\left(\left(123000 + \frac{ct_e}{R_S}\right)^3 - \left(90 + \frac{ct_e}{R_s}\right)^3\right)} \frac{f^2}{e^f - 1}(1 - e^{-\tau(f(E))})T\left(f(E)\right) df\right) \times \left(\frac{\triangle t_L}{s}\right). \tag{3.39}$$

Upon rearrangement we obtain

$$\left(123000 + \frac{ct_e}{R_S(M)}\right)^3 - \left(90 + \frac{ct_e}{R_s(M)}\right)^3 = \left(7.9 \times 10^{29} \int_0^\infty \frac{f^2}{e^f - 1}(1 - e^{-\tau(f(E))})T\left(f(E)\right) df\right) \times \frac{\triangle t_L}{s}. \tag{3.40}$$

The the term within parenthesis on the RHS of equation (3.40) is simply equal to the product between equation (3.33d) and $\left(123000^3 - 90^3\right)$. Therefore

$$LHS = (123000^3 - 90^3) \times \frac{d\tau}{dt}\triangle t_L \tag{3.41a}$$

$$\approx \left(3.7 \times 10^7 s^{-1} \times \left(123000^3 - 90^3\right) \times 1s\right) \tag{3.41b}$$

$$= 6.9 \times 10^{22} \tag{3.41c}$$

where $\Delta t_L = 1$. Upon inserting the LHS value into equation (3.40) and solving the cubic equation we notice that terms of $O(t_e^3)$ cancel and thus utilize the quadratic formula to obtain solutions to the expansion time, t_e, given by

$$t_{e_{+,-}} = \frac{-\frac{3c}{R_S(M)}\left(123000^2 - 90^2\right)}{6\left(\frac{c}{R_S(M)}\right)^2(123000 - 90)} \tag{3.42a}$$

$$\pm \frac{\sqrt{\left[\frac{3c}{R_S(M)}\left(123000^2 - 90^2\right)\right]^2 - 12\left(\frac{c}{R_S(M)}\right)^2(123000 - 90)\left(123000^3 - 90^3 - 6.9 \times 10^{22}\right)}}{6\left(\frac{c}{R_S(M)}\right)^2(123000 - 90)}$$

$$\approx \pm 5.5 \times 10^{-17} s. \tag{3.42b}$$

3.4 Annihilation Rates

Only considering the positive time solution given in equation (3.42b) it is evident that expansion time for the positron/electron fireball to reach an optical depth of unity is extremely fast. According to our model, using a loading time of $\triangle t_L = 5 \times 10^{-20} s$ produces an optically thin fireball while using $\triangle t_L = 1s$ produces an optically thick fireball only briefly. We must also consider the annihilation rate density, $< r_{e^+e^- \to \gamma\gamma} >$, for positron/electron annihilation to determine the dynamics of our expanding fireball. We seek to calculate total annihilation total number of annihilations for our pair fireball as it is expanding. Let us first define useful quantities necessary for our calculations. Below we define the four velocity for the positron/electron within the center of momentum frame as (Svensson 1982)

$$v_\pm = \gamma_{cm} \left(1, \pm \vec{\beta}_{cm}\right), \tag{3.43}$$

within the lab frame as

$$v_\pm = \gamma_\pm \left(1, \vec{\beta}_\pm\right) \tag{3.44}$$

and within the positron(electron) rest frame as

$$v_+ = \gamma_r (1, \vec{\beta_r})[v_- = \gamma_r (1, \vec{\beta_r})]. \tag{3.45}$$

Utilizing the property of Lorentz invariance of four products yields

$$v_+ \cdot v_- = \gamma_+ \gamma_- \left(1 - \beta_+ \beta_- \mu\right) = 2\gamma_{cm}^2 - 1 = \gamma_r \tag{3.46a}$$

where we obtain a relationship between the center of momentum velocity, the rest frame velocity, and the electron/positron lab frame velocites. We also define the annihilation cross section as (Svensson 1982)

$$\sigma_{e^+e^-}(\gamma_{cm}) = \pi r_e^2 \frac{1}{4\gamma_{cm}(\gamma_{cm}^2 - 1)^{1/2}} \left(\frac{\gamma_{cm}}{(\gamma_{cm}^2 - 1)^{1/2}} \left(2 + \frac{2}{\gamma_{cm}^2} - \frac{1}{\gamma_{cm}^4} \right) ln \left[\left(\frac{\gamma_{cm} + (\gamma_{cm}^2 - 1)^{1/2}}{\gamma_{cm} - (\gamma_{cm}^2 - 1)^{1/2}} \right) \right] - 2 - \frac{2}{\gamma_{cm}^2} \right) \tag{3.47a}$$

$$= \frac{3}{8} \sigma_T \Gamma(\gamma_{cm}). \tag{3.47b}$$

We define the annihilation rate per unit volume as (Svensson 1982)

$$r_{e^+e^- \to \gamma\gamma} = n_+ n_- \frac{\left[(v_+ \cdot v_-)^2 - 1 \right]^{1/2}}{\gamma_+ \gamma_-} c \sigma_{e^+e}(\gamma_{cm}) \tag{3.48}$$

and utilizing equation (3.37) we rewrite the reaction rate density in terms of γ_{cm} as

$$r_{e^+e \to \gamma\gamma} = n_+ n_- \frac{\gamma_{cm}^2}{\gamma_+ \gamma_-} 2\beta_{cm} c \sigma_{e^+e} = n_+ n_- \frac{\gamma_{cm}}{\gamma_+ \gamma_-} 2 \left(\gamma_{cm}^2 - 1\right)^{1/2} c \sigma_{e^+e}(\gamma_{cm}) \tag{3.49}$$

where $n_+ = n_- = \frac{1}{2}\frac{dn_+}{dt}\triangle t_L$ and thus utilizing equation (3.38) the total reaction rate is given by

$$R_{e^+e^-\to\gamma\gamma} = r_{e^+e^-\to\gamma\gamma}V_{\pm}(t_e). \tag{3.50}$$

We will first calculate the annihilation rate per unit volume for the simple case that the interacting positron and electron possess similar lab frame velocities within the radial direction taking $\mu = cos\theta \approx 1$ as well as considering a static fireball with a loading time $\triangle t_L = 5\times 10^{-20}s$. Due to the incredibly high temperature of our BH which is at $kT_{bh} \approx 130MeV \approx 260\times m_ec^2$ the emerging photons possess significantly higher energies than the individual positron/electron rest energies and therefore impart the positron/electron pairs with large kinetic energies. We approximate the lab frame lorentz factors as

$$\gamma_+ \approx \frac{E_+}{m_ec^-}, \gamma_- \approx \frac{E_-}{m_ec^2} \tag{3.51}$$

where $E_+ = \frac{E_1+\epsilon_1}{2}$ and $E_- = \frac{E_2+\epsilon_2}{2}$ where E_1 and ϵ_1 represent the photon energies involved in producing the positron and E_2 and ϵ_2 represent the photon energies responsible for producing the electron. We note that the interacting positron and electron originate from different photon-photon collisions. We calculate the center of momentum frame gamma factor as a function of lab frame gamma factors of the interacting pairs utilizing equation (3.46a) as

$$\gamma_{cm} = \left[\frac{1+\gamma_+\gamma_-}{2} - \frac{\left[\left(\gamma_+^2-1\right)\left(\gamma_-^2-1\right)\right]^{1/2}}{2}\right]^{1/2}. \tag{3.52}$$

Selecting a positron and electron whos energies are comprable given by $E_+ = 800MeV$ for the positron and $E_- = 700MeV$ for the electron we calculate can calculate an annihilation rate density. Utilizing equations (3.33b), (3.47a), (3.49), (3.51), (3.52) and taking $\triangle t_L = 5\times 10^{-20}s$ yields

$$r_{e^+e\to\gamma\gamma} = \frac{1}{4}\left(\frac{dn_\pm}{dt}\right)^2 (\triangle t_L)^2 \frac{\gamma_{cm}\left(\frac{E_+}{m_ec^2},\frac{E_-}{m_ec^2}\right)}{\frac{E_+}{m_ec^2}\frac{E_-}{m_ec^2}} 2\left(\gamma_{cm}^2\left(\frac{E_+}{m_ec^2},\frac{E_-}{m_ec^2}\right)-1\right)^{1/2} c\sigma_{e^+e}\left(\gamma_{cm}\left(\frac{E_+}{m_ec^2},\frac{E_-}{m_ec^2}\right)\right) \tag{3.53a}$$

$$\approx 7.7\times 10^{30}m^{-3}s^{-1}. \tag{3.53b}$$

We now consider a distribution of positron/electron gamma factors with radial velocities where we obtained 1000 monte carlo realizations of target photon energies E_i and 1000 realizations of field photon energies ϵ_j based off of a probability distribution function derived from photon number. From the generated photon energies which were normalized by the electron rest energy we then calculated the distribution of gamma

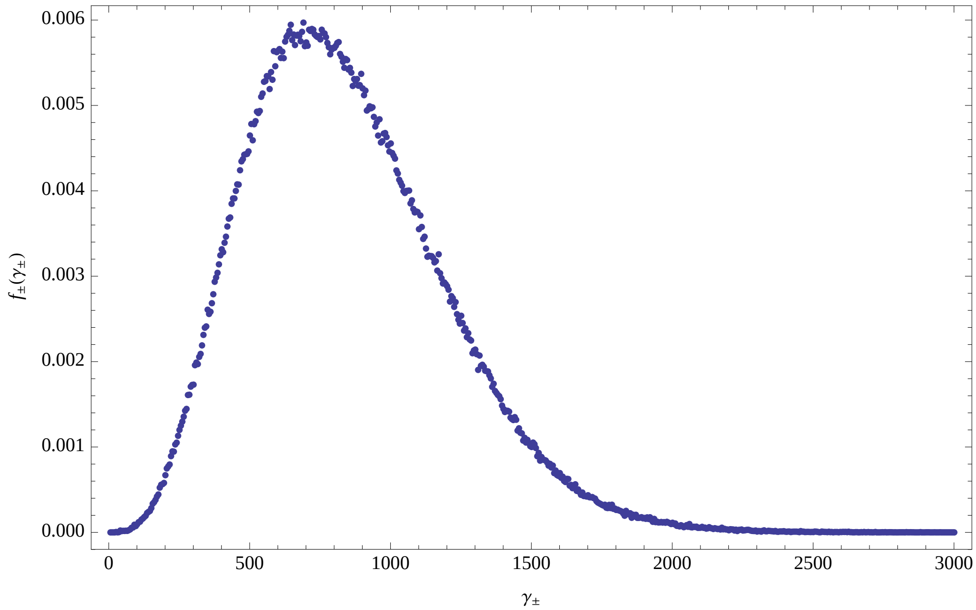

Figure 3.1: Here we plot $f_\pm(\gamma_\pm)$ vs. $\gamma_\pm$ where the bin sizes were incremented by $5\gamma_\pm$ and each point represents the number of positron or electron gamma factors within that respective bin. Each respective bin therefore represents a monoenergetic group of electrons and positrons.

factors given by

$$\gamma_+^{i,j} = \gamma_-^{i,j} = \frac{E_i + \epsilon_j}{2m_e c^2} \tag{3.54}$$

where $i, j \in \mathbb{N}$ are indices for the target and field photon energies. We then binned the gamma factors using increments of $5\gamma_\pm$ each increment representing a group of monoenergetic positrons or electrons with that particular gamma factor and from that obtained their probability distribution function. Since our approximation imparts the created positrons and electrons with the same energies their distribution functions are identical. Their distribution functions are then given by

$$f_+(\gamma_{+,k}) = f_-(\gamma_{-,k}) = \frac{N_k}{N} \tag{3.55}$$

where N=10^6 represents the total number of positrons or electrons, $k \in [1, M]$ where M=600 is the maximum number of gamma bins and N_k represents the number of positrons or electrons within the k'th gamma bin where each k represents an increment of $5\gamma_\pm$ beginning at $\gamma_\pm = 1$. For example k=1 corresponds to the bin containing the number of positrons/electrons with $\gamma_\pm$ between $1\gamma_\pm - 6\gamma_\pm$. In Figure 3.1 we plot the electron/positron velocity distribution function. We also consider time dependent positron and electron number densities where the fireball is now expanding at approximately the speed of light. We write the average annihilation rate density (Svensson 1982) for a distribution of generated positron and electron velocities as

$$< r_{e^+e^-\to\gamma\gamma} > = \int_{\beta_{+,min}}^{\beta_{+,max}} d\beta_+ f_+ (\gamma_+) \int_{\beta_{-,min}}^{\beta_{-,max}} d\beta_- f_- (\gamma_-) \Bigg(n_+ (t)\, n_- (t)\, \frac{\gamma_{cm} (\gamma_+, \gamma_-)}{\gamma_+\gamma_-} 2 \left(\gamma_{cm}^2 (\gamma_+, \gamma_-) - 1\right)^{1/2} \tag{3.56a}$$

$$\times\, c\sigma_{e^+e^-} (\gamma_{cm} (\gamma_+, \gamma_-)) \Bigg)$$

$$= \int_{\gamma_{+,min}}^{\gamma_{+,max}} \frac{1}{\left(\gamma_+^2 - 1\right)^{1/2} \gamma_+^2} d\gamma_+ f_+ (\gamma_+) \int_{\gamma_{-,min}}^{\gamma_{-,max}} f_- (\gamma_-) \frac{1}{\left(\gamma_-^2 - 1\right)^{1/2} \gamma_-^2} d\gamma_- \tag{3.56b}$$

$$\times \left(n_+ (t)\, n_- (t)\, \frac{\gamma_{cm} (\gamma_+, \gamma_-)}{\gamma_+\gamma_-} 2 \left(\gamma_{cm}^2 (\gamma_+, \gamma_-) - 1\right)^{1/2} \right) c\sigma_{e^+e} (\gamma_{cm} (\gamma_+, \gamma_-)) \Big)$$

$$= n_+(t) n_-(t) < \sigma v > \tag{3.56c}$$

where

$$< \sigma v > \; = \; \int_{\gamma_{+,min}}^{\gamma_{+,max}} d\gamma_+ f_+ (\gamma_+) \int_{\gamma_{-,min}}^{\gamma_{-,max}} f_- (\gamma_-)\, d\gamma_- 2 \frac{\gamma_{cm} (\gamma_+, \gamma_-)}{\left(\gamma_+^2 - 1\right)^{1/2} \left(\gamma_-^2 - 1\right)^{1/2} \gamma_+^3 \gamma_-^3} \tag{3.57}$$

$$\times \left(\gamma_{cm}^2 (\gamma_+, \gamma_-) - 1\right)^{1/2} c\sigma_{e^+e} (\gamma_{cm} (\gamma_+, \gamma_-))$$

is the distribution averaged reaction rate where $\gamma_{+,min} = \gamma_{-,min} = 6$ and $\gamma_{+,max} = \gamma_{-,max} = 3001$. Also we note the change in integration variables from velocities to gamma factors given by

$$d\beta_- = \frac{d\gamma_-}{\left(\gamma_-^2 - 1\right)^{1/2} \gamma_-^2}, d\beta_+ = \frac{d\gamma_+}{\left(\gamma_+^2 - 1\right)^{1/2} \gamma_+^2}. \tag{3.58}$$

If we replace the static volume used within equation (3.32b) with equation (3.37) we obtain the following number rate density equations for the positrons and electrons,

$$n_+(t) = n_-(t) = \left(\frac{2.3 \times 10^{63}}{\left(123000 + \frac{ct_e}{R_S}\right)^3 - \left(90 + \frac{ct_e}{R_s}\right)^3} \right) \left(\frac{\triangle t_L}{s}\right) m^{-3} \tag{3.59a}$$

$$= \left(4.7 \times 10^{63}\right) p(t)^{1/2} \left(\frac{\triangle t_L}{s}\right) m^{-3} \tag{3.59b}$$

where $p(t) = \frac{1}{\left(\left(123000+\frac{ct_e}{R_S}\right)^3 - \left(90+\frac{ct_e}{R_s}\right)^3\right)^2}$. Performing the integration in equation (3.57) over the probability distribution functions of the positron/electron gamma factors we obtain

$$< r_{e^+e\to\gamma\gamma} > = n_+(t) n_-(t) \times \left(1.2 \times 10^{-39} m^3 s^{-1}\right) \tag{3.60}$$

where $< \sigma v >= 1.2 \times 10^{-39} m^3 s^{-1}$. In order calculate the number of annihlations, $N_{e^+e^-\to\gamma\gamma}(t, \Delta t_L)$ we multiply $< r_{e^+e\to\gamma\gamma} >$ by V(t) and integrate over time,

$$N(t) = \int_0^t < r_{e^+e\to\gamma\gamma} > \left(t^{'}, \triangle t_L\right) V\left(t^{'}\right) dt^{'} \tag{3.61a}$$

$$= \left(2.3 \times 10^{63} m^{-3}\right)^2 < \sigma v > \left(\frac{\triangle t_L}{s}\right)^2 \int_0^t p(t^{'}) V\left(t^{'}\right) dt^{'} \tag{3.61b}$$

$$= \left(6.5 \times 10^{87} s^{-1} \times m^{-3}\right) \left(\frac{\triangle t_L}{s}\right)^2 \int_0^t p(t^{'}) V\left(t^{'}\right) dt^{'}. \tag{3.61c}$$

Substituting our expressions for p(t) and V(t) which is given in equation (3.37) yields

$$N(t, \triangle t_L) = \left(6.5 \times 10^{87} s^{-1}\right) \left(\frac{\triangle t_L}{s}\right)^2 \left(\frac{\frac{4}{3}\pi R_S^3}{m^3}\right) \int_0^t \frac{1}{\left(123000 + \frac{ct^{'}}{R_S}\right)^3 - \left(90 + \frac{ct}{R_s}\right)^3} dt^{'} \tag{3.62a}$$

$$= \left(4.9 \times 10^{40} s^{-1}\right) \left(\frac{\triangle t_L}{s}\right)^2 \int_0^t \frac{1}{\left(123000 + \frac{ct^{'}}{R_S}\right)^3 - \left(90 + \frac{ct^{'}}{R_s}\right)^3} dt^{'}. \tag{3.62b}$$

We then expand the denominator and upon simplifying our equations the integrand becomes

$$I = \frac{R_S}{3c}\left(\frac{1}{Ax^2 + Bx + C}\right) \tag{3.63}$$

where

$$A = (123000 - 90) \tag{3.64}$$

$$B = \left(123000^2 - 90^2\right) \tag{3.65}$$

$$C = \frac{\left(123000^3 - 90^3\right)}{3} \tag{3.66}$$

$$x' = \frac{ct^{'}}{R_S}. \tag{3.67}$$

Inserting the above expressions into equation (3.62b) and performing the correct variable substitutions yield

$$N(t, \triangle t_L) = \left(4.9 \times 10^{40}\right) \left(\frac{\triangle t_L}{s}\right)^2 \left(\frac{\frac{R_S}{3c}}{s}\right) \int_0^x \frac{1}{Ax'^2 + Bx' + C} dx^{'} \tag{3.68a}$$

$$= \left(6.6 \times 10^{15}\right) \left(\frac{\triangle t_L}{s}\right)^2 \int_0^x \frac{1}{Ax'^2 + Bx' + C} dx^{'} \tag{3.68b}$$

where $x = \frac{ct}{R_S}$. Utilizing an integral table (Spiegel 1968) we obtain

$$N(t, \triangle t_L) = \left(6.6 \times 10^{15}\right) \left(\frac{\triangle t_L}{s}\right)^2 \frac{2}{\sqrt{4AC - B^2}} \arctan\left[\frac{2Ax + B}{\sqrt{4AC - B^2}}\right] \tag{3.69}$$

and after restoring necessary constants and variables we obtain the final form of our solution to be

$$\begin{aligned} N(t, \triangle t_L) &= \left(7.6 \times 10^5\right) \left(\frac{\triangle t_L}{s}\right)^2 \left(\arctan\left[\frac{6.1 \times 10^{29} t + 1.5 \times 10^{10}}{8.7 \times 10^9}\right] - \arctan\left[1.73\right]\right) && (3.70a) \\ &= \left(7.6 \times 10^5\right) \left(\frac{\triangle t_L}{s}\right)^2 \left(\arctan\left[7 \times 10^{19} t + 1.73\right] - \arctan\left[1.73\right]\right). && (3.70b) \end{aligned}$$

Setting $\Delta t_L = 5 \times 10^{-20} s$ we obtain the expression

$$N\left(t, \triangle t_L = 5 \times 10^{-20} s\right) = 2 \times 10^{-33} \left(\arctan\left[7 \times 10^{19} t + 1.73\right] - \arctan\left[1.73\right]\right). \tag{3.71}$$

Arctangent asympotically approaches $\frac{\pi}{2}$ as the argument approaches infinity therefore for $t \in [0, \infty)$ we have $N\left(t, \triangle t_L = 5 \times 10^{-20} s\right) \in \left[0, \times 10^{-33}\right)$ and therefore we see no pair annihilation and conclude that the positrons and electrons freely stream radially. Using the number rate, equation (3.29e), we can calculate the number of freely streaming positrons and electrons over a time interval in which our emission spectrum changes negligibly. Based on the size of our PBH and the radial velocity ($\gamma_\pm$) profile of our positrons and electrons an optically thick positron/electron fireball does not form but rather as soon as pairs are generated they stream freely outward. This also implies that the photon spectrum emerging directly from the black hole is allowed to propagate to infinity because an optically thick fireball is never generated.

3.5 Variable Mass Evaporating Black Holes

If we were to consider a primordial black hole evaporating today, one that had an initial mass of 5×10^{11} kg, and an initial temperature of 2.5×10^{11} K, then the conditions and temperatures during the final stages of the evaporation process would be extreme and many considerations would have to be taken into account in order to predict the blackbody spectrum that would emerge which has been investigated by Carr and Macgibbon. Therefore let us assume, for simplicity, higher mass black holes and determine their spectra. We assume three different black holes that are presently a 1 MeV , 500 MeV, and 1 GeV source. Therefore we have three gamma ray sources that we would expect to observe. By constraining the energy of the black hole today and using equation (1.1) we can set the mass of the black hole normalized to the hubble mass which is given by

$$\left(\frac{M}{M_H}\right) = \frac{\hbar c^3}{8\pi G E M_H} \tag{3.72a}$$

$$= \left(\frac{E_H}{E}\right) \tag{3.72b}$$

where E_H represents the energy of a hubble mass black hole and assuming formation shortly after the big bang we can determine its initial mass normalized to the Hubble mass by rearranging equation (2.6b) and obtaining

$$\left(\frac{M_0}{M_H}\right) = \frac{1}{M_H}\left(M^3 + \frac{3\alpha\hbar c^4 t_H}{G^2}\right)^{1/3} \tag{3.73a}$$

$$= \left(\left(\frac{M}{M_H}\right)^3 + \frac{3\alpha\hbar c^4 t_H}{G^2 M_H^3}\right)^{1/3} \tag{3.73b}$$

$$= \left(\left(\frac{M}{M_H}\right)^3 + 1\right)^{1/3} \tag{3.73c}$$

where $t_H \approx 13.7 \times 10^9 yrs$ (Spergel et. al. 2003). From this initial mass we can determine the evaporation time normalized to t_H, by again rearranging equation (2.6b) and setting M = 0 obtaining

$$\left(\frac{t_{evap}}{t_H}\right) = \frac{G^2}{3\alpha t_H \hbar c^4} M_0^3 \tag{3.74a}$$

$$= \left(\frac{M_0}{M_H}\right)^3. \tag{3.74b}$$

We can also determine the radius of the current black hole by using the relation $R = \frac{2GM}{c^2}$. Also utilitizing equation (3.72a) we can write the Schwarzschild radius in terms of the energy of the black hole normalized to $R_H = \frac{2GM_H}{c^2}$ as

$$\left(\frac{R}{R_H}\right) = \frac{\hbar c}{4\pi E R_H} \tag{3.75a}$$

$$= \frac{E_H}{E}. \tag{3.75b}$$

For comparison we note that a Solar mass BH $\left(M_\odot = 2 \times 10^{30}\right)$ kg would have a lifetime of $\frac{t_{evap}}{t_H} = 6.4 \times 10^{55}$ or $t_{evap} = 8.8 \times 10^{56}$ Gyr which is much longer than the age of the universe and a temperature of $T = 6.2 \times 10^{-8}$ K which is almost absolute zero whereas a Planck mass BH ($m_P = 2.18 \times 10^{-8}$ kg) would have a lifetime of $\frac{t_{evap}}{t_H} = 8.3 \times 10^{-59}$ or $t_{evap} = 3.6 \times 10^{-41} s \approx 660 t_P$ (where $t_P = 5.4 \times 10^{-44} s$) and a temperature of $T = 5.7 \times 10^{30}$ K $\approx 4 \times 10^{-2} T_P$ (where $T_P = 1.42 \times 10^{32}$ K is the Planck temperature). A summary of the properties of the three different black holes is given in Table 3.1,

Table 3.1: Characteristics of 1, 500, 1000 MeV BH

Energy	**1 MeV**	**500 MeV**	**1 GeV**
Schwarzschild Radius (current normalized to Hubble Radius)	21.2177	4.24355 x 10^{-2}	2.12177 x 10^{-2}
Mass (current, normalized to Hubble Mass)	21.2177	4.24355 x 10^{-2}	2.12177 x 10^{-2}
Initial Mass (normalized to Hubble Mass)	21.184	1.00003	1.0
Evaporation time (normalized to Hubble Time)	9.442 x 10^{3}	1.00009	1.0

In Fig. 3.2 we plot an SED for three different black holes to show how the spectrum shifts with temperature

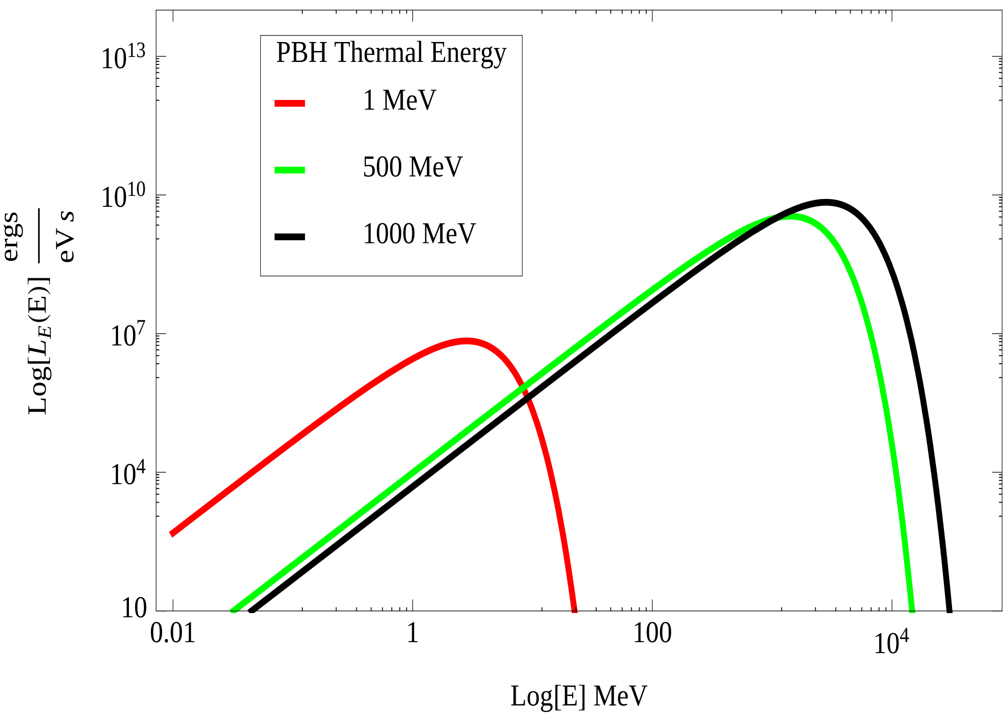

Figure 3.2: SED Plot for 1 MeV, 500 MeV, and 1 GeV Evaporating Black Hole

Traditionally for stars, treating their surface as a blackbody emitter, it is well known that as the temperature increases the maximum shifts to higher luminosities, and the tail, described by Rayleigh-Jeans behavior undergoes an shift upward, proportional to the temperature. In the case of BH's, as you increase their temperature their peak undergoes a blueshift but the tail undergoes a downward shift inversely proportional to temperature. Within the Rayleigh Jeans limit the BH luminosity is inversely proportional to its energy whereas for stars the luminosity is directly proportional to the temperature. In the Rayleigh Jeans limit, ($\epsilon \ll k_B T(M)$), the equation for luminosity per unit energy given as

$$L_\epsilon(T(M), \epsilon) = \frac{32\pi^2 G^2 M^2}{c^6 h^3} \frac{\epsilon^3}{e^{\frac{\epsilon}{k_B T(M)}} - 1}, \tag{3.76}$$

becomes

$$L_\epsilon(T(M), \epsilon) = \frac{\epsilon^2}{16\pi^2 E h}, \tag{3.77}$$

where E represents the thermal energy of the black hole. Therefore as the temperature increases we see a decrease in luminosity and conversely as the temperature decreases we see a increase in luminosity. This is because the mass of the black hole is inversely proportional to the temperature. If primordial black holes existed today at these characteristic temperatures we would expect them to possess an emission spectra similar to Figure 2.

Chapter 4

Detection of Primordial Black Holes

4.1 Detection Limits of Gamma Ray Bursts from PBH's

Detection of BH evaporation is very difficult and elusive. The upper limit on number density of PBH today is quite small. The Fermi LAT, a space-based telescope is cabable of detecting high energy gamma rays from a range of approximately 200 MeV-300 GeV sources. The flux threshold (LAT sensitivity for E>100 MeV) required to detect a high energy photon is $5 \times 10^{-4} \gamma m^{-2} s^{-1}$ (Atwood et. al.). We are interested in the minimum distance at which a BH at a given mass, M, can be detected. The flux in units of $eVm^{-2}s^{-1}eV^{-1}$ is given as

$$F_E = \frac{L_E}{4\pi D^2} \tag{4.1}$$

where $\mathrm{L_E}$ is given in Eqn. (4.2) and D is the distance from the BH to the detector. We convert from $eVm^2s^{-1}eV^{-1}$ to $\gamma m^{-2}s^{-1}eV^{-1}$

$$F_{E,\gamma} = \frac{\dot{N}_E}{4\pi D^2} = \frac{L_E/E}{4\pi D^2}. \tag{4.2}$$

We must convert to bolometric flux so we integrate both sides equation (3.9) over the energy and we obtain

$$F_\gamma = \frac{\dot{N}}{4\pi D^2} \tag{4.3}$$

where $\dot{N} = \int \frac{L_E}{E} dE$. Inserting Eqn. (67) into the previous expression yields

$$\dot{N} = \int_0^\infty \frac{32\pi^2 G^2 M^2}{c^6 h^3} \frac{E^2}{e^{\frac{E}{k_B T(M)}} - 1} dE. \tag{4.4}$$

Normalizing our integration variable, $x = \frac{E}{kT_{bh}}$, we obtain the the following expression

$$\dot{N} = \frac{c^3}{64\pi^4 G M_H} \left(\frac{M_H}{M}\right) \int_0^\infty \frac{x^2}{e^x - 1} dx \tag{4.5}$$

Upon integration, we obtain

$$\dot{N} = 3.1 \times 10^{20} \left(\frac{M_H}{M}\right) s^{-1}. \tag{4.6}$$

Inserting equation (4.6) into (4.3) and solving for distance yields

$$D(M) = \left(\frac{3.1 \times 10^{20} s^{-1}}{4\pi F_\gamma} \left(\frac{M_H}{M}\right)\right)^{1/2} \tag{4.7a}$$

$$\approx \left(\frac{2.47 \times 10^{19} s^{-1}}{F_\gamma}\right)^{1/2} \left(\frac{M_H}{M}\right)^{1/2}. \tag{4.7b}$$

Therefore the minimum distance for which to detect Hawking Radiation is inversely proporational to the square root of its mass. The Fermi LAT has a photon flux threshold of $5 \times 10^{-4} \gamma m^{-2} s^{-1}$. Inserting this photon flux into equation (4.7b) yields

$$D\left(M\right) = 2.2 \times 10^{11} \left(\frac{M_H}{M}\right)^{1/2} m \tag{4.8a}$$

$$= 7.2 \times 10^{-6} \left(\frac{M_H}{M}\right)^{1/2} pc. \tag{4.8b}$$

For the 500 MeV and 1000 MeV black holes which lie within the Fermi LAT detection range of 200 MeV - 300 GeV the corresponding maximum distances, respectively, are 7.2 au's (3.5×10^{-5} pc), and 10.2 au's (4.9×10^{-5} pc). These distances are relatively nearby considering that the sun is approximately 1 au from earth. David Cline has been searching for gamma ray transient events that would possibly emerge during the final stages of black hole evaporation. He argues that the peculiar signatures of VSB (Very Short Burst; $T_{90} < 100ms$) events have been detected as an emerging hard photon spectra in comparison to longer gamma ray events as well as an angular assymetry. T_{90} denotes the duration of time for which 90 percent of the counts have been detected. From BATSE data, the number of events observed from a region near the galactic anticenter was 20 which is significantly higher than the average of 51/8 where probability of observing 20 out of 51 events within a given region is 0.00007 which he argues is evidence against just a statistical fluctuation (D.B. Cline et al. 2007). He also notes that KONUS data shows also an excess of events with T90<100ms. This peculiar excess of events could be evidence for PBH evaporation. Because PBH's are free moving dark matter components they should cluster within galaxies and trace dark matter paths (Cline 1998). Cline claims that detection of the Galactic component of the diffuse gamma ray background from PBH evaporation would also provide a good argument presence of PBHs and proposes two possibilites. He first argues that there could be a glow within the halo due to PBH evaporation possibly detected by EGRET. The second possibility is that their could exist an anisotropy of gamma rays due to sufficient density within the halo which have been detected as well he claims. The relaxed Page-Hawking bound, a modification of their original approximation that PBH evaporation would produce a diffuse gamma ray background due to modification of radiation behavior from Q-G phase transition and QCD calculations (Cline 1998), predicts a density 10^4 PBHs pc^{-3} within the universe. Using the density of PBHs along with the clustering factor of 5×10^5 Cline calculated that the halo "glow" flux was $\approx 0.1 \gamma m^{-2} s^{-1} sr^{-1}$ (Cline 1998).

4.2 Measurement of 511 keV Line

There is an unresolved and onging search to find the sources of positron production within the galactic center. There are several candidate sources. One being radioactive decay of unstable nuclei resulting from thermonuclear supernovae, massive stars, hypernovae, γ-ray bursts, and classical novae. The second being high energy cosmic ray and compact object processes stemming from p-p collisions, galactic cosmic rays, pulsars, milli-second pulsars, magnetars, X-ray binaries, microquasars, and galactic black holes. Lastly, positron generation is predicted from from dark matter and "non-standard" model processes (Prantzos et. al. 2010). The presence of these positrons is evidenced by the 511 keV emission line emerging from annihilation processes, the first signal being detected from the galactic center in the early 1970's by balloon-borne instruments of low energy resolution (Prantzos et. al. 2010). Years later it was identified with high resolution Ge detectors being the most intense γ-ray line ever detected that originated beyond our solar system. The measured flux ($10^{-3} cm^{-2} s^{-1}$) and the distance to the Galactic center ($\approx 8kpc$) suggests an annihilation rate of $2 \times 10^{43} e^{+} s^{-1}$ (Prantzos et. al. 2010). We investigate an alternative source for positron production and annihilation through the evaporation of primordial black holes. Using our production rate given in equation (3.29e), the predicted annihilation given previously, and assuming steady state where the annihilation rate is equal to the average production rate of positrons during their lifetime within the ISM we can calculate the number of evaporating black holes that must exist within the galactic center as

$$N_{bh} = \frac{\dot{N}_{e^+,gc}}{\dot{N}_{e^+,bh}} \tag{4.9a}$$

$$= \frac{\dot{N}_{e^+,gc}}{\frac{1}{2}\dot{N}_{e^+e^-,bh}} \tag{4.9b}$$

$$\sim 10^{27}. \tag{4.9c}$$

Under these simplified assumptions the calculation suggest that the galactic center must possess a large number of 8.2×10^{10} kg BH's. This is a factor of 10^5 more PBH's than the number predicted by Cline which was $\left(\sim 10^{22}\right)$. Furthermore the peak regarding the observed 511 keV signal resembles the annihilation of positronium where the positrons energy is estimated to be less than 2 MeV (Prantzos et. al. 2010) which along with the vast number of PBH's required (an excess of $\sim 10^5$) eliminates them as a possible source for the 511 keV line emerging from the galactic center. The majority of the positrons emerging from PBH evaporation possess energies on the order of a half of a GeV.

4.3 Detection of High Energy Cosmic Positrons

Here we analyze a third possibility for the confirmation of black hole evaporation by analyzing high energy cosmic ray positrons. The PAMELA satellite-borne experiment which took place between July 2006 and February 2008 obtained trigger events for 9,430 positrons between the ranges of 1.5 - 100 GeV (Adriani et al). The distribution given in Fig 3.1 suggests that the positrons created by pair production from the evaporating black hole possess energies lower than the minimum energy signuture detected by the PAMELA experiment of 1.64 GeV. The low energy regime of cosmic positrons have been explained by known processes within the galaxy. Therefore we rule out this possibility for confirming the existance of PBH's.

Chapter 5

Conclusions and Discussion

5.1 Fireball Model Analysis

We hypothesized that the an evaporating PBH with initial mass $\approx 5 \times 10^{11}$ kg would form an optically thick positron/electron fireball under several assumptions. We assume only photon-photon production from the black hole and neglect any scattering back into the black hole. The number density of photons was calculated by considering a spherical blackbody emitting surface instead of a point source. This gave us an isotropic photon field. We performed our relativisitic kinematic calculations on top of a locally Minkowski geometry instead of Schwarzschild. We neglected the CMB field due to its low temperature. Its contribution was negligible. Using the compactness parameter $L/R \geq 10^{31} \frac{ergs}{cm \times s}$ we constrained the mass of the black hole at which pair creation becomes relevant to be 1.3×10^{11} kg. However when calculating the optical depth due to positron/electron creation from photon-photon collisions an optical depth of unity was never reached and therefore we allowed our black hole to evaporate until the optical depth reached unity for any given energy or radial distance from the black hole. This mass was found to be 8.2×10^{10} kg. We then calculated the thickness of our fireball by propagating a field photon through the radiation field and calculated the corresponding distance from the BH at which the optical depth reached unity. The inner and outer radius was given as $90R_S$ and $123000R_S$ respectively. From this we were able to calculate relevant quantities such as the rate of positron/electron production, the positron/electron fireball volume and from that the number rate density as well as the optical depth rate of change. We note that the number densities of our positrons and electrons were homogeneous throughout the shell. Because of the nearly radial profile of the generated positrons and electrons and the ultra-relativistic energies at which they were streaming forth from the black hole the loading time for which our fireball remained static was extremely short and thus an optically thick fireball due to Compton scattering was never reached. We then calculated annihilation rates per unit volume for our generated positrons and electrons. We used the free streaming approximation in order to estimate the generated positron/electron energies and therefore their respective gamma factors. In order to calculate the correct annihilation rates and the numbers of annihilations within a given time frame we needed to know the velocity distribution. Using our photon distribution we generated random velocities and generated a distribution function by creating bins of $5\gamma_{\pm}$ from $6\gamma_{\pm} - 3001\gamma_{\pm}$. Using our generated distribution function which is identical for both positrons and electrons we calculated the annihilation rates per particle to be $1.2 \times 10^{-39} m^{-3} s^{-1}$. We then calculated the number of annihilations as a function of time using a loading time of $5 \times 10^{-20} s$. This never reached above 10^{-33} for any time. From this we conclude that, given our extremely short loading time, there are no annihilations that take place within the generated posititron/electron fireball but instead freely streaming pairs emerging from the vicinity of the PBH. We note that coulomb scattering was neglected within our calculations as a simplification. From this we also conclude that subsequent photons are able to propagate freely as they never see an optical depth greater than one. The emerging spectrum obtained after the pair production processes is able to be predicted from the actual emission spectrum at the event horizon. We note that our analysis neglected massive particle creation from the event horizon as well as QCD, quantum gravity, and backscattering effects all of which must be included in order to fully explain the evaporation process.

5.2 Connections and Conclusions

We found that when considering only photon-photon production, neglecting QCD, QG, and backscattering back into the BH that evaporating PBH's do not produce optically thick positron/electron fireballs but rather freely streaming pairs propagating from the vicinity of the BH. From this we concluded that subsequently emitted photons created at the event horizon would propagate freely to infinity because of optical depth transparency within the fireball. We then calculated for different current mass (thermal energy) black holes their corresponding SED which under our simplified assumptions would provide an accurate spectra of their emission. We then discussed three possible detection methods for which to confirm the existence of evaporating black holes and concluded that the two least viable possibilities were the contribution to measured 511 keV line emerging from the galactic center the production of high energy cosmic ray positrons. For the spectrum emerging directly from the BH we investigated David Clines work in searching for evaporation events within the galactic halo. He concluded that of BH evaporation signatures could be evidenced by halo glows, anisoptropies of the diffuse gamma ray background, and excess VSB events near the galactic anticenter.

5.3 Theoretical Implications and Recommendations for Further Research

The implications for the detection of evaporating PBH's are significant. Their detection would prove the work devoloped by Stephen Hawking which showed that BH's have a characteristic temperature and radiate a blackbody spectrum of various species of fermions and bosons. In confirming this the correspondence between black hole physics and classical thermodynamics would also be confirmed. Furthermore their detection could provide theoretical laboratories for which to study early phases of Big Bang physics where early epochs such as the Plank era, Electroweak era, QG phase era among others could be further intestigated. Further research possibilities beyond this work include a more accurate analysis of pair creation and optical depth around the BH by also including electron/positron production as well as various flavors of neutrino and antineutrino production. We could also examine these processes within a Schwarzschild instead of Minkowski geometry.

Appendices

.1 Photon-Photon Interaction using Minkowksi Metric

The photon-four momentum is given by

$$p_\mu = \left(\frac{E+\epsilon}{c}, \frac{E+\epsilon cos\theta}{c}, 0, \frac{\epsilon sin\theta}{c} \right) \tag{1}$$

The inner product of the rour-momentum using the Minkowski Metric is given by

$$p_\mu p^u = \left(\frac{E+\epsilon}{c} \right)^2 - \left(\frac{E+\epsilon cos\theta}{c} \right)^2 - \left(\frac{\epsilon sin\theta}{c} \right)^2 = \frac{4E_{cm}^2}{c^2} \tag{2}$$

where E_{cm} represents the total energy of the electron (and positron) within the center of momentum frame. Squaring all terms we obtain

$$E^2 + \epsilon^2 + 2\epsilon E - E^2 - \epsilon^2 cos^2\theta - 2\epsilon E cos\theta - \epsilon^2 sin^2\theta = 4E_{cm}^2. \tag{3}$$

Using a trig. identity and solving for ϵ we obtain the following expression for the field photon energy,

$$\epsilon = \frac{2E_{cm}^2}{E\left(1 - cos\theta\right)}. \tag{4}$$

.2 Magnitude of Positron/Electron Velocity

Here we calculate the velocity of a created electron (and positron) utilized in equation (3.22). From conservation of energy we have

$$E_{cm} = \gamma_{cm} m_e c^2 \tag{5}$$

where $\gamma_{cm}(\beta_{cm})$ represents the familiar relativistic gamma factor. Solving for β and utilizing equations (4) and (5), we obtain

$$\beta_{cm} = \left(1 - \frac{1}{\gamma_{cm}}\right)^{1/2} \tag{6a}$$

$$= \left(1 - \frac{m_e^2 c^4}{E_{cm}^2}\right)^{1/2} \tag{6b}$$

$$= \left(1 - \frac{2m_e^2 c^4}{E\epsilon(1 - cos\theta)}\right)^{1/2}. \tag{6c}$$

Utilizing equation (5) we find that the minimum energy required by a field photon to produce an electron (and positron), taking $\beta_{cm} = 0$, is then given by

$$\epsilon_{min} = \frac{2m^2c^4}{E\,(1 - cos\theta)}. \tag{7}$$

Bibliography

[1] Atwood, W. B. et. al., 2009, *The Large Area Telescope On The Fermi Gamma-Ray Space Telescope Mission*, arXiv:0902.1089v1 [astro-ph.IM]

[2] Cline D., 1998, *A Gamma-Ray Halo "Glow" From Primordial Black Hole Evaporation*, The Astrophysical Journal, **501**, L1-L3

[3] Cline, D., et. al., 2007,*The Search for Primordial Black Holes Using Very Short Gamma Ray Bursts*, arXiv.org:07042398

[4] Goodman, J., 1986, *Are Gamma-Ray Bursts Optically Thick*, The Astrophysical Journal, **308**, L47-L50

[5] Gould, R.J., Schreder, G., 1967, *Pair Producion in Photon-Photon Collisions*, Physical Review **155**, 5

[6] Hawking, S. W., 1975, *Particle Creation by Black Holes*, Commun. math. Phys **43**, 199-220

[7] Lightman, R., Rybicki, A., 1979, *Radiative Processes in Astrophysics*

[8] Page, D., 1976, *Particle Emission Rates From a Black Hole: Massless Particles From an Uncharged Nonrotating Black Hole*, Physical Review D, **13**, 2, 198-206

[9] Page D., Hawking S., 1976, *Gamma Rays From Primordial Black Holes* The Astrophysical Journal **206**, 1-7

[10] Rohrlich, F., Jauch, J.M., 1959, *The Theory of Photons and Electrons*

[11] Spiegel, M. R., 1968, *Math Handbook of Formulas and Tables*

[12] Spergel, D. N. et. al., 2003, *First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters*, The Astrophysical Journal Supplement Series **148**, 1, 175-194

[13] Stecker, F.W., 1996, *Absorption of High Energy Gamma-Rays By Low Energy Intergalactic Photons*, SSRV

[14] Svensson, R., 1982, *The Pair Annihilation Process in Relativistic Plasmas*, **258**, 321-334