# CONTINGENCY ANALYSIS USING SYNCHROPHASOR MEASUREMENTS 

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# CONTINGENCY ANALYSIS USING SYNCHROPHASOR MEASUREMENTS 

\(\left.\begin{array}{c}A Thesis <br>
Presented to <br>
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Clemson University\end{array}\right]\)| In Partial Fulfillment |
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| of the Requirements for the Degree |
| Master of Science |
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#### Abstract

The rapid progress of synchrophasor technology greatly promotes many applications of wide-area measurement systems. Traditionally, contingency analysis is based on off-line studies conducted long in advance. This is becoming increasingly unreliable for real-time operations. New technologies, which rely on accurate, high resolution, and real-time monitoring of actual system conditions using phasor measurements are needed to support the real-time operations. In this research an algorithm is developed, using phasor measurements, that allows real-time analysis and correction of contingencies in power systems. The focus is specifically on overloaded lines. An off-line study performed with POWERWORLD software is used for contingency analysis, and contingency indicative phasor limits are investigated using current magnitude and voltage angle. These limits are applied to a rotating phasor chart. An algorithm which predicts sensitivity is applied in the off-line analysis in order to determine the buses that need to be monitored. An actual system with available real-time PMU data is used to verify the phasor chart obtained using off-line data. The chart is completed for on-line data, and the off-line and on-line charts are compared for further verification.


## DEDICATION

This thesis is dedicated to my family, my friends, and my husband. Thank you all for your unwavering love and encouragement over the last two years.

## ACKNOWLEDGMENTS

Throughout the course of this research many issues have come to light involving the politics associated with releasing PMU data to parties outside the utility. I would like to thank the utilities that were able to provide me with data; without them the completion of this thesis would not have been possible.

I would also like to thank my advisor, Dr. Elham Makram, and all of my professors here at Clemson for their support, guidance, and dedication to my success, both in and out of the classroom.

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## CHAPTER ONE

## INTRODUCTION AND LITERATURE REVIEW

### 1.1 Overview of System Overload

The ability to manipulate a power network to efficiently supply the power demands of all loads within a network is a necessity in power system operation. In daily periods of high demand the loading of a network peaks. This may cause an overflow in the transmission lines and may lead to an abnormal operating condition. One of the solutions to relieve the overloaded lines is to remove a line from a system. Certain lines within the system are very sensitive to changes in power flow, these are critically loaded, but others may be able to handle excess flow. In many cases forcing outages of specific lines within the system will properly redistribute power flow in these more lightly loaded lines and allow alleviation of the overload while still supplying the power demanded by all loads. Contingency analysis must be performed to determine the effects of increased system loading as well as the lines that are most likely to overload.

### 1.2 Contingency Analysis - Current Practices and Problems

Current contingency analysis methods rely heavily on off-line data, limiting the effectiveness of these methods. Real-time, or on-line, data deals with the constantly updating system at any given moment, while off-line data is created in a simulator program and has a "base case" foundation for the system in which all voltage and overload operating limits are met. Off-line data provides a good base for contingency analysis, but also has some inherent problems when extended to on-line analysis.

The first problem involves unexpected losses within the system [1]. System planning practices use off-line data for analysis of whether the system will operate within its limits if a predefined contingency, an outage or loss expected at some point in the future, occurs. Due to the time intensive process, this analysis may not cover losses or outages that occur unexpectedly in the system, assuming the predefined contingencies account for most of the contingencies that may occur within the system. This is an inaccurate assumption according to Balu, et al. [1], who says at any given time an undetermined assortment of components within the system could be out for maintenance, repair, upgrade, etc., and exposed to environmental conditions that may be very different from the implicitly normal conditions present in off-line planning. The lack of a predetermined solution to an unexpected contingency can force operators to make quick decisions without having any indication of possible solutions or their effectiveness.

This undermines the ultimate purpose of contingency analysis, which is to allow the operator, if forewarned, to prevent problems within a system by taking some sort of remedial action before or after an outage occurs [1]. However, to solve for all possible contingencies, usually anywhere from a few hundred to a few thousand cases for a large network, by running a load flow for each case is impractical. These numbers can reach millions if double contingencies are observed. Using off-line data, usually only the most severe contingencies are recorded, as most contingencies do not cause voltage violations or overloads. The most severe cases may not even be initially determined by running a load flow, but by more approximate and faster methods such as distribution factor calculations. The distribution factor method uses generation shift factors and line shift
factors to approximately determine the post-contingency flows along lines in a system due to a specific generator outage or line outage by determining how power will redistribute based on its line and transformer impedances. This is beneficial in quickly determining the most likely threats to the system, but still depends on the "base case" foundation, and means in the event of an unexpected outage a new load flow must be performed on-line to identify any problems that may occur. This may lead to a second problem.

The second problem with extending contingency analysis to on-line data is the time intensity. Unexpected outages require the use of on-line data for analysis. Current practice, as stated above is to run the power flow solution with the new configuration. The power flow solution may not be suitable for on-line applications because it relies on state estimators, which normally take minutes to update snapshots of the power system [2]. This provides enough time for a system to collapse while waiting for a solution.

### 1.3 Phasor Measurements - Attributes and Applications

Phasor measurements, commonly referred to as phasors, are currently being greatly researched in the power industry. A phasor is a vector representation of the magnitude and phase angle of an AC waveform [3]. Phasor data has been found to give a good indication of a system's state in numerous areas of study. The data comes from phasor measurement units, PMUs, installed on the lines. Phasor measurement units are defined by Nuqui [4] as power system devices that provide synchronized measurements of real-time phasors of bus voltages and line currents. A brief history of the development of the PMU is presented in [3]. Figure 1.1 gives a summary of a PMU and its
measurements. Many PMUs are currently installed in systems around the world for various applications in many different areas such as monitoring, control, protection, security, stability, and state estimation. A detailed discussion of these applications will be presented later in the chapter.


Figure 1.1 - Phasor Measurements from a PMU
where:
$\mathrm{I}_{\mathrm{L}, 1,2,3,4}$ are the magnitudes of line or injected currents
$\Phi_{\mathrm{L}, 1,2,3,4}$ are the line or injected current angles
$\mathrm{V}_{\mathrm{B}}$ is the magnitude of positive sequence bus voltage
$\Phi_{\mathrm{B}}$ is the bus voltage angle
Also, the complex voltages of buses incident to the PMU bus can be calculated by applying Ohm's law at each of the incident lines [5].

Undetermined currents can be found using the equation

$$
I_{12} \cong \frac{\left(V_{1} \angle \theta_{1}\right)-\left(V_{2} \angle \theta_{2}\right)}{j X_{12}}
$$

where
$\mathrm{I}_{12}$ is the branch current along the line between buses 1 and 2
$\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are the bus voltage magnitudes at buses 1 and 2 respectively $\theta_{1}$ and $\theta_{2}$ are the bus voltage phase angles at buses 1 and 2 respectively $\mathrm{j} \mathrm{X}_{12}$ is the reactance of the line impedance between buses 1 and 2

The real portion of the line impedance, the resistance $R$, is neglected since $X \gg R$. This should be determined for each individual system before any assumptions are made. The ability to quickly determine phasor quantities through direct reading or indirect calculation has made the PMU a valuable tool in the power industry.

The accuracy of the PMU, as compared to current data acquisition, is shown in Figure 1.2, which provides a comparison between synchronous data and asynchronous data. Asynchronous data is found in most power utilities and is referred to as SCADA data, or $\underline{\text { Supervisory }} \underline{\text { Control }} \underline{\text { and }} \underline{\text { Data }} \underline{\text { Acquisition data [6]. }}$

It can clearly be seen that voltage angles cannot be compared in the asynchronous data sample, as the waveforms are observed at different times and therefore do not have the same reference. This can provide large error in a system if the angles are simply approximated, providing a small error, but then those approximated angles are used in determining angles further out, allowing the small error to propagate.


Figure 1.2 - Comparison of Synchronous and Asynchronous Data Measurements
The PMU uses GPS (Global Positioning System) satellites to time synchronize its measurements to within $0.2 \mu \mathrm{~s}$ of the world standard Coordinated Universal Time, providing a measurement reference point that is beneficial in many aspects [7]. First, it indicates an exact time when measurements were observed, allowing for even more clarity in real-time system operation. Second, it assures that a reference point for the measurements can be taken and all angles from a single reference can be compared with little to no error.

Phadke, et al. [5] suggests a stressed power system is determined by widening angular separation of bus voltage angles. He uses voltage phase angle measurements from PMUs and the power angle curve, as shown in Figure 1.3, to create "decision trees" to determine whether or not a system will remain in a secure state after a contingency. If the voltage angle ( $\delta$ ) goes beyond $90^{\circ}$, the system is considered unstable.


Figure 1.3 - Power-Angle Bifurcation Diagram
He also extends the method to include angular differences in the above P- $\delta$ curve. The following equation is used to determine angular difference for a transmission line between buses i and j .

$$
\delta_{\mathrm{ij}}=\delta_{\mathrm{i}}-\delta_{\mathrm{j}}
$$

System angles are measured in degrees rather than radians. The difference between two bus voltage angles was used because angular difference between two buses is a more accurate measurement than a singular angle as the latter depends on a reference bus within the system $[5,8]$. This is not difficult for off-line studies where the reference bus is fixed, but in on-line studies problems may occur. If the reference bus (slack bus) is not functioning correctly or even removed from the system the angles will not have a correct reference.

Phadke, et al. [5] also states that bus voltage magnitude was not used as an indicator of security for the following reasons:

1) There are inherent bias errors on voltage transformers as their effective turns ratio drift due to aging or short circuiting over time.
2) Voltage magnitudes become poor measures of voltage security when reactive support is being continuously switched into the system to maintain the voltage profile.
3) PMUs are usually installed at high voltage buses where voltage magnitudes are quite insensitive to the loading of the system.

Bus voltage magnitudes (in per-unit) typically have around a 5\% range. Most transmission lines operate within 0.95 to 1.05 p.u. voltage at the buses and are designed to keep the voltages as close to within those limits as possible. The bus voltage phase angle, however, has been determined to be a good indicator of stress or loading in a system $[9,10]$ and can change drastically if there is a contingency introduced.

### 1.4 PMU Applications

There are many currently implemented and proposed future applications of PMUs. The major applications reported in the literatures and mentioned here are state estimation, system and model validation, measurement precision, system monitoring, stability, protection, and fault recording. PMU applications in contingency analysis are presented in the next section.

1. State Estimation

PMU obtained phasors are being researched in state estimation, where more phasor data from the buses allows for greater accuracy in the estimate of the system [8]. Phadke, et al. [5] gives three reasons as to why PMU direct measurement is better than state estimation based measurements. These are:

1) Direct measurement of selected states is faster than extracting the same states from the state estimator.
2) Synchronized phasor measurements provide the "real" snapshot of selected states compared to state estimator where measurements are asynchronous.
3) In the state estimator, voltage measurement errors will propagate into the PQ flow measurements resulting in less accurate state estimates.

In addition, depending on the number of PMUs in the system, phasors can provide redundancy and practically eliminate all bad data from the system [11,12].

## 2. System and Model Validation

Many aspects of power system operation rely on imprecise computer models and simulations. Model parameters, if unknown, must be estimated. This allows the possibility for errors in engineering judgment to propagate into system models and simulations. Burnett, et al. [13] states it is now possible to verify the data obtained through these simulations using actual field data. In the past, field tests for model verification would have been time consuming, impractical, or even impossible. This model verification is extremely useful in older power systems, where many components may no longer precisely follow the specifications provided by the vendors. Many errors can arise when slight changes in parameters such as generator time constants, saturation curves, etc. no longer match the specifications with which the system was modeled.

## 3. Measurement Precision

The precision of PMU data is one of the major advances in power system operation. Burnett, et al. [13] discusses commissioning of a power system stabilizer on a large generator unit. The only available means to asses the stabilizer was fault recording equipment at the generator location. This equipment did not have the ability to detect the effects of the stabilizer due to changes of less than 1 percent to the system. The PMU
however, gave a very accurate portrayal of the system during these small changes and the subsequent effect the stabilizer had on the system.

On a larger scale, precision in system parameters can allow operators to run the system closer to its limits, or to anticipate when power should be rerouted to allow for heavier loading. This feature is currently being used in Scandinavia, where smart control based on phasor measurements is being used as an alternative to adding new transmission lines to a heavily loaded grid $[12,14]$. If load demand keeps increasing new lines will eventually be necessary, but smart control can provide a temporary solution for an overstressed grid.

## 4. System Monitoring

The precision of a PMU also lends itself to system monitoring. Parameters that provide indication of system state, such as undervoltage, overvoltage, overcurrent, and undercurrent can be read directly from the PMU. Standardized frequency can be calculated based on the reference point and angular measurements. On a higher level, thermal monitoring of transmission lines that depend heavily on resistance can be enhanced using actual measurements and calculated line parameters based on PMU data [15].

## 5. Stability

The ability of a PMU to monitor voltages and angles in real-time is a valuable tool for stability analysis. Many times a system heading for instability can be identified by separating angular values or voltage levels approaching a critical point. If the operator is given warning of instability, remedial action may be taken to possibly avoid insecure
system condition. Angular deviation can be directly observed from the phasor data. Voltage collapse requires additional computation and current research $[2,16]$ suggests using a stability index (sometimes called a stability margin) for real time observation. In the case of frequency deviations, remedial action involves separating the system into sections with common frequencies, called "islands". The goal is to separate the system while still maintaining system load-generation balance. The best separation point candidates are located through off-line analysis, which is based on pre-calculated system behaviors [17]. In a system that is becoming unstable this assumed system is likely inaccurate. The PMU measurements can give the operator real-time parameters from the system, giving a much clearer picture of the consequences of separation. Often, once stability is restored in a system the "islands" are reconnected to return the system to its normal operating configuration. When two buses, separated by a line with open breakers, are to be electrically reconnected, it is important that the angles at these two buses are similar. If the angular difference is too large, the system may again become unstable. PMUs can monitor precise angle data from the buses, making reconnection safer and more reliable.

## 6. Protection

The improvements in protection schemes available from PMUs are somewhat obvious -- the more precise the voltage and current measurements, the more accurate the protection. Adaptive relaying optimizes system protection by allowing a relay to receive communication, from a remote location, to change its settings based on different system configurations $[17,18]$. It follows easily that these settings will be more accurate if they
are based on actual rather than approximated system measurements. High impedance faults often have low fault currents that may not cross detection thresholds in protective devices [19]. PMUs can provide very accurate phasor representations to aid in detection of a high impedance fault. This detection, paired with communication devices, allows for proper isolation of high impedance faults. On a secondary level, if there are two PMUs installed on a line, one at each end, the exact impedance of that line can be calculated. Many protective device settings rely heavily on line impedances, as these impedances are used to determine fault currents.

## 7. Fault Recording

The PMU can actually provide 60 Hz wave form and magnitude data in faulted conditions using individual channels [13]. This feature helps to give the operator a realtime visual representation of the system during a fault, possibly even allowing for realtime transient analysis. The PMU measurements can also be used to more accurately determine the location, time, and consequences of any faults that may be difficult to simulate, proving PMU applications to be valuable in post-disturbance analysis.

### 1.5 Contingency Analysis Using Phasor Measurements

In addition to faults, other system contingencies caused by losses of generators, loads, or lines can be observed by the PMU. These disturbances usually require more in depth and/or longer observation of the system while still requiring system data in relatively frequent intervals, making PMU a favorable alternative to repeatedly running a load flow on the system. As mentioned earlier, load flows in contingency analysis rely on state estimators, which take minutes to obtain results, for system phasor measurements.

PMUs can also provide these measurements, and multiple system PMU locations along with high speed communications networks make it possible to create wide-area monitoring systems that can update a snapshot of the power system within as little as one second [20,21].

This gives the system operator more time to determine a solution to any violations introduced by the contingency as measurements are taken at frequent intervals to insure that the conditions of the system have not drastically changed over the last interval [1]. Efficient corrective action, in preventative and restorative capacities, by the system operator reduces stress on the system, allowing the network to run closer to "base case" conditions for longer periods of time. Given the advantages of direct PMU measurement over state estimation procedures, phasor measurements and their applications are a more realizable tool for the power industry today. Phasors are currently being heavily researched and their advantages over current practices are becoming increasingly apparent in many areas of study.

### 1.6 System Integration of PMUs

Though the PMU has obvious advantages for current practices in both data acquisition and system applications, many utilities have not yet fully integrated PMUs into their systems. If a system has older protection schemes, placing a new PMU into a pre-existing power system can be both tedious and expensive. The objective then becomes obtaining the largest possible quantity of accurate information from a minimum number of installed PMUs. If a system has newer protection schemes, the PMU capabilities are already built into the protection devices. The objective now becomes
obtaining the largest quantity of accurate information from a minimum amount of PMU data observation. Most of the literature reviewed addresses PMU placement rather than PMU observation, so the remainder of this section focuses on PMU placement objective. Note that if the PMU data is already available, "observation" can be substituted for "placement" without loss of generality, i.e. the PMU placement locations simply become the locations where PMU data should be observed.

A system is said to be observable if it contains enough PMUs to determine the complete set of bus voltages. Current research in PMU placement suggests PMUs need to be installed on $1 / 5$ to $1 / 3$ of the total number of system buses to be observable [22]. This is still very intensive for larger systems. Nuqui, et al. [4] provides an algorithm to determine the "depth-of-unobservability" of buses within the system. This algorithm, based in linear programming, determines depths for each bus, where increasing depths correspond to more approximated bus measurements. For example, a bus with depth 0 has a PMU, depth 1 is one bus away from the PMU and can be accurately calculated as previously mentioned. Depth of 2 is two buses away from the PMU, and one bus away from the previous calculated depth of 1 bus, etc. This algorithm is a helpful approach to understanding the trade-off between system state accuracy and installation time/cost of the PMUs.

Many researchers $[15,23]$ claim that placement of PMUs should depend on the desired network applications, many of which may not require complete observability [4]. PMU placement is then based on a subset of buses or lines which indicate the general state of a system. These subsets are determined using multiple "sensitivity" approaches,
which depend on the desired application. The sensitive buses and/or lines are the ones which give the most information about the system state, and are good candidates for PMU locations.

## 1. Line Sensitivity and Selectivity Methods

Sometimes it is preferred to choose a subset of lines in a system that gives the most information about system state. The selection of lines could consist of:

1) Lines which are already heavily loaded or operating close to limits, and therefore are prone to overloading.
2) Lines which are connecting different areas, i.e. tie lines, which provide support between areas.
3) Lines which are supporting a radial load feeder, whose disconnection would cause load shedding to occur.
4) Lines which are incident to one or more of the sensitive buses found using the methods described in the below section.

Depending on the application, the converse of the latter could also be true, where the sensitive buses are those most connected to the sensitive lines. Though the research almost exclusively favors bus sensitivity over line sensitivity, a voltage phasor approach was used [24] to determine a transmission path stability index which identifies the critical transmission paths within the system. This approach directly determines which lines in the system are sensitive in terms of stability.

## 2. Bus Sensitivity Methods

Placing PMUs for voltage stability monitoring involves identifying buses where instability may be a problem [15]. As mentioned previously, a stability margin is often a good indicator of proximity to voltage collapse. In [25] the stability margin is calculated
by determining the highest changes in voltage with respect to changes in active and reactive loading within the system. A similar approach, taken by [26] characterizes the margin two ways. First, how much the reactive loss increases when the reactive load increases, second, how much the voltage changes when injected reactive power changes. The authors in [27] determine the sensitivity of node voltages to changes in line parameters by calculating the derivative of the voltage angle with respect to the node self admittance.

Tso, et al. [28] uses history of bus behavior from a large quantity of previous disturbances to statistically determine which buses in the system are most sensitive to load changes. In [29], decoupled Newton-Raphson load flow is extended to indicate which buses in the system are the most sensitive to changes in angle or voltage magnitude at other points in the system. The authors also discuss the importance of placing enough PMUs so that the system can be monitored both in normal state and in an isolated state. Islanding is mentioned as an important consideration for PMU placement [29-31], both in stability and state estimation applications.

For state estimation, it is unnecessary to replace all SCADA data with PMU data in order for the estimate to be valid. The PMUs should be placed wherever they provide the most benefit to estimate accuracy. One method for determining sensitive buses with respect to state estimation involves numerical analysis. This numerical method determines redundancy levels if a measurement is taken at different points in the system [32,33]. If the redundancy level at a particular location equals zero, there must be a measurement unit there. This method utilizes the dependency of measurements on one
another. Jiang, et al. [34] expands the previous algorithm by separating the buses into two groups. The first group contains the locations of critical measurements and measurements with low accuracy. The second group contains the locations of measurements with low redundancy. PMUs are then installed at the most connected buses appearing in both lists.

A different approach is to base PMU placement on how well they are distributed within a system. Zhao, et al. [35] suggests dividing a system into small areas where state estimation is performed, with a PMU in each area. The results will then be sent to a central location to obtain the full system state. The idea of distributed state estimation is expanded upon in [36]. The system is divided as before, with one PMU in each area. The method then uses the residual sensitivity matrix to determine which buses internal to one area are sensitive to changes in a different area.

It is important to note that obtaining the number of required PMUs through sensitivity analysis alone does not ensure efficient system monitoring. Communications channels are of vital importance in PMU implementation, as there must be a way for information to get from the PMU to the control center [37]. If additional expense is not desired and communications are not present at the sensitive buses, the presence of an available communications channel may be considered as an additional constraint in sensitivity determinations. This constraint is not considered in this thesis as much of the communication availability is unknown.

### 1.7 Thesis Overview

This thesis examines a new approach to solve contingencies, especially those involving overloaded lines, within a large power system. The main goal is to observe
problem contingencies before they occur. In addition, the approach may provide fast solutions to any contingencies that may occur unexpectedly by providing a list of critical lines to prevent blackouts under peak loading conditions. This is done by examining the off-line data at each bus of a large scale system and attempting to find predictive patterns in voltage and/or current phasors in the system using load flow solutions when the system is under heavy loading conditions. The use of specific phasors, in particular the voltage magnitude and current angle, from the system is suggested to provide the most accurate predictive pattern of system condition. The off-line results are then compared with online data provided by a local utility for verification. A general algorithm is then developed using on-line data and phasors (obtained through the use of PMUs). This may eliminate the need for a load flow solution and provide a faster resolution to the problem, in turn preventing blackouts.

## CHAPTER TWO

## OFF-LINE ANALYSIS

### 2.1 System Overview and Software

Off-line analysis is a valuable tool in becoming familiar with a power system and its characteristics. Building a base case scenario and then varying loads, removing lines, applying faults, etc. gives the operator insight into how the system will react in different situations. This is not always helpful in real-time applications because of speed requirements, as mentioned earlier, but provides basic understanding into the fundamental properties and characteristics of an individual system.

The system considered for off-line analysis in this thesis is the IEEE 37-bus system with a base of $138 \mathrm{kV}, 100 \mathrm{MVA}$. The base case contains fifty-two lines, fifteen loads, one regulating transformer, and six generators. All bus and line data for the base case is specified. The bus data includes bus number, assumed voltage at the bus, generation in MW and MVAR, and load in MW and MVAR. Line data includes location (from bus, to bus) impedance and resistance in per unit, and line charging in per unit.

The system is simulated using PowerWorld. This program allows simulation of any power system and is capable of performing power flow analysis for up to 100,000 bus systems. Fault analysis and contingency analysis are available. Networks are built in the edit mode and solutions calculated and determined in the run mode. A concise listing of all bus voltages, bus criterion, line flow, and line criterion is available throughout simulation. Contingency analysis is readily available for large scale studies, and the user can easily simulate breaker trips for studies of outages on a smaller scale. This software is
chosen above other simulators, such as PSCAD, PSSE, etc, for its user interface and output data configurations.

### 2.2 Method Outline

In the course of this research the power flow output of the system is analyzed offline and the voltage constraints on the buses are verified. The total load, losses, and generation are determined as well as the generation at the slack bus. Four areas are created to divide the system. The areas are created by dividing the number of buses in the system into four equally sized subsections, and each area is guaranteed at least one generator. The base loading condition one-line diagram is shown in Figure 2.1.

As a final step in preparing the system for analysis, all loads in the system are increased in increments of 5\%. Any violated criteria are fixed by modifying the design of the system. It is found that the system can sustain an overall $20 \%$ load increase before any lines violate $>100 \%$ of their MVA limit. The one-line diagram of the overloaded system is shown in Figure 2.2.

In PowerWorld, total MVA of the line is shown as a percentage of the MVA limit of the line on pie-charts for each line in the one line diagram of the system. A blue pie chart shows the line is operating under $80 \%$ of its total MVA limit, an orange pie chart means the line is operating between 80 and $100 \%$ of its total MVA limit, and a red pie chart means the line is operating at or above $100 \%$ of its total MVA limit. A line within the system may operate safely under $80 \%$ conditions, for a short time in $80-100 \%$ conditions, and any line over $100 \%$ loading condition is not allowed and must be relieved immediately to avoid any problems.


Figure 2.1 - One-Line Diagram of Base Loading Condition in Off-line Analysis


Figure 2.2 - One-Line Diagram of 20\% Increased Loading Condition in Off-line Analysis

Once all areas are defined, contingency analysis is performed. First, a specified line in an area of the system is intentionally removed to define the overloaded lines. Line outage(s) may occur due to a fault or to relieve overloads at certain operating conditions. Second, lines within the system are ranked based on their contribution to overloaded conditions when forced outage is implemented. The critical and limited lines are identified. This allows the system to rank the lines that may relieve the overloading situation. The off-line data is also analyzed in cases of single or multiple forced outages to get familiar with several cases that may cause severe overloading conditions. These cases involve both the
base case and maximum allowable loading that the system may have at the precontingency cases. In the case of contingencies, different line(s) within the system are intentionally removed and load flow data is observed for the new system configurations.

Voltage and current phasors are observed in both pre- and post-contingency cases.

### 2.3 Phasor Application in Contingency Evaluation

Voltage and current phasors have previously been studied for different applications. One such application is contingency evaluation. The voltage and current phasors are available if phasor measurement units (PMUs) are installed in the system. Power can easily be calculated from the voltage and current measurements.

As mentioned in the previous chapter, voltage magnitude is often not a good indicator of a stressed system. This can be verified using off-line simulations. Voltage measurements are taken at all buses after the load is increased by $20 \%$. Nine lines within the system are slightly overloaded. Bus voltages are compared with the base case loading condition and the results are shown below (Figure 2.3).


Figure 2.3 - Off-line Bus Voltages Before and After Load Increase
Figure 2.3 shows the change in voltage is extremely small, with the median change as .004 per unit. Several cases are studied and it is concluded that the voltage magnitude will not be considered as an overload indicative phasor measurement.

In contrast, widening angular difference is very apparent between the base loading case and $20 \%$ increased loading case, as shown in Figure 2.4. Many of the voltage angle
differences are almost doubled. This concludes the voltage angle difference $\left(\delta_{\mathrm{ij}}\right)$ is a good indicator for system behaviors during different loading conditions.



Figure 2.4 -Angle Differences ( $\delta \mathrm{ij}$ ) Before and After Load Increase
The real power in heavily loaded lines is also studied. Loads are gradually increased and real power is observed at each loading condition. The system is manipulated to get each line as close to $80 \%$ and as close to $100 \%$ as possible to capture the exact limits. The power is then plotted versus delta for each of the lines, as the power is a function of the voltage angle. The results are shown in Figure 2.5.


Figure 2.5 - Power vs. Voltage Angle on Heavily Loaded Lines

The desired pattern is to set a vertical limit which indicates significant changes in power flow. As the data has different starting values and different incremental changes, a percent increase is used for uniformity. There are still, however, lines which fall outside of a pattern. It is noticed that in line 20-19, the power flow crosses back and forth over $100 \%$ capacity. The graphs clearly show a trend, but not all of the graphs follow the same patterns. The most obvious cause for this is that the power still depends on the voltage magnitude. As noted earlier, the bus voltage magnitude is not a good indicator of system overload. This magnitude may or may not change depending on the properties of the bus in the system (i.e. whether it is close to a generator, capacitor, or regulating transformer).

The data seems close to adhering to a pattern however, so the next logical step is to consider the current and its angle. These values are also included in the power calculations. Again, a percent increase is used to quantify the changes as the values do not start off at the same values nor increase by the same numerical amounts. The percent increase limits for the current and its angle are also considered separately. Again, the most heavily loaded lines are considered, and the base capacity to $80 \%$ capacity and base capacity to $100 \%$ capacity increase percentages are calculated. These percentages in currents and their angles are shown in Figure 2.6.


Figure 2.6 - Percent Increases in Current Magnitude and Current Angle
A vertical limit can clearly be determined using the current magnitude, as the largest base capacity to $80 \%$ increase is smaller than the smallest base capacity to $100 \%$ increase. The current angle does not follow this pattern though, and is therefore not a good phasor indicator. From these results, it is decided that a similar percent increase chart will be created for the voltage angle difference, $\delta \mathrm{ij}$. The percent increase limits for voltage angle and current are considered separately, as they do not directly depend on one another. Indicative phasor limits will be established using the voltage angle and current magnitude.

### 2.4 Initial Off-line Solution

From the off line study, an initial solution is obtained to indicate any problems that might occur due to line outage(s). This initial solution involves taking a measurement of current magnitude and voltage angle when the power through each line increases to $80 \%$ and $100 \%$ of the line limit. The system is manipulated to get the lines as close to these values as possible, through the use of single contingencies, multiple contingencies, system load increase, area load increase, and spot load increase. The percent increase (or decrease) in current magnitude and voltage angle are determined for base case to $80 \%$ capacity and base case to $100 \%$ capacity. Lines at similar base case capacities are compared to verify accuracy. The percent increases are found using the equation

$$
\% \text { inc } \cong\left|\frac{\left(\mathrm{x}_{\text {capacity }}-\mathrm{x}_{\text {initial }}\right)}{\mathrm{x}_{\text {initial }}} * 100\right|
$$

where $\mathrm{x}_{\text {capacity }}$ is the value of $\delta \mathrm{ij}$ or Iij at $80 \%$ or $100 \%$, depending on the desired limit, and $\mathrm{x}_{\text {initial }}$ is the value in the base case. This equation can be compared to the generation shift factors and line distribution factors presented in [38]. Inherent assumptions in these factors include complete linearity and propagated changes. In other words, a change somewhere in the system will have an affect on other lines within the system if there is a path between them. If no affect is noticed, the result of the percent increase equation above will simply be zero, compensating for the assumption. The numerical data for the percent increases of lines around $73 \%$ capacity and $65 \%$ capacity can be found in Tables 2.1-2.4.

Table 2.1-\% Increases of Voltage Angle for Lines at 73\% Initial Capacity

| Voltage Angle Difference (8ij in <br> Degrees) | Initial <br> Capacity | $\mathbf{8 0 \%} \%$ <br> Capacity | $\mathbf{1 0 0 \%}$ <br> Capacity |
| :---: | :---: | :---: | :---: |
| Line 1-10, 72\% Initial Capacity | $2.44^{\circ}$ | $2.68^{\circ}$ | $3.295^{\circ}$ |
| Line 20-19, 73\% Initial Capacity | $0.79^{\circ}$ | $0.895^{\circ}$ | $1.13^{\circ}$ |
| Line10-11, 73\% Initial Capacity | $2.21^{\circ}$ | $2.7^{\circ}$ | $3.31^{\circ}$ |
| Line 12-20,74\% Initial Capacity | $0.67^{\circ}$ | $0.74^{\circ}$ | $0.9^{\circ}$ |
| \% Increase in Voltage Angle | Initial to 80\% | Initial to 100\% |  |
| Line 1-10, 72\% Initial Capacity | $9.386 \%$ | $35.041 \%$ |  |
| Line 20-19,73\% Initial Capacity | $13.291 \%$ | $43.038 \%$ |  |
| Line10-11,73\% Initial Capacity | $22.172 \%$ | $49.774 \%$ |  |
| Line 12-20,74\% Initial Capacity | $10.448 \%$ | $34.328 \%$ |  |

Table 2.2 - \% Increases of Current Magnitude for Lines at 73\% Initial Capacity

| Current Magnitude (I $\mathbf{I}_{\mathbf{i j}}$ in p.u.) | Initial <br> Capacity | $\mathbf{8 0 \%}$ <br> Capacity | $\mathbf{1 0 0 \%}$ <br> Capacity |
| :---: | :---: | :---: | :---: |
| Line 1-10, 72\% Initial Capacity | 2.21 pu | 2.45 pu | 2.99 pu |
| Line 20-19,73\% Initial Capacity | 1.43 pu | 1.60 pu | 1.99 pu |
| Line10-11, 73\% Initial Capacity | 1.48 pu | 1.73 pu | 2.07 pu |
| Line 12-20, 74\% Initial Capacity | 1.44 pu | 1.58 pu | 1.93 pu |
| \% Increase in Current <br> Magnitude | Initial to 80\% | Initial to 100\% |  |
| Line 1-10, 72\% Initial Capacity | $11.104 \%$ | $35.571 \%$ |  |
| Line 20-19,73\% Initial Capacity | $12.327 \%$ | $39.755 \%$ |  |
| Line10-11,73\% Initial Capacity | $16.635 \%$ | $39.280 \%$ |  |
| Line 12-20, 74\% Initial Capacity | $10.093 \%$ | $33.896 \%$ |  |

Table 2.3-\% Increases of Voltage Angle for Lines at 65\% Initial Capacity

| Voltage Angle Difference (iij in <br> Degrees) | Initial <br> Capacity | $\mathbf{8 0 \%}$ <br> Capacity | $\mathbf{1 0 0 \%}$ <br> Capacity |
| :---: | :---: | :---: | :---: |
| Line 9-14, 63\% Initial Capacity | $0.88^{\circ}$ | -- | $1.5^{\circ}$ |
| Line 16-15, 65\% Initial Capacity | $1.31^{\circ}$ | $1.68^{\circ}$ | $2.23^{\circ}$ |
| Line 21-22, 65\% Initial Capacity | $1.0^{\circ}$ | $1.45^{\circ}$ | $1.86^{\circ}$ |
| Line 1-24, 67\% Initial Capacity | $4.25^{\circ}$ | $5.35^{\circ}$ | $6.69^{\circ}$ |
| \% Increase in Voltage Angle | Initial to 80\% | Initial to 100\% |  |
| Line 9-14, 63\% Initial Capacity | -- | $70.455 \%$ |  |
| Line 16-15, 65\% Initial Capacity | $28.244 \%$ | $70.229 \%$ |  |
| Line 21-22, 65\% Initial Capacity | $45.000 \%$ | $86.000 \%$ |  |
| Line 1-24, 67\% Initial Capacity | $25.882 \%$ | $57.412 \%$ |  |

Table 2.4-\% Increases of Current Magnitude for Lines at 65\% Initial Capacity

| Current Magnitude (I $\mathbf{I}_{\text {ij }}$ in p.u.) | Initial <br> Capacity | $\mathbf{8 0 \%}$ <br> Capacity | $\mathbf{1 0 0 \%}$ <br> Capacity |
| :---: | :---: | :---: | :---: |
| Line 9-14, 63\% Initial Capacity | 1.15 pu | -- | 1.78 pu |
| Line 16-15, 65\% Initial Capacity | 0.54 pu | 0.63 pu | 0.84 pu |
| Line 21-22, 65\% Initial Capacity | 0.37 pu | 0.38 pu | 0.47 pu |
| Line 1-24, 67\% Initial Capacity | 0.35 pu | 0.42 pu | 0.52 pu |
| \% Increase in Current <br> Magnitude | Initial to 80\% | Initial to 100\% |  |
| Line 9-14, 63\% Initial Capacity | -- | $54.860 \%$ |  |
| Line 16-15, 65\% Initial Capacity | $15.274 \%$ | $54.525 \%$ |  |
| Line 21-22, 65\% Initial Capacity | $2.510 \%$ | $29.252 \%$ |  |
| Line 1-24, 67\% Initial Capacity | $20.688 \%$ | $46.761 \%$ |  |

Figure 2.7 gives a summary of the percent increase in voltage angle and current magnitude of lines initially operating around $73 \%$ capacity and Figure 2.8 gives a summary for lines initially operating around $65 \%$ capacity.


Figure 2.7 - Percent Increases in Current Magnitude and Voltage Angles for 73\%


Figure 2.8 - Percent Increases in Current Magnitude and Voltage Angles for 65\% Once the percent increases are completed, an average is found over all percent increases through the system on lines moving to $80 \%$ and $100 \%$ capacity. This allows for a limit in percent increase to indicate overloads and/or contingencies within the system. The limits, as computed for each group of lines and for the overall system, are shown in Table 2.5.

Table 2.5 - Percent Increase Limits

|  | Base to 80\% |  | Base to $100 \%$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Lines Included | ¢ij | \|Iij| | Sij | \|Iij| |
| Around 65\% | $33.042 \%$ | 17.981\% | 71.024\% | 52.049\% |
| Around 73\% | 13.824\% | 12.540\% | 40.545\% | 37.126\% |
| Averaged Over All | 22.052\% | 14.354\% | 55.785\% | 43.521\% |

For this particular system, lines initially operating above $65 \%$ capacity are averaged to obtain the percent increase limits. The nine lines that overload when the overall system load is increased to $20 \%$ and their percent increases are numerically given in Table 2.6.

Table 2.6 -Percent Increases of Overloaded Lines

| Line | \% Capacity | \% Increase $\delta \mathrm{ij}$ | \% Increase \|Iij| |
| :---: | :---: | :---: | :---: |
| $12-20$ | 87 | 22.39 | 21.20 |
| $10-11$ | 87 | 29.41 | 23.76 |
| $21-22$ | 87 | 61.00 | 12.37 |
| $11-12$ | 85 | 31.25 | 13.33 |
| $19-20$ | 86 | 20.25 | 19.09 |
| $14-12$ | 80 | 12.71 | 16.24 |
| $1-24$ | 96 | 52.00 | 42.23 |
| $1-10$ | 92 | 25.00 | 23.80 |
| $3-5$ | 83 | 13.33 | 4.29 |

The last three lines in the list are incident to generator buses, and are the lines that adhere the least to the limits. This follows from the fact that generator buses are controlled buses, allowing smaller changes in parameters. If the lines incident to generator buses are neglected, the resulting subset (Figure 2.9) shows much less deviation. Note, however, that some limits are not crossed even though the lines are above $80 \%$ capacity. This is due to the fact that the average limits are used. Most of the lines that overloaded are closer to the $73 \%$ capacity range than the $65 \%$ range. If the limits for $73 \%$ are applied all
thresholds are crossed. This implies the averaging of limits should be based on the desires of the operator, and that the limits are valid in the off-line analysis.


Figure 2.9 -Percent Increases of Overloaded Lines
It should also be noted that actual numerical limits will have to be determined for each individual system depending on its initial "base case" flow, line limits, etc.

### 2.5 Application of Off-line Solution

Once the limits are established for a certain system, an interface must be created so that the operator can observe the limits and violations. The idea of a visual representation of system state is not new. Phadke [39] discusses the importance of realtime displays using PMU data in system operation. The authors in [40] give many realtime display options, ranging from images similar to the PowerWorld one-line flow diagrams, to bar graph representations of voltage magnitudes, to contour plots displaying
voltage angle differences. The representation chosen for this thesis is proposed by Xie, et al. [41]. The authors impose limits on a rotating phasor pie chart, with different color zones to indicate the severity. Though the original use of this chart is for stability and dynamic security, the same idea can be applied to system overloads.

The proposed rotating phasor chart is generalized below in Figure 2.10. Note that the current magnitude and voltage angle limits work jointly to indicate overload, meaning both limits must be reached for the line to cross over the threshold from one color to another. For this reason, the operator display will consist of two pie charts, one for voltage angle, and one for current magnitude. Further benefits of this representation will be discussed later in the thesis. Figure 2.11 provides an example of the display representation from the $20 \%$ increased loading system, with an $87 \%$ capacity overload of line 10-11.


Figure 2.10 - Generalized Rotating Phasor Pie Chart


Figure 2.11 - Implementation of Phasor Chart
An additional application of the off-line analysis involves placement of the PMUs, and therefore limit charts, with in the system. This placement relies on sensitivity analysis, and will be discussed further in the next chapter.

### 2.6 Conclusion of the Off-line Phasor Study

This chapter examines a power system as well as many of its characteristics when the system is operating at heavy loading conditions. Lines which are critical to the system are identified. Off-line phasor data is gathered both for entire system load increases and area load increases. Any lines which do not cause critical power flow when removed cause other minor overloads in the system. This allows for a small scale change in phasors as opposed to a critical overloaded line. Contingency analysis using phasors can then be performed on lines still operable within the system. The data taken off-line is analyzed and a pattern is found which can now be applied to the on-line data. Limits are established using the percent increase in current magnitude jointly with the percent
increase in voltage angle difference over the line. Further investigation of the phasor pattern, as well at its applications in sensitivity analysis will be discussed in the following sections. Applications in on-line studies will be discussed in the next chapter.

### 2.7 Review of Sensitivity

Sensitivity analysis methods, as discussed in Chapter 1, are very important when analyzing a large power system. Systems with older protection schemes may not have PMU data available at all locations in the system. Sensitivity methods help to determine the most efficient placement of PMUs. Newer protection devices come equipped with the ability to obtain phasor measurements. Now data processing, rather than location, is an issue. Multiple phasor measurements are taken by each PMU. If many PMUs are located in the system, the amount of resulting data will be enormous. Operators need an efficient way to process only the data that correlates strongly to system state. This reduces both the communication required, as there is less data to move from the PMU to the operator, and also the time necessary for data evaluation.

### 2.8 Derivation of Sensitivity Equation

The equation for determination of sensitivity in this thesis is developed using a premise from the previous chapter. The current magnitude and voltage angle are shown to be good indicators of system state, and are therefore used in determining the sensitivity to overload as well. The equation for current, as mentioned in Chapter 1, is

$$
\begin{equation*}
\left|I_{i j}\right|=\left|\frac{\left(V_{i} \angle \delta_{i}\right)-\left(V_{j} \angle \delta_{j}\right)}{\left(Z_{i j} \angle \theta_{i j}\right)}\right| \tag{2.8.1}
\end{equation*}
$$

Substituting the admittance, $\mathrm{Y}_{\mathrm{ij}}$ for the impedance, $\mathrm{Z}_{\mathrm{ij}}$, the equation becomes

$$
\begin{align*}
\left|I_{i j}\right| & =\left|\left(\left(V_{i} \angle \delta_{i}\right)-\left(V_{j} \angle \delta_{j}\right)\right)\left(Y_{i j} \angle \theta_{i j}\right)\right| \\
& =\left|\left(V_{i} \angle \delta_{i}\right)-\left(V_{j} \angle \delta_{j}\right)\right|\left|Y_{i j}\right| \tag{2.8.2}
\end{align*}
$$

Then the Law of Cosines is used to obtain the current in terms of $\delta_{\mathrm{ij}}$. A review of the law of cosines is shown in Figure 2.12, and the resulting equation is


Figure 2.12 - Law of Cosines

$$
\begin{equation*}
\left|\vec{V}_{i}-\vec{V}_{j}\right|^{2}=\left|\vec{V}_{i}\right|^{2}+\left|\vec{V}_{j}\right|^{2}-2\left|\vec{V}_{i}\right|\left|\vec{V}_{j}\right| \cos \left(\delta_{i}-\delta_{j}\right) \tag{2.8.3}
\end{equation*}
$$

The equation for the current is now

$$
\begin{equation*}
\left|I_{i j}\right|=\sqrt{V_{i}^{2}+V_{j}^{2}-2 V_{i} V_{j} \cos \left(\delta_{i j}\right)} *\left|Y_{i j}\right| \tag{2.8.4}
\end{equation*}
$$

Taking the derivative with respect to $\delta \mathrm{ij}$,

$$
\begin{equation*}
\frac{\partial\left|I_{i j}\right|}{\partial \delta_{i j}}=\frac{1}{2 \sqrt{V_{i}^{2}+V_{j}^{2}-2 V_{i} V_{j} \cos \left(\delta_{i j}\right)}} * 2 V_{i} V_{j} \sin \left(\delta_{i j}\right) *\left|Y_{i j}\right| \tag{2.8.5}
\end{equation*}
$$

Here the derivative of the current magnitude with respect to the voltage angle is used to determine line sensitivities. The final sensitivity equation is then

$$
\begin{equation*}
\frac{\partial\left|I_{i j}\right|}{\partial \delta_{i j}}=\frac{V_{i} V_{j} \sin \left(\delta_{i j}\right) *\left|Y_{i j}\right|}{\sqrt{V_{i}^{2}+V_{j}^{2}-2 V_{i} V_{j} \cos \left(\delta_{i j}\right)}} \tag{2.8.6}
\end{equation*}
$$

### 2.9 Off-line Application of Sensitivity Equation

This equation is applied in the off-line analysis and the sensitive lines are determined. Lines with a sensitivity value above 100 are further considered. The equation is applied again after the $20 \%$ loading increase and the same lines appear in the sensitive line list. The results of the equation both in the base loading and increased loading cases are shown in Table 2.7.

Table 2.7 - Line Sensitivities Before and After Off-line Loading Increase

| Line | Sensitivity - Base Loading | Sensitivity - 20\% Increased Loading |
| :---: | :---: | :---: |
| $29-33$ | 776.25 | 772.45 |
| $19-34$ | 424.56 | 418.81 |
| $12-17$ | 181.38 | 180.74 |
| $12-20$ | 121.18 | 119.77 |
| $19-20$ | 101.54 | 100.33 |

Note that the sensitive lines are, in most cases, not the overloaded lines. However, the buses which are most likely to be incident to overloaded lines, referred to as sensitive buses, can be established. The bus sensitivity will be further investigated. This sensitivity will be based on voltage collapse analysis.

### 2.10 Conclusion of Off-line Sensitivity Study

In this chapter, offline sensitivity analysis is applied to the system using Equation 2.8.6. This equation shows consistency over different loading conditions. This equation will be applied in the on-line analysis to monitor and analyze the PMU data. This greatly reduces the evaluation time necessary for on-line data collection.

## CHAPTER THREE

## ON-LINE ANALYSIS

### 3.1 Consistency Study

A consistency study is performed before the on-line study for further verification. This study is performed using real system data from an EMS snapshot, but not obtained from a PMU. The observed system contains 1800 buses, with a 230 kV transmission system. All necessary data is not available to perform complete analysis. Multiple cases of contingencies, most involving outages, are observed. Percent increases in both current magnitude and voltage angle are obtained for the provided system subset. These percentages are shown in Table 3.1. No line MVA limits are breached, meaning no lines become overloaded after these outages. However, some line MVA limits seem to be arbitrarily set to very large values implying the actual percent loading of the line could be inaccurate. This contributes to the inability to perform complete analysis.

Table 3.1 - Percent Increases for Consistency Study

| Outage | Remaining Lines | \% Increase IIijl | \% Increase $\delta \mathrm{ij}$ |
| :---: | :---: | :---: | :---: |
| 1.230 kV Line | 1 | 77.78 | 25.0 |
|  | 2 | 15.15 | 20.0 |
|  | 3 | 45.16 | 28.57 |
|  | 4 | 65.58 | 34.93 |
| 2.230 kV Line | 1 | 108.77 | 97.11 |
|  | 2 | 30.46 | 89.06 |
|  | 3 | 18.6 | 25.0 |
|  | 4 | 40.7 | 68.92 |
| 3.69 kV Line | 5 | 20.33 | 28.38 |
|  | 1 | 1.08 | 12.5 |
|  | 2 | 81.48 | 41.67 |
|  | 3 | 0.47 | 2.74 |
|  | 4 | 0.38 | 2.78 |
|  | 5 | 0.88 | 4.76 |

The phasors are observed before and after each outage, and the percent increases are consistent with the previously mentioned pie chart. The important conclusion here is that a percent increase in current magnitude corresponds to a proportional percent increase in voltage angle and vice versa. There are very few cases when the percent increase in one phasor quantity changed significantly without a proportional percent increase in the other. This leads to the conclusion that the rotating limit pie chart is applicable in the on-line analysis as well. Complete on-line analysis of a system with PMUs is presented in the following sections.

### 3.2 On-line Analysis Overview

On-line analysis is a vital component to this research. Off-line analysis of power systems is useful in determining and verifying patterns. These patterns will be applied to systems analyzed on-line and must be further verified for use in real-time. The actual system contains 7476 buses. It is divided into forty-seven areas, with the area of interest containing 1289 buses. Data from nine PMUs is available for this area, with most of the PMUs located on the 500 kV transmission system. The window of PMU data includes a three phase fault occurring in the low voltage side of the system.

### 3.3 System Reduction

Data for this system is provided in table format, with no visual aid. In order to verify the PMU data and system behavior, the system configuration is necessary. Attempting to determine this configuration solely by tracing out line and bus connections, found in the data table provided by the utility, requires an enormous amount of time. The
system must be reduced to a manageable size for analysis. The approaches investigated in this thesis are discussed below.

1. Parallel and Series Equivalents

A MATLAB computer program is written to reduce lines in parallel or series combinations. The generator buses and PMU buses are kept, as well as buses with loads over 5\% of the maximum load. All other buses are candidates for removal. If two or more lines are in series, the connecting buses are eliminated and the new line impedance is

$$
Z_{e q}=Z_{1}+Z_{2}+\cdots+Z_{n}
$$

If two or more lines are in parallel, the equivalent single line is drawn with new impedance

$$
Z_{e q}=\left(\frac{1}{Z_{1}}+\frac{1}{Z_{2}}+\ldots+\frac{1}{Z_{n}}\right)^{-1}
$$

The program cycles through series and parallel equivalents until no more reductions are possible, either due to load/generator/PMU constraints or connectivity constraints. The system is passed through this program, and the number of buses is reduced to 656. This is still much too large for visual analysis. It is concluded that this method of reduction alone is not efficient for a dense system of this size.

## 2. Lumped Generation and Load Equivalents

The premise of this method uses Thévenin equivalents (obtained from the Ybus) to replace areas that are only loads or only generators. In the case of a lumped load, one bus acts as the top of a feeder for all the loads below. This load area is removed and the equivalent Thévenin impedance of the area is placed on the feeder bus. The concept is
similar for generation. If a system is mostly radial, consolidating generation and loads can reduce the system rapidly. In the case of a loop system however, the system reduction is much smaller. For this particular system, 409 buses have either a generator, a significant load, or both. Due to the loop nature of the system, many of these can not be consolidated. Lumping loads and generation only reduces the system by about 100 buses, leaving around 550 buses. This is still too large for analysis. It is concluded that the system contains too many loops for this method to be efficient.

## 3. Sparse Matrix Equivalents

This method is generally regarded as the most accurate equivalent [42, 43], but also takes a long time for set up and computation. The method is based on the sparsity of the Ybus. Again, in a loop system, the Ybus will be much less sparse than in a radial system. As this system has many loops, the Ybus will have many non-zero entries. It is decided that the tradeoff in accuracy and time is not significant enough for this method to be efficient for the system.

## 4. Power-Injection Equivalents

Most of the PMUs in the on-line analysis are on the major transmission lines. As this thesis focuses on transmission, it is decided that the transmission system will be separated from medium and low voltages. This means that all 500 kV buses within the area are kept. A map of the system is shown in Figure 3.1. The majority of the 500 kV transmission lines are shown in red. The majority of the 161 kV lines are shown in green, and voltages less than 161 kV are not shown. It can clearly be seen that the system needs to be reduced for accurate visual analysis.


Figure 3.1 - Map of System Analyzed On-line
Power flow injections are observed for lines connecting high voltage to lower voltage, and for tie lines connecting 500 kV buses in the area of study to 500 kV buses in other areas. Any buses that are incident to the above lines are referred to as "boundary buses". Buses in lower voltage or other areas are "external buses", and 500 kV buses in
the area of study that are connected only to other 500 kV buses within the area are "internal buses". When the equivalent is complete, the network consists of internal and boundary buses, along with the 500 kV transmission lines connecting them. Any power injection going out of a boundary bus is modeled as a load, any power injection coming into a boundary bus is modeled as a generator, and if there is a comparable exchange of power (i.e. in and out) the boundary bus has both a generator equivalent and load equivalent. Examples of these concepts are shown in Figures 3.2-3.4. The reduced system now contains only 47 buses, allowing for more straightforward visual analysis.


Figure 3.2 - Power Injection Out (Equivalent Load)


Figure 3.3 - Power Injection In (Equivalent Generator)


Figure 3.4 - Power Injection Exchange (Equivalent Generator + Equivalent Load)

### 3.4 Comparison of Reduced System and PMU Data

System reduction is performed for three cases. The first is the pre-fault condition, the second is the faulted condition, and the third is the post-fault condition. The reduced system one-line diagrams for each case are shown in Figures 3.5-3.7, respectively. System changes occur on the entire system and the power injections are recalculated for each individual case. As the fault occurs on the low voltage side, this provides the most accurate reduction. In all cases, the total generation is verified to be larger than the total load. The pre-fault slack bus is the bus with the highest generation, numbered bus 1 here, and remains the slack bus in all cases.


Figure 3.5 - One-Line Diagram of the Reduced System Pre-Fault


Figure 3.6 - One-Line Diagram of the Reduced System, Faulted


Figure 3.7 - One-Line Diagram of the Reduced System Post-Fault

The line current magnitudes and $\delta_{\mathrm{ij}}$ 's are determined in each reduced case. These results are then compared to the actual PMU data for verification. The PMU data for Bus 5, which is the closest 500 kV bus to the fault, is shown below in Figure 3.8. The data for the remaining high voltage PMUs (listed in order of distance from the fault) is shown in Figures 3.9-3.12.


Figure 3.8 - Bus 5 PMU Data



Figure 3.9 - PMU Data for Bus 6


Figure 3.10 - PMU Data for Bus 16



Figure 3.11 - PMU Data for Bus 26


Figure 3.12 - PMU Data for Bus 45

A three-phase bus fault occurs at time 0.00 on the time scale of the PMUs. The fault is cleared from the system after 29 cycles (about half a second) when the two lines incident to the faulted bus are outaged. It is clearly seen that the current magnitude spikes in either an upward or downward direction at the time of the fault, and a transient decay in current occurs after the fault is cleared. The current eventually settles at a value slightly higher or lower than the pre-fault value. The PMUs at a distance further away from the fault have less of a response to the fault, which is expected. The effect of the fault on voltage angle difference can most clearly be seen in the PMUs closest to the fault as well. Notice that at buses 5 and 6, the voltage angle differences increase when the fault occurs. The post-fault angle difference then returns to a level comparable to the pre-fault difference. Due to the length of the fault, however, the angular difference continues to increase after the fault is cleared. This may lead to a stability problem in the system even though no lines are overloaded.

The reduced system data follows closely with the PMU data, with the main sources of error provided by the transient nature of the system after the fault. PowerWorld models the system in steady state, and therefore shows no transient values. The reduced post-fault system current values match best with the PMU current data after transient decay, and the continuous increase in voltage angle difference cannot be observed. This highlights the importance of monitoring actual system data in real time as opposed to simply simulating events on the system, as well as understanding the capabilities of the chosen simulation program.

Note that as voltage phasor magnitudes are affected by many factors, the current magnitude is also affected by other factors such as errors in current transformers (CT's). In the given PMU data, these errors are not found to be large enough to affect the applicability of the pie chart. In actual system implementation the contribution of these errors should be investigated further.

### 3.5 Results of On-line Analysis

The purpose of the pie chart limit display is to give the operator a real-time indication of system state, with particular emphasis on overloading. As the rotation approaches a limit threshold, the operator is given some warning that a line is about to become overloaded. In the case of a fault, however, the change is instantaneous. This has been seen in the previous PMU measurements. The operator has no warning that the fault is about to occur. In this regard, the PMU data cannot be used to verify the rotation in the direction of overload. The PMU data can, however, support the indication of system state both during and after the fault.

In the faulted case, one line (1-25) becomes overloaded. This line is operating at $49 \%$ capacity in the pre-fault case. For this line, there is a percent increase of $90.72 \%$ between the pre-fault and faulted current magnitude, and a percent increase of $98.53 \%$ between the pre-fault and faulted voltage angle difference. These percentages, along with the initial capacity of the line, will clearly pass the limit thresholds in the pie chart. In the post-fault case, when the line is operating at $47 \%$ capacity, the percent increases are $4.89 \%$ for the current magnitude and $4.16 \%$ for the voltage angle difference. These values will not cross the limit thresholds, indicating the line is no longer overloaded.

It is important to note that the pre-fault case given is very lightly loaded. The most heavily loaded line is operating at $58 \%$ capacity. This indicates that the fault occurred when the system was not at peak loading conditions. The result is that the system is more tolerant to increases in power through the lines when the fault is introduced, and fewer overloads occur. In order to set the limits for the pie chart, the system needs to be more heavily loaded. If the limits are set when the system is lightly loaded, a high percentage change may not indicate an overload, but simply an increase in power flow through a line. To verify that limits can be established if the system given had included heavier loading, the overall system load in the reduced case is increased by $25 \%$. This brings the highest line capacity to $70 \%$. Lines operating between $60 \%$ and $70 \%$ capacities are manipulated as in the offline study. Current magnitude and voltage angle measurements are obtained for the line in the initial state, when the line is operating at $80 \%$ capacity, and at $100 \%$ capacity. The results are shown in Figure 3.13.


Figure 3.13 - On-line Limit Study
It is seen that limits can again clearly be established, as the percent increases are easily divided into independent subsets. The allowable percent increases are largest for the line with the lowest initial capacity, as expected. It is also noticed that lines with lower capacity restrictions change loading percentages faster than lines with larger capacity restrictions. This indicates the charts should not only be observed on the lines that are heavily loaded initially, but also on the lines with the lowest MVA capacity restrictions. This study shows the pie chart limit can be established for the online system if the operator observes a higher loading condition than is given in pre-fault data. Note that the
limits established above do not actually apply to the given pre-fault system, but are merely mentioned to provide assurance that limit setting is still achievable.

Even though only one case of overload can actually be observed in the given data, additional values of the phasor limit chart can be seen in the faulted case. The percent increases in current at the time of the fault are very high while the voltage angle limits are not as significant. As mentioned previously in the thesis, the thresholds for both current magnitude and voltage angle must be crossed jointly to indicate an overload. However, if one threshold is crossed while another is not, a system disturbance is still indicated. In the case of the fault, the current magnitude limit is crossed during the fault. The PMUs observe the transients as well, allowing the limits to be crossed repeatedly for a short period of time. Depending on the time sampling of the PMUs and the phasor pie chart, transients can be roughly observed. Once the fault is cleared, the voltage angle difference slowly continues to increase. The percent increase in the current is not significant any longer, but the percent increase in voltage angle approaches and eventually crosses limit thresholds. Now the phasor pie chart indicates problems with system stability. These additional indications lead to the conclusion that the phasor pie chart has multiple uses for the operator.

### 3.6 On-line Sensitivity Analysis

The sensitivity equation $\left(\partial \mathrm{I}_{\mathrm{ij}} \mathrm{I} / \partial \delta_{\mathrm{ij}}\right)$ is applied to all lines in the reduced system, and the results are shown in Table 3.2.

Table 3.2 - Sensitive Lines in the Reduced System

| Line | Sensitivity - <br> Pre-Fault | Sensitivity - <br> Fault | Sensitivity - <br> Post-Fault |
| :---: | :---: | :---: | :---: |
| $2-3$ | 383.46 | 268.36 | 383.46 |
| $30-28$ | 332.60 | 332.59 | 332.60 |
| $15-14$ | 332.59 | 266.07 | 332.59 |
| $43-44$ | 305.08 | 277.35 | -- |
| $23-22$ | 302.47 | -- | 302.47 |
| $1-35$ | 262.34 | 262.34 | 262.34 |
| $44-45$ | 224.05 | 203.68 | 203.67 |
| $26-25$ | 216.92 | 216.86 | 216.92 |
| $6-5$ | 207.93 | 145.55 | 207.93 |
| $36-35$ | 184.67 | 184.67 | 184.68 |
| $37-38$ | 179.94 | 163.58 | 163.58 |
| $10-11$ | 166.29 | 133.04 | 166.30 |
| $3-5$ | 163.58 | 114.50 | 163.58 |
| $19-22$ | 159.41 | 142.57 | 158.41 |
| $19-24$ | 155.92 | -- | 155.91 |
| $18-17$ | 155.77 | 155.77 | 155.77 |
| $12-16$ | 131.26 | 118.12 | 131.27 |
| $42-43$ | 129.05 | 117.31 | 129.04 |
| $6-13$ | 126.22 | -- | 126.22 |
| $25-28$ | 116.00 | 115.99 | 115.99 |
| $1-36$ | 115.99 | 115.99 | 115.99 |
| $20-19$ | 110.85 | -- | 110.85 |
| $7-5$ | 109.61 | -- | 109.61 |
| $5-10$ | 107.30 | -- | 107.30 |
| $15-12$ | 102.81 | -- | 102.81 |

The results of the sensitivity study are comparable across all three system cases. Many of the lines that are determined to be sensitive in the pre-fault case are sensitive in the postfault case as well. Even though many of these same lines are sensitive in the faulted case, this case provides the most discrepancies. These discrepancies from the other cases are due to the behavior of the voltages in the faulted case. This supports the conclusion that an overall system sensitivity analysis should not be performed when the system is in a faulted condition.

### 3.7 Conclusion of the On-Line Data Study

This chapter first observes a consistency study which verifies applicability of percent increase limits to real system data. An actual system with PMU data available is then examined. The system is reduced to a manageable study size through the use of power injection equivalents. The reduced system is then verified using the PMU data before, during, and after the system fault. The pie chart limits determined off-line are verified on-line through a supplementary study in which a manipulated system operates in heavier loading conditions. The possibility for additional uses of the pie chart limits in varying system states, particularly faults, is observed. The sensitivity equation is again verified in different system cases. It is concluded that the percent increases in current magnitude and voltage angle are successfully used to create thresholds in a phasor pie chart. This pie chart indicates system state both in off-line and on-line applications.

## CHAPTER FOUR

## COMPARISONS AND CONCLUSIONS

### 4.1 Off-line and On-line Comparisons

In the course of this research, a phasor pie chart is developed. Limit thresholds are created using percent increases in voltage angle and current magnitude. These limits are developed off-line and then applied in on-line analysis of a system with PMU data. It is determined that limits can be established both in off-line and on-line analysis, as long as the system is in a moderate to heavy loading condition. Tendencies to overload are observed in both off-line and on-line analysis. In the off-line analysis, the line capacities are all similar values. In this system it is seen that the lines that start out heavily loaded are the first to overload. In the on-line analysis this is not always the case. The tendencies to overload here depend on a number of different factors, such as line capacities, initial line loading conditions, and locations of increased loads. This is more realistic for real applications, and indicates that the operator must observe not only heavily loaded lines, but those with strong capacity restrictions as well.

Limit thresholds must be crossed jointly to indicate an overload, but can indicate disturbances when crossed individually. This is seen in the off-line study, in the consistency study, and in the on-line study. It is important to observe when one parameter has large changes and the other does not, supporting the display proposed in Chapter 2. This is also the main premise for sensitivity analysis. The lines most likely to overload cannot be constantly predicted, as the tendency to overload may change with different system configurations. The potential of a line to have large individual parameter changes
with small changes in another parameter, however, is predicted using the sensitivity equation. The results of this sensitivity equation are shown to be consistent in both the off-line and on-line analyses across different system loading conditions and configurations.

Possible expansions of this research involve dynamic stability studies, further development of the solution algorithm, a system implementation study, an application study in the distribution system, and continued research in sensitivity analysis. The study in this thesis has been a static study, and may be expanded to a dynamic study if machine properties are included. The dynamic study could be built off of the $\delta_{\mathrm{ij}}$ portion of the phasor chart. The study in this thesis also investigates an overload warning system for operators, but does not go into detail about particular lines that should be outaged to remove the possible overload. System implementation of the chart has been introduced on a fundamental level, but a detailed implementation plan for installment in a system has not been developed. Particular attention should be paid to any errors introduced into current magnitude readings by current transformers. This research involves a study of a high voltage in on-line analysis of transmission systems. For this reason, the system is considered balanced. The cross from high voltage to low voltage, i.e. to the distribution system, does not maintain balance. Most loads are unbalanced, leaving an opportunity for chart application analysis in unbalanced distribution systems. Finally, a development of line sensitivities is presented. The opportunity exists for the sensitivity equation to be expanded for determination of bus sensitivities as well, leading to potential voltage collapse studies.

### 4.2 Overall Conclusions

This research examines many important manipulations of the power system. Increased load can become a serious problem for a sensitive power network because of overloaded lines. The increased loading can cause lines to become heavily loaded and even to fail, leaving more overloading on lines or loads in the network not receiving the power they demand. Many times, alleviation of overload can be accomplished by simply forcing outages, allowing power flow to be redistributed to lines which can handle the increase. The applications of this research are numerous in that every power distributor will have to deal with excess loading as well as unintentional power line outages. This research examines which lines in a system are most beneficial (to restoring the network to normal operating conditions) during these periods of overloading through ranking the lines which can be more heavily loaded.

Using the limits on percent increase of current magnitude and voltage angle along with PMU readings from the system and the chart shown previously, it can be determined when lines are approaching overload or instability. As long as forced outages allow the system to stay within the percent increase limits, distributors can simply force outages on some of their transmission lines during the heavily loaded portion of the day. This allows for the most efficient and effective method of transmission and distribution for the power company. The developed chart is very important in examining a system after any change in operating conditions occurs, allowing a method of correction without running load flow. This leads to a faster solution of a problem that may cause a blackout.

## REFERENCES

[1] N. Balu, T. Bertram, A. Bose, V. Brandwajn, G. Cauley, D. Curtice, A. Fouad, L. Fink, M.G. Laouby, B.F. Wollenberg, J.N. Wrubel, "On-Line Power System Security Analysis", Proceedings of the IEEE, vol. 80 no. 2, Feb 1992, pp. 262282.
[2] Y. Gong, N. Schulz, and A. Guzman, "Synchrophasor-Based Real-Time Voltage Stability Index," Western Protective Relay Conference, 32 annual, Spokane, WA, 2005.
[3] A.G. Phadke and J.S. Thorp, "History and Applications of Phasor Measurements," IEEE Power Systems Conference and Exposition, pp. 331-335, 2006.
[4] R.F. Nuqui, A.G. Phadke, "Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability," IEEE Transactions on Power Delivery, vol. 20, no. 4, Oct. 2005, pp. 2381-2388.
[5] R.F. Nuqui, A.G. Phadke, R.P. Schulz, and N. Bhatt, "Fast On-Line Voltage Security Monitoring Using Synchronized Phasor Measurements and Decision Trees," 2001 IEEE/PES Winter Meeting, vol. 3, pp. 1347 - 1352.
[6] S. Skok, I. Ivankovic, and Z. Cerina, "Applications Based on PMU Technology for Improved Power System Utilization," IEEE Power Engineering Society General Meeting, June 2007.
[7] K.E. Martin, G. Benmouyal, M.G. Adamiak, and others, "IEEE Standard for Synchrophasors for Power Systems,"IEEE Transactions on Power Delivery, Jan. 1998, vol. 13, no. 1, pp. 73-77.
[8] R. Zivanovic and C. Cairns, "Implementation of PMU technology in state estimation: an overview," in Proc. IEEE 4th AFRICON, Sep. 1996, vol. 2, pp. 1006-1011.
[9] L.H. Fink, ed., Proceedings: Bulk Power System Voltage Phenomena - Voltage Stability and Security, EPRI EL-6183, in [5], Jan. 1989.
[10] M. Chau, R. Gutman, B.M. Pasternack, "Understanding Voltage Collapse in Bulk Power Transmission Systems," in [5], pp. 3-45 to 3-55.
[11] J. Chen and A. Abur, "Placement of PMU's to Enable Bad Data Detection in State Estimation," IEEE Transactions on Power Systems, vol. 21, no. 4, Nov. 2006.
[12] M.B. Filho, J.C. Souza, F.M. de Oliveira, M.T. Shilling, "Identifying Critical Measurements \& Sets for Power System State Estimation," 2001 IEEE Porto Power Tech Conference, Portugal, Sep. 2001.
[13] Burnett, R., Jr., Butts, M., Sterlina, P., "Power System Applications for Phasor Measurement Units," IEEE Computer Applications in Power, vol. 7, no. 11, pp. 8-13, Jan. 1994.
[14] M. Akke, D. Karlsson, "Phasor Measurement Applications in Scandinavia," IEEE/PES Transmission and Distribution Conference and Exposition, 2002.
[15] E. Price, "Practical Considerations for Implementing Wide Area Monitoring, Protection and Control," Conference for Protective Relay Engineers, 59 annual, Apr. 2006.
[16] B. Milosevic, M. Begovic, "Voltage-Stability Protection and Control Using a Wide-Area Network of Phasor Measurements," IEEE Transactions on Power Systems, vol. 18, no.1, pp. 121-127, Feb. 2003.
[17] J.S. Thorp, A.G. Phadke, S.H. Horowitz, M.M Begovic, "Some Applications of Phasor Measurements to Adaptive Protection," IEEE Transactions on Power Systems, vol. 3, no. 2, pp. 791-798, May 1998.
[18] D. Novosel and K. Vu, "Benefits of PMU Technology for Various Applications," International Council on Large Electric Systems Symposium On Power System Management, 7 annual, Croatia, Nov. 2006.
[19] D. Hou and N. Fischer, "Deterministic High-Impedance Fault Detection and Phase Selection on Ungrounded Distribution Systems," Schweitzer Engineering Laboratories, 2005.
[20] G. Benmouyal, E. O. Schwietzer, and A. Guzman, "Synchronized Phasor Measurement in Protective Relays for Protection, Control, and Analysis of Electric Power Systems," Western Protection Relay Conference, 29 Annual, Spokane, WA, Oct. 2002.
[21] C.W. Taylor, "The Future in On-Line Security Assessment and Wide-Area Stability Control," Proceedings of the 2000 IEEE/PES Winter Meeting, vol. 1, Jan. 2000.
[22] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power System Observability with Minimal Phasor Measurement Placement," IEEE Transactions on Power Systems., vol. 8, no. 2, pp. 701-715, May 1993.
[23] J. Rasmussen and P. Jorgensen, "Synchronized Phasor Measurements of a Power System Event in Eastern Denmark," IEEE Transactions on Power Systems, vol. 21, no. 1, Feb. 2006.
[24] V. Balamourougan, T.S. Sidhu, and M.S. Sachdev, "Technique for Online Prediction of Voltage Collapse," IEE Proceedings - Generation, Transmission, and Distribution, vol. 151, no. 4, July 2004.
[25] M. Begovic and A. Phadke, "Control of Voltage Stability using Sensitivity Analysis," IEEE Transactions on Power Systems, vol. 7, no. 1, Feb. 1992.
[26] N. Flatabo, R. Ognedal, T. Carlsen, "Voltage Stability Condition in a Power Transmission System Calculated by Sensitivity Methods," IEEE Transactions on Power Systems, vol. 5, no. 4, Nov. 1990.
[27] Z. Hong-Shan, L. Ying, M. Zeng-qiang, Y. Lei, "Sensitivity Constrained PMU Placement for Complete Observability of Power Systems," IEEE/PES Transmission and Distribution Conference and Exhibition, 2005.
[28] S. Tso, J. Lin, H. Ho, C. Mak, K. Yung, and Y. Ho, "Data Mining for Detection of Sensitive Buses and Influential Buses in a Power System Subjected to Disturbances," IEEE Transactions on Power Systems, vol. 19, no. 1, Feb. 2004.
[29] G. Mueller, P. Komarnicki, Z. Styczynski, C. Dzienis, and others, "PMU Placement Method Based on Decoupled Newton Power Flow and Sensitivity Analysis," Electrical Power Quality and Utilization International Conference, 9 annual, Barcelona, Oct. 2007.
[30] B. Gou and A. Abur, "A Direct Numerical Method for Observability Analysis," IEEE Transactions on Power Systems, vol. 15, no. 2, May 2006.
[31] A. Monticelli and F. Wu, "Network Observability: Theory," IEEE Transactions on Power Apparatus and Systems, vol. PAS-104, no. 5, May 1985.
[32] J. London, L. Albero, and N. Bretas, "Network Observability: Identification of the Measurements Redundancy Level," Proceedings - PowerCon, Australia, 2000.
[33] N. Bretas and J. London, "Measurement Placement Design and Reinforcement for State Estimation Purposes," IEEE Porto Power Tech Conference, Portugal, Sep. 2001.
[34] W. Jiang and V. Vittal, "Optimal Placement of Phasor Measurements for the Enhancement of State Estimation," IEEE PES Power Systems Conference and Exposition, Atlanta, GA, 2006.
[35] L. Zhao, A. Abur, "Multiarea State Estimation Using Synchronized Phasor Measurements," IEEE Transactions on Power Systems, vol. 20, no. 2, May 2005.
[36] W. Jiang, V. Vittal, and G. Heydt, "A Distributed State Estimator Utilizing Synchronized Phasor Measurements," IEEE Transactions on Power Systems, vol. 22, no. 2, May 2007.
[37] R. Moxley, "Practical Application of Synchronized Phasor Measurement," Schweitzer Engineering Laboratories, 2006.
[38] B. Wollenberg and A. Wood, Power Generation, Operation, and Control, $2^{\text {nd }}$ ed., Wiley Interscience, New York, 1984.
[39] A.G. Phadke, "Synchronized Phasor Measurements ~ A Historical Overview," IEEE/PES Transmission and Distribution Conference and Exposition, Oct., 2002.
[40] R. Klump, R. Wilson, and K. Martin, "Visualizing Real-Time Security Threats Using Hybrid SCADA / PMU Measurement Displays," Proceedings of the Hawaii International Conference on System Sciences, 38 Annual, 2005.
[41] J. Xiao, X. Xie, H. Li, J. Wu, and Y. Han, "Power System Dynamic Security Assessment Via Synchrophasor Monitoring," International Conference on Power System Technology, Singapore, 21-24 Nov., 2004.
[42] W. Tinney and J. Bright, "Adaptive Reductions for Power Flow Equivalents," IEEE Transactions on Power Systems, vol. PWRS - 2, no. 2, May 1987.
[43] Y. Chen and A. Bose, "Choosing the Appropriate Boundary for Adaptive Reduction," IEEE Transactions on Power Systems, vol. 3, no. , May 1988.

## APPENDIX

## MATLAB Code for Series and Parallel Reduction

function [ final_to_bus, final_from_bus, final_bus_num, final_src, final_ld, final_TVA_Area, TVA_tie_line, Not_TVA_tie_line, final_line_Z ] = Loop( to_bus, from_bus, bus_num, Gen_MW, Gen_MVAR, Ld_MW, Ld_MVAR, TVA_Area, line_Z )
\%LOOP Function Performs multiple passes, reducing system until the results of two
\%consecutive passes are the same
\%Calls TEST function to perform individual passes
output_to_bus = [];
output_from_bus = [];
new_bus_num = [];
new_src = [];
new_ld = [];
output_TVA_Area = [];
TVA_tie_line = [];
Not_TVA_tie_line = [];
temp_to_bus = to_bus;
temp_bus_num = bus_num;
ld = zeros(size(Ld_MW,1),1);
src= zeros(size(Ld_MW,1),1);
for $\mathrm{i}=1: 1:$ size(Ld_MW,1)
$\operatorname{ld}(\mathrm{i})=\operatorname{sqrt}\left(\operatorname{Ld} \_M W(\mathrm{i})^{\wedge} 2+\operatorname{Ld} \_\operatorname{MVAR}(\mathrm{i})^{\wedge} 2\right)$;
$\operatorname{src}(\mathrm{i})=\operatorname{sqrt}($ Gen_MW(i)^ $2+$ Gen_MVAR(i)^2);
end
while((size(output_to_bus, 1) ~= size(temp_to_bus, 1)) || (size(new_bus_num, 1) ~= size(temp_bus_num, 1)))
[ output_to_bus, output_from_bus, new_bus_num, new_src, new_ld, output_TVA_Area, TVA_tie_line, Not_TVA_tie_line, output_line_Z ] = Test( to_bus, from_bus, bus_num, src, ld, TVA_Area, line_Z );
temp_to_bus = to_bus;
temp_bus_num = bus_num;
to_bus = output_to_bus;
from_bus = output_from_bus;
bus_num = new_bus_num;
src = new_src;
ld = new_ld;
TVA_Area = output_TVA_Area;
line_Z = output_line_Z;
end
final_to_bus = output_to_bus;
final_from_bus = output_from_bus;
final_bus_num = bus_num;
final_src = new_src;
final_ld = new_ld;
final_TVA_Area = output_TVA_Area;
final_line_Z = output_line_Z;
return
function [ output_to_bus, output_from_bus, output_bus_num, output_src, output_ld, output_TVA_Area, TVA_tie_line, Not_TVA_tie_line, output_line_Z ] = Test( to_bus, from_bus, bus_num, src, ld, TVA_Area, line_Z )
\%TEST Function performs a single pass, eliminating series combinations first, then \%parallel combinations. TEST is called multiple times by LOOP.
keep_bus = zeros(size(bus_num,1),1);
TVA_tie_line $=$ zeros(size(bus_num,1),1);
Not_TVA_tie_line = zeros(size(bus_num,1),1);
\% Keep if Tie Line Bus
line_del = zeros(size(from_bus, 1), 1);
$\mathrm{m}=1$;
for $\mathrm{i}=1: 1$ :size(from_bus, 1 )
for $\mathrm{j}=1$ :1:size(bus_num, 1)
if((from_bus(i) == bus_num(j)) || (to_bus(i) == bus_num(j))) for $\mathrm{k}=1: 1$ :size(bus_num, 1)
if((from_bus(i) == bus_num(k)) || (to_bus(i) == bus_num(k)))
if((TVA_Area(j) $==1) \& \&($ TVA_Area(k) $==0)$ )
keep_bus(j) = 1 ;
keep_bus(k) $=1$;
TVA_tie_line (m, 1) = bus_num(j);
Not_TVA_tie_line(m, 1) = bus_num(k);
$\mathrm{m}=\mathrm{m}+1$;
end
if((TVA_Area $(\mathrm{j})==0) \& \&($ TVA_Area $(\mathrm{k})==0))$
line_del $(\mathrm{i}, 1)=1$; end
end
end end
end
end
TVA_tie_line = TVA_tie_line(1:(m-1), 1);
Not_TVA_tie_line $=$ Not_TVA_tie_line(1:(m-1), 1);
area_to_bus = zeros(size(to_bus, 1) - sum(line_del), 1);
area_from_bus = zeros(size(from_bus, 1) - sum(line_del), 1);

```
% Put lines which weren't removed into new to and from buses
j = 1;
for i= 1:1:size(from_bus, 1)
    if(line_del(i, 1) == 0)
        area_to_bus(j, 1) = to_bus(i, 1);
        area_from_bus(j, 1) = from_bus(i, 1);
        j = j + 1;
    end
end
to_bus = area_to_bus;
from_bus = area_from_bus;
% Remove buses that are non TVA and not tie line buses
k=1;
for i = 1:1:size(keep_bus, 1)
    if((keep_bus(i) == 1) | (TVA_Area(i) == 1))
        area_bus_num(k, 1) = bus_num(i);
        area_src(k, 1) = src(i);
        area_ld(k, 1) = ld(i);
        area_TVA_Area(k, 1) = TVA_Area(i);
        area_keep_bus(k, 1) = keep_bus(i);
        k=k + 1;
    end
end
bus_num = area_bus_num;
src = area_src;
ld = area_ld;
TVA_Area = area_TVA_Area;
keep_bus = area_keep_bus;
conn_1 = -ones(size(bus_num,1),1);
conn_2 = -ones(size(bus_num,1),1);
% Series Combination
% Is there a bus in series that can be deleted?
for j = 1:1:size(bus_num,1)
    if ((ld(j)> 100) || (src(j)> 0) || (bus_num(j) == 18556) |
        (bus_num(j) == 18557) | (bus_num(j) == 18873)| (bus_num(j) ==
        18413) || (bus_num(j) == 18009) || (bus_num(j) == 18008) |
        (bus_num(j) == 18425) || (bus_num(j) == 18929) || (bus_num(j) ==
        18468) || (bus_num(j) == 18644) || (bus_num(j) == 18773) |
    (bus_num(j) == 43))
        keep_bus(j) = 1;
```

```
    else
        for k = 1:1:size(from_bus,1)
        if (from_bus(k) == bus_num(j))
            if conn_1(j) == -1
                conn_1(j) = to_bus(k);
            else if conn_1(j) == to_bus(k)
                        keep_bus(j) = 1;
                else if conn_2(j) == -1
                        conn_2(j) = to_bus(k);
                    else
                        keep_bus(j) = 1;
                            end
                end
            end
        else if (to_bus(k) == bus_num(j))
                if conn_1(j) == -1
                    conn_1(j) = from_bus(k);
                else if conn_1(j) == from_bus(k)
                    keep_bus(j) = 1;
                        else if conn_2(j) == -1
                                    conn_2(j) = from_bus(k);
                                    else
                                    keep_bus(j) = 1;
                                    end
                    end
                end
            end
        end
        end
        if conn_2(j) == -1
            keep_bus(j) = 1;
        end
    end
end
% Deletion of Series Bus and Creation of New Lines
num_kept = sum(keep_bus, 1);
num_removed = size(bus_num, 1) - num_kept;
remove = zeros(num_removed, 1);
remove_bus = zeros(num_removed, 1);
remove_check = zeros(num_removed, 1);
```

```
% Build list of buses that could be removed due to "in series"
j = 1;
for i = 1:1:size(keep_bus, 1)
    if(keep_bus(i) == 0)
        remove(j) = i;
        remove_bus(j) = bus_num(i);
        j = j + 1;
    end
end
% Mark as keep buses which are connected to a bus to be deleted
for i = 1:1:size(remove, 1)
    for j = 1:1:size(remove, 1)
                if(((conn_1(remove(i)) == remove_bus(j)) && (keep_bus(remove(j))
                == 0)) Il ((conn_2(remove(i)) == remove_bus(j)) &&
                (keep_bus(remove(j)) == 0)))
            remove_check(i) = remove_check(i) + 1;
        end
    end
    if(remove_check(i) > 0)
        keep_bus(remove(i)) = 1;
    end
end
% Rebuild and recount num kept, num removed, and remove list
num_kept = sum(keep_bus);
num_removed = size(bus_num, 1) - num_kept;
remove = zeros(num_removed, 1);
remove_bus = zeros(num_removed, 1);
output_bus_num = zeros(num_kept, 1);
output_src = zeros(num_kept, 1);
output_ld = zeros(num_kept, 1);
output_TVA_Area = zeros(num_kept, 1);
% Build list of buses that could be removed due to "in series"
% and new bus_num list
j = 1;
k=1;
for i = 1:1:size(keep_bus, 1)
    if(keep_bus(i) == 0)
        remove(j) = i;
        remove_bus(j) = bus_num(i);
```

```
        j = j + 1;
    else
        output_bus_num(k) = bus_num(i);
        output_src(k) = src(i);
        output_ld(k) = ld(i);
        output_TVA_Area(k) = TVA_Area(i);
        k=k+1;
    end
end
line_del = zeros(size(from_bus, 1), 1);
% Mark lines to a bus which is removed
for i = 1:1:size(from_bus, 1)
    if(size(remove_bus, 1) > 0)
        for j = 1:1:size(remove_bus, 1)
            if((from_bus(i) == remove_bus(j)) || (to_bus(i) ==
                        remove_bus(j)))
                        line_del(i) = 1;
            end
        end
    end
end
new_to_bus = zeros(size(to_bus, 1) - sum(line_del), 1);
new_from_bus = zeros(size(from_bus, 1) - sum(line_del), 1);
new_line_Z = zeros(size(from_bus, 1) - sum(line_del), 1);
% Put lines which weren't removed into new to and from buses
j = 1;
for i= 1:1:size(from_bus, 1)
    if(line_del(i) == 0)
        new_to_bus(j) = to_bus(i);
        new_from_bus(j) = from_bus(i);
        new_line_Z(j) = line_Z(i);
        j = j + 1;
    end
end
```

```
% Add new lines to make up for removed buses
for i = 1:1:size(remove, 1)
    new_from_bus(size(new_from_bus, 1) + 1) = conn_1(remove(i));
    new_to_bus(size(new_to_bus, 1) + 1) = conn_2(remove(i));
    temp_Z = 0;
    for j = 1:1:size(from_bus, 1)
        if(((from_bus(j) == conn_1(remove(i))) && (to_bus(j) ==
                                    remove(i))) II ((from_bus(j) == remove(i)) && (to_bus(j) ==
                                    conn_1(remove(i)))) Il ((from_bus(j) == conn_2(remove(i)))
                                    && (to_bus(j) == remove(i))) II ((from_bus(j) == remove(i))
                                    && (to_bus(j) == conn_2(remove(i)))))
            temp_Z = temp_Z + line_Z(j);
        end
    end
    new_line_Z(size(new_to_bus, 1) + 1) = temp_Z;
end
% Remove Duplicate entries in to and from buses (parallel lines)
line_del = zeros(size(new_from_bus, 1), 1);
% Mark lines which are repeated
for i = 1:1:size(new_from_bus, 1)
    if(line_del(i) == 0)
        for j = 1:1:size(new_from_bus, 1)
                if(i ~= j)
                    if((new_from_bus(i) == new_from_bus(j)) II
                                    (new_from_bus(i) == new_to_bus(j)))
                if((new_to_bus(i) == new_from_bus(j)) |
                                    (new_to_bus(i) == new_to_bus(j)))
                                    line_del(j) = 1;
                                    new_line_Z(i) = (1 / ((1 / new_line_Z(i)) + (1
                                    / new_line_Z(j))));
                end
                end
            end
        end
    end
end
```

```
output_to_bus = zeros(size(new_to_bus, 1) - sum(line_del), 1);
output_from_bus = zeros(size(new_to_bus, 1) - sum(line_del), 1);
output_line_Z = zeros(size(new_to_bus, 1) - sum(line_del), 1);
% Put lines which weren't removed into new to and from buses
j = 1;
for i = 1:1:size(new_from_bus, 1)
    if(line_del(i) == 0)
            output_to_bus(j) = new_to_bus(i);
            output_from_bus(j) = new_from_bus(i);
            output_line_Z(j) = new_line_Z(i);
            j = j + 1;
    end
end
return;
```

