8-25-1967

Particle-Induced Electromagnetic De-Excitation of Nuclei in Stellar Matter

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(Received 31 March 1967)

The rate for electromagnetic de-excitation of excited nuclei by inelastic scattering with electrons and ions in stellar matter is calculated as a function of temperature, density, transition energy, and multipole type. The results of this paper indicate that for temperatures in the range 10^9 °K $<T<10^{10}$ °K and densities in the range 10^9 g/cm^3 $<\rho<10^{12}$ g/cm^3, particle-induced electromagnetic de-excitations compete favorably with spontaneous radiative transitions. As an example, a C^{12} nucleus, put in the 7.65-MeV 0^+ excited state and imbedded in an electron-helium plasma with a density of 10^{11} g/cm^3, will be de-excited by electromagnetic interaction with the plasma 40 times faster at $T=10^9$ °K and 500 times faster at $T=10^{10}$ °K than its natural radiative rate. At the lower temperature the de-excitation is dominated by the electrons, and at the higher temperature by the helium ions. The conditions for the applicability of the present work to modern astrophysical problems are discussed.

I. INTRODUCTION

IN calculations of nuclear reaction rates in stellar matter it has traditionally been assumed that radiative transitions in nuclei occur at the rate measured in the terrestrial laboratory. This assumption has been reasonable in that the temperatures and densities required to stimulate characteristic electromagnetic transitions in nuclei have exceeded those traditionally discussed in the evolution of stars. But the modern problems of the implosion-explosion mechanism for supernovas, the expansion of the universe from a hypothetical initially condensed state, and the unknown nature of quasars all suggest the possibility of nuclear reactions at very high temperature, very high density, or both. One or all of these problems may involve the fusion of nuclei during the expansion of highly condensed matter as the temperature falls from near 10^{10} °K, and in turn many of the most important fusion reactions are accompanied by the emission of γ rays. The subject we consider in this paper is the extent to which electromagnetic de-excitation of nuclear states may be accelerated by collisions with constituents of the environment.

The most extreme densities encountered in stellar nucleo synthesis are found in the imploded matter of supernovas. In this terminal stage of stellar evolution the stellar core collapses due to an instability triggered either by inverse β decay or by photodisintegration. The overlying layers initially follow the implosion, until a core stiffening leads to subsequent ejection of the outer portions of the imploded star. The matter near the mass cut dividing the ultimate collapsed remnant from the redispersed matter achieves densities in the range 10^{12}–10^{13} g/cm^3 before the expansion. The constituents, predominantly neutrons and α particles, reinitiate fusion reactions during the expansion with densities and temperatures in the gross range of 10^9–10^{12} g/cm^3 and 10^9–10^{10} °K, respectively.[1] We will use this range of density and temperature as a guideline for this investigation, although the detailed applicability of this paper to such an event depends upon other details that have not yet been delineated with precision.

There appear to be three mechanisms capable of inducing the electromagnetic de-excitation of nuclear states in this temperature range. The first of these mechanisms is the interaction between the internal nuclear electromagnetic current and that of a passing electron, leading to a transition in the nuclear state and in the state of the continuum electron. This inelastic-scattering process is analogous to the familiar internal conversion process studied in the laboratory except that the initial state of the electron lies in the continuum—a free-free internal conversion.[2] Because of the high density required for this process to compete with spontaneous emission, the electron gas will be degenerate. Secondly, the more massive ions constitute a Maxwellian gas and are dominated in their collisions by the Coulomb repulsion between positive charges. Nuclear states may de-excite in such collisions, nonetheless, by the electromagnetic process of Coulomb de-excitation, in which the time-varying electric field produced at the excited nucleus during the collision causes an internal transition in the excited nucleus. The third electromagnetic process is the familiar stimulated emission, which may play a role in some transitions at temperatures in this range.

In Sec. II of this paper we calculate rates for the first

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[1] W. D. Arnett, Can. J. Phys. 44, 2553 (1966); S. A. Colgate and R. H. White, Astrophys. J. 143, 626 (1966).

[2] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), Chap. XI.

two processes and include the third as a matter of course. The results will largely be displayed in the form of graphs showing the competition with spontaneous emission for several multipolarities. In Sec. III we consider the specific example of the particle-induced electromagnetic de-excitation of C^{12} imbedded in an electron-α particle plasma. Finally, in Sec. IV we discuss other features of nuclear reactions in astrophysics that contribute to the determination of the relevence of the processes studied here.

II. CALCULATION OF PARTICLE-INDUCED ELECTROMAGNETIC DE-EXCITATION RATES

A. De-Excitation by Electrons

The rate for inelastic scattering of electrons by nuclei

$$e^- + (Z,A) \rightarrow e^- + (Z,A)' \tag{1}$$

can be calculated with sufficient accuracy in Born approximation (one-photon exchange) provided

$$Z\alpha/v \ll 1\,, \tag{2}$$

where Z is the nuclear atomic number, $\alpha = 1/137$, and v is the speed of the incident electron in units of the speed of light. The value of v for a degenerate electron gas is effectively the speed at the Fermi surface and is approximately unity at densities for which the electron-induced de-excitation competes favorably with radiative de-excitation. The condition in Eq. (2) is then a restriction to values of $Z \ll 137$. From perturbation theory the amplitude for inelastic electron scattering is given in Born approximation, with $\hbar = c = 1$, by

$$F = 4\pi e \frac{\bar{u}_{\sigma'}(\mathbf{p}')\gamma_\nu u_\sigma(\mathbf{p})\langle J'M'|J_\nu(\mathbf{\Delta})|JM\rangle}{\omega^2 - \Delta^2}\,, \tag{3}$$

where the initial electron has momentum $\mathbf{p}$, energy E, and Dirac spinor $u_\sigma(\mathbf{p})$ and the final electron has momentum $\mathbf{p}'$, energy E', and Dirac spinor $u_{\sigma'}(\mathbf{p}')$. The electron 3-momentum transfer is denoted by $\mathbf{\Delta} = \mathbf{p}' - \mathbf{p}$, and by conservation of energy the nuclear transition energy is given by $\omega = E' - E$.

In Eq. (3) the electromagnetic interaction has been approximated by using an unperturbed photon propagator. This vacuum propagator is clearly insufficient in a dense plasma, where dispersion due to the plasma must, in principle, be taken into account.[3] In the present case, nuclear transitions with frequencies less than or approximately equal to the plasma frequency cannot be treated adequately with the vacuum photon propagator. For transition frequencies somewhat larger than the plasma frequency, however, the vacuum approximation should be sufficient. The degree of accuracy can be estimated crudely by the deviation from unity of the plasma dielectric constant, which is adequately given by $1 - \omega_0^2/\omega^2$ for both longitudinal and transverse excitations.[3] For relativistic electrons $\hbar\omega_0 \simeq 0.1\ \rho_9^{1/3}$ MeV and $\rho_9 = 10^{-9}\rho$ with the density ρ in g/cm^3. In the present work we make no attempt to include plasma effects for an accurate treatment of the case $\omega \lesssim \omega_0$.

In Eq. (3) the quantity $\langle J'M'|J_\nu(\mathbf{\Delta})|JM\rangle$ is the Fourier transform of the nuclear current matrix element[4]

$$\langle J'M'|J_\nu(\mathbf{\Delta})|JM\rangle = \int d^3x \exp(-i\mathbf{\Delta}\cdot\mathbf{x})\,\langle J'M'|j_\nu(\mathbf{x})|JM\rangle\,, \tag{4}$$

where

$$\langle J'M'|\rho(\mathbf{x})|JM\rangle \equiv \langle J'M'|j_0(\mathbf{x})|JM\rangle = \sum_{r=1}^{A} e_r \int \prod_{s\neq r} d^3x_s\, \psi_{J'M'}^*(\cdots\mathbf{x}_r=\mathbf{x}\cdots)\psi_{JM}(\cdots\mathbf{x}_r=\mathbf{x}\cdots) \tag{5}$$

and

$$\langle J'M'|\mathbf{j}(\mathbf{x})|JM\rangle = \langle J'M'|\mathbf{j}_c(\mathbf{x})|JM\rangle + \mathrm{curl}\langle J'M'|\mathbf{m}(\mathbf{x})|JM\rangle\,, \tag{6}$$

with

$$\langle J'M'|\mathbf{j}_c(\mathbf{x})|JM\rangle = \sum_{r=1}^{A} \frac{e_r}{2N}\int \prod_{s\neq r} d^3x_s \Big\{\psi_{J'M'}^*(\cdots\mathbf{x}_r=\mathbf{x}\cdots)[-i\nabla\psi_{JM}(\cdots\mathbf{x}_r=\mathbf{x}\cdots)] + [-i\nabla\psi_{J'M'}(\cdots\mathbf{x}_r=\mathbf{x}\cdots)]^*\psi_{JM}(\cdots\mathbf{x}_r=\mathbf{x}\cdots)\Big\} \tag{7}$$

and

$$\langle J'M'|\mathbf{m}(\mathbf{x})|JM\rangle = \sum_{r=1}^{A} \mu_r \frac{e}{2N} \int \prod_{s\neq r} d^3x_s\, \psi_{J'M'}^*(\cdots\mathbf{x}_r=\mathbf{x}\cdots)\boldsymbol{\sigma}_r\psi_{JM}(\cdots\mathbf{x}_r=\mathbf{x}\cdots)\,. \tag{8}$$

In the above expressions e_r and μ_r are the charge and magnetic moment of the nucleon in question, N is the proton mass, and $\boldsymbol{\sigma}$ is the Pauli spin operator. The charge is expressed in unrationalized units ($e^2 = \alpha = 1/137$). The initial and final nuclear wave functions, ψ_{JM} and $\psi_{J'M'}$, respectively, are given with all quantum numbers suppressed except for those pertaining to the total angular momentum of the nucleus.

[3] V. N. Tsytovich, Zh. Experim. i Teor. Fiz. **40**, 1775 (1961) [English transl.: Soviet Phys.—JETP **13**, 1249 (1961)].
[4] J. M. Blatt and V. F. Weiskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1956), Chap. XII.

The current matrix elements can be decomposed into multipole components by the expansions

$$\langle J'M'|J_0(\mathbf{\Delta})|JM\rangle=\sum_{\lambda\mu}\langle J'M'|J_0(\Delta;\lambda\mu)|JM\rangle Y_{\lambda\mu}(\hat{\Delta}) \tag{9}$$

and

$$\langle J'M'|\mathbf{J}(\mathbf{\Delta})|JM\rangle=\sum_{\lambda\mu}\hat{\Delta}\frac{\omega}{\Delta}\langle J'M'|J_0(\Delta;\lambda\mu)|JM\rangle Y_{\lambda\mu}(\hat{\Delta}) +\langle J'M'|J_E(\Delta;\lambda\mu)|JM\rangle\hat{\Delta}\times\mathbf{X}_{\lambda\mu}(\hat{\Delta})+\langle J'M'|J_M(\Delta;\lambda\mu)|JM\rangle\mathbf{X}_{\lambda\mu}(\hat{\Delta}), \tag{10}$$

where $\hat{\Delta}=\mathbf{\Delta}/\Delta$, $Y_{\lambda\mu}(\hat{\Delta})$ is a spherical harmonic, and $\mathbf{X}_{\lambda\mu}(\hat{\Delta})$ is a vector spherical harmonic defined by

$$\mathbf{X}_{\lambda\mu}(\hat{\Delta})=\mathbf{L}Y_{\lambda\mu}(\hat{\Delta})/\lambda(\lambda+1), \tag{11}$$

with $\mathbf{L}=-i\mathbf{\Delta}\times\text{grad}$ where the gradient is with respect to $\mathbf{\Delta}$. In Eq. (10) we have used current conservation in the form

$$\mathbf{\Delta}\cdot\langle J'M'|\mathbf{J}(\mathbf{\Delta})|JM\rangle=\omega\langle J'M'|J_0(\mathbf{\Delta})|JM\rangle \tag{12}$$

to eliminate the longitudinal component of the current. Using orthonormality properties of the spherical harmonics, the current multipole matrix elements are found to be

$$\begin{aligned}\langle J'M'|J_0(\Delta;\lambda\mu)|JM\rangle&=4\pi(-i)^\lambda\int d^3x\, j_\lambda(\Delta r)Y_{\lambda\mu}{}^*(\hat{x})\langle J'M'|\rho(\mathbf{x})|JM\rangle,\\ \langle J'M'|J_E(\Delta;\lambda\mu)|JM\rangle&=\frac{4\pi(-i)^\lambda}{\lambda(\lambda+1)}\frac{1}{\Delta}\int d^3x\, j_\lambda(\Delta r)Y_{\lambda\mu}{}^*(\hat{x})\,\text{div}(\mathbf{x}\times\text{curl}\langle J'M'|\mathbf{j}(\mathbf{x})|JM\rangle),\\ \langle J'M'|J_M(\Delta;\lambda\mu)|JM\rangle&=\frac{4\pi(-i)^\lambda}{\lambda(\lambda+1)}\int d^3x\, j_\lambda(\Delta r)Y_{\lambda\mu}{}^*(\hat{x})\,\text{div}(\mathbf{x}\times\langle J'M'|\mathbf{j}(\mathbf{x})|JM\rangle),\end{aligned} \tag{13}$$

where $r=|\mathbf{x}|$, $\hat{x}=\mathbf{x}/r$, and $j_\lambda(\Delta r)$ is the regular spherical Bessel function of order λ. By means of the Wigner-Eckart theorem these multipole matrix elements can be written in reduced form

$$\langle J'M'|J_0(\Delta;\lambda\mu)|JM\rangle=\frac{\langle JM\lambda\mu|J\lambda J'M'\rangle}{2J'+1}\langle J'\|J_0(\Delta;\lambda)\|J\rangle, \tag{14}$$

where $\langle JM\lambda\mu|J\lambda J'M'\rangle$ is a Clebsch-Gordan coefficient. Similar expressions exist for the electric and magnetic current multipole matrix elements.

The reaction rate is formed in the standard manner by integrating the transition probability per unit time over the electron Fermi distribution $f(E)$. For complete degeneracy (which is adequate for the temperatures and densities considered in this paper) the Fermi function is given by a step function

$$f(E)=\theta(E_F-E), \tag{15}$$

where E_F is the Fermi energy and

$$\begin{aligned}\theta(x)&=1 \quad \text{for} \quad x>0\\ &=0 \quad \text{for} \quad x<0.\end{aligned} \tag{16}$$

Furthermore, because of the exclusion principle an electron cannot be scattered into a state that is already occupied. Sufficient energy must be transferred to the scattered electron in the nuclear de-excitation to lift the electron above the Fermi surface. This possible reduction in the final electron phase space is taken into account by inclusion of the factor $1-f(E')$. The resulting reaction rate is then given by

$$R_e=2\pi\int\frac{d^3p}{(2\pi)^3 2E}\frac{d^3p'}{(2\pi)^3 2E'}f(E)[1-f(E')]\delta(E'-E-\omega)\sum_{\text{spins}}|F|^2/2J+1, \tag{17}$$

where

$$\sum_{\text{spins}}|F|^2=16\pi\alpha\sum_{\lambda>0}\left\{\frac{(2E+\omega)^2-\Delta^2}{2\Delta^4}|\langle J'\|J_0(\Delta;\lambda)\|J\rangle|^2 +\frac{E^2-m^2-\frac{1}{4}(2\omega E+\omega^2-\Delta^2)^2/\Delta^2-\frac{1}{2}(\omega^2-\Delta^2)}{(\omega^2-\Delta^2)^2}(|\langle J'\|J_E(\Delta;\lambda)\|J\rangle|^2+|\langle J'\|J_M(\Delta;\lambda)\|J\rangle|^2)\right\}. \tag{18}$$

It is now convenient to introduce multipole transition moments of order λ and to define form factors through the prescription

$$|\langle J'\|J_0(\Delta;\lambda)\|J\rangle|^2=\frac{(4\pi)^2\Delta^{2\lambda}}{[(2\lambda+1)!!]^2}|\langle J'\|Q_\lambda\|J\rangle|^2|F_{C\lambda}(\Delta)|^2,$$

$$|\langle J'\|J_E(\Delta;\lambda)\|J\rangle|^2=\frac{(4\pi)^2\Delta^{2\lambda}}{[(2\lambda+1)!!]^2}\left(\frac{\omega}{\Delta}\right)^2\frac{\lambda+1}{\lambda}|\langle J'\|Q_\lambda\|J\rangle|^2|F_{E\lambda}(\Delta)|^2, \quad (19)$$

$$|\langle J'\|J_M(\Delta;\lambda)\|J\rangle|^2=\frac{(4\pi)^2\Delta^{2\lambda}}{[(2\lambda+1)!!]^2}\frac{\lambda+1}{\lambda}|\langle J'\|M_\lambda\|J\rangle|^2|F_{M\lambda}(\Delta)|^2,$$

where the reduced electric multipole transition moment $\langle J'\|Q_\lambda\|J\rangle$ and the magnetic multipole transition moment $\langle J'\|M_\lambda\|J\rangle$ can be obtained from the expressions

$$\langle J'M'|Q_{\lambda\mu}|JM\rangle=\int r^\lambda Y_{\lambda\mu}{}^*(\hat{x})\langle J'M'|\rho(\mathbf{x})|JM\rangle d^3x,$$

$$\langle J'M'|M_{\lambda\mu}|JM\rangle \quad (20)$$

$$=-\frac{1}{\lambda+1}\int r^\lambda Y_{\lambda\mu}{}^*(\hat{x})\,\mathrm{div}(\mathbf{x}\times\langle J'M'|\mathbf{j}(\mathbf{x})|\,JM\rangle)d^3x$$

by use of the Wigner-Eckart theorem. The functions $F_{C\lambda}(\Delta)$, $F_{E\lambda}(\Delta)$, and $F_{M\lambda}(\Delta)$ are the charge, electric, and magnetic form factors, respectively, and are normalized to unity at $\Delta=0$.

The above electron-induced transition rate is to be compared with the *vacuum* spontaneous radiative transition rate,[5] which is given in terms of the multipole transition moments and form factors by

$$R_\gamma{}^0=\frac{8\pi}{2J+1}\sum_{\lambda>0}\frac{\lambda+1}{\lambda}\frac{\omega^{2\lambda+1}}{[(2\lambda+1)!!]^2}\times[|J'\|Q_\lambda\|J\rangle|^2|F_{E\lambda}(\omega)|^2+|\langle J'\|M_\lambda\|J\rangle|^2|F_{M\lambda}(\omega)|^2]. \quad (21)$$

Under the assumption that only one multipole contributes to the current matrix element and with the approximation

$$F_{E\lambda}(\omega)\simeq F_{E\lambda}(0)=1\,,\quad F_{M\lambda}(\omega)\simeq F_{M\lambda}(0)=1\,, \quad (22)$$

we obtain for the electric transitions

$$R_e(E\lambda)/R_\gamma{}^0(E\lambda)=\frac{2\alpha}{\pi}\int dE\,f(E)[1-f(E+\omega)]\times\int_{p'-p}^{p'+p}d\Delta\left(\frac{\Delta}{\omega}\right)^{2\lambda-1}\left[\frac{G(E,\omega,\Delta)|F_{E\lambda}(\Delta)|^2}{(\omega^2-\Delta^2)^2}+\frac{\lambda}{\lambda+1}\frac{H(E,\omega,\Delta)|F_{C\lambda}(\Delta)|^2}{\omega^2\Delta^2}\right] \quad (23)$$

and for the magnetic transitions

$$R_e(M\lambda)/R_\gamma{}^0(M\lambda)=\frac{2\alpha}{\pi}\int dE\,f(E)[1-f(E+\omega)]\times\int_{p'-p}^{p'+p}d\Delta\left(\frac{\Delta}{\omega}\right)^{2\lambda+1}\frac{G(E,\omega,\Delta)|F_{M\lambda}(\Delta)|^2}{(\omega^2-\Delta^2)^2}, \quad (24)$$

where

$$G=E^2-1-\tfrac{1}{4}(2\omega E+\omega^2-\Delta^2)^2/\Delta^2 \quad (25)$$

and

$$H=\tfrac{1}{2}[(2E+\omega)^2-\Delta^2]. \quad (26)$$

In the above expressions all energies and masses are expressed in units of the electron rest energy and mass, $p=(E^2-1)^{1/2}$, and $p'=[(E+\omega)^2-1]^{1/2}$.

For values of momentum transfer for which $\Delta R_N\ll 1$ (where R_N is the nuclear radius) the form factors in Eqs. (23) and (24) can also be replaced by unity. With this approximation, which is reasonably well satisfied for the average momentum transfer at densities considered in this paper, we have

$$R_e/R_\gamma{}^0=\frac{\alpha}{\pi}[I_\lambda(\omega,E_F)-\theta(E_F-1-\omega)I_\lambda(\omega,\,E_F-\omega)], \quad (27)$$

where $\theta(E_F-1-\omega)$ is defined in Eq. (16) and $I_\lambda(\omega,E_F)$ is of the form

$$I_\lambda(\omega,E_F)=\int_1^{E_F}L_\lambda(\omega,E)dE. \quad (28)$$

For electric transitions the first few $L_\lambda(\omega,E)$ are given by

$$\omega^3L_1(\omega,E)=\omega^2(1/g_+-1/g_-)+[h(\omega,E)+1]\ln(g_+/g_-)-(g_+-g_-)/4\,, \quad (29)$$

$$\omega^5L_2(\omega,E)=\omega^4(1/g_+-1/g_-)+\omega^2h(\omega,E)\ln(g_+/g_-)+\{\omega^2/4+4[h(\omega,E)+1-\tfrac{1}{2}\omega^2]/3\}(g_+-g_-)-(g_+{}^2-g_-{}^2)/6\,,$$

[5] To a good approximation the actual radiative rate in the plasma will vanish for $\omega<\omega_0$ and will be reduced by a factor $(1-\omega_0{}^2/\omega^2)^{\lambda+\frac{1}{2}}$ for $\omega>\omega_0$.

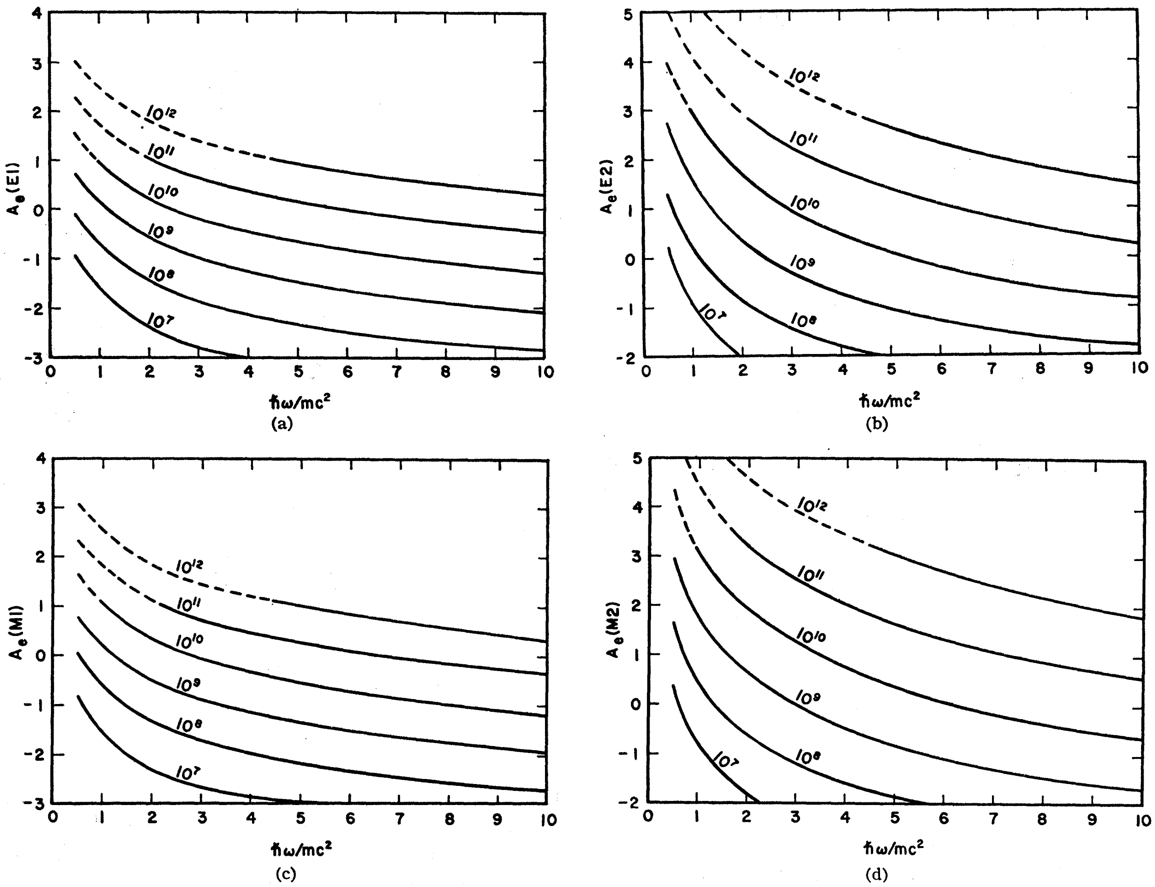


FIG. 1. $A_e{}^0=\log_{10}(R_e/R_\gamma{}^0)$ as a function of nuclear transition energy for different values of density. The number of atomic mass units per electron is assumed to be two. R_e and $R_\gamma{}^0$ are the relevant electron-induced and spontaneous radiative transition rates for (a) $E1$, (b) $E2$, (c) $M1$, and (d) $M2$ transitions. Curves are labeled with density in g/cm³, and are dashed for $\omega\leq$plasma frequency.

while for magnetic transitions

$$\omega^3 L_1(\omega,E)=\omega^2(1/g_+-1/g_-)+h(\omega,E)\ln(g_+/g_-)+(g_+-g_-)/4, \quad (30)$$

$$\omega^5 L_2(\omega,E)=\omega^4(1/g_+-1/g_-)+\omega^2[h(\omega,E)-1]\ln(g_+/g_-)+[h(\omega,E)+\tfrac{1}{4}\omega^2](g_+-g_-)+(g_+{}^2-g_-{}^2)/8,$$

where $g_\pm=(p'\pm p)^2-\omega^2$ and $h(\omega,E)=E^2-1+E\omega+\frac{1}{2}\omega^2$. For the two limiting cases of (1) large transition energy and (2) large Fermi energy, the ratio $R_e/R_\gamma{}^0$ is given approximately by

$$R_e/R_\gamma{}^0\simeq\frac{\alpha}{2\pi}\left(\frac{E_F}{\omega}\right)^2, \quad \omega\gg E_F\gg 1 \quad (31)$$

and

$$R_e/R_\gamma{}^0\simeq\frac{\alpha}{2\pi\lambda}\left(\frac{2E_F}{\omega}\right)^{2\lambda}, \quad E_F\gg 1,\omega. \quad (32)$$

These results are independent of multipole type (electric or magnetic). For general values of density and transition energy, the integrals must be done numerically. The results for $E1$, $E2$, $M1$, and $M2$ transitions are given in Figs. 1(a)–1(d) as a function of transition energy for various values of density in the range 10^7 g/cm³$<\rho<10^{12}$ g/cm³. The ratio $R_e/R_\gamma{}^0$ has the same qualitative behavior as the bound-free internal conversion coefficient; it is an increasing function of multipolarity and a decreasing function of transition energy. Finally, under the assumption that there are two atomic mass units per electron in the stellar matter the Fermi energy is related to the density by $E_F\simeq 0.8\,\rho_6{}^{1/3}$, where $\rho_6=10^{-6}\,\rho$ in g/cm³. Then for a *fixed* value of $\omega\ll E_F$ we have from Eq. (32)

$$A_e=\log_{10}(R_e/R_\gamma{}^0)\simeq\text{constant}+\tfrac{2}{3}\lambda\log_{10}\rho_6, \quad (33)$$

and neighboring curves in Figs. 1(a)–(d) will differ in their values of A_e by

$$\Delta A_e\simeq\tfrac{2}{3}\lambda, \quad E_F\gg 1,\omega \quad (34)$$

which explains the nearly constant separation of these curves.

In all of the above discussion, the electric monopole transition has been omitted, as this transition cannot occur by radiation. The transition can occur by inelastic electron scattering,[6] however, and is important in the

[6] L. I. Schiff, Phys. Rev. 96, 765 (1954).

de-excitation of C^{12} as will be discussed in Sec. III. The monopole current matrix element is given by

$$\langle JM|J_{mp}(\mathbf{\Delta})|JM\rangle$$

$$=(4\pi)^{1/2}\int d^3x\langle JM|\rho(\mathbf{x})|JM\rangle \sin(\Delta r)/(\Delta r), \quad (35)$$

where

$$\langle JM|\rho(\mathbf{x})|JM\rangle=\sum_r e_r\int \prod_{s\neq r} d^3x_s$$

$$\times\psi_{J'M'\alpha'}^*(\cdots\mathbf{x}_r=\mathbf{x}\cdots)\psi_{JM\alpha}(\cdots\mathbf{x}_r=\mathbf{x}\cdots). \quad (36)$$

We have included additional state labels α and α' to distinguish between the initial and final nuclear states ($\alpha\neq\alpha'$). Upon expanding the function $\sin(\Delta r)/(\Delta r)$ in a power series the first term does not contribute to the integral in Eq. (35) because the initial and final nuclear states are orthogonal. The leading term in the monopole current matrix element is thus

$$-\frac{(4\pi)^{1/2}}{6}\Delta^2\int d^3x\, r^2\langle JM|\rho(\mathbf{x})|JM\rangle$$

which resembles the electric quadrupole current matrix element. Defining

$$\langle J\|Q_{mp}\|J\rangle=\int d^3x\, r^2\langle J\|\rho(\mathbf{x})\|J\rangle, \quad (37)$$

we have

$$|\langle J\|J_{mp}(\Delta)\|J\rangle|^2=4\pi\frac{\Delta^4}{36}|\langle J\|Q_{mp}\|J\rangle|^2|F_{mp}(\Delta)|^2, \quad (38)$$

where $F_{mp}(\Delta)$ is the monopole form factor with $F_{mp}(0)=1$. For electron-induced monopole and quadrupole transitions of the *same* energy it then follows that

$$R_e(E0)/R_e(E2)=\frac{225}{144\pi}\frac{|\langle J\|Q_{mp}\|J\rangle|^2}{|\langle J\|Q_2\|J\rangle|^2}, \quad (39)$$

where we assume that $J\geq 1$ (so that both transitions can take place), and where the transverse part of the current in the quadrupole transition has been neglected and the form factors have been set equal to unity. We return to a discussion of the monopole transition in Sec. III.

B. De-Excitation by Ions

In this section we calculate the rate for inelastic Coulomb scattering by ions in the semiclassical theory, which has been thoroughly discussed in several reviews.[7] According to this theory, which is valid for small relative ion velocity v and small transition energy ω, the transition probability for inelastic Coulomb scattering is given by

$$\sigma v=2M_r{}^3W'\int|F|^2d\Omega, \quad (40)$$

where σ is the de-excitation cross section, M_r is the reduced mass of the system, W' and W are the final and initial relative kinetic energy, respectively, with $W'=W+\omega$, and F is the amplitude for de-excitation given by

$$|F|^2\simeq|f_C|^2p. \quad (41)$$

Here f_C is the *elastic* Coulomb scattering amplitude for a mean energy $\bar{W}=(WW')^{1/2}$, and P is the probability that the target nucleus undergoes a transition:

$$P=\frac{1}{2J+1}\sum_{MM'}|b_{fi}|^2, \quad (42)$$

with

$$b_{fi}=-i\int_{-\infty}^{\infty}dt\,\langle f|H|i\rangle e^{-i\omega t}, \quad (43)$$

where

$$\langle f|H|i\rangle=\int d^3x\,[\Phi(\mathbf{x},t)\langle J'M'|\rho(\mathbf{x})|JM\rangle - \mathbf{A}(\mathbf{x},t)\cdot\langle J'M'|\mathbf{j}(\mathbf{x})|JM\rangle]. \quad (44)$$

Here the target nucleus current matrix elements are the same as those given in Eqs. (5) and (6), and the electromagnetic potentials due to the (classical) motion of the projectile nucleus are given by

$$\Phi(\mathbf{x},t)=\frac{Z_pe}{|\mathbf{x}-\mathbf{x}(t)|}-\frac{Z_pe}{r_p(t)},\quad \mathbf{A}(\mathbf{x},t)=\frac{Z_pe\mathbf{v}(t)}{|\mathbf{x}-\mathbf{x}(t)|}. \quad (45)$$

In these expressions Z_pe is the projectile charge, $\mathbf{x}(t)$ traces out the classical hyperbolic trajectory with $r(t)=|\mathbf{x}(t)|$, and $\mathbf{v}(t)=d\mathbf{x}(t)/dt$. In calculating the classical orbit for the relative motion, the kinetic energy loss is neglected and we again use the mean kinetic energy $\bar{W}$. In ABHMW the electric and magnetic multipole transition probabilities are given by

$$\sigma_{E\lambda}v=\frac{1}{2J+1}Z_p{}^2\alpha\left(\frac{M_r}{2W}\right)^{1/2}\left(\frac{4WW'}{Z_p{}^2Z^2\alpha^2}\right)^{\lambda-1}\times|\langle J'\|Q_\lambda\|J\rangle|^2f_{E\lambda}(\xi) \quad (46)$$

and

$$\sigma_{M\lambda}v=\frac{1}{2J+1}Z_p{}^2\alpha\left(\frac{2W'}{M_r}\right)^{1/2}\left(\frac{4WW'}{Z_p{}^2Z^2\alpha^2}\right)^{\lambda-1}\times|\langle J'\|M_\lambda\|J\rangle|^2f_{M\lambda}(\xi), \quad (47)$$

where

$$\xi=ZZ_p\alpha\left(\frac{M_r}{2}\right)^{1/2}\left(\frac{1}{W^{1/2}}-\frac{1}{W'^{1/2}}\right),$$

$\alpha=1/137$, and Z is the target atomic number. The

[7] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28, 432 (1956), henceforth referred to as ABHMW; G. Breit and R. L. Gluckstern, in *Handbuch der Physik*, edited by S. Flügge (Springer Verlag, Berlin, 1959), Vol. XLI/1, p. 496.

Coulomb de-excitation functions $f_{E\lambda}(\xi)$ and $f_{M\lambda}(\xi)$ have been tabulated by ABHMW, and the multipole transition moments have been defined previously in Eq. (20). It should be pointed out that the parameter ξ introduced above is actually the negative of the corresponding parameter introduced by ABHMW, but that the positive argument in the Coulomb de-excitation function above is correct because of the symmetry relations

$$f_{E\lambda}(\xi)=f_{E\lambda}(-\xi)\,,$$
$$f_{M\lambda}(\xi)=f_{M\lambda}(-\xi)\,. \tag{48}$$

To obtain the thermonuclear de-excitation rate per target nucleus we multiply the transition probabilities in Eqs. (46) and (47) by the Boltzmann factor $2\pi N_p(\beta/\pi)^{3/2}W^{1/2}\exp(-\beta W)$ (where N_p is the projectile number density, $\beta=1/kT$, k is the Boltzmann constant, and T is the absolute temperature), and integrate over the initial relative kinetic energy. As in the case of inelastic electron scattering, we compare the resulting de-excitation rates by dividing the Coulomb de-excitation rate by the radiative rate (neglecting dispersion effects)

$$R_\gamma=R_\gamma{}^0[1-\exp(-\beta\omega)]^{-1}\,, \tag{49}$$

where $R_\gamma{}^0$ is given by Eq. (21). The factor

$$[1-\exp(-\beta\omega)]^{-1}$$

is inserted to take stimulated emission into account. We thus obtain

$$R_i(E\lambda)/R_\gamma(E\lambda)=6.93\frac{\lambda[(2\lambda+1)!!]^2}{\lambda+1}\left(\frac{A}{A_p(A+A_p)}\right)^{1/2}\times\frac{\beta^{3/2}Z_p{}^2\rho_{10}(p)}{(Z_p{}^2Z^2\alpha^2/4)^{\lambda-1}}\frac{(1-e^{-\beta\omega})}{\omega^{2\lambda+1}}K_{E\lambda}\,,$$

$$R_i(M\lambda)/R_\gamma(M\lambda)=1.48\times10^{-2}\frac{\lambda[(2\lambda+1)!!]^2}{\lambda+1}\times\left(\frac{A+A_p}{AA_p{}^3}\right)^{1/2}\frac{\beta^{3/2}Z_p{}^2\rho_{10}(p)}{(Z_p{}^2Z^2\alpha^2/4)^{\lambda-1}}\frac{(1-e^{-\beta\omega})}{\omega^{2\lambda+1}}K_{M\lambda}\,, \tag{50}$$

where R_i is the appropriate ion Coulomb de-excitation rate, and

$$K_{E\lambda}=\int_0^\infty dWe^{-\beta W}f_{E\lambda}[\xi(W)][W(W+\omega)]^{\lambda-1}\,,$$
$$K_{M\lambda}=\int_0^\infty dWe^{-\beta W}f_{M\lambda}[\xi(W)][W(W+\omega)]^{\lambda-1/2}\,. \tag{51}$$

Again, all energies and masses are measured in units of the electron rest energy and mass, $\rho_{10}(p)=10^{-10}\,\rho(p)$, where $\rho(p)$ is the projectile density in g/cm^3, and A, A_p are the atomic weights of the target and projectile nuclei, respectively.

The quantity $A_i=\log_{10}[R_i/R_\gamma]$ has been calculated by numerical integration of Eq. (51) and the results have been plotted against nuclear transition energy for a density of 10^{10} g/cm^3 and for various values of multipolarity, temperature, and Z with $Z=A/2$. The projectile nuclei were chosen to be α particles since significantly larger values of Z_p lead to values of R_i/R_γ that, at temperatures considered in this paper ($T<10^{10}$ °K), are small in comparison with the corresponding quantity for electron de-excitation. Furthermore, a plasma with a large density of helium at high temperature is of astrophysical interest. While a proton projectile would lead intrinsicly to a larger de-excitation rate (because of the smaller Coulomb barrier), the large proton density needed to achieve significant values of R_p/R_γ seems of little astrophysical interest at such high temperatures. For this reason the case of projectile protons has not been included in the graphs. For the sake of comparison we have included the appropriate electron de-excitation curves in each of Figs. 2(a)–2(c). It is evident that for magnetic transitions, electron de-excitation always dominates ion de-excitation for temperatures considered in this paper. This is to be expected since the ion velocities are much smaller than the electron velocities. In the case of electric multipole transitions, the relative importance of electron and ion de-excitation depends upon the temperature. In general, for a given density there is a critical temperature $T_c(\rho)$ such that for $T>T_c$ ion de-excitation is dominant, while for $T<T_c$ electron de-excitation is dominant. This delineation is shown explicitly in Sec. III in which the de-excitation of C^{12} is considered.

Finally, it should also be pointed out that the accuracy of the semiclassical theory decreases with increasing transition energy. However, for values of ω considered in Figs. 2(a)–2(c), the error is at most 10–15% in the ratio R_i/R_γ and is considerably better over most of the range.

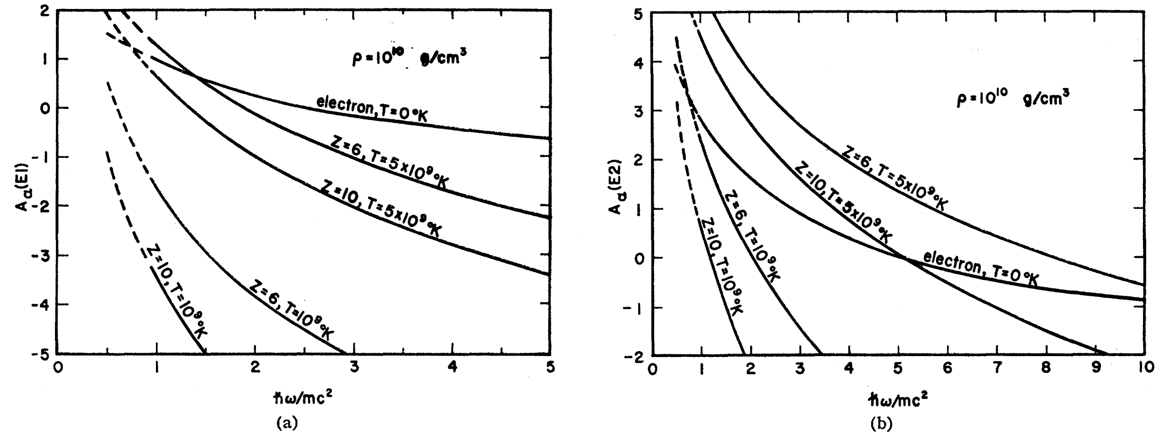
III. DE-EXCITATION OF C^{12}

A particular reaction of astrophysical interest involving a radiative transition is the helium burning process

$$He^4+Be^8\to C^{12*}(0^+)\to C^{12*}(2^+)+\gamma\,, \tag{52}$$

in which a small fraction of the C^{12} produced in the 7.65-MeV 0^+ state by the resonant He^4-Be^8 interaction decays to the 4.43-MeV 2^+ state by an electric quadrupole γ-ray emission. The extent to which the He^4 plasma enhances the radiative transition rate by the Coulomb de-excitation of $C^{12*}(0^+)$ has been discussed previously by the authors.[8] The result of that work is contained in the numerical fit to the calculated de-

[8] D. D. Clayton and P. B. Shaw, Astrophys. J. 148, 301 (1967).



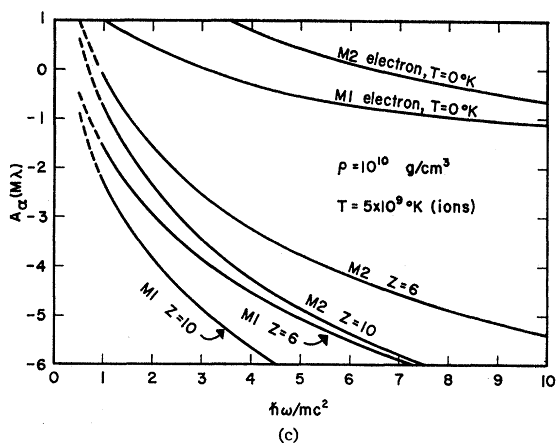


FIG. 2. $A_\alpha=\log_{10}(R_\alpha/R_\gamma)$ as a function of nuclear transition energy for a helium density of 10^{10} g/cm³ and for various temperatures and target atomic number. The projectile nuclei are α particles. R_α and R_γ are the relevant helium-induced and radiative (spontaneous plus stimulated) transition rates for (a) $E1$, (b) $E2$, and (c) $M1$ and $M2$ transitions. The curves are dashed for $\omega\lesssim$ plasma frequency.

excitation rate

$$R_\alpha(E2)/R_\gamma(E2)=2.22\times10^3\rho_9(\mathrm{He})\exp(-19.32/T_9)\,, \tag{53}$$

where $T_9=10^{-9}T(°\mathrm{K})$ with T_9 in the range $1<T_9<10$ and in which stimulated emission has been included. In obtaining Eq. (53) the quantum-mechanical theory of Coulomb de-excitation was employed rather than the simpler semiclassical theory discussed in the previous section, although the difference in the two theories is at most 10–15% over the indicated temperature range. In this section we determine the extent to which a degenerate electron plasma can de-excite the $C^{12*}(0^+)$ state at high density. To obtain significant results, we restrict the density to the range 10^9 g/cm³$<\rho(\mathrm{He})<10^{12}$ g/cm³, where we assume that essentially all electrons come from the ionization of helium.

There are two ways in which the de-excitation can proceed, either by the electric quadrupole transition as in the helium de-excitation or by a monopole transition from the 0^+ 7.65-MeV level to the 0^+ ground state. The inverse laboratory process of monopole excitation of the 7.65-MeV level has been discussed by a number of authors.[6,9] Neglecting the transverse part of the $E2$ current matrix element we have from Eq. (23)

$$R_e(E2)/R_\gamma{}^0(E2)=\frac{1}{3}\frac{1}{\omega^5}\int_1^{E_F}\left[1-f(E+\omega)\right]\times\int_{(p'-p)^2}^{(p'+p)^2}d\Delta^2\left[(2E+\omega)^2-\Delta^2\right]|F_{E2}(\Delta)|^2, \tag{54}$$

where ω is the transition energy between the 0^{+*} and the 2^{+*} levels. The charge form factor is determined from *elastic* electron-carbon scattering data by following a theory due to Helm.[10] For the values of momentum transfer at densities under consideration the form factor can be represented to good approximation by

$$|F_{E2}(\Delta)|^2\approx1-(1/7)\tilde{R}^2\Delta^2\,, \tag{55}$$

where $\tilde{R}$ is an effective charge radius and is numerically equal to 8.95×10^{-3} in the dimensionless units used here. From Eqs. (18) and (38) the monopole de-excitation rate relative to the $E2$ radiative transition rate is

[9] J. H. Fregeau, Phys. Rev. **104**, 225 (1954).

[10] R. D. Helm, Phys. Rev. **104**, 1466 (1956).

given by

$$R_e(E0)/R_\gamma^0(E2)=\frac{mc^2}{\Gamma(0^{+*}\rightarrow 2^{+*})}\frac{\alpha^2}{2\pi}\frac{|\mathrm{ME}|^2}{\lambda_c^4}\times\int_1^{E_F}dE\,[1-f(E+\omega)]\int_{(p'-p)^2}^{(p'+p)^2}d\Delta^2\times[(2E+w')^2-\Delta^2]F_{mp}(\Delta)|^2, \quad (56)$$

where ω' is the transition energy between the 0^{+*} and ground levels. The monopole form factor was taken from the experiment of Gudden and Strehl[11] as was the matrix element $|\mathrm{ME}|$. Again the form factor is of the form

$$|F_{mp}(\Delta)|^2 \simeq 1-\tfrac{1}{10}R^2\Delta^2, \quad (57)$$

where in dimensionless units $R=9.56\times10^{-3}$. The values of the other parameters in Eq. (56) are

$$\begin{aligned}\Gamma(0^{+*}\rightarrow 2^{+*}) &\simeq 2.4\times10^{-3}\ \mathrm{eV},\\ mc^2 &= 5.11\times10^5\ \mathrm{eV},\\ \lambda_c &= 3.86\times10^{-11}\ \mathrm{cm},\\ |\mathrm{ME}| &= 55.7\times10^{-27}\ \mathrm{cm}^2,\end{aligned} \quad (58)$$

where $\Gamma(0^{+*}\rightarrow 2^{+*})$ is taken from Seeger and Kavanaugh.[12] The quadrupole and monopole rates were evaluated numerically, and can be expressed as a function of helium density by

$$\begin{aligned}R_e(E2)/R_\gamma^0(E2) &\simeq 1.66\times10^{-2}\,\rho_9^{4/3}(\mathrm{He}),\\ R_e(E0)/R_\gamma^0(E2) &\simeq 6.96\times10^{-2}\,\rho_9^{4/3}(\mathrm{He})\end{aligned} \quad (59)$$

over the range $1<\rho_9(\mathrm{He})<10^3$ with good accuracy. It is interesting to note that the monopole de-excitation is more important than the quadrupole de-excitation by a factor of 4.2.

In Fig. 3 we have plotted $\log_{10}[(R_e(E0)+R_e(E2)+R_\alpha(E2))/R_\gamma(E2)]$ as a function of temperature for various values of helium density. For helium densities above 10^{10} g/cm^3 particle-induced de-excitation is more important than spontaneous radiative de-excitation. The competition between the electron and ion modes of de-excitation is also indicated in Fig. 3 with the electrons dominating at the lower temperatures and the ions at the higher temperatures. Finally, we point out that plasma dispersion effects are relatively unimportant for the C^{12} problem (except perhaps for $\rho\simeq10^{12}$ g/cm^3) since the transition energies are rather large. As an example, ω_0 is about 1 MeV at a density of 10^{11} g/cm^3, while ω for the dominant monopole transition is 7.65 MeV.

IV. DISCUSSION

An overwhelming majority of the nuclear reactions in astrophysics occur by way of resonances in the compound nucleus. In most cases the cross section is adequately represented by a sum over resonances of the single-level Breit-Wigner formula, whereupon the average of the transition probability σv over a Maxwellian distribution of velocities becomes for radiative capture[13]

$$\langle\sigma v\rangle=\left(\frac{2\pi\hbar^2}{MkT}\right)^{3/2}\sum_r\left(\frac{\omega\Gamma_1\Gamma_\gamma}{\hbar\Gamma}\right)_r\exp(-E_r/kT), \quad (60)$$

where the sum is over resonances designated by r. The resonance energy relative to the mass of the resonating particles is E_r and the resonance width Γ is assumed to be smaller than kT. Many reactions of astrophysical interest, especially at moderate temperatures, are so dominated by the Coulomb barrier in the charged-particle channel that the incident-particle width is much smaller than the radiative width, in which case $\Gamma_1\Gamma_\gamma/\Gamma\rightarrow\Gamma_1$. For such reactions we see that any enhancement of Γ_γ by particle de-excitation leads to no increase in reaction rate; in this limit the compound nucleus always de-excites so that the reaction rate depends only upon the rate of formation of the compound nucleus.

But in several special reactions such as C^{12} formation, radiative neutron capture, or charged-particle capture at high temperature, the particle widths dominate the widths of the relevant resonant states. In this case, $\Gamma_1\Gamma_\gamma/\Gamma\rightarrow\Gamma_\gamma$ and any enhancement of Γ_γ by particle de-excitation enters linearly into the reaction rate. The astrophysical applications of this paper are thus restricted to reactions of the second type.

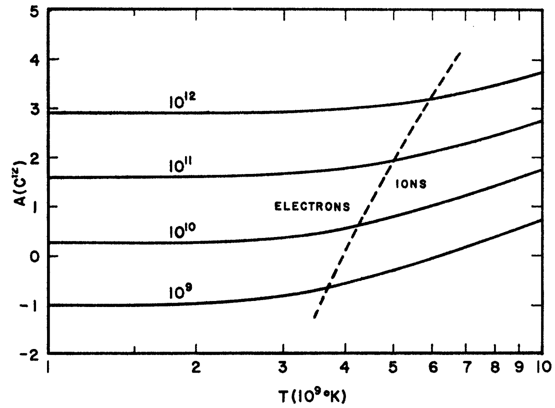


FIG. 3. $A(C^{12})=\log_{10}[(R_e(E0)+R_e(E2)+R_\alpha(E2))/R_\gamma(E2)]$ as a function of temperature for different values of helium density. The electrons are assumed to come solely from the ionization of helium. $R_e(E2)$, $R_\alpha(E2)$, and $R_\gamma(E2)$ are the electron-induced, helium-induced, and radiative (spontaneous plus stimulated) electric quadrupole transition rates, respectively, for the 7.65-MeV $(0^+)\rightarrow$4.43-MeV (2^+) transition in C^{12}. $R_e(E0)$ is the electron-induced electric-monopole transition rate for the 7.65-MeV $(0^+)\rightarrow$ground-state (0^+) transition in C^{12}. Curves are labeled with helium density in g/cm^3.

[11] F. Gudden and P. Strehl, Z. Physik **185**, 111 (1965).

[12] P. A. Seeger and R. W. Kavanaugh, Astrophys. J. **137**, 704 (1962).

[13] W. A. Fowler and F. Hoyle, Astrophys. J., Suppl. IX **201**, (1964), Appendix C.

At high temperature and density it frequently happens that the values of the rates come to be unimportant. This situation occurs when the environment exists for a sufficient length of time for radiative reactions to equilibrate with the inverse photodisintegration reactions, in which case nuclear abundance ratios depend only upon binding energies but not upon individual rates. The astrophysical problem, therefore, is to compare the time scale for an event, say the shocked compression following a supernova implosion, with the relevant nuclear lifetimes for equilibration. Interest in the processes described in this paper are restricted to the nonequilibrium conditions. The outstanding example is the rapid flow to heavy nuclei that can occur following the fusion of C^{12} from α particles in a dense gas.

If the density is great enough that radiative capture is enhanced by collisional de-excitation, then the inverse photodisintegration rate is also enhanced. The average photodisintegration rate with respect to a Planck spectrum of photons is, in direct correspondence with Eq. (60),

$$\lambda_\gamma=\frac{\exp(-Q/kT)}{2J+1}\sum_r \frac{(2J_r+1)\Gamma_{1r}\Gamma_{\gamma r}}{\hbar\Gamma_r}\exp\left(-\frac{E_r}{kT}\right), \quad (61)$$

where the statistical weight of each nuclear particle as obtained by the sum over its states has been approximated, as is reasonable in practical cases, by the statistical weight of the ground state. In this equation J is the ground-state spin of the compound nucleus, J_r is the spin of the compound nuclear resonances having center-of-mass energy E_r for the resonating particles as in Eq. (60), and Q is the binding energy of the photoejected particle in the ground-state of the compound nucleus. The point to be noticed is that if the disintegration occurs through excited states of the compound nucleus whose widths are dominated by the particle widths, then excitation of that state by inelastic particle collisions may enhance the photodisintegration rate. This point should not be overlooked inasmuch as photodisintegration rates play key roles in several nonequilibrium applications in high-temperature astrophysics. If, for example, the environment is such that the de-excitation of the 7.65-MeV level of C^{12} is predominantly by Coulomb de-excitation to the 4.43-MeV level, it follows that the photodisintegration of C^{12} in the same environment proceeds by the Coulomb excitation of the 4.43-MeV level to the 7.65-MeV level, which then disintegrates into three α particles.

We have intentionally limited ourselves to the question of the de-excitation of nuclear states by *electromagnetic* interactions with passing particles, but it must be noted that at extreme conditions of temperature and density the de-excitation by *nuclear* interactions may also occur. That is, a compound nuclear state, once formed, may be destroyed by nuclear interactions more rapidly than by electromagnetic de-excitation. Because of the Coulomb barrier, the most likely interactions of this type will be with neutrons, protons, or α particles. For the example of C^{12} discussed earlier a rough calculation shows that the nuclear interaction of the 7.65-MeV level with α particles fails by only about one order of magnitude to compete with the Coulomb de-excitation by α particles. Neutrons are even more efficient because of the complete absence of a Coulomb barrier. Another rough calculation shows that a neutron density of order 10^{-5} of the α particle density at $T_9=5$ is sufficient to render inelastic neutron scattering as efficient as Coulomb de-excitation. Thus it appears that for thermal environments sufficiently extreme for application of the principles of this paper, one must also compute the rates of the available nuclear interactions with excited states.

It does not appear to be possible to say at the present time whether de-excitations of the type mentioned in this paper will be important in astrophysical problems. By all odds the most likely application seems to be in fusion reactions in supernovas, but the details of the history of the various mass zones in the supernova event are not yet known with sufficient accuracy to determine the applicability of the present work. When the center implodes to nuclear densities, the overlying layers tend to follow. The maximum temperature and density encountered during the shock when the implosion is reversed decreases as one moves outward in mass. What one requires is the time profile of the temperature and density in each mass zone. The current results[1] indicate that temperatures and densities in the range considered in this paper are encountered in the expansion following the implosion, and if the expansion is fast enough that nuclear equilibrium is not maintained, our results will be necessary for a complete analysis of the problem.

ACKNOWLEDGMENTS

The authors wish to thank H. C. Goldwire, Jr. and S. Guzzardo for their assistance with some of the numerical work in this paper. This research was in part sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. 855-65 and in part by the National Aeronautics and Space Administration under Grant No. NGR 44-006-033.