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NEUTRON TUNNELING IN $^3He(^3He,2p)^4He$*

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ABSTRACT

We investigate the possibility that in the astrophysically interesting energy range (i.e., well below the Coulomb barrier), the reaction $^3He(^3He,2p)^4He$ proceeds predominantly by a neutron tunneling mechanism. Rough calculations of the magnitude and energy dependence of the total cross-section and of the shape of the proton differential cross-section are made on this basis. The results are in moderate agreement with recent experiments, and tend to justify a smooth extrapolation of these laboratory results down to the lower energies of astrophysical significance.

I. INTRODUCTION

The cross-section for the nuclear reaction $^3He(^3He,2p)^4He$ at an energy of about 10–20 keV is of considerable astrophysical interest, as this reaction plays a significant role in the proton-proton chain which generates energy in stars (see, e.g., Parker, Bahcall, and Fowler 1964). In particular, incomplete knowledge of this reaction rate is probably the major nuclear-physics uncertainty in the prediction of the ^8B neutrino flux from the Sun (Bahcall 1966).

The cross-section for the reaction down to a laboratory incident energy of around 2 MeV is well determined, and the dominant reaction mechanism is identified (Bacher and Tombrello 1968) as being via the ground state of ^5Li as an intermediate state; i.e., $^3He + {}^3He \rightarrow {}^5Li(g.s.) + p \rightarrow {}^4He + p + p$. However, recent experiments (Bacher and Tombrello 1968; Neng-Ming *et al.* 1966; Winkler and Dwarakanath 1967*a*, *b*) down to an energy of about 300 keV (lab.) show a change in the energy dependence of the total cross-section (see Fig. 1, p. 861), along with a change in the character of the proton energy spectrum, which indicate that some new mechanism is dominating the reaction at these low energies. Since one wishes to extrapolate these laboratory results downward to the energies of astrophysical interest, an identification of the new reaction mechanism is clearly desirable.

In the present work we suggest the possibility that well below the Coulomb barrier the cross-section is dominated by a neutron tunneling mechanism. In this process, first suggested by Breit and Ebel (1956*a*, *b*) for $^{14}N(^{14}N,^{13}N)^{15}N$, the neutron "tunnels" from its initial to its final state even though target and incident nuclei are significantly distant. At incident energies below the Coulomb barrier, when it becomes harder and harder for incident and target nuclei to get close together, such a mechanism will tend to dominate other processes (Breit and Ebel 1956*a*).

In exploring this suggestion further, we make two approximations: first, in the wave function for the relative motion of the incident and target nuclei, we include only the long-range Coulomb forces, neglecting nuclear forces which should be small at those separations which contribute predominantly to the neutron tunneling matrix element; second (and more crudely), we treat the outgoing two protons as if they were temporarily a single particle, a "diproton." This latter approximation is made in order to render the calculation feasible; its justification is discussed in § II. For the initial and

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final neutron states, we use asymptotic wave functions or Hulthen wave functions (with the Hulthen parameter chosen to fit the "size" of the ^{3}He or ^{4}He nucleus). As one consequence of all these assumptions, *no* free parameters enter the calculation, except for the initial and final neutron reduced widths[1] (or spectroscopic factors), which appear simply as an over-all multiplicative constant.

With the above assumptions, our calculation is necessarily only a rough one. However, it should be adequate to describe the character of the results to be expected from a neutron tunneling mechanism for the reaction at low energies.

The theoretical results are compared with recent experiments with respect to (i) the energy dependence of the cross-section, (ii) the shape of the differential cross-section, and (iii) the initial and final neutron spectroscopic factors needed to fit the absolute magnitude of the cross-section. These results are discussed in § IV, where it is concluded that the neutron tunneling mechanism leads to results which are in at least qualitative agreement with present experiments. Any agreement is, however, by no means conclusive, since the present experiments are largely at energies which are sufficiently high to emphasize the crudities in our present calculations.

In summary, we believe that the present work represents a useful first step toward elucidating the reaction mechanism for ^{3}He(^{3}He,2p)^{4}He at low energies, and that it lends confidence to a smooth extrapolation of the present experimental results downward to energies of astrophysical interest.

II. ESTIMATE OF THE TUNNELING CROSS-SECTION

We now list the steps taken to estimate the consequences of the neutron tunneling hypothesis.

One central approximation underlies all that follows: For the purplse of calculating a transition matrix element, we treat the diproton as a particle; that is, the final outgoing nucleus is a "diproton." This is done in order to make a calculation possible. Moreover, it seems a reasonable assumption, since the diproton only just fails to be bound (having a resonance at about 0.6 MeV), and as the two final protons will carry about 8 MeV, the virtual diproton should live long enough (after the neutron has tunneled out of the ^{3}He) to carry it out of the region of space contributing most to the integrals in the matrix element. Eventually, to get a differential cross-section for the outgoing protons, we simply convolve the differential cross-section for the final-state virtual diproton with the angular distribution of protons from the breakup of the diproton. This virtual diproton is taken to be unbound by 0.6 MeV, but it is encouraging to note that the results are essentially unaltered if we instead take it to be unbound by 0.0 or by 1.0 MeV.

If this approximation is *not* made, one must include the appropriate wave functions for the two individual protons, not only in their final outgoing states, but also in the incident ^{3}He nucleus. The consequent calculation would be very much more complicated than the present one.

Consider now the general process whereby a target nucleus of charge Z' and mass number B is bombarded by particles of charge Z and mass number $A + 1$, with the target capturing a neutron to give a final nucleus of mass $B + 1$ and an outgoing particle of mass number A.

If the incident energy is well below the Coulomb barrier for the process, we may make the further approximation of using pure Coulomb wave functions for the scattering states (i.e., neglecting nuclear forces). The amplitude for the process is now, in what is, essentially, distorted-wave Born approximation,

$$T = S_i^{1/2} S_f^{1/2} \int \int d\boldsymbol{r}_n d\boldsymbol{r} \chi_f^*(\boldsymbol{r}_n) \phi_f^{(-)}(\boldsymbol{k}_f, \boldsymbol{r} - \boldsymbol{r}_n/(B+1)) \times [V_{\mathrm{ni}} \text{ or } V_{\mathrm{nf}}] \chi_i(\boldsymbol{r} - \boldsymbol{r}_n) \phi_i^{(+)}(\boldsymbol{k}_i, \boldsymbol{r} + (\boldsymbol{r}_n - \boldsymbol{r})/(A+1)) . \quad (1)$$

[1] These quantities are defined and discussed in, e.g., Macfarlane and French (1960).

Here r_n is the neutron coordinate referred to the target nucleus B, and r is the separation between the nuclei A and B. The nuclear internal coordinates have been integrated out in the usual manner, so that χ_i and χ_f are the initial and final bound-neutron wave functions, and S_i and S_f the usual spectroscopic factors for these neutron states (that is, S_i and S_f measure how much the initial and final nuclei like to be written as core + neutron). The Coulomb wave functions $\phi_i^{(+)}$ and $\phi_i^{(-)}$ are normalized to asymptotically outgoing and incoming spherical waves, respectively. The initial and final center-of-mass wavenumbers are k_i and k_f; thus the incident center-of-mass kinetic energy is $E_i = \hbar^2 k_i^2/2mM_i$, where M_i is the reduced mass number $M_i = (A+1)B/(A+B+1)$, and the final kinetic energy is $E_f = \hbar^2 k_f^2/2mM_f$, with $M_f = A(B+1)/(A+B+1)$.

Finally, the potential in equation (1) is *either* the potential binding the neutron in the final nucleus, $V_{nf}(r_n)$, yielding the so-called "post" form of the amplitude, T_{post}, *or* the potential binding the neutron in the initial nucleus, $V_{ni}(r - r_n)$, yielding the "prior" form T_{prior}. In a fully exact calculation, post and prior forms give identical answers, and it is of interest to keep track of *both* forms in our approximation.

The expression for $\phi^{(+)}(k,r)$ is

$$\phi^{(+)}(k,r) = C(\eta)e^{ik\cdot r}\, {}_1F_1(-i\eta,1,ikr - ik\cdot r)\ , \tag{2}$$

where η is the Coulomb parameter, which in this case is

$$\eta_i = ZZ'e^2 mM_i/(\hbar^2 k_i)\ . \tag{3}$$

When we say "below the Coulomb barrier," we mean $\eta > 1$. The normalization constant C has the usual magnitude $|C(\eta)|^2 = 2\pi\eta\,[\exp(2\pi\eta) - 1]^{-1}$.

We obtain $\phi^{(-)}$ by the usual procedure (Bethe, Low, and Maximon 1953) of letting $k \to -k$. Notice that for the final state "diproton" in the ^{3}He(^{3}He,2p)^{4}He reaction, η_f is small, and the results are the same whether we use Coulomb waves or plane waves for the final state.

We first present the reduced form of equation (1) when asymptotic wave functions are used for the initial and final neutron states; that is,

$$\chi(r) = \frac{N}{r}\, e^{-\gamma r}\ . \tag{4}$$

Here γ is the binding-energy wavenumber, $\hbar^2\gamma_i^2/2\mu_i m$ being the initial binding energy in the nucleus $(A+1)$, and $\hbar^2\gamma_f^2/2\mu_f m$ being the final binding energy in $(B+1)$; μ_i and μ_f are the neutron reduced mass numbers, $\mu_i = A/(A+1)$ and $\mu_f = B/(B+1)$. N is the normalization constant $(\gamma/2\pi)^{1/2}$. Subsequently we shall generalize the results to include Hulthen wave functions (eq.[15]) for the neutron states.

The use of a zero-range potential for V_{ni} or V_{nf} in equation (1) is of course logically concomitant with the use of the wave function (4) for all radii; the integral in equation (1) is thereby reduced to a three-dimensional one.

By use of integral representations for the hypergeometric functions, equation (1) can finally be written in closed form as

$$T = -4\pi S_i^{1/2} S_f^{1/2} N_i N_f (\hbar^2/2m) C(\eta_i) C(\eta_f) B(p_i,p_f,\gamma) I(\theta)\ , \tag{5}$$

where B is an angle-independent factor (defined below), and the angle dependence is contained in $I(\theta)$:

$$I(\theta) = \frac{4\pi}{\mu(\gamma^2+\Delta^2)}\, e^{-i\eta_i \ln(\gamma^2+\Delta^2)}\, {}_2F_1\left(1 + i\eta_i, -i\eta_f; 1; \frac{2(p_i p_f - p_i\cdot p_f)}{\gamma^2+\Delta^2}\right). \tag{6}$$

Here $\Delta = |p_i - p_f|$ is the momentum transfer. The relation between the quantities p_i, p_f, γ, μ, and the previously defined quantities k_i and k_f, γ_i and γ_f, μ_i and μ_f depend on whether we use the post or the prior form for the matrix element T. These relationships

	p_i	p_f	γ	μ
Prior. . . .	k_i	$\mu_f k_f$	γ_f	$\mu_i = A/(A+1)$
Post. .	$\mu_i k_i$	k_f	γ_i	$\mu_f = B/(B+1)$

are set out in the accompanying table. The factor B is defined to be

$$B = \exp\,[\eta_i\phi_1 + \eta_f\phi_2 + i(\eta_i + \eta_f)\psi_1 + i(\eta_i - \eta_f)\psi_2]\,, \tag{7}$$

$$\phi_1 = \arctan\,[2\gamma p_i/(\gamma^2 + p_f^2 - p_i^2)]\,, \qquad 0 < \phi_1 < \pi\,, \tag{8a}$$

$$\phi_2 = \arctan\,[2\gamma p_f/(\gamma^2 + p_i^2 - p_f^2)]\,, \qquad 0 < \phi_2 < \pi\,, \tag{8b}$$

$$2\psi_1 = \ln\,[(p_i + p_f)^2 + \gamma^2]\,, \tag{8c}$$

$$2\psi_2 = \ln\,[(p_i - p_f)^2 + \gamma^2]\,. \tag{8d}$$

Expressions (5) and (6), or closely related ones, have been derived independently in various contexts by many authors in the past: in particular, they have been presented for neutron tunneling by Buttle and Goldfarb (1966). The approximations listed above (with the exception of the diproton approximation) have been discussed in more detail by these authors.

The difference ensuing from the use of post or prior form for the original matrix element (eq. [1]) can readily be seen from equations (5) and (6). However differently the approximations (particularly the zero-range approximation) may appear to enter the different post and prior forms, we note that the angle dependences of the resulting expressions are identical, i.e.,

$$I_{\text{post}}(\theta) \equiv I_{\text{prior}}(\theta)\,. \tag{9}$$

This remark follows from the definitions of p_i, p_f, and γ set forth above, along with energy conservation for the reaction, which can be written as

$$[\mu(\gamma^2 + \Delta^2)]_{\text{post}} = [\mu(\gamma^2 + \Delta^2)]_{\text{prior}}\,. \tag{10}$$

The angle-independent factors $B(p_i,p_f,\gamma)$ do differ for post and prior forms, leading to a difference in the magnitude of the total cross-sections; this difference, however, amounts to only about 1 per sent for our ^{3}He(^{3}He,2p)^{4}He reaction, which is a cheering fact.

To proceed from equation (5) to expressions for the differential and total cross-sections, it is necessary to take account of spin and also (since for ^{3}He(^{3}He,2p)^{4}He the incident and target nuclei are identical) statistics. This is easily done following the method set out in detail by Breit and Ebel (1956*a*).

The end result for the differential cross-section for our reaction is

$$\frac{d\sigma}{d\Omega} = \frac{M_i M_f}{4}\frac{k_f}{k_i}(S_i S_f N_i^2 N_f^2)\,|C(\eta_i)|^2\,|C(\eta_f)|^2\,|B|^2\,|I(\theta) + I(\pi - \theta)|^2\,, \tag{11}$$

where B is defined by equation (7) and $I(\theta)$ by equation (6).

When dealing with reactions below the Coulomb barrier, it is convenient and conventional to extract the systematic energy dependences, defining a quantity $S(E)$ as follows:

$$\sigma(E) = \frac{S(E)e^{-2\pi\eta_i}}{E} . \tag{12}$$

Doing this, and inserting the numerical constants pertinent to $^3He(^3He,2p)^4He$, we get

$$\frac{dS(E)}{d\Omega} = (0.106)(S_i S_f N_i^2 N_f^2)(1 - e^{-2\pi\eta_i})^{-1}(e^{2\pi\eta_f} - 1)^{-1} \times |B|^2 \, |I(\theta) + I(\pi - \theta)|^2 \tag{13}$$

in units of MeV barns per steradian. The total cross-section is, of course, just

$$S(E) = \int \frac{dS}{d\Omega} d\Omega = 4\pi \int_0^{\pi/2} \frac{dS}{d\Omega} \, d(\cos\theta) . \tag{14}$$

These expressions have been obtained by using asymptotic neutron wave functions throughout. It would be more realistic to treat the neutron potentials as being of finite range, which can be done by replacing equation (4) by Hulthen wave functions for the neutron states (e.g., Sachs 1953):

$$\chi(r) = \frac{N'}{r} (e^{-\gamma r} - e^{-\beta r}) . \tag{15}$$

Here N' is related to the N of equation (4) by $N' = N[\beta(\beta + \gamma)]^{1/2}/(\beta - \gamma)$, and the parameter β can be chosen by several essentially equivalent methods.[2] We take $\beta_i = 3\gamma_i$ for the initial neutron state in ^3He, and $\beta_f = 1.5\,\gamma_f$ for the final neutron state in ^4He. Equation (4) is, of course, the limit $\beta \rightarrow \infty$.

The generalization of equation (5) to the case when only *one* neutron state has a Hulthen wave function is trivial. If, after first using a zero-range potential in either post or prior form of equation (1), together with equation (4) for the accompanying neutron wave function, we then use equation (15) for the *remaining* neutron wave function, we get, in place of equation (5), the expression

$$T \rightarrow \frac{N'}{N} [T(\text{with } \text{"}\gamma\text{"} = \gamma) - T(\text{with } \text{"}\gamma\text{"} = \beta)] . \tag{16}$$

The appropriate alterations to equations (11) and (13) are correspondingly simple.

We would, of course, prefer to use Hulthen wave functions for *both* neutron states, along with a Hulthen potential for V_{ni} or V_{nf} in equation (1). In the limit where the colliding nuclei are infinitely massive, i.e., A, $B \rightarrow \infty$, this can readily be done (May 1968); unfortunately, in our case, $A = 2$ and $B = 3$, so that this limiting result is inapplicable. However, to a first approximation, we may estimate the result obtained using Hulthen wave functions for *both* neutron states as differing from the result for asymptotic wave functions (eq. [5]) by the product of the two correction factors obtained by replacing the asymptotic wave functions by Hulthen functions one at a time (according to eq. [16]). In the limiting case mentioned above, when A, $B \rightarrow \infty$, this rough estimate can be compared with an exact result and is found to be reasonably accurate.

[2] One method relates β to the range of the potential (e.g., Sachs 1953). Alternatively, one can assume that neutrons and protons have a similar distribution about the center of mass, and then relate β to the rms radius of the charge distribution, which is 1.61 f for ^4He and 1.87 f for ^3He. For ^3He, one is led to a value of β/γ which lies between 2.4 and 4; and for ^4He between 1.1 and 2.0. In § III we discuss the sensitivity of our results to the choice of β/γ within these ranges.

Thus we end up with four sets of results: (A) both initial ^{3}He and final ^{4}He neutron states represented by asymptotic wave functions (eq. [5]); (B) asymptotic wave functions for ^{3}He neutron state, Hulthen wave functions for ^{4}He state (eq. [16] in prior form); (C) asymptotic ^{4}He and Hulthen ^{3}He wave functions (eq. [16] in post form); (D) both ^{3}He and ^{4}He neutron states represented by Hulthen wave functions (crude estimate based on results A, B, and C).

Finally, we list the various pertinent numerical quantities for our reaction: $k_i = 0.190[E_i \text{ (lab.) MeV}]^{1/2}$ f^{-1}; $k_f = 0.889\ [1 + 1.122k_i^2]^{1/2}$ f^{-1}; $\gamma_i = 0.517$ f^{-1}; $\gamma_f = 0.864$ f^{-1}; $\eta_i = 0.208/k_i$, and $\eta_f = 0.185/k_f$.

III. RESULTS

In this section we present our estimates of $S(E)$ and compare them with experimental results.

Bearing in mind that our results contain the product of initial and final spectroscopic factors as a normalizing constant, we can express our results in numerical form for very small values of the incident energy as follows:

A. Assuming asymptotic wave functions for both initial and final neutron states,

$$S(E) = 18.2S_iS_f[1 - 0.073E_{\text{cm}}(\text{MeV}) + \ldots] \text{ MeV barns} . \tag{17a}$$

B. Using asymptotic wave functions for the initial ^{3}He neutron state, and a Hulthen wave function (with $\beta_f = 1.5\ \gamma_f$) for the final ^{4}He state,

$$S(E) = 72S_iS_f[1 - 0.15E_{\text{cm}}(\text{MeV}) + \ldots] \text{ MeV barns} . \tag{17b}$$

C. Using asymptotic functions for the final ^{4}He neutron state, and a Hulthen function (with $\beta_i = 3.0\gamma_i$) for the initial ^{3}He state,

$$S(E) = 29.8S_iS_f[1 - 0.13E_{\text{cm}}(\text{MeV}) + \ldots] \text{ MeV barns} . \tag{17c}$$

D. Finally, as discussed in § II, on the basis of results A, B, and C we can make an enlightened guess at the result when Hulthen wave functions are used throughout:

$$S(E) = 118S_iS_f[1 - 0.21E_{\text{cm}}(\text{MeV}) + \ldots] \text{ MeV barns} . \tag{17d}$$

The rather large differences between the leading terms in these four results stem mainly from the fact that the asymptotic behavior of the neutron wave functions (which contributes predominantly to the integrals for the neutron tunneling matrix elements) is altered by the normalization factor N'/N (see eq. [15]) when Hulthen wave functions are employed.

It is worth commenting on the sensitivity of results B and C to the choice of the parameter β/γ in the relevant Hulthen wave functions. In case C above (Hulthen wave functions for the initial ^{3}He neutron state), the results vary by only a few per cent as β/γ is varied within the reasonable range of values from 2.4 to 4.0. The sensitivity is greater in case B above (Hulthen wave functions for the final ^{4}He neutron state): at one end of the range of reasonable values, $\beta/\gamma = 1.2$ leads to $S(E) = 83S_iS_f[1 - 0.16E_{\text{cm}} + \ldots]$; and toward the other end, $\beta/\gamma = 2.0$ leads to $S(E) = 58S_iS_f[1 - 0.13E_{\text{cm}} + \ldots]$. Even here, however, the *slope* of the curves is not very sensitive to the parameter choice.

In Figure 1 we display the experimental results of Neng-Ming *et al.* (1966) and of Bacher and Tombrello (1968) for $S(E)$ as a function of incident energy E. Also shown are the earlier results of Good, Kunz, and Moak (1954), which bear testimony to the difficulties of this experiment. Above 2 MeV or so, the reaction is well understood as proceeding via the ground state of ^{5}Li; below about 1 MeV, this is no longer so. Figure 2 shows the recent experimental results of Winkler and Dwarakanath (1967*b*).

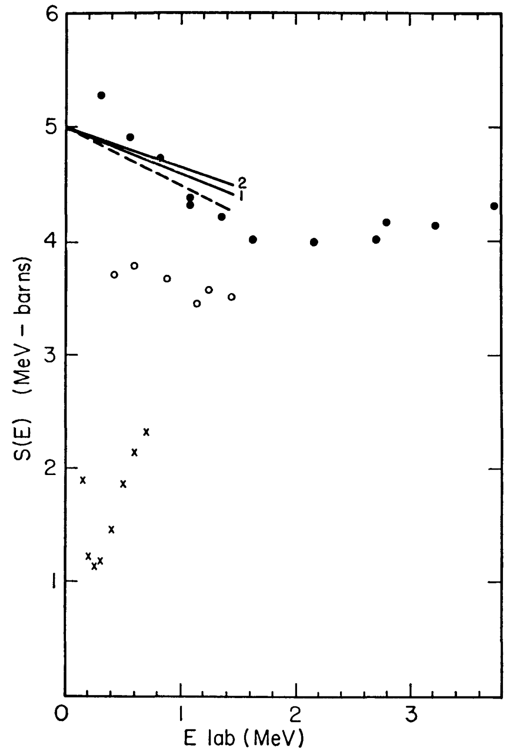

FIG 1.—Cross-section $S(E)$ as a function of incident (lab.) energy. The open circles are the experimental points of Neng-Ming *et al.* (1966), the closed circles those of Bacher and Tombrello (1968), and the crosses are the earlier experiments of Good *et al.* (1954). The solid theoretical curves labeled *1* and *2* are obtained by using Hulthen wave functions for one neutron state and asymptotic wave functions for the other state, as described in the text. The dashed curve is the theoretical estimate with Hulthen wave functions throughout. All theoretical curves have fixed shape but arbitrary normalization.

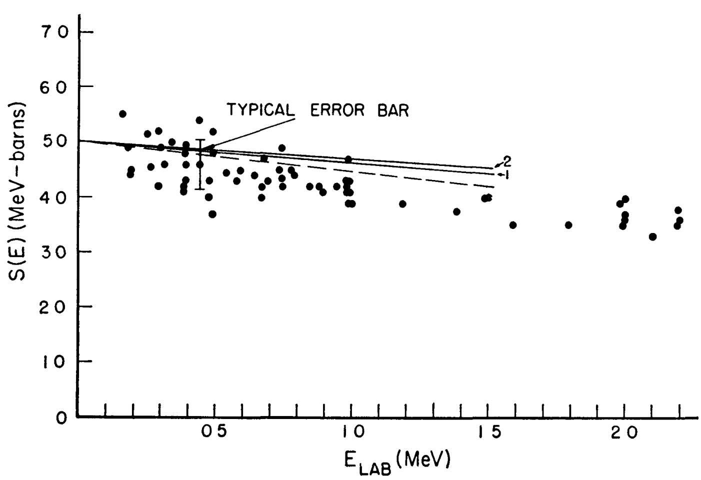

FIG. 2.—Theoretical curves are as in Fig 1; experimental points are the recent results of Winkler and Dwarakanath (1967*b*).

In Figures 1 and 2, the solid curve labeled *1* is result B where Hulthen wave functions are used for the final neutron state (only), and the solid curve labeled *2* is result C where Hulthen functions are used for the initial state (only). The broken curve is our estimate for the case when Hulthen wave functions are used throughout. Since the assumptions underlying the theory are justifiable only for $\eta_i > 1$, the theoretical curves are drawn up only to about 1.5 MeV.

The *shapes* of these curves are fixed. In all cases the normalization (i.e., $S_i S_f$) is chosen to give $S(0) = 5$ MeV barns. This implies $S_i S_f$ to be $\sim$0.07 for case B, $\sim$0.16 for case C, and $\sim$0.04 for case D. Although there are at present no reliable calculations bearing on this number, the values to which we have been led lie in the range of reasonable values for the product $S_i S_f$.

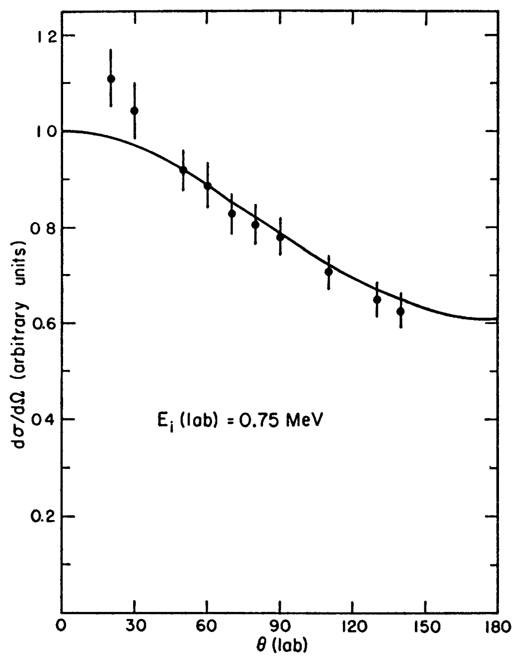

FIG. 3.—Outgoing proton differential cross-section as a function of laboratory proton angle for an incident laboratory energy of 0.75 MeV. Both theoretical curve and experimental points have arbitrary normalization.

As a further exploration of our model, in Figure 3 we show the shape of the differential cross-section for outgoing protons, as a function of proton laboratory angle. The experimental points are those of Winkler and Dwarakanath (1967*b*), and the theoretical curve is obtained from equation (6) by first transforming from center-of-mass to laboratory coordinates for the "diproton" and then convolving the resultant differential cross-section with the angular distribution of protons from the diproton decay. Since the diproton differential cross-section is a smooth function, this calculation is insensitive to the explicit amount (between 0 and 1 MeV) by which our "diproton" is unbound. (The curve in Fig. 2 corresponds to the virtual diproton being unbound by 0.6 MeV, the value of the diproton resonance.)

IV. CONCLUSION

The main hope of the present investigation is to understand the low-energy behavior of the reaction $^3He(^3He,2p)^4He$ well enough to make a reliable extrapolation to obtain $S(0)$ for astrophysical purposes.

The physics of the situation makes it plausible that the neutron tunneling mechanism should dominate the reaction well below the Coulomb barrier, and our rough calculations on this basis appear to be in qualitative agreement with experiments.

To go beyond this preliminary investigation, it would be desirable (i) to abandon the "diproton" approximation and retain the wave functions for the individual two protons; (ii) at the energies where comparison with experiment is possible, to include the effects of nuclear forces between bombarding and target nuclei; and (iii) to use realistic (Hulthen) wave functions for both the initial and final states of the "tunneling" neutron. These are all non-trivial projects.

Nonetheless, it seems reasonable to use the present work to justify a smooth extrapolation from the currently available experimental results; on this basis, one observes from Figure 1 that $S(0)$ is about 5 (± 1) MeV barns.

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