1-2015

Operating Principles, Common Questions, and Performance Data for an Atmospheric Driven Atmos Clock

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Abstract: *The elegance of the Atmos clock and the curiosity of mankind in self-operational mechanical systems have propelled this time device into our collective desire for more knowledge. The search for a self-winding time piece, based on normal atmospheric fluctuations, was pursued for centuries by horologists with the well-known clock proposed by J. L. Reutter and commercialized by Jaeger LeCoultre. This clock has generated numerous discussions throughout the years as noted in past Bulletin articles and other correspondences within the time keeping community. In this paper, the operating principles of the Atmos clock will be reviewed using fundamental science and engineering principles. Next, key questions and experimental observations will be discussed in light of the operating concepts to clarify the clock's performance. Finally, an extensive database will be introduced which was gathered through physical measurements and data recording of an Atmos 540 clock.*

Introduction

The first commercially successful atmospheric driven clock was created and patented by Jean Leon Reutter in 1928 (France) and 1930 (Switzerland). The concept was later refined by Reutter and Jaeger LeCoultre and fully commercialized with Atmos clock production continuing today [1]. In the first design (1928-1938), two glass bulbs containing mercury and a small amount of ammonia were connected to form a U-tube. These fluids responded to atmospheric changes and changed the assembly's center of gravity so that it pivoted back-and-forth to wind a flat spiral spring using a ratchet. In contrast, the second design (1938 to present) utilized ethyl chloride within a sealed bellows to wind the mainspring using a ratchet and linear displacement of the end plate. The Atmos clock represents a long effort to develop a time piece that requires no human interaction or electrical power source [2]. In 1740, Pierre de Rivaz demonstrated a clock that operated solely on variations of temperature and air pressure [3]. In essence, changes in atmospheric pressure cause mercury column height variations which may be converted into rotary motion for clock power. James Cox and John Merlin created a clock driven by barometric pressure changes in the

1760s [4]. A wristwatch has been developed by Phillips [5] which uses a bimetallic coil which operates on temperature differences to wind the mainspring.

A detailed history of the Atmos clock development process and manufactured products has been authored by Lebet [6]. In an on-line posting, Hubby [7] discussed the Atmos clock operation with both mercury filled U-tube and ethyl chloride charged bellows. The prototype Reutter clock design operated only on temperature variations due to the hermetically sealed and rigid container which did not experience significant pressure changes due to external barometer variations. The Horological Journal [8] offered an operational analysis of the U-tube configured Atmos clock with attention focused on power generation, gear train design, and temperature driven performance. In contrast the refined Atmos clock bellows were affected by both temperature and pressure variations, although the latter had a smaller effect [9]. Deith and Aked [10] offered an overview of the Atmos clock movement and temperature/barometric pressure motor. An Atmos Calibre 58 clock had been discussed by Monk [11-13] with a discussion of static and dynamic pendulum balancing, lubrication, and mechanical dismantling/reassembly. For many years, the NAWCC Bulletin had hosted discussions in "The Answer Box" championed by Henry Fried [14] and Paul Heffner [15] as well as "Vox Temporis" [16] regarding Atmos clock operation. Clearly, great interest remains in the fundamental operation of this thermodynamically driven mechanical time keeping mechanism.

A detailed technical analysis of an Atmos 540 clock (refer to Figure 1) will be undertaken to address a number of poised questions over the past five decades. The article is organized as follows. First, the Atmos clock components will be identified with photographs and a physical database listed in Appendix A. The mechanical and thermodynamic operations will be discussed to establish a technical basis to understand the ethyl chloride properties, bellows motion, stored energy in the mainspring, motion works transmission of spring energy through the gears to the escapement, and torsional pendulum behavior. Second, analytical calculations and experimental measurements will be presented to support a detailed energy analysis of the clock's performance. Some of the topics addressed regarding the clock include power usage, energy harvesting, effect of surrounding temperature and pressure variations, and elevation changes. Third, the conclusion offers some final thoughts on the innovative Atmos clock and the many efforts over several centuries to realize an atmospheric driven clock. A complete Nomenclature List is contained in Appendix B as the mathematical symbols may not be defined in the text.

Figure 1: Atmos 540 in protective glass enclosure to minimize pendulum air currents, and enclosure removed with bellows container mounted on backside

Atmos Clock Components and Mechanical / Thermodynamic Operation

The Atmos 540 clock principal components are the energy harvesting bellows and mainspring, motion works, escapement, and torsional pendulum as listed in Table 1 and labeled in Figure 2 [17]. In brief, the two-piece bellows canister, housing the ethyl chloride filled bellows and stiff coiled return spring, attaches to the circular retainer on the back side of the rear support plate with a twisting motion. A winding chain, pinned to an end-cap supported by a soft coil winding spring, follows and transmits the contraction motion of the bellows to mainspring winding through a one way ratchet mechanism. Note that only chain displacement, no energy harvesting, occurs when the bellows expands since the chain winding drum spring maintains tension in the chain and the ratchet does not allow rotation in the opposite direction.

The stored potential energy in the mainspring, contained within a barrel, drives the motion works in a manner similar to a pocket watch. A series of six arbors, featuring a number of wheels and pinions,

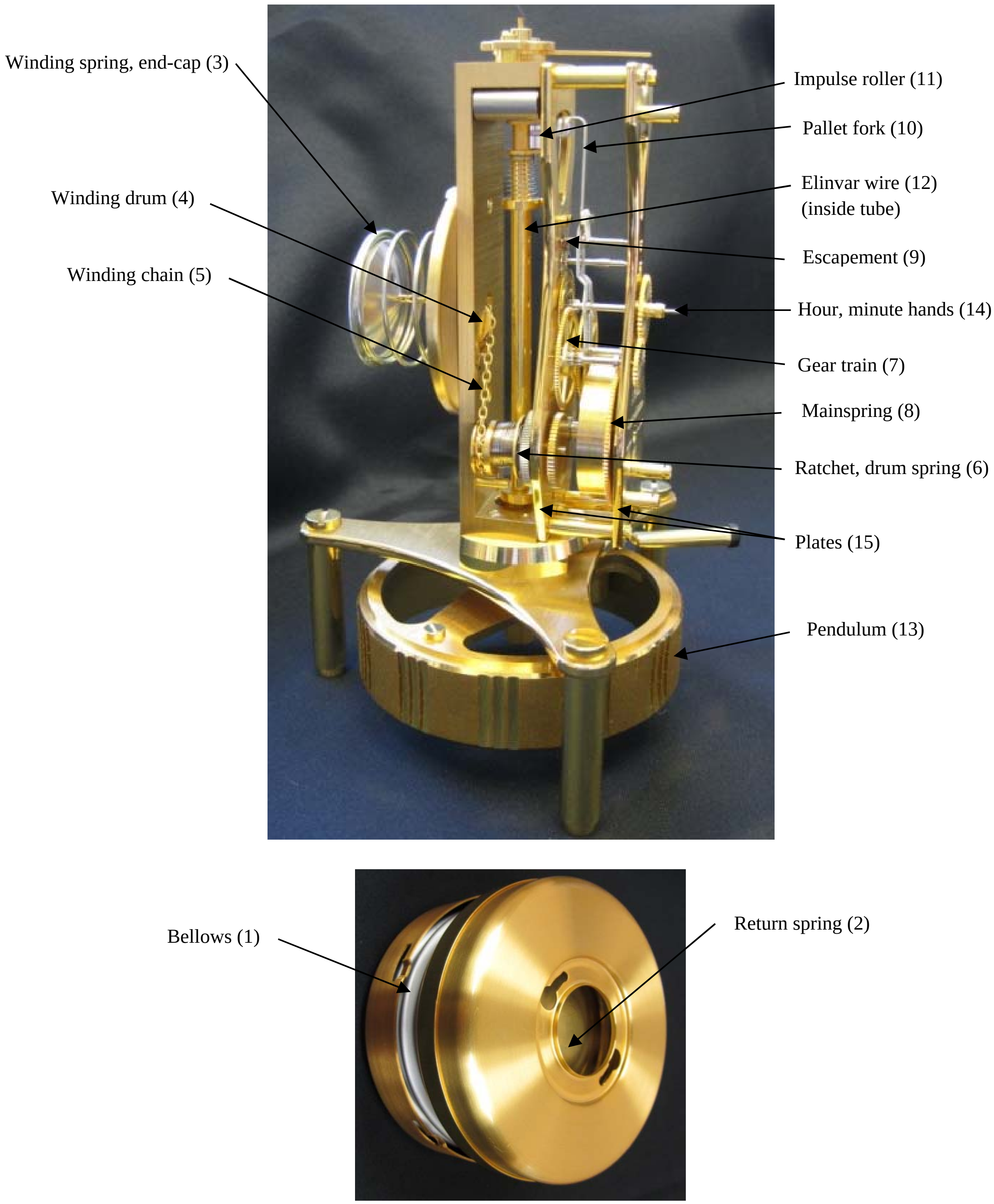

Figure 2: Atmos 540 mechanism with bellows and dial removed for identification [20]

reduce the torque and increase the angular rotations to drive the escape wheel. The escape wheel interfaces with the lever escapement which contains twin pallets and counterbalance. The end of the pallet fork interfaces with the impulse roller to apply forces every 30 seconds in alternating directions, thus impulsing the torsion pendulum twice per minute as it rotates suspended by Elinvar wire.

The operation of the Atmos clock may be analyzed using a systems engineering approach [18] which considers the power flow through the mechanism. It is an integration of three subsystems - energy harvesting and storage, motion works and escapement, and a torsional pendulum time base. The first two subsystems listed distinguish it from other torsion pendulum clocks such as a 400 day clock [19] which had been on the market for nearly 30 years prior to Reutter's invention. Reutter recognized that the torsion pendulum clock required very little power, but then invented an impulse mechanism that further reduced the energy consumption and improved accuracy. Reutter coupled this with a relatively simple air-filled actuated energy harvesting bellows that was later refined to use the thermodynamic behavior of ethyl chloride in the form we know today.

Table 1: Summary of Atmos 540 clock components and accompanying descriptions

No.	Component	Description
1	Bellows	Clock power source with ethyl chloride gas
2	Return spring	Spring force opposes bellows expansion; aids contraction
3	Winding spring, end-cap	End-cap maintains contact with bellows due to coil spring force
4	Winding drum	Conversion of translational motion into rotational motion
5	Winding chain	Bellows contraction allows chain to move ratchet and wind spring
6	Ratchet, drum spring	Ratchet, drum spring to wind mainspring per bellows motion
7	Gear train	Transfers rotational motion from mainspring to escape wheel
8	Mainspring	Winds on bellows contraction; maintains position on expansion
9	Pallet fork	Accelerates pendulum through impulse roller based on escape wheel
10	Escapement	Converts rotational motion of great train into pallet fork motion
11	Impulse roller	Transfers impulses from pallet fork to drive Elinvar wire
12	Elinvar wire	Steel alloy wire that suspends pendulum
13	Torsional pendulum	Balance wheel suspended by a thin gauge Elinvar wire
14	Hour, minute hands	Display of time on front dial
15	Plates	Front, back and support plates hold clock components

Ethyl Chloride Fluid in Bellows

Ethyl chloride, C_2H_5Cl, is a gas at room temperatures but can exist in both liquid and gaseous states when confined within the sealed bellows. The vapor pressure, heat capacity, and heat of fusion properties for ethyl chloride have been experimentally studied under different conditions by Gordon and Giauque [21]. The pressure in the bellows is dictated by the vapor pressure of ethyl chloride (refer to Figure 3) which follows Antoine's equation [22] based on the approximate relationship

$$\log(p_b) = \left(6.24 - \left[\frac{1062}{T+237.63}\right]\right) \tag{1}$$

where p_b and T denote the bellows pressure (kPa) and temperature (C). The coefficients of this equation are found empirically using experimental data [23] with a residual error of 0.2% or less over the range shown. A similar curve has been published by Martt [24] with an accompanying discussion of vapor pressure verses temperature behavior.

The significance of the phase-change diagram shown in Figure 3 will be briefly discussed. The ethyl chloride exists as a liquid at pressure and temperature conditions above the red curve, and as a vapor for conditions below the curve. Thus at room temperature and atmospheric pressure (e.g., 101 kPa), ethyl chloride is a vapor: a quantity of liquid ethyl chloride with boil away in an open container. The boiling temperature for ethyl chloride is just below 55°F at atmospheric pressure. A bellows removed from the clock will collapse to its minimum volume at this temperature. In a confined container such as the Atmos bellows, both liquid and vapor ethyl chloride can exist if there is enough ethyl chloride to supply the vapor needed to maintain pressure. If ethyl chloride has been leaking, there is a point at which there isn't enough to supply the gas to fill the design volume. The bellows then is no longer able to expand as it used to against the bellows spring. This is the effect that is noted by measuring the "depth" of the bellows to check its condition. When conditions are such that no liquid ethyl chloride exists in the bellows, the pressure, P, will follow the ideal gas law [25]

$$\mathrm{PV} = \mathrm{nRT} \tag{2}$$

and will be lower than the vapor pressure. In this expression, $R = 8.31434$ (J/mol·K) is the gas constant, V is the volume (m^3), and n is the number of moles.

Bellows Motion

Winding of the Atmos clock mainspring happens in accordance with the motion of the end-cap in the bellows assembly (refer to Figure 4) The pressure, p_b, of the ethyl chloride within the bellows creates a driving force, $f_b = a_p p_b$ where a_p denotes the bellows surface area. In opposition, a large coil spring force, $f_{s1} = k_{s1}x$, and small coil spring force, $f_{s2} = k_{s2}x$, exist to restore the bellows and maintain contact between the end-cap and the bellows. In these equations, x, k_{s1}, and k_{s2} represent the end-cap displacement and the large and small coil spring stiffness, and preload is not included. An atmospheric force, $f_{atm} = a_p p_{atm}$, exists due to the atmospheric pressure, p_{atm}, which acts over the bellows surface area, a_p. The bellows itself is a mechanical structure with compliance that can create a restoring force given as $f_{bs} = k_{bs}x$ where k_{bs} is the bellows compliance and preload is again neglected. When the bellows contracts, the small spring force will cause the end-cap to follow the bellows, pulling the winding chain off the winding drum and winding the clock through the ratchet engaged at the click wheel. When the bellows expands, the end-cap no longer pulls the chain; instead, the drum spring between the winding drum and ratchet, now disengaged, rotates the winding drum on arbor 0 to take up the slack in chain. The chain force, f_c, can be expressed as $f_c = k_c x + f_{click}(\theta_0)$ where k_c denotes the drum spring, again without preload. The nonlinear f_{click} function is given by $f_{click}(\theta_0) = \left\{ \begin{array}{c} \frac{1}{r_r}\tau_{11};\ \theta_0 > 0 \\ 0\ ;\ \theta_0 \leq 0 \end{array} \right\}$ where τ_{11}, $\theta_0 = x/r_p$, and r_p denote the mainspring torque, the angular rotation of arbor 0, and the winding drum radius.

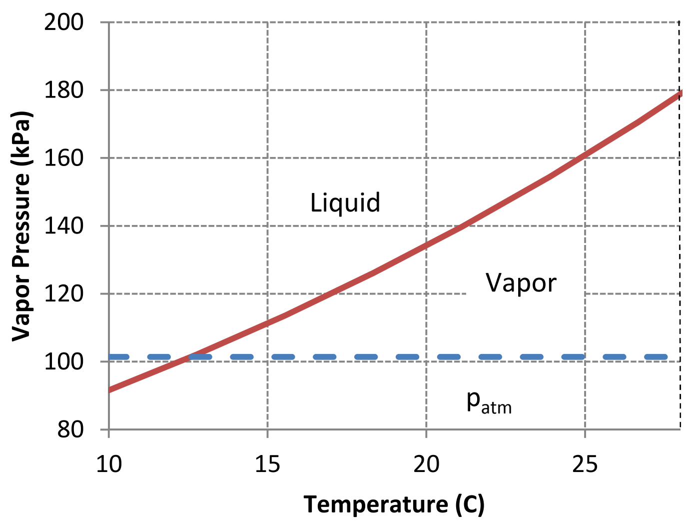

Figure 3: Pressure variation of ethyl chloride due to ambient temperature changes [23]

The net force, f_{cap}, acting on the end-cap is what drives its motion and may be expressed as

$$f_{cap} = f_b + f_{bs} - f_{atm} - f_{s1} - f_{s2} - f_c \tag{3}$$

This equation may be expanded using the individual component hips into the form

$$f_{cap} = a_p(p_b - p_{atm}) - (k_c - k_{bs} + k_{s1} + k_{s2})x - f_{click}(\theta_0) \tag{4}$$

When assembled, the preload of the three springs and bellows cancel each other, but some operating point must be selected to establish a value for x. Therein lays the difficulty of Equation (4): the uncompressed lengths of the springs are known, but the bellows parameters may only be found by destructive testing methods.

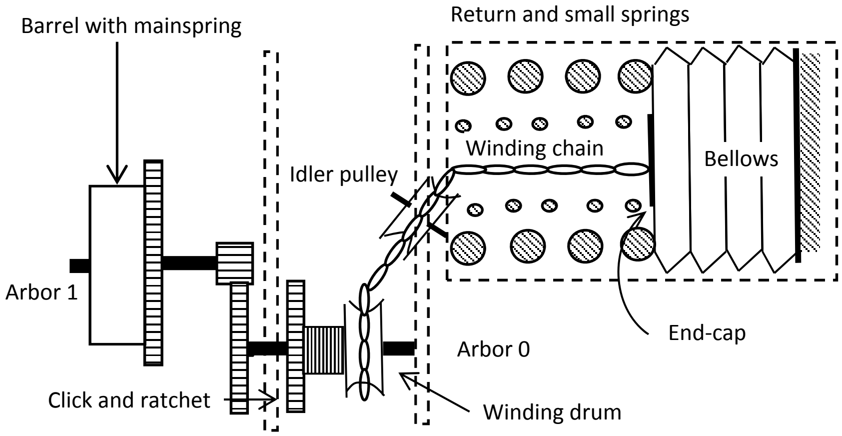

Figure 4: Atmos winding system consisting of the bellows, springs, winding chain and drum which act through a ratchet and click wheel to wind the mainspring.

Mainspring in Barrel

The mainspring within the barrel stores the bellow's harvested energy to drive the clock works for time keeping. The theoretical drive torque produced by the mainspring can be calculated as [26]

$$\tau_{11} = \frac{E b_{ms} h_{ms}^3 (\theta_1 - \theta_{10})}{12L} + \tau_{10} \tag{5}$$

where E is the spring steel modulus of elasticity, b_{ms} is the mainspring width, h_{ms} is the mainspring thickness, and L is the spring length. The variable θ_1 is the angular rotation of Arbor 1 and θ_{10} is the initial angular position of Arbor 1. The parameter τ_{10} denotes the initial mainspring torque due to spring construction or assembly, which may be empirically determined. The experimental measurement of the available torque of an Atmos 540 spring and barrel assembly removed from a clock shows the theoretical value to agree well with measurements, after letting τ_{10} be 0.75 N-cm (refer to Figure 5). For these measurements, the spring arbor was held fixed in a vise while the drum was wound. The torque required to hold the drum after every other revolution was calculated by measuring the tension in a string looped around the drum, preventing it from rotating. When inspecting the barrel, the spring occupies a small percentage of the barrel's volume when fully unwound along the outer edge. As the spring is wound, a portion of the spring fills part of the barrel and offers a nearly linear torque characteristic that varies by ±40% of the mean torque value of 1.33 N-cm.

Clock Motion Works

The motion works of the Atmos 540 clock transmits the stored potential energy from the mainspring to the escapement to impulse the torsional pendulum. In a similar fashion to watches, the gears change the

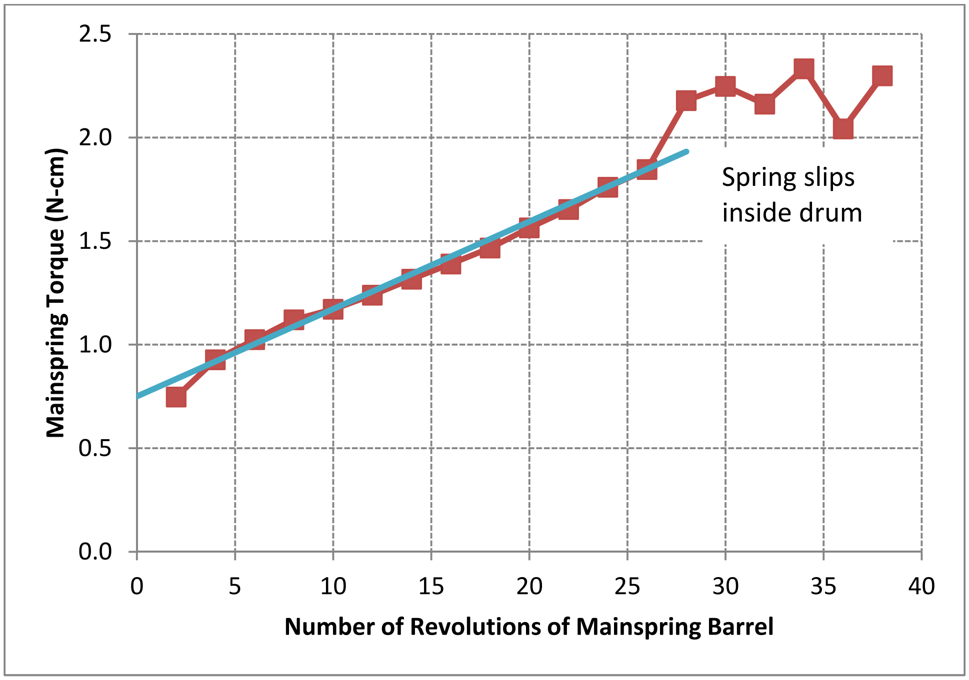

Figure 5: Torque characteristic of mainspring as it is wound – experimental (red line with markers) and theoretical (gray line)

speed and torque between the input and output shafts. Let the ratio, R_w, between two intermeshed gears be defined as the ratio of their number of teeth, N, pitch diameters, D, torque, τ, and/or angular velocities, ω, which may be expressed as

$$R_w = \frac{N_{out}}{N_{in}} = \frac{D_{out}}{D_{in}} = \frac{\tau_{out}}{\tau_{in}} = \frac{\omega_{in}}{\omega_{out}} \quad (6)$$

In this equation, the subscripts "in" and "out" denote the input and output motions, respectively of one wheel driving another on another arbor. For wheels and pinions which share a common arbor, they will have the same angular velocity. For the Atmos 540 clock, the number of teeth and diameters for each wheel and pinion are listed in Table 2 to accompany the diagram which defines the arbors and gears in

Figure 6.

Overall, the mechanical ratio between the great wheel, W_{11}, and the hour hand, R_h, minute hand, R_m, and the escapement, R_e, may be stated as

$$R_h = \left(\frac{N_{22}}{N_{11}}\right)\left(\frac{N_{32}}{N_{21}}\right)\left(\frac{N_{43}}{N_{33}}\right) = \left(\frac{11}{103}\right)\left(\frac{10}{87}\right)\left(\frac{40}{29}\right) = \frac{4{,}400}{259{,}869} \quad (7)$$

$$R_m = \left(\frac{N_{22}}{N_{11}}\right)\left(\frac{N_{32}}{N_{21}}\right)\left(\frac{N_{42}}{N_{31}}\right) = \left(\frac{11}{103}\right)\left(\frac{10}{87}\right)\left(\frac{10}{87}\right) = \frac{1{,}100}{779{,}607} \quad (8)$$

$$R_e = \left(\frac{N_{22}}{N_{11}}\right)\left(\frac{N_{32}}{N_{21}}\right)\left(\frac{N_{42}}{N_{31}}\right)\left(\frac{N_{52}}{N_{41}}\right) = \left(\frac{11}{103}\right)\left(\frac{10}{87}\right)\left(\frac{10}{87}\right)\left(\frac{20}{80}\right) = \frac{22{,}000}{62{,}368{,}560} \quad (9)$$

As expected, a 12:1 ratio (R_h / R_m =12) exists between the hour and minute hands (i.e., minute hand rotates 12 times for one full rotation of the hour hand). Similarly, the 4:1 ratio (R_m / R_e =4) denotes that the escape wheel rotates 4 times per hour. The escape wheel has 15 teeth and advances 1 tooth per minute (2 beats per minute), thus one turn of the great wheel takes R_e/N_{51} or $R_e/15$ or 42,524 minutes or 708.7 hours or 29.5 days. As it takes about 26 turns of the great wheel arbor to fully wind the mainspring, a full winding yields 768 days of running or 2.1 years.

Table 2: Atmos 540 motion works parameter summary

Arbor	**Wheel**	**Symbol**	**N**	**D (mm)**
0	1	W_{01}	90	18.90
	2	W_{02}	55	18.50
1	1	W_{11}	103	37.03
	2	W_{12}	18	6.20
2	1	W_{21}	87	25.07
	2	W_{22}	11	4.28
3	1	W_{31}	87	25.07
	2	W_{32}	10	3.17
	3	W_{33}	29	12.37
4	1	W_{41}	80	18.21
	2	W_{42}	10	3.09
	3	W_{43}	40	16.75
5	1	W_{51}	15	8.95
	2	W_{52}	20	4.70

Torsional Pendulum

The torsional pendulum is the time base of the Atmos, forming a rotational spring-mass oscillating system. A mathematical analysis of this motion would generate a second order differential equation from which we could predict the natural frequency of the clock. (This same method is what tells us that period of an ideal pendulum involves the square root of the ratio of the pendulum length and the gravitational acceleration constant.) The governing differential equation, subject to the escapement transmitted impulse torque, $\tau_{impulse}$, may be written as

$$I\frac{d^2\theta}{dt^2} + k\theta = \tau_{impulse} \qquad (10)$$

In this expression, the natural frequency in radians/second becomes $\omega_n = \sqrt{k/I}$. The natural frequency in cycles/second (hertz) is this number divided by 2π, and he natural frequency is related to the period as $T = \frac{2\pi}{\omega_n}$. The pendulum period is 60 seconds. If the twin pallets of the anchor are considered, then the impulse rate is 120 beats per hour. Since the period is known, the stiffness, k, can be determined only if the torsional pendulum's inertia is established through analytical or experimental means.

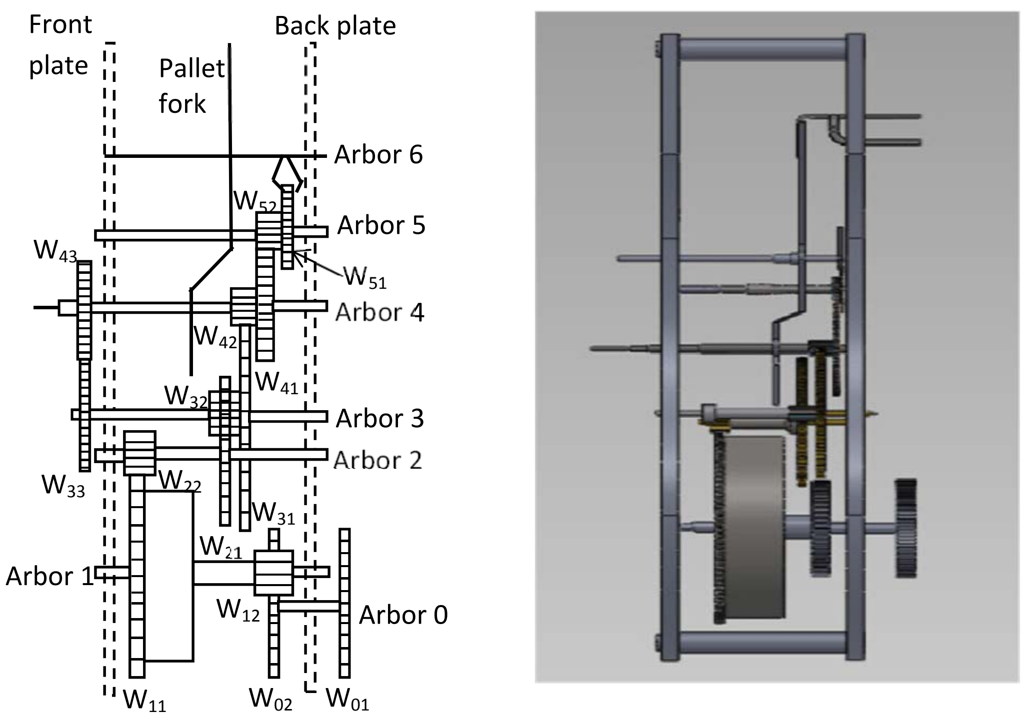

Figure 6: Atmos 540 motion works with labeled parts [27] and CAD diagram

Energy Analysis and Measurements

Over many decades, a number of statements and questions have been pondered and put forth by horologists and others regarding the behavior of Atmos clocks. After reviewing past editions of the NAWCC Bulletin and other references on this subject, some insights will be shared on the subject matter. Roehrich [28] has summarized the general information regarding Atmos clocks from both historical and production perspectives including a description of the conversion of small temperature variations into energy to wind the clock. The Atmos clock was initially developed by J. L. Reutter in the late 1920s. As the air temperature changes surrounding the bellows, the pressure of the ethyl chloride will change and thereby expand or contract the bellows. This mechanical motion can be harvested to drive the clock mechanism through displacement and winding of the mainspring when the bellows contracts (i.e., temperature falls). In contrast, the clock designed by Cox [2] can wind when the ambient pressure rises or falls.

Statement #1: *"The energy needed to light a 15 Watt bulb would be sufficient to power 60 million Atmos clocks" [28].*

Measurements of the mainspring torque can be used to estimate the power consumption of the Atmos clock. For one turn of the great wheel, the energy stored or removed (depending on direction) may be calculated as the product of the torque and the angle rotated, here 360 degrees or 2π radians, giving

$$E = \left(1.33(N \cdot cm)\left(\frac{1\ m}{100\ cm}\right)\right)\left(\frac{2\pi}{turn}\right) = 8.36\ (Joule) \tag{11}$$

As the clock motion expends 1/42,524th of this per minute, the power consumed by the clock becomes

$$P = 8.36(Joule)\left(\frac{1/42524}{1\ minute}\right)\left(\frac{1\ minute}{60\ second}\right) = 0.0327\ \mu W \tag{12}$$

Thus the average rate of power consumption is about 0.033 microwatts. Thus the power for a 15 Watt bulb would in fact drive 450 million clocks.

The quality factor, Q, gives us a way to estimate the power lost by the pendulum motion alone. The ratio of the amplitude from one cycle to the next is found by $e^{-\frac{\pi}{Q}}$, where *e* is the natural logarithm base, 2.71828. The quality factor for an Atmos calibre 540 clock pendulum was estimated at several amplitudes by noting the decay over several consecutive cycles with the escapement completely removed from the clock (refer to Figure 7). Measurements of the peak amplitude at each extreme of rotation were made by using a microscope to read the angle from a printed scale attached to the pendulum as it moved behind a "pointer" formed by a dark thread attached to the clock.

For these measurements, the angular resolution was nominally ¼ degree, but there was some variable parallax error equal to that amount or less at times, and, of course, pendulum wobble must be at a minimum for usable measurements. Consequentially, measurements at low amplitudes showed higher standard deviation in the estimate of Q and required 6 complete cycles as a minimum for calculation of Q, as opposed to 4 cycles for the other measurements taken. Computing Q over a larger number of cycles returned values within 1 standard deviation of those charted.

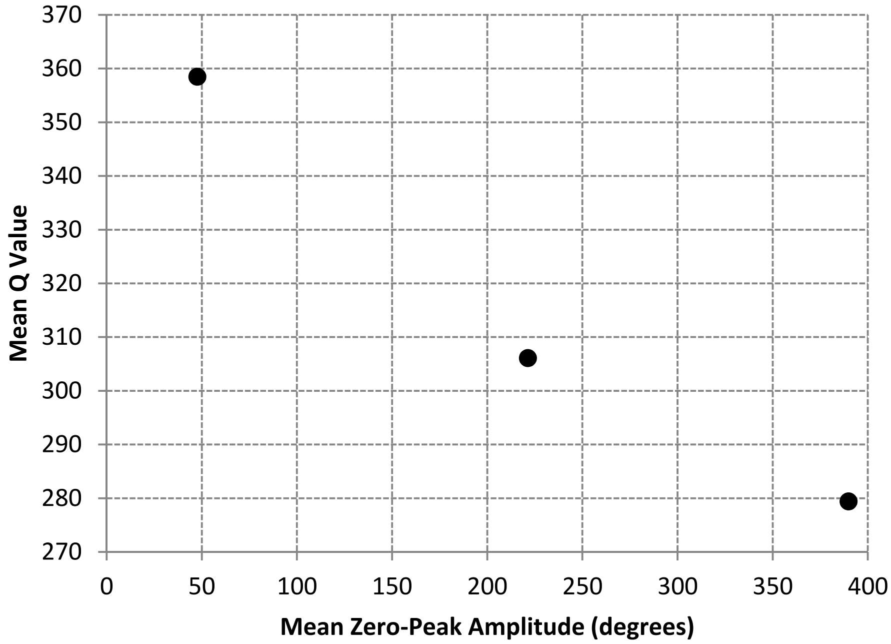

Figure 7: Estimate of quality factor Q over 4 to 6 oscillation cycles at several amplitudes of rotation.

These results show a dependence on amplitude that is probably attributable to increased aerodynamic drag corresponding to the higher velocity of the pendulum's rim. From these results, the Q for this particular Atmos clock was estimated to be Q=300 at typical running amplitudes. Thus, the amplitude of each successive oscillation may be stated as $e^{-\frac{\pi}{300}}$, or 0.990, that of the previous one. That is, approximately 1% of the amplitude is lost per minute, i.e. one oscillation. Likewise, the peak velocity of successive oscillations is reduced by the same ratio, and the kinetic energy of each successive oscillation is further reduced as the square of this ratio (from velocity squared), or $(0.99)^2$ =0.98.. Thus, 2% of the kinetic energy is lost per minute, i.e. per one oscillation cycle.

For a rotating oscillating mass of rotational inertia, I, oscillating at given peak amplitude, A, and frequency of oscillation, ω, the peak angular velocity will be $A\omega$ and the kinetic energy of the rotating mass at a moment of peak velocity is given by

$$K.E. = \frac{1}{2} I (A\omega)^2 \tag{13}$$

In the case of this Atmos clock, the inertia of the pendulum was computed by CAD software to be I=1.675x10-4 (kg·m^2), the amplitude of the oscillations was about A=225 degrees (zero to peak), or $\frac{5}{4}\pi$ radians at a frequency of 1 cycle per minute, or 2π rad/minute. This gives the kinetic energy as

$$K.E. = \frac{1}{2}(1.675 \times 10^{-4}\ kg \cdot m^2)\left(\frac{5}{4}\pi\ rad \cdot \frac{2\pi}{60}\ {}^{rad}\!/_{s}\right)^2 = 14.2 \times 10^{-6} Joule \tag{14}$$

A 2% loss of this energy per minute equates to an average power “consumption” rate of the pendulum of

$$P = (0.02)\left(\frac{14.2\times10^{-6} Joule}{60\ seconds}\right) = 0.0048\ \mu W \tag{15}$$

In other words, the pendulum consumes about 15%, or 1/7th, of the power coming from the mainspring, the remainder being lost in the gear train and escapement.

Statement #2: *What are the forces on the movement? Will it wear out without lubrication? 100 years versus 600 year life span?*

To answer these questions precisely would require careful analysis of the escapement geometry, but some estimates can be made. While running, an Atmos balance wheel receives nominally 150 times fewer impulses per hour than a common wristwatch. The Atmos clock Q factor indicates that the Atmos balance wheel is consuming almost 0.0048 microwatts. For comparison, an inexpensive wristwatch having an estimated Q of 200 will lose about 3% of its energy per oscillation. From measurements of the geometry and motion of an inexpensive watch balance wheel, we calculate the kinetic energy to be

$$K.E. = \frac{1}{2}(1.39 \times 10^{-9}\ kg \cdot m^2)\left(\frac{5}{8}\pi\ rad \cdot \frac{11\pi}{2}\ {}^{rad}\!/_{s}\right)^2 = 0.80 \times 10^{-6} Joule \tag{16}$$

so 3% of this energy is lost per oscillation period, which at $\frac{11\pi}{2}$ rad/s, or 2.75 Hz, is 3% lost per 1/2.75 seconds or

$$P = 0.03 \times \frac{0.8\times10^{-6} Joule}{1/2.75\ seconds} = 0.066 \times 10^{-6} W = 0.066\ \mu W \tag{17}$$

Hence, the Atmos pendulum is consuming approximately 7% of the power for a common wristwatch balance wheel.

The difference in power consumption does not answer the question of forces and wear, however. Lacking a formal analysis, some approximations and deductions can be made. The Atmos delivers the power to the balance wheel twice per minute, whereas the wristwatch has nominally 300 beats per minute. The average work, W, done per beat for each escapement is then the product of power and the duration of the beat. For the Atmos, we have

$$W = (0.0048\ \mu W)(30\ seconds) = 1.44 \times 10^{-7}\ Joule \tag{18}$$

For the wristwatch, we have for one beat

$$W = (0.066\ \mu W)\left(\frac{1/2.75\ seconds}{2}\right) = 0.12 \times 10^{-7}\ Joule \tag{19}$$

Thus we estimate that the escapement of the Atmos clock must do about 12 times more work per beat to maintain oscillations. Recall from the discussion of Equation (11) that work done winding the mainspring was the product of torque and an angular displacement. Since the watch, in this case, and the Atmos have the same number of teeth on the escape wheel and therefore rotate the same angle per beat, we conclude that the torque delivered to the escapement by the escape wheel is 12 times greater in the Atmos. Were the Atmos pivots and wheels built at the same scale of the wristwatch, this would cause correspondingly higher pressures and wear. However, a review of Appendix A will show the wheels of the Atmos clock to be larger and thicker than what would be found in a wristwatch, indicating they are designed with the higher torque in mind.

How long an Atmos can run without lubrication can only be speculated here. The forces that would cause wear are probably not 12 times greater, considering the relative size of the wheels and pivots. For example, the escape wheel in the Atmos is nearly twice the diameter of that in the watch in the comparison above, thereby reducing the forces on the pallets from the drive torque such that the forces in the Atmos are only 6 times greater than the watch. Perhaps the most important aspect in assuring the longevity of the movement would be the slow beat rate: it would take an Atmos nearly 3 years to beat as many times as a common wristwatch would in one week. Friction aside, how long could a wristwatch run without lubrication before damage would be done? Is predicting the life of an Atmos as simple as multiplying that time estimate by 150?

Statement #3: *"Clock requires a temperature variation of only 1° C over a 2 day time interval to remain in operation indefinitely" [6].*

This statement appears to be a perpetuation of a misunderstanding of an article published about the clock Reutter invented. In that paper [19], it is the prototype clock that is described, which used the pressure of air contained in a sealed 10 liter bell jar to drive a bellows as the air temperature changed. Reutter found that the bell jar configuration generated 36 g-cm of energy per 1°C temperature change, which was enough to power the clock for 2 days. It was also noted that for this prototype that an atmospheric pressure change of 3 mm was equivalent to a 1°C temperature change. Later models would eliminate this bell jar and use ethyl chloride in the bellows to achieve the needed pressure changes with temperature.

Interestingly, Callaway [3] uses Reutter's data to compute the power consumed by the prototype clock. The power calculation, with the conversion for the vintage units, g-cm, of stored energy to N-m, goes as

$$P = 36(g \cdot cm)\left(\frac{1\,m}{100\,cm}\right)\left(\frac{1\,kg}{1000\,g}\right)\left(\frac{9.81\,N}{1\,kg}\right)\left(\frac{1}{48\,hours}\right)\left(\frac{1\,hour}{3600\,seconds}\right) = 0.0204\,\mu W \qquad (20)$$

This reported rate is comparable to the rate calculated in Equation (12) from the mainspring torque in the Atmos 540 clock measured extensively for the current article.

The bellows of an Atmos 540 was removed from the clock and tested in two configurations over a range of temperatures. Figure 8 shows the motion of the bellows, interpreted as "depth" of the bellows inside the canister, over a limited range of temperatures measured in a residential setting in which in a room was opened to the outdoors and allowed to cool over the course of about 2 hours until the bellows collapsed. During this cooling period, a fan was use to keep air moving over the bellows to ensure uniform cooling of the entire bellows assembly. At 70° F (21.1°C), the motion is 1.63 mm/degree C, or 0.036 in/degree F. The motion of the bellows moves the chain on the winder pulley, the circumference of which can hold 34 mm of chain. Thus, we have winder rotation of

$$\left(\frac{360\,°}{34\,mm}\right)\left(\frac{1.63\,mm}{deg.C}\right) = \left(\frac{17.3\,°}{deg.C}\right) \qquad (21)$$

From the winder to the mainspring arbor there is a 55:18 gear ratio; thus, the spring arbor rotation per degree C is

$$\left(\frac{17.3\,°}{deg.C}\right)\left(\frac{55}{18}\right) = \left(\frac{52.7\,°}{deg.C}\right) \quad (22)$$

Since a full turn of the great wheel takes 29.5 days, we calculate the number of days of running per degree C temperature change as

$$\left(\frac{52.7\,°}{deg.C}\right)\left(\frac{29.5\ days}{360°}\right) = \left(\frac{4.32\ days}{deg.C}\right) \quad (23)$$

Note: This will vary with the condition of the bellows, the operating temperature, and atmospheric pressure.

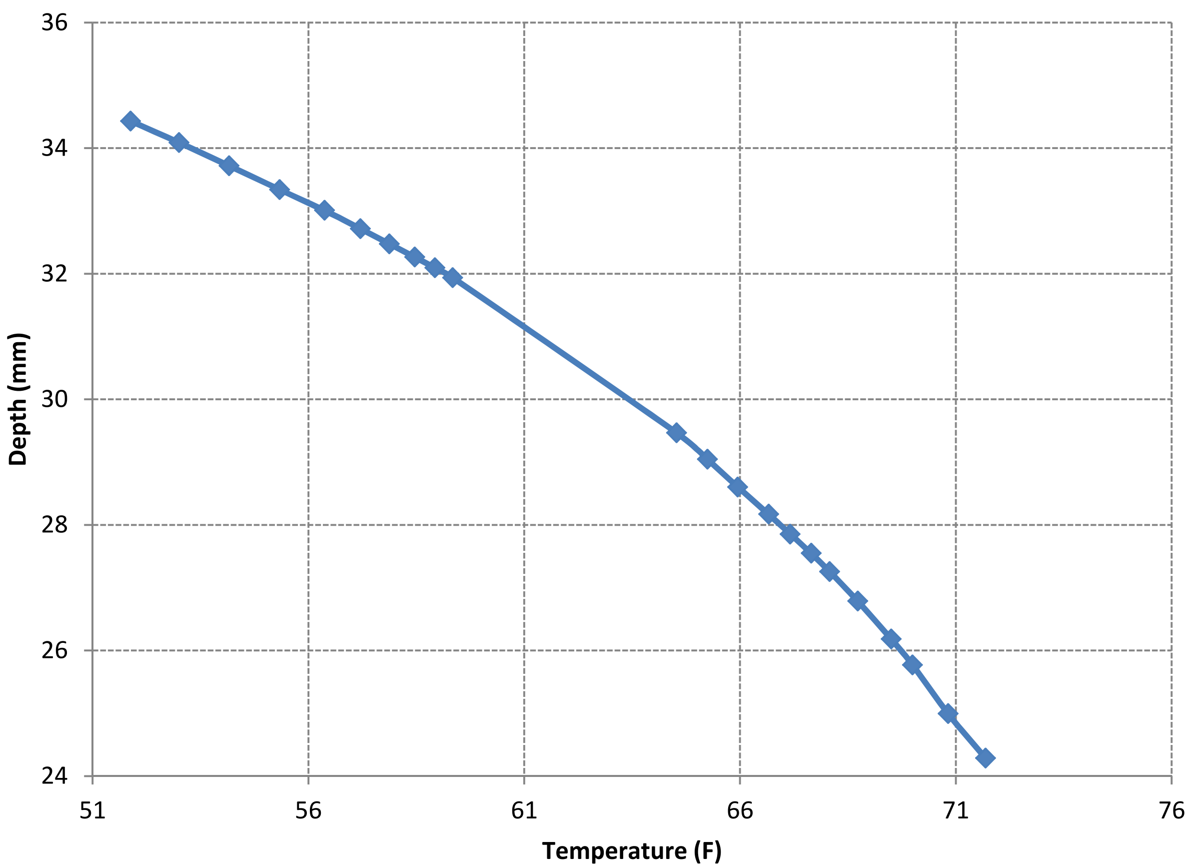

Figure 8: Depth of bellows and bellows spring assembled in bellows canister vs temperature of the surface of the bellows.

Alternately, consider that the winder ratchet has 90 teeth and 2 clickers spaced at ½ that pitch. Thus there are 180 clicks per revolution of the winder, or 2 degrees rotation per click. Given the gear ratio between the winder and the mainspring, we have the following rotation of the arbor per click of the winder

$$\left(\frac{2°}{click}\right)\left(\frac{55}{18}\right) = \left(\frac{6.11°}{click}\right) \quad (24)$$

This is the minimum incremental "input" to the mainspring arbor. Given 29.5 days of running per full turn of the arbor, we calculate the days of running per click as

$$\left(\frac{6.11\,°}{click}\right)\left(\frac{29.5\;days}{360°}\right) = \left(\frac{0.50\;days}{click}\right) \tag{25}$$

The bellows was also removed from the canister and tested in the open air, measuring the "height" of the top of the bellows dome (refer to Figure 9). As expected, the bellows collapses near 54°F (12.2°C). In this configuration we have only the pressure of the gas expanding against the stiffness of the thin metal bellows. A curve fit to this data shows the height to be following approximately the square root of the temperature, suggesting an ideal gas law condition. As this bellows is over 25 years old, this would be expected. (Note: vapor pressure at these temperatures would suggest a fully charged bellows would be damaged if allowed to expand outside of the canister.)

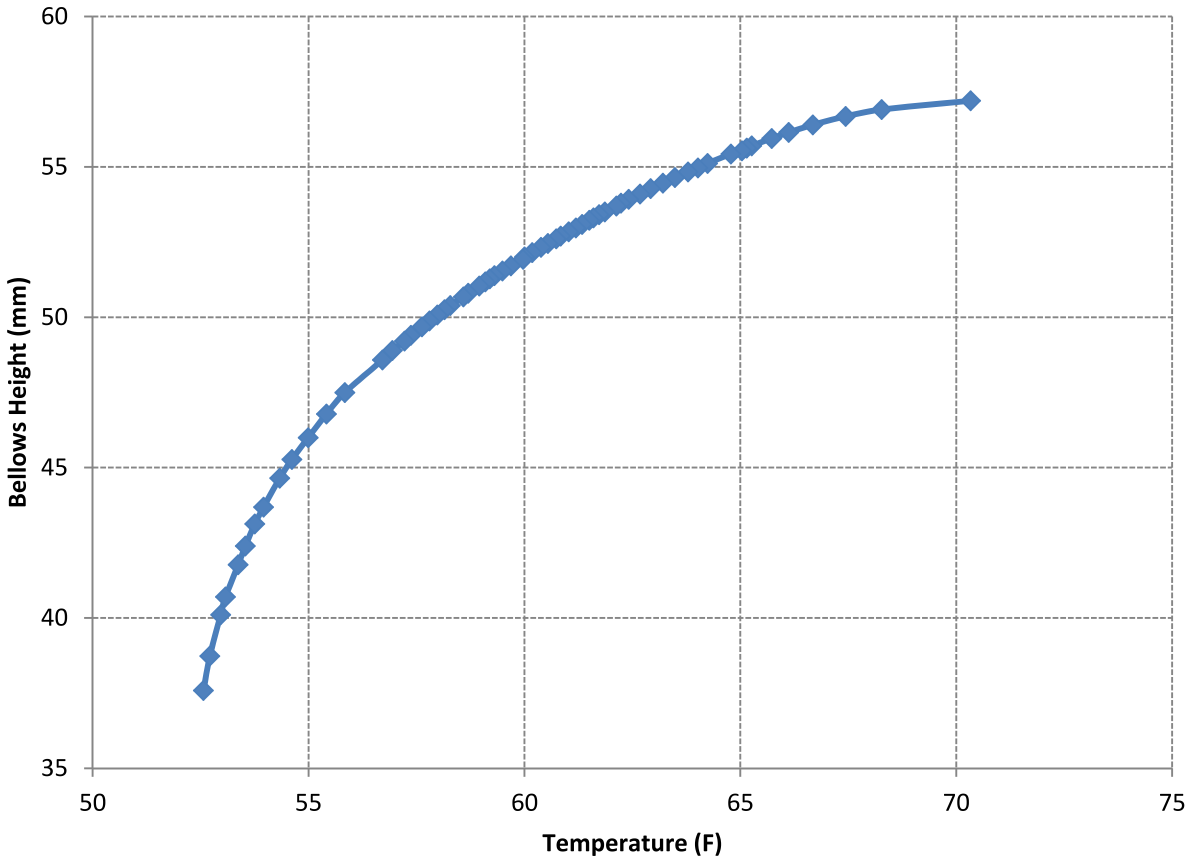

Figure 9: Expansion of bare bellows removed from canister, note collapse near 52.5°F (11.4°C)

Statement #4: *Can an Atmos clock successfully function located in a modern heated and air-conditioned indoor environment?*

The temperature inside an Atmos case in a residential situation was measured with a MicroSet™ environmental sensor over a 24 hour period and is shown in Figure 10. The case provides some insulation, as separate measurements with the same sensor show outside temperature to vary 2.5° during the daytime heating system cycle, resulting in the daytime variation in the case being 0.75° F or less. However, the temperature variation of the metal bellows is lower still because of the restricted heat input from the room as the air in the case transmits heat to the metal parts, resulting in temperature variation of only about 0.3° F, or 0.17° C, as measured with a thermistor mounted to the inside of the bellows. As the winder gets 17.3° of rotation per degree C, the number of clicks advanced at the winder per degree C becomes

$$\left(\frac{click}{2\,°}\right)\left(\frac{17.3\,°}{deg.C}\right) = \left(\frac{8.65\ click}{deg.C}\right) \tag{26}$$

The estimated number of clicks advanced per "heating cycle" has been determined to be 0.17°C as 1.5 clicks. Only whole clicks count, so one heating cycle as observed would wind the clock enough to run ½ day. This would be expected to be repeated several times a day.

It should be noted that an home HVAC (heating, ventilation, and air conditioning) system with a narrower temperature control band would likely cause temperature variations at the bellows that are too low to cause enough motion to advance the winder. The minimum temperature change required to advance the winder one click, in the case of our clock, is 1/8.65, or 0.12°C, or 0.22°F.

Statement #5: *Contribution of surrounding air temperature (90% to power) change versus atmospheric pressure or barometric (10%) change to power time device*.

According to the vapor pressure characteristic near 21.1°C (70° F), a change of barometric pressure of 37 mm Hg is comparable to a temperature change of 1°C. However, typical variations of atmospheric pressure rarely exceed 5 mm over several days. In the case of a strong storm, the weather data for one week showed an excursion of 15 mm, peak to peak over 4 days, which would achieve less than half the clock winding needed to run over that time period. Consequently, barometric pressure changes due to severe weather might generate up to 40% of the winding energy, while normal weather patterns would contribute a nominal 10% of the required operational power.

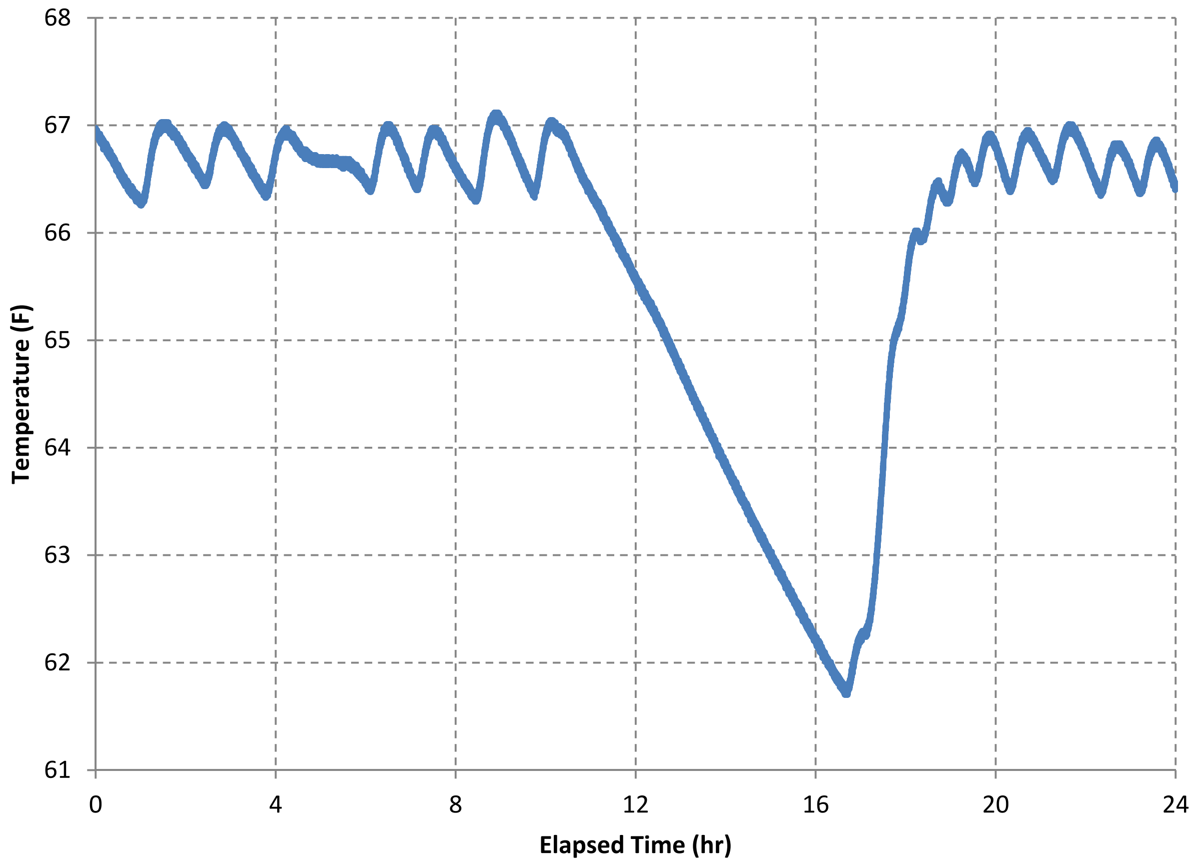

Figure 10: Temperature inside an Atmos case for a 24 hour time period beginning at noon; night time room thermostat setting is 5°F lower than daytime. The plateau near 4.5 hours is caused by the presence of a family of four having dinner in the room with the thermostat and Atmos.

Statement #6: *What happens if the Atmos clock is moved from sea level to mountain top and vice-versa?*

Changes in elevation affect the Atmos clock by shifting the temperature range required to achieve bellows motion for winding the mainspring. Consider again Figure 3, which is a best case scenario of a theoretically fully charged bellows. It is the surplus pressure of the ethyl chloride above atmospheric pressure, acting on the effective area of the bellows, that opposes the spring force to move the winding chain. The bellows theoretically collapses at 12.8°C, at sea level. At 914 meters (3,000 feet) elevation, the atmospheric pressure is reduced by 10%, moving the "collapse" temperature to just under 10°C, where the vapor pressure is equal to the atmospheric pressure at 914 meters. Likewise, we would expect the temperature at which the bellows is fully extended, i.e. maximum bellows spring compression, to be reduced by about 3°C.

At 2,438 meters (8,000 feet), atmospheric pressure is reduced by 25%, and we can expect our operating temperature envelope to shift by about 7°C, or nearly 13°F, lower than at sea level. A clock at this altitude and a room temperature of 22.2°C (72°F) would show a bellows extension comparable to 29.4°C (85°F) at sea level. A bellows that is weak at sea level might in fact benefit from a move to higher elevation, now being able to extend itself against the force of the spring at normal temperatures. Simply put, a periodic 3° temperature difference is required for the Atmos clock to operate continually. However, a lower temperature range will be required at high elevations as the bellows will be fully expanded and not capable of winding the mainspring if the ambient temperature is too high.

Conclusion

The Atmos clock represents a realization of an atmospheric driven time piece that was envisioned for centuries by many individuals in the time keeping community and commercially developed by J. L. Reutter in the 1920s. Although the clock has undergone three design upgrades (e.g., ammonia to ethyl chloride, u-tube to bellows), the mechanism operates on atmospheric temperature and pressure variations to wind the mainspring driven clock escapement and torsional pendulum. In this article, the power consumption, energy harvesting, and impact of elevation changes has been examined using both analytical and experimental methods. An important operational consideration is the minimization of frictional effects and other system losses as the Atmos clock must operate solely on the environmental changes typical in residences with modern heating and air conditioning systems that maintain the temperature within a specified range. The current ethyl chloride filled bellows design, with high precision clock works and escapement, drives the torsional pendulum at a 60 second time period to regulate the displayed time to within a few seconds a day. The Atmos clock represents a wonderful contribution to both society and the global horology community in terms of innovation and persistence to overcome technical challenges and create an artistic solution to the realization of a pseudo-perpetual time keeping device. Lastly, the clock represents a “green technology” which harvests power from naturally occurring events and whose operation can be sustained for decades.

Acknowledgements

The authors wish to thank Errol Ger (Wilmington, DE), Helen Bunting (Seneca, SC), and NAWCC Western Carolina Chapter #126 (Asheville, NC) for the donated Atmos clocks.

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References

1. Pritchard, K. H., Swiss Timepiece Makers 1775-1975, vol. 2 (L-Z), Phoenix Publishing: West Kennebunk, ME, 1997.
2. Smith, A., Clocks & Watches, *The Connoisseur*: London, 1975.
3. Callaway, Jr., E., “Wireless Sensor Networks - Architecture and Protocols”, CRC Press - Auerbach Publications: Boca Raton, FL, 2003.
4. Bruton, E., History of Clocks & Watches, New York: Rizzoli International Publications, 1979.
5. Phillips, S., “Temperature Responsive Self-Winding Timepieces”, US Patent Office and Trademark Office, Patent 6,457,856, 1 October 2002.
6. Lebet, J., Living on Air: History of the Atmos Clock, Jaeger Le Coultre: Le Sentier, Switzerland, 1997.
7. Hubby, J., “Thread: How does ATMOS Clock Work”, National Association of Watch and Clock Collectors (NAWCC), Horological Education Forum - 400-Day, Atmos, and other Torsion Clocks, March 8, 2009, http://mb.nawcc.org/showthread.php?47947-How-does-ATMOS-clock-work&highlight=How+does+ATMOS+clock+work, accessed February 2013.
8. Anonymous, “The Atmos Clock”, *The Horological Journal*, pp. 14,16, 56, 58, May and June 1934.
9. Dean, J. (editor), “The Atmos Clock”, *The Watchmakers’ Journal*, vol. 5, no. 25, pp. 1, 2, 5, November – December 1951.

10. Deith, C., and Aked, C. (editors), "A Weather Driven Clock", *The Clockmaker Magazine*, vol. 1, no. 1, pp. 13, April/May 1990.
11. Monk, S. C., "The Essence of Clock Repair – The LeCoultre Atmos Clock, Part A", *AWI News*, vol. 15, no. 6, pp. 13,14, November - December 1974.
12. Monk, S. C., "The Essence of Clock Repair – The LeCoultre Atmos Clock, Part B", *AWI News*, vol. 16, no. 1, pp. 44, 48, January - February 1975a.
13. Monk, S. C., "The Essence of Clock Repair – The LeCoultre Atmos Clock, Part C", *AWI News*, vol. 16, no. 2, pp. 20, 21, March - April 1975b.
14. Fried, H., "The Answer Box", *NAWCC Bulletin*, vol. 38, no. 163, pp. 1138-1139, April 1973.
15. Heffner, P., "The Answer Box", *NAWCC Bulletin*, vol. 29, no. 247, pp. 129-131, April 1987.
16. Anonymous, "Vox Temporis - What Does My Atmos Fear?", *NAWCC Bulletin*, vol. 18, no. 181, pp. 206-209, April 1976.
17. Jaeger Le Coultre, Atmos 540 Repair Guide, Jaeger Le Coultre: Le Sentier, Switzerland, 1993.
18. Moline, D., Wagner, J., and Volk, E., "Model of a Mechanical Clock Escapement", *American Journal of Physics,* vol. 80, no. 7, pp. 599-606, July 2012.
19. Reverchon, L., "A Perpetual Clock", *Swiss Horological Journal*, no. 1, pp. 1-4, January 1928.
20. Fritz, M., Reverso – The Living Legend, Braus: Berlin, Germany, 1992.
21. Gordon, J., and Giauque, W., "The Entropy of Ethyl Chloride. Heat Capacity from 13 to 287K. Vapor Pressure. Heats of Fusion and Vaporization", *Journal of the American Chemical Society*, vol. 70, no. 4, pp.1506-1510, April 1948.
22. National Institute of Standards and Technology (NIST), Chemistry Web Book – Standard Reference Database No. 69, *http://webbook.nist.gov/cgi/cbook.cgi?ID=C75003&Units=SI& Mask=1EFF*, accessed April 2012.
23. Office of Response and Restoration (ORR), National Oceanic and Atmospheric Administration, U.S. Department of Commerce, "Ethyl Chloride", June 1999, http://cameochemicals.noaa.gov/ chris/ECL.pdf, accessed March 2013.
24. Martt, E., "The Atmos Clock Ethyl Chloride as the Bellows Gas", National Association of Watch and Clock Collectors, Horological Science Chapter #161, Horological Science Newsletter, Issue 3, pp. 16, May 1998.
25. Van Wylen, G. and Sonntag, R., Fundamentals of Classical Thermodynamics, John Wiley & Sons: New York, 1991.
26. Norton, R., Machine Design - An Integrated Approach, Prentice Hall: Upper Saddle River, NJ, 2000.
27. Patel, S., Moline, D., and Wagner, J., "Modeling and Analysis of an Atmospheric Driven Atmos Clock with Mechanical Escapement Control", proceedings of the European Control Conference, WeA9.5, Zurich, Switzerland, July 2013.
28. Roehrich, R. L., "Atmos - The Perpetual Clock", *NAWCC Bulletin*, vol. 30, issue 257, pp. 494-500, December 1988.

Operating Principles, Common Questions, and Performance Data for an Atmospheric Driven Atmos Clock

David Moline and John Wagner (Chapter 126 - Western Carolinas members)

Appendix A: Atmos 540 Clock Component Database

Figure A.1: Thermodynamic driven bellows with top and bottom canister shells

Table A.1: Bellows parameters

Description	Value	Units
Inner diameter	87.0	mm
Outer diameter	113.0	mm
Height fully expanded (contracted)	70.0 (24.0)	mm
Weight	204.0	g

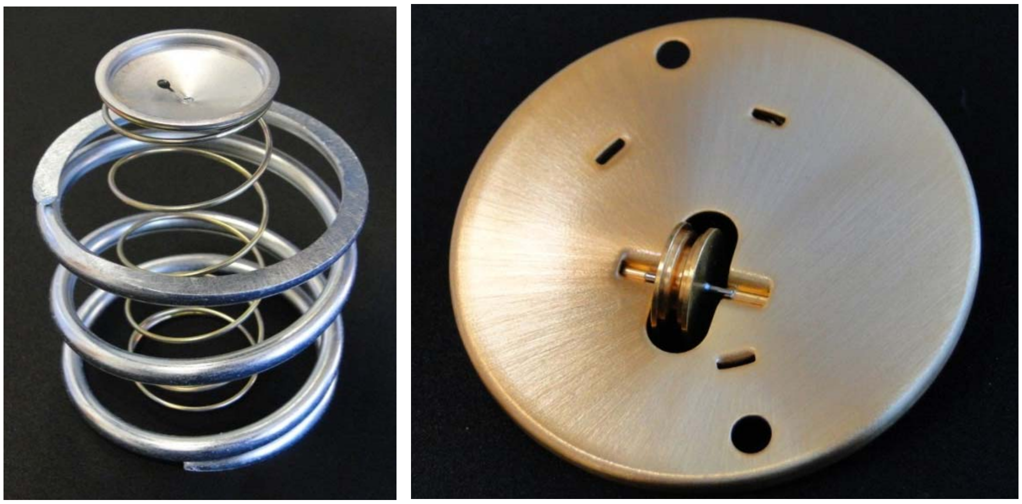

Figure A.2: Bellow and coil springs with end-cap, pulley housed in coil spring end plate

Table A.2: Bellows and coil spring parameters

Description	Bellows	Coil	Units
Inner diameter	60.4	45.4	mm
Outer diameter	71.1	48.0	mm
Wire coil diameter	5.9	1.3	mm
Axial length (uncompressed)	55.8	79.3	mm
Stiffness coefficient	16.28	0.18	N/mm
Weight	138.6	7.5	g

Figure A.3: (a) Idler and (b) winding drum (with spring and chain) pulleys

Table A.3: Idler and winding drum (with spring and chain) pulley parameters

Description	Idler Pulley	Winding Drum	Units
Outside diameter	13.0	13.97	mm
Shaft, wire diameter	0.94	-	mm
Width	4.03	4.8	mm
Weight	3.5	9.4	g
Arbor, chain length	10.0	80.0	mm
Spring wire diameter	-	0.34	mm

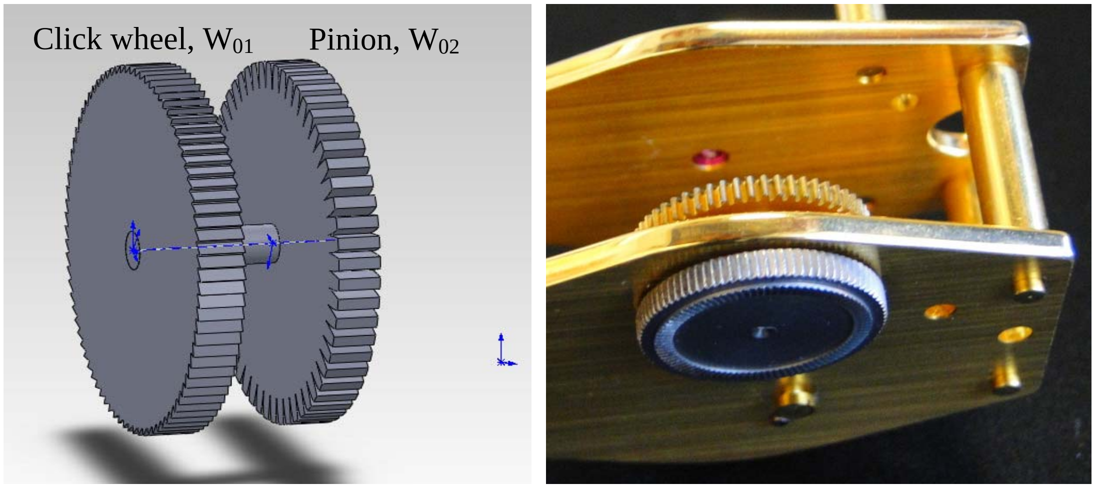

Figure A.4: Click wheel, W_{01}, and great wheel pinion, W_{02} - (a) SolidWorks image, (b) actual components mounted on plate

Table A.4: Click wheel, W_{01}, and great wheel pinion, W_{02}, parameters

Description	Value	Units
Click wheel, W_{01}, diameter	18.9	mm
Click wheel, W_{01}, thickness	2.2	mm
Great wheel pinion, W_{02}, diameter	18.5	mm
Great wheel pinion, W_{02}, thickness	1.89	mm
Combined weight	1.10	g
Total moment of inertia about rotational axis	43.0	g-mm^2
Shaft diameter connecting wheels	1.96	mm
Shaft length connecting wheels	9.75	mm

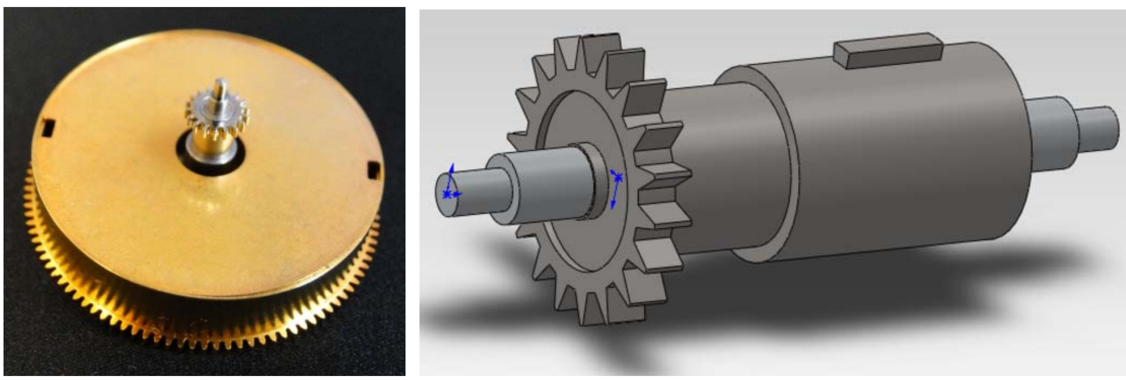

Figure A.5: Great wheel, W_{11}, with attached barrel and inserted winding stem - (a) actual component, (b) SolidWorks image of winding stem

Table A.5: Great wheel, W_{11}, and winding stem, W_{12}, parameters

Description	Value	Units
Great wheel diameter, W_{11}	37.03	mm
Great wheel thickness, W_{11}	1.02	mm
Weight of great wheel and barrel	14.8	g
Barrel moment of inertia about rotational axis	1,261.0	g-mm^2
Winding stem gear diameter, W_{12}	6.2	mm
Winding stem gear thickness, W_{12}	1.2	mm
Weight of winding stem assembly	1.34	g
Stem's moment of inertia about rotational axis	6.0	g-mm^2
Winding stem arbor length	20.6	mm

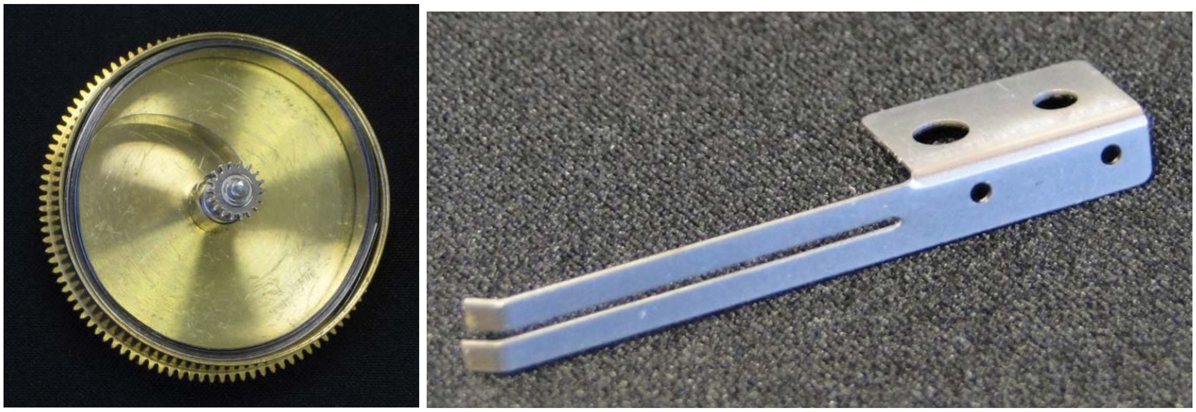

Figure A.6: Main torsional spring inside barrel, and mainspring click

Table A.6: Main torsional spring parameters

Description	**Value**	**Units**
Length (unwound)	1,079.5	mm
Width	5.97	mm
Thickness	0.09	mm
Weight	4.8	g

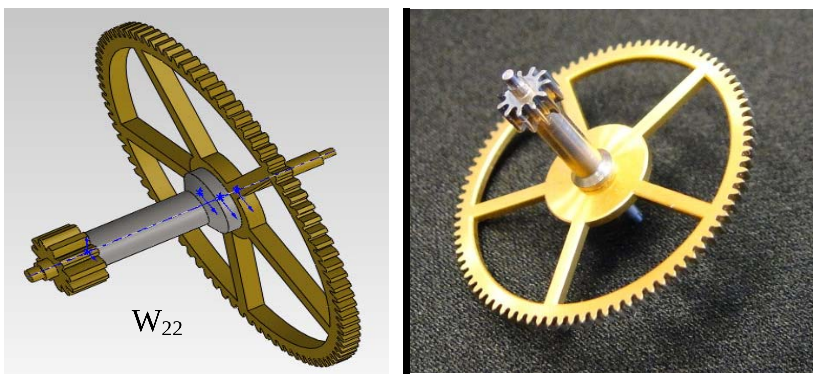

Figure A.7: Second wheel, W_{21}, and pinion, W_{22} - (a) SolidWorks image, (b) actual component

Table A.7: Second wheel, W_{21}, and pinion, W_{22}, parameters

Description	**Value**	**Units**
Wheel diameter, W_{21}	25.07	mm
Wheel thickness, W_{21}	1.0	mm
Wheel diameter, W_{22}	4.28	mm
Wheel thickness, W_{22}	2.16	mm
Weight	2.1	g
Moment of inertia about rotational axis	114.0	g-mm^2
Arbor diameter	2.42	mm
Arbor length	8.7	mm

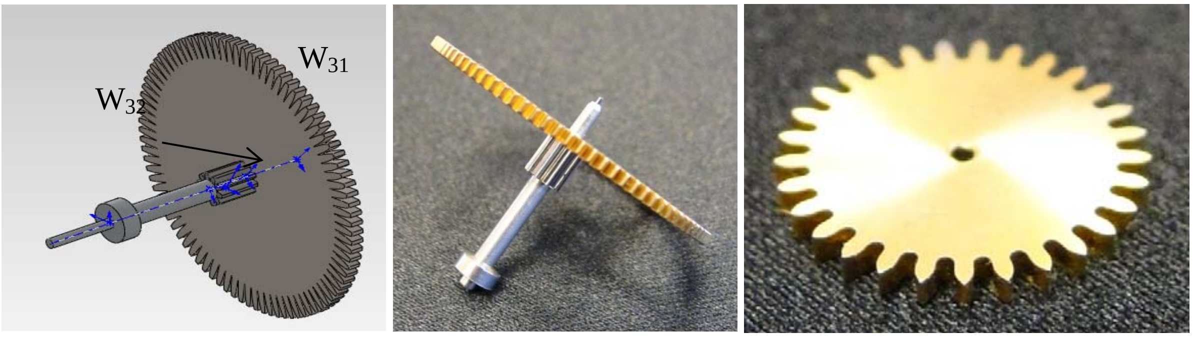

Figure A.8: Third wheel, W_{31}, pinion, W_{32}, and wheel, W_{33} on arbor 3 - (a) SolidWorks image, (b) actual component, (c) actual component

Table A.8: Third wheel, W_{31}, pinion, W_{32}, and wheel W_{33} parameters

Description	**Value**	**Units**
Wheel diameter, W_{31}	25.07	mm
Wheel thickness, W_{31}	1.0	mm
Wheel diameter, W_{32}	3.17	mm
Wheel thickness, W_{32}	3.25	mm
Weight wheel W_{31}, pinion, W_{32}, arbor 3	2.0	g
Moment of inertia about rotational axis	108.0	g-mm^2
Arbor 3 diameter	1.75	mm
Arbor 3 length	11.79	mm
Weight of W_{33}	0.8	g
Wheel diameter, W_{33}	12.37	mm
Wheel thickness, W_{33}	1.02	mm

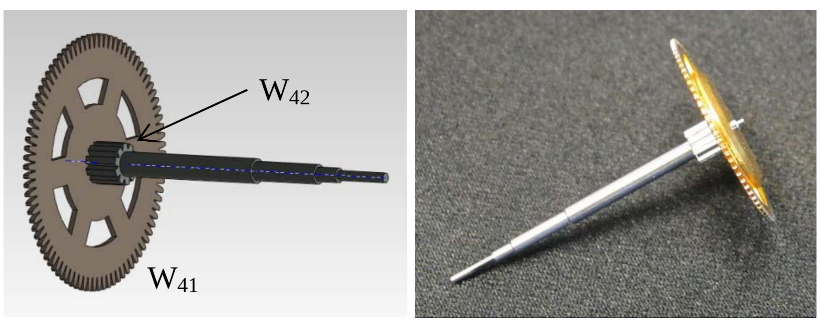

Figure A.9: Fourth wheel, W_{41}, and wheel, W_{42} - (a) SolidWorks image, (b) actual component

Table A.9: Fourth wheel, W_{41}, and wheel, W_{42}, parameters

Description	**Value**	**Units**
Wheel diameter, W_{41}	18.21	mm
Wheel thickness, W_{41}	0.46	mm
Wheel diameter, W_{42}	3.09	mm
Wheel thickness, W_{42}	2.87	mm
Weight	1.85	g
Moment of inertia about rotational axis	84.0	g-mm^2
Maximum arbor diameter	1.75	mm
Arbor length	25.31	mm

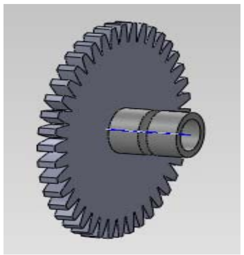
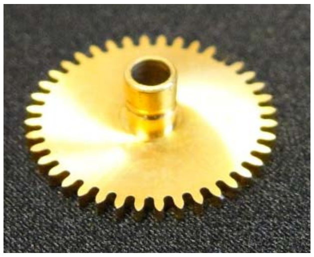

Figure A.10: Hour wheel, W_{43}, with canon - (a) SolidWorks image, (b) actual component

Table A.10: Hour wheel, W_{43}, with canon parameters

Description	**Value**	**Units**
Wheel diameter, W_{43}	16.75	mm
Wheel thickness, W_{43}	1.0	mm
Weight	1.77	g
Moment of inertia about rotational axis	59.0	$g*mm^2$
Canon outside diameter	3.49	mm
Canon inside diameter	2.4	mm
Canon length	3.95	mm

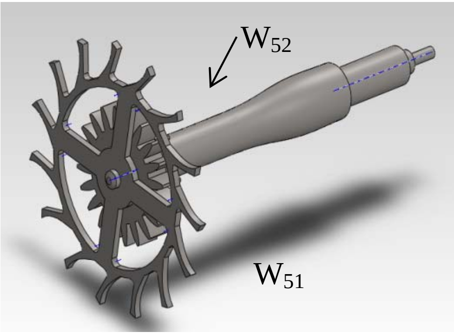

Figure A.11: Escape wheel, W_{51}, and wheel, W_{52} - (a) SolidWorks image, (b) actual component

Table A.11: Escape wheel, W_{51}, and pinion, W_{52}, parameters

Description	**Value**	**Units**
Wheel diameter, W_{51}	8.95	mm
Wheel thickness, W_{51}	0.26	mm
Wheel diameter, W_{52}	4.70	mm
Wheel thickness, W_{52}	0.90	mm
Weight	0.280	g
Moment of inertia about rotational axis	6.0	$g\text{-}mm^2$
Maximum arbor diameter	1.86	mm
Arbor length	14.39	mm

Figure A.12: Pallet with twin jewels and staff - (a) SolidWorks image, (b) actual component; note the counter balance on the forks to help oscillation behavior

Table A.12: Pallet with twin jewels and staff parameters

Description	**Value**	**Units**
Weight	0.6	g
Moment of inertia about rotational axis	149.0	g-mm^2
Length	56.0	mm
Maximum width	12.1	mm
Maximum depth at impulse roller interface	14.0	mm
Arbor diameter	1.43	mm
Arbor length	18.0	mm

Figure A.13: Torsional pendulum rod and impulse roller – (a) actual component, (b) SolidWorks image, (c) impulse roller photograph

Table A.13: Torsional pendulum rod measurements

Description	**Value**	**Units**
Weight	28.9	g
Moment of inertia about rotational axis	92,100	g-mm^2
Tube Length	140.0	mm
Elvinar wire width	0.203	mm
Elvinar wire thickness	0.051	mm
Tube diameter	5.0	mm
Impulse roller diameter	5.64	mm
Impulse roller height	5.75	mm

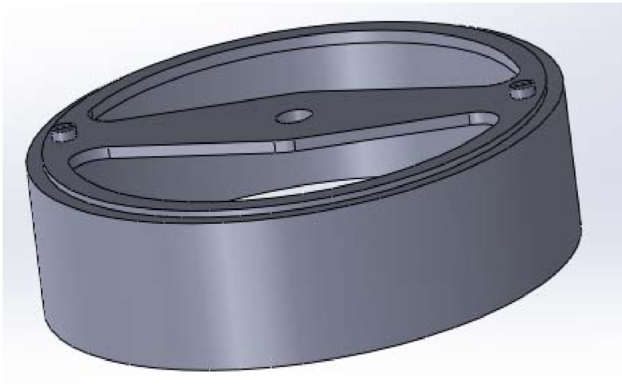

Figure A.14: Torsional pendulum (vertical axis rotation) - (a) SolidWorks image, (b) actual view

Table A.14: Torsional pendulum parameters

Description	Value	Units
Weight	208.0	g
Moment of inertia about rotational axis	167,500	$g\text{-}mm^2$
Outside diameter	88.6	mm
Inside diameter	81.8	mm
Height	22.9	mm

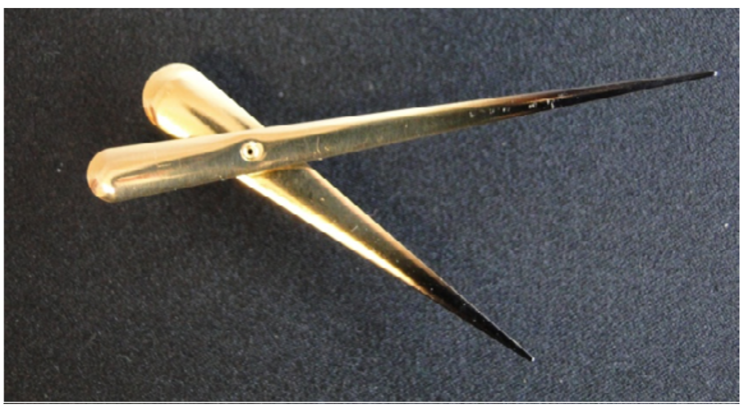

Figure A.15: Minute and hour hands; counter weight on arbor 4 to help balance

Table A.15: Minute and hour hands parameters

Description	Minute Hand	Hour Hand	Units
Hand weight	1.4	1.7	g
Hand length	67.4	51.3	mm

Appendix B. Nomenclature List (symbol, units, description)

a_p	mm^2	Bellows surface area
A	rad	Amplitude of oscillation
b_{ms}	mm	Spring steel width
D	mm	Diameter
E	N-cm, N/mm^2	Energy, modulus of elasticity
f_{atm}	N	Atmospheric force acting on bellows
f_{bs}	N	Force acting on bellows surface area
f_{cap}	N	Net force acting on the end-cap
f_{click}	N	Click force
f_c	N	Force of chain attached to bellows
f_b	N	Bellow force from ethyl chloride
f_{s1}	N	Large coil spring return force
f_{s2}	N	Force of small coil spring
h_{ms}	mm	Main spring cross section thickness
I	$N\text{-}mm^2$	Torsional pendulum inertia
in	-	Input
$K.E.$	Joules	Kinetic energy
k	N-m/rad	Torsional spring stiffness
k_{bs}	N/mm	Bellows compliance
k_c	N/mm	Winding drum spring compliance
k_{s1}	N/mm	Stiffness of large coil spring
k_{s2}	N/mm	Small coil spring stiffness
L	mm	Spring length
n	-	Number of moles
N	-	Number of gear teeth
out	-	Output
P	Watt, Pa	Power, pressure
p_{atm}	bar	Atmospheric pressure
p_b	bar	Bellows pressure
Q	-	Quality factor
R	J/mol·K	Gas constant
R_e	-	Escapement gear ratio
R_h	-	Hour hand gear ratio
R_m	-	Minute hand gear ratio
r_p	mm	Winding pulley radius
r_r	mm	Ratchet wheel radius
R_w	-	Wheel ratio
T	K,sec	Temperature, period
t	sec	Time
V	m/s, m^3	Velocity, volume
W_{01}	-	Ratchet wheel on arbor 0 with click
W_{02}	-	Pinion on arbor 0 interfaces to arbor 1
W_{11}	-	Great wheel on arbor 1 interfaces to arbor 2

W_{12}	-	Pinion on arbor 1 interfaces to arbor 0
W_{21}	-	Wheel on arbor 2 interfaces to arbor 3
W_{22}	-	Pinion on arbor 2 interfaces to arbor 1
W_{31}	-	Wheel on arbor 3 interfaces to arbor 4
W_{32}	-	Pinion on arbor 3 interfaces to arbor 2
W_{33}	-	Minute wheel on arbor 3 interfaces to arbor 4
W_{41}	-	Wheel on time arbor 4 interfaces to arbor 5
W_{42}	-	Pinion on time arbor 4 interfaces to arbor 3
W_{43}	-	Hour canon interfaces to arbor 3
W_{51}	-	Escape wheel on arbor 5
W_{52}	-	Pinion on arbor 5 interfaces to arbor 4
x	mm	Displacement of end plate
τ	N-mm	Torque
θ	rad	Angular rotation
θ_{10}	rad	Initial angular rotation of Arbor 1
ω	rad/s	Rotational speed
ω_n	rad/s	Natural frequency