1988

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Mon. Not. R. astr. Soc. (1988) **234**, 1–36

Nuclear cosmochronology within analytic models of the chemical evolution of the solar neighbourhood

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Accepted 1987 December 18. Received 1987 December 1; in original form 1987 September 28

Summary. This paper investigates whether the age of the Galaxy can be deduced from natural radioactivity. I demonstrate that two recent influential claims (by Butcher and by Fowler) that such observations set the age at $T_G=10$ Gyr depend on special assumptions that run counter to existing astrophysical theory, so that greater ages are possible. I derive exact analytic time-dependent linear models of the chemical evolution of the solar neighbourhood to illustrate the extent to which continuous growth of the local mass density by early additions of metal-poor matter greatly lengthens the galactic ages inferred from the Solar System abundances of natural radioactive nuclei and of their stable daughters. I argue that such time-dependent infall of local matter is plausible and supported by several arguments, and I demonstrate that its past magnitude is the greatest uncertainty in nuclear cosmochronology, which I argue at length to be now the province more of the astronomer than of the nuclear physicist. I attempt here to enlarge the scientific community of these concerns by a very detailed treatment that makes explicit the dependence of age on the parameters describing the infall. In addition to these general aims I present many specific new results, especially: (i) new exact analytic models of chemical evolution; (ii) analysis of the five cosmoradiogenic chronologies (Clayton) in addition to the three based on U and Th; (iii) useful expressions for exact analytic secondary metallicity showing that the magnitude of the secondary component of *s*-process nucleosynthesis influences the Th/Nd observations in G-dwarfs more than galactic age does; (iv) a new argument in stellar evolution based on the ^{13}C neutron source that may finally resolve the old puzzle of apparent contemporaneous growth of *r*-process and *s*-process abundances, an argument that would if correct lend support to Butcher's extreme assumption of equal growth rates; (v) an explicit derivation of the distinction between the rate of growth of local metallicity in the gas and the age spectrum of its imbedded nuclei. An unbiased look at all eight methods together favours a galactic age $12<T_G<20$ Gyr rather than 10 Gyr, although no single method is

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reliable. I point out how each nuclear method is still amenable to further improvement, but argue that they alone will not be able to determine the Galaxy's age. Only a detailed and specific and correct model for the growth and chemical evolution of the solar neighbourhood can enable the galactic age to be correctly inferred from radioactivity.

1 Introduction

The concepts of nuclear chronology were introduced by Rutherford six decades ago. The known rates of decay of radioactive species enable the calculation of the age of a closed system either from the known decrease in those radioactive-decay intensities or from the ratio of the increase in daughter abundance to the final parent abundance. Rutherford pioneered both techniques. He even attempted to calculate the time of nucleosynthesis with his measurements of the observed relative activities of ^{235}U and ^{238}U; however, his astrophysical model was hopelessly incorrect so he got the wrong answer (Rutherford 1929). That is our first lesson.

Following their collaboration in the development of a general theoretical framework for nucleosynthesis in stars (Burbidge *et al.* 1957), Fowler & Hoyle (1960) realized that the *r*-process responsible for the stellar synthesis of ^{235}U, ^{238}U and ^{232}Th would populate respectively what Fowler (1987) today counts as 6.0, 3.35 and 5.8 alpha-emitting transuranic parents, so that their relative production rates in the *r*-process should stand roughly in those ratios, from which they calculated the duration of continuous galactic nucleosynthesis, which they also advocated as being continuous in place of the simpler idea of a single creation event. They suggested that a galaxy having age $T_G=12$ Gyr would fall most comfortably into the middle of the phase space of possible solutions. It is remarkable that they got such a good answer in the first attempt. This calculation has been repeated a large number of times, and in his latest update Fowler (1987) has urged most probable production ratios $y(235)/y(238)\equiv p_{58}=1.34\pm0.19$ and $y(232)/y(238)\equiv p_{28}=1.71\pm0.07$, leading him to assert with considerable emphasis that star formation in the Galaxy had occurred for 5.4 ± 1.5 Gyr before the Sun was born, setting the galactic age $T_G=10.0\pm1.6$ Gyr, with profound cosmological consequences.

It was also advances in nucleosynthesis theory that made it possible to consider daughter-nucleus cosmochronologies. In his description of how to do this Clayton (1964) more than tripled the number of long-lived chronometric techniques with detailed prescriptions for five parent–daughter chronologies, which he named *cosmoradiogenic* chronologies: $^{187}Re \rightarrow {}^{187}Os$ ($\tau=61.7$ Gyr), $^{87}Rb \rightarrow {}^{87}Sr$ ($\tau=69.2$ Gyr), $^{235}U \rightarrow {}^{207}Pb$ ($\tau=1.015$ Gyr), $^{238}U \rightarrow {}^{206}Pb$ ($\tau=6.446$ Gyr), and another independent technique from the daughter ratio $^{207}Pb/^{206}Pb$. Each of these decay schemes was known in applications to geochronology, for which standard techniques can extract that special portion of the daughter abundance that has accrued from terrestrial decay. The novel ingredient for cosmoradiogenic chronology was Clayton's method for determining that portion of the natural abundance that was the specific product of cosmoradiogenic decay *before* the solar system formed, a method dependent upon the blossoming of *s*-process theory (Clayton *et al.* 1961; Seeger, Fowler & Clayton 1965; Clayton & Rassbach 1967; Clayton & Ward 1974; Ward, Newman & Clayton 1976). The $^{232}Th \rightarrow {}^{208}Pb$ decay is not useful because the cosmoradiogenic daughter is not determinable, whereas the so-called 'extinct radioactivities' have half-lives too short to constrain the duration of galactic nucleosynthesis. The present work follows Clayton (1964) in designating the cosmoradiogenic component of the abundances by $^{187}Os_c$, $^{87}Sr_c$, $^{207}Pb_c$ and $^{206}Pb_c$. The only one of these yielding a numerical estimate at that time was the ^{187}Re decay, which seemed to suggest a considerably older galaxy ($T_G \approx 18$ Gyr) than the estimate of Fowler & Hoyle (1960). Despite the attendant uncertainties, the cosmoradiogenic decays offered new opportunities for determining the galactic age.

A new and useful technique has just been introduced by Butcher (1987). He concludes that the Galaxy is relatively youthful (T_G<9.6 Gyr) from the near constancy of the Th/Nd line-strength ratio in the surfaces of old G-dwarfs of differing photometric ages. This conclusion seems to strengthen Fowler's (1987) argument against an old Galaxy, although Clayton (1987b) challenged the conceptual foundations of Butcher's conclusion. But cosmological enthusiasm runs high because of the connection of galactic age with the questions of the Hubble constant and the openness of the universe (e.g. Fowler 1987). Cosmology must remain vigilant to the dangers of circumstantial agreement, however.

In the present work I will show that both Butcher's and Fowler's arguments do not rule out an old Galaxy because they rely on special circumstances that may be implausible. I will present very explicit calculations and arguments so that the reader can himself verify my conclusions easily. But first I must continue with some negative subjective evaluations of the state of nuclear cosmochronology and indicate clearly the new tools that I bring to the attempt to improve the astrophysical context of such calculations.

I allege, and a careful perusal of the voluminous literature will confirm, that one can get almost any answer one wants from nuclear cosmochronology by choosing critical parameters well within their range of uncertainties, especially with U and Th, and by following an unproductive practice of arbitrarily specifying the nucleosynthesis history. I will demonstrate this explicitly. I argue that the only useful nuclear cosmochronology must be evaluated within the confines of those specific models of the chemical evolution of the solar neighbourhood of the Galaxy that satisfy a large body of astronomical data and astrophysical argument. Tinsley (1977, 1980) has made this point already with examples of three vastly different models for chemical evolution that reveal that infall history makes a huge difference to age estimates. Her point has not been widely appreciated. Yokoi, Takahashi & Arnould (1983) presented a very explicit numerical evolution model that reached conclusions compatible with Tinsley's and with those to be developed in this work. I will follow Clayton's (1985a) approach in making the relationships explicit by inventing time-dependent analytic models of the chemical evolution of the Galaxy whose parameters can be chosen in such a way as to conform to the observational and theoretical constraints. I will add new features of such analytic models in this present work, and then use those models to support my statements above. I will not attempt myself the huge task of fitting those constraints and in arguing over the correct historical model, preferring instead to provide an explicit mathematical canvas for such surveys in the form of entire families of exact solutions to the simplified equations of chemical evolution.

The major shortcoming of a plethora of arbitrary parameterizations of nucleosynthesis history (e.g. Fowler & Hoyle 1960; Clayton 1964; Cowan, Thielemann & Truran 1987) is their neglect of the growth of the galactic disc. That growth requires material to fall on to the disc, probably over substantial periods of time at the solar position 8–10 kpc distant from the Galactic Centre. Larson (1976) pioneered dynamic models of such collapse. His model 9, which seems more relevant than most to the Galaxy, concludes that the star formation rate and the gas mass in the solar annulus both increase for about 5 Gyr before beginning their decline toward today's values. He and Tinsley & Larson (1978) argued that many important observational features of the evolution of the solar annulus are dominated by this infall history. Lynden-Bell (1975) had also argued elegantly that an infall rate that increased the local gas mass and star formation rate for several Gyr before declining should assume an important role in understanding the smallness of the percentage of old dwarfs having low metallicity (the G-dwarf problem). Gunn (1987) has shown reasons to expect in the cold dark matter theory of Galaxy formation that continuing infall should become increasingly important over long times as one's attention moves outward from the Galactic Centre. I agree with these papers while admitting that wide variations in the infall history of the local interstellar medium can be considered. But it seems to me artificial to still formulate arguments in terms of the closed-box model of chemical evolution for the solar neighbourhood. It is judged by many to conflict with too many facts (e.g. Tinsley 1977, 1980). And yet, as I will confirm with explicit calculations below, the infall of gas into the solar neighbourhood importantly disturbed the cosmochronometers even if it no longer continues. We seem to be bound to evaluate local cosmochronology within the context of a model in which infall greatly increased the disc mass locally over the first couple of Gyr after disc formation, and continued to increase it significantly for at least 5 Gyr, perhaps dying away to small infall rates over the past 5 Gyr. What I present here are time-dependent analytic families of models with just these desired properties. The radioactive chronologies can be analytically evaluated for these families, of which I discuss here two. I will describe several new results of this approach while extending the initial efforts of Clayton (1985a, 1984a) in this direction. At all times I will attempt to 'demystify' this topic, showing carefully the logical steps so that they are easily reproduced by the reader.

2 Specific realistic family of analytic models of chemical evolution

I will paint a very specific picture in order to be well understood and in order that meaningful variations of that picture can be identified. I consider a galactic annulus containing well mixed matter at the solar galactocentric radius, and I ignore radial transport through that annulus. The gas mass M_G contained within it is reduced by star formation $\psi(t)$ and increased by both infall $f(t)$ of material from the same initial perturbation of the intergalactic medium and by ejection $E(t)$ of matter from stars within the annulus:

$$\frac{dM_G}{dt}=-\psi(t)+E(t)+f(t). \tag{1}$$

The mass ZM_G of a nucleus in that gas phase changes as

$$\frac{d}{dt}(ZM_G)=-Z\psi+Z_eE+Z_ff-\lambda ZM_G \tag{2}$$

where Z_e and Z_f are the concentrations in the stellar ejecta and in the infall respectively, and λ is the radioactive decay rate.

The analytic models that I will construct employ the instantaneous recycling approximation, which follows from the assumption that the stars dominating chemical evolution evolve so rapidly in comparison with the galactic time-scales that their ejecta can be approximated as immediately returned, namely $E(t)=R\psi(t)$, where R is the return fraction. This simplification is satisfactory unless one is interested in very early evolution when the differential evolution times of the first stars matter or in late evolution when M_G is so small and the Galaxy so old that the return is dominated by old dwarfs. Clayton & Pantelaki (1986) present several graphs showing the accuracy of this approximation for species produced in stars of specific mass. The nuclear cosmochronometers are r-process products, which I take to come from common massive stars rather than from some special low-mass scenario of length incubation. The strong overabundance of r-process nuclei relative to s-process nuclei in the most metal-deficient stars (Sneden & Parthasarathy 1983; Sneden & Pilachowski 1985) certainly supports that assumption.

I also will assume that the initial mass function is constant in time and that stellar evolution is metallicity independent. That allows me to consider the yield of a given generation of stars to be constant for primary nucleosynthesis. It also allows the return fraction R to be treated as constant. For the purpose of this paper I state explicitly that I regard the r-process nuclei as being primary nucleosynthesis; i.e. having a yield independent of the initial composition of the stars. The early rise of r-process abundances in stars mentioned above supports this assumption. The r-element Eu rises in step with Fe even in the very metal-deficient stars (e.g. Lambert 1987). I also none the less remind the reader that in their initial formulation of the physics of the *r*-process, Burbidge *et al.* (1957) supposed that it built up out of initial iron seed nuclei, similarly to the *s*-process, in which case both processes would be secondary. I take it that the neutronization occurs near the mass cut in massive stars (e.g. Schramm 1982) where the abundant heavy element build-up destroys any memory of small amounts of initial metallicity. These statements must be explicitly made, because one of the possible ways of invalidating nuclear cosmochronology would be the construction of a special argument for non-constancy of *r*-process yield.

The third assumption of my analytic models is a linear dependence of star formation in the solar annulus on the gas mass within it. This assumption is supported by plausibility only. The physics of star formation is not understood but is certainly very complicated and involves non-linear *physical processes*. If, however, the mass of molecular clouds in this annulus is proportional to the total gas mass within it (plausible), and if star formation proceeds at a fixed average rate within molecular clouds (plausible), the linearity follows. Note also that this is not a universal suggestion, but only one applied within the solar annulus itself, where the ambient conditions and density wave perturbations may be relatively fixed. This assumption also merits emphasis because abandonment of it is one way of altering nuclear cosmochronology, a point to which I will return. But he who would do this should probably also propose a physical reason for his assumption that star formation in the solar annulus is not proportional to the gas mass within, and what its temporal controls are.

Using these assumptions and defining the gas consumption constant ω by the relation $\omega M_G=(1-R)\psi$, equations (1) and (2) take their familiar linear form:

$$\frac{dM_G}{dt}=-\omega M_G+f(t) \tag{3}$$

and

$$\frac{dZ}{dt}=y\omega-(Z-Z_f)\,\frac{f}{M_G}-\lambda Z \tag{4}$$

where the yield y of any nucleus has the usual definition as the ratio of newly created mass of that nucleus to newly created mass of permanent stellar remnants. The solution of equation (3) is

$$M_G(t)=e^{-\omega t}\left[M_{G0}+\int_0^t f(t')\,e^{\omega t'}\,dt'\right] \tag{5}$$

where M_{G0} is the initial gas mass in the disc at the solar position. It is, as I argued in the Introduction, likely to be considerably smaller at that large radial distance than the total mass that will be accumulated from infall over galactic time. The solution of equation (4), on the other hand, demands some specification of $Z_f(t)$ in the infalling material. Clayton (1985a) discovered that the solution for *constant* Z_f is

$$(Z-Z_f)=e^{-\lambda t-\theta(t)}\left[(y\omega-\lambda Z_f)\int_0^t e^{\lambda t'+\theta(t')}\,dt'+Z_0-Z_f\right] \tag{6}$$

where Z_0 is the initial disc metallicity and the physically meaningful concept of 'cycle number' $\theta(t)$ defined by

$$\frac{d\theta}{dt}=\frac{f(t)}{M_G(t)} \tag{7}$$

is the rate at which the instantaneous gas mass is being replenished by the instantaneous rate of infall. Chemically it is a mixing function describing how extensively the disc gas has been diluted by infall. In terms of $\theta(t)$ equation (5) can be written

$$M_G(t)=M_{G0}\,e^{\theta(t)-\omega t}. \tag{8}$$

The problem with solution (6) is the restriction Z_f=constant, which is not likely to be the case for radioactive nuclei. I therefore describe a solution for $Z_f=Z_0\,e^{-\lambda t}$, which will apply if the early burst of star formation (Larson 1976; Cox 1985) has produced a spike of initial metallicity Z_0 in halo gas, which, for radioactive nuclei, then decays exponentially in latter infalling material. Furthermore, I envision for definiteness and simplicity a model in which the initial disc metallicity Z_0 is the same as the initial metallicity of the infall. That is, in the absence of nucleosynthesis the disc concentration would also be $Z_0\,e^{-\lambda t}$ for the nucleus with decay rate λ, regardless of the rate of infall of material carrying the same concentration. This is a very specific but plausible model within which to evaluate the nuclear cosmochronometers. For stable nuclei, the initial metallicity Z_0 has been much discussed for its potential role in limiting the metal deficiency observed in the oldest disc stars (Truran & Cameron 1971). The analytic models to be presented share the merit of time-decaying inflow models, however, in avoiding the need of a large prompt initial enrichment, so that I regard Z_0 as an unjustified parameter in the solutions. Whether it or infall is the correct solution of the G-dwarf problem is an important issue for the galactic age, because the solutions will show explicitly that a resolution of G-dwarf data by infall (Lynden-Bell 1975) will lead to an older radioactive age. With the initial metallicity described above, the solution of equation (4) takes the even simpler form

$$Z-Z_0\,e^{-\lambda t}=y\omega\,e^{-\lambda t-\theta(t)}\int_0^t e^{\lambda t'+\theta(t')}\,dt'. \tag{9}$$

What Clayton (1985a), and subsequently Clayton (1984a), achieved with this approach was the discovery of specific representations in which all abundances are analytically expressible and in which the infall $f(t)$ can be shaped as desired to conform to astrophysical requirements. I will employ here two specific families that are well suited not only to the chronological investigation but also to the fitting of astronomical data on chemical evolution and to the fitting of astronomical arguments on the shape of the infall function.

2.1 CLAYTON (1985A) STANDARD MODEL

Clayton (1985a) showed that if one takes a specific family of monotonically declining ratios of infall to gas mass constrained by

$$\frac{d\theta}{dt}=\frac{f(t)}{M_G(t)}=\frac{k}{t+\Delta} \tag{10}$$

with k an integer (=0 if infall is omitted) and Δ an arbitrary time constant, the resulting solutions have very nice physical properties:

$$\text{infall:}\qquad f(t)=\frac{kM_{G0}}{\Delta}\left(\frac{t+\Delta}{\Delta}\right)^{k-1}e^{-\omega t} \tag{11}$$

which declines smoothly toward zero (exponentially if k=1) following a single maximum:

$$\text{gas mass:}\qquad M_G(t)=M_{G0}\left(\frac{t+\Delta}{\Delta}\right)^{k}e^{-\omega t} \tag{12}$$

which first rises to a maximum at $t(\max)=(k/\omega)-\Delta$ before declining monotonically toward zero:

stable abundance: $$Z-Z_0=\frac{y\omega\Delta}{k+1}\left[\frac{t+\Delta}{\Delta}-\left(\frac{t+\Delta}{\Delta}\right)^{-k}\right] \quad (13)$$

which produces a quasilinear growth in time from the initial metallicity Z_0 but with slope smaller by $(k+1)^{-1}$ than in the closed-box ($k=0$) model;

radioactivity: $$Z_\lambda-Z_0\,e^{-\lambda t}=y\omega\,e^{-\lambda t}\left(\frac{t+\Delta}{\Delta}\right)^{-k} I_k(t,\lambda) \quad (14)$$

where the integrals

$$I_k(t,\lambda)\equiv\int_0^t [(t'+\Delta)/\Delta]^k\,e^{\lambda t'}\,dt'$$

are simple products of polynomials in time with $e^{\lambda t}$. Their form for several values of integer k are succinctly summarized by writing $x\equiv(t+\Delta)/\Delta$ and $d\equiv(\Delta\lambda)^{-1}$ as

$k=0$: $\lambda I_0=e^{\lambda t}-1$

$k=1$: $\lambda I_1=e^{\lambda t}(x-d)-(1-d)$

$k=2$: $\lambda I_2=e^{\lambda t}(x^2-2dx+2d^2)-(1-2d+2d^2)$

$k=3$: $\lambda I_3=e^{\lambda t}(x^3-3dx^2+6d^2x-6d^3)-(1-3d+6d^2-6d^3)$

and a general recursion relation

$\lambda I_k=x^k\,e^{\lambda t}-1-(k/\Delta)I_{k-1}=x^k\,e^{\lambda t}-1-k\,d\lambda I_{k-1}$.

These solutions express analytically the galactic features commonly addressed in chemical evolution studies; the age–metallicity relation for stars; the stellar and gas mass; the $S(<Z)$ relation; the mean age of dwarfs and their age distribution. The parameters k and Δ can be chosen to approximate these constraints (Clayton 1985a), so that evaluating the cosmochronometers becomes a more meaningful exercise than it is with arbitrary parameterization of the 'rate of nucleosynthesis', a common euphemism for the age spectrum of Solar System nuclei. The appropriate parameters certainly cannot be uniquely determined in this way, but Clayton (1985a) showed that $k=1$, $\Delta=1$ Gyr and $\omega=0.3\ \mathrm{Gyr}^{-1}$ are reasonable numbers producing much better fits to metallicity data then does a simple closed-box model unless it is given large prompt initial enrichment.

It will be helpful to comment on the physical significance of the parameters of this analytic family. The gas consumption constant measures how rapidly star formation would deplete the gas, and is commonly estimated near $\omega=0.3\ \mathrm{Gyr}^{-1}$, corresponding to an e-fold time for consumption of 3 Gyr. This parameter will *not*, however, enter into the nuclear cosmochronology, as I show below. The parameters Δ and k determine the initial rate for increasing disc mass; i.e. $k/\Delta=f(0)/M_{G0}$. So $k=1$ and $\Delta=1$ Gyr corresponds to an initial infall rate that would double the local disc mass in 10^9 yr. The parameter k is also related to the time $t(\max)=(k/\omega)-\Delta$ at which M_G reaches its maximum, and also to the total infall over all time, whose value

$$\int_0^\infty f\,dt/M_{G0}=1/\Delta\omega \quad (k=1), \qquad 2/\Delta\omega[1+(\Delta\omega)^{-1}] \quad (k=2), \qquad \text{etc.,} \quad (15)$$

so that a $k=1$, $\omega=0.3\ \mathrm{Gyr}^{-1}$, $\Delta=1$ Gyr model has the property that the total integrated infall in our neighbourhood is 3.3 times the initial mass of disc locally. Increasing k increases infall and delays further the maximum of M_G. For $k>1+\omega\Delta$ the infall rate first increases before beginning its monotonic decline, a useful feature if astrophysical argument should support that behaviour. Considerable flexibility exists, even within this rather restricted model family; but the advantage of having exact analytic solutions to the differential equations is obvious. One is then discussing the solutions of a model rather than of arbitrary prescriptions. Everything has an immediately obvious physical meaning.

Rather than using equations (13) and (14) in their present form, however, it is more meaningful to eliminate the unknown yields y by demanding that the chemical evolution be such that for stable nuclei $Z(t)$ passes through $Z_\odot$ at $t=t_\odot$, where $t_\odot$ is the Solar System formation time, the time whose value is the objective of the chronometric search. Similarly letting $Z_0=\alpha Z_\odot$, so that α defines the initial stable metallicity in terms of the solar value, the stable abundance in equation (13) can clearly be expressed instead

$$\frac{Z}{Z_\odot}=\alpha+(1-\alpha)\,\frac{x-x^{-k}}{x_\odot-x_\odot^{-k}} \qquad \text{(primary)} \tag{16}$$

where $x=(t+\Delta)/\Delta$ and where $Z_\odot=[y\omega\Delta/(k+1)(X_\odot-X_\odot^{-k})]/(1-\alpha)$. I note that this result is for a primary nucleosynthesis product. A similar result can be expressed for a secondary nucleosynthesis product, defined as one whose yield $y_s=\beta Z_1$ where Z_1 is the abundance of a primary seed nucleus, from equation (19) of Clayton & Pantelaki (1986), who solved this problem. Their general solution for zero secondary abundance in initial disc or infall is, with y_1 the yield of the primary seed nucleus.

$$Z_s=e^{-\theta(t)}\left[\beta\omega Z_1(0)\int_0^t e^{\theta(t')}dt'+\beta y_1\omega^2\int_0^t\int_0^{t'} e^{\theta(t'')}\,dt''\,dt'\right] \tag{17}$$

and does apply to the model under discussion. For this family of analytic solutions it is rather easily shown to pass through $Z_{s\odot}$ at $t=t_\odot$ if

$$Z_{ks}/Z_{\odot s}=\frac{[\alpha/(k+1)](x-x^{-k})+[(1-\alpha)/(k+2)][x^2-(k+2)x^{-k+1}+(k+1)x^{-k}]/(x_\odot-x_\odot^{-k})}{[\alpha/(k+1)](x_\odot-x_\odot^{-k})+[(1-\alpha)/(k+2)][x_\odot^2-(k+2)x_\odot^{-k+1}+(k+1)x_\odot^{-k}]/(x_\odot-x_\odot^{-k})}. \tag{18}$$

Equation (18) is that particular solution having $Z_{ks}\to 0$ as $t\to 0$ $(x\to 1)$ and $Z_{ks}\to Z_{\odot s}$ as $t\to t_\odot$ $(x\to x_\odot)$. It is also (Clayton & Pantelaki 1986) the exact solution of equation (4) for a yield $y_s=\beta Z_1(t)$ where the seed nucleus with abundance $Z_1(t)$ is a primary product described by equation (16). In addition to being a result of general interest for chemical evolution surveys, equation (18) will be needed for the discussion of the Th/Nd ratio in the surface of G dwarfs, because a portion of the Nd abundance is *s*-process rather than *r*-process and would accordingly be expected to have in part the temporal growth of a secondary nucleosynthesis product.

For the radioactive nuclei that are the main topic of this work we also need a form of equation (14) in which Z_0 and the yield are not in evidence. The chronological information is carried not in the abundance of the radioactive nucleus (proportional to its unknown yield) but in the ratio of that abundance to the abundance it would have had were it stable (a ratio independent of yield). Of such logical utility in this concept that I follow Clayton (1985a) in defining that ratio as *the remainder* $r(t)=Z_\lambda(t)/Z_{\lambda=0}(t)$. The remainder is the analogue of the function $e^{-\lambda t}$ that applies to a closed ensemble of nuclei created at $t=0$. The radioactive abundance can be written in terms of its yield and initial metallicity α as

$$Z_\lambda=\frac{\alpha}{1-\alpha}\,\frac{y\omega\Delta}{k+1}\,(x_\odot-x_\odot^{-k})\,e^{-\lambda t}+y\omega\,e^{-\lambda t}x^{-k}I_k(t,\lambda) \tag{19}$$

whereas the ratio of equation (19) to its value were $\lambda=0$ is

$$\text{remainder:} \qquad r_k(t)=\frac{Z_\lambda(t)}{Z_{\lambda=0}(t)}=e^{-\lambda t}\,\frac{[\alpha/(1-\alpha)](x_\odot-x_\odot^{-k})+[(k+1)/\Delta]x^{-k}I_k(t,\lambda)}{[\alpha/(1-\alpha)](x_\odot-x_\odot^{-k})+(x-x^{-k})}. \tag{20}$$

Although this result appears cumbersome, its explicit form has arisen from algebraic manipulations that have eliminated both the yield y of the radioactive nucleus and the absolute abundance $Z_\lambda(t_\odot)=Z_\odot$ of that radioactive species in the initial Solar System. All of the chronological information concerning each radioactive nucleus is contained in the value of its remainder. That value depends on both the galactic age $t_\odot$ at that time and upon the model of chemical evolution. Its second factor shows the deviation from exponential owing to continuous nucleosynthesis and infall. For the standard-model family under discussion, one chooses the solar birthdate $t_\odot$, the index k, and the arbitrary time Δ and from them calculates r_k. The first example of this, shown in Fig. 1, is the one Rutherford first discussed in 1929 and which has been discussed more than any other – the ratio of ^{235}U to ^{238}U. The individual remainders cannot presently be utilized in this case because that would require knowing the abundances that they would have had if stable, which would in turn require the ability to compute from theory the production ratio of U to a stable nucleus. It seems doubtful that this can be achieved to the accuracy needed (±25 per cent) to be

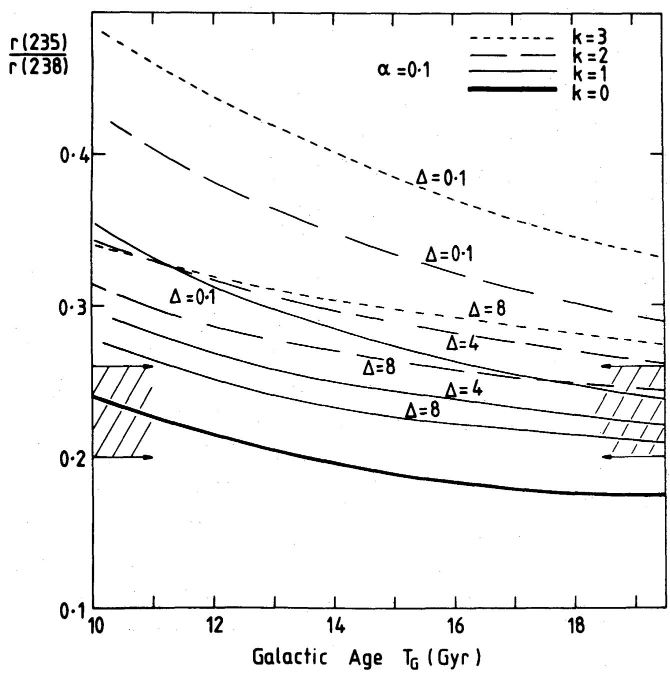

Figure 1. The ratio of remainders for the uranium isotopes at the time $t=t_\odot$ when the Sun formed as a function of the galactic age $T_G=t_\odot+4.5$ Gyr. Each curve corresponds to an analytic standard model (Clayton 1985a) of the chemical evolution of the solar neighbourhood from equation (20). The different orders $k=0, 1, 2, 3$ of infall functions are distinguished by different line formats. The closed infall-free model is distinguished here and in subsequent figures by a doubly heavy curve. Several different values for the time parameter Δ are shown for each k, thereby mapping the remainder on to the infall function. Increasing k increases the importance of infall, and increasing Δ diminishes it. Shown as an incompletely hatched horizontal band is the remainder ratio 0.23 ± 0.03 that must be satisfied by the initial solar abundance ratio $^{235}\text{U}/^{238}\text{U}=0.315$, which is known, and the assumption of an r-process production ratio $y(235)/y(238)=1.34\pm0.19$ as advocated by Fowler (1987). Although the closed model could tolerate a Galaxy as young as 10 Gyr, it has no discrimination against an older Galaxy, whereas even modest past infall demands an older Galaxy. Each curve as shown is for initial metallicity $\alpha=0.1$ times solar. Increase to $\alpha=0.3$ makes each curve flatter and lowers the right-hand ends slightly.

chronologically useful. But the ratio of U isotopes at all times in the interstellar gas can be written

$$\frac{^{235}\mathrm{U}}{^{238}\mathrm{U}}=\frac{r(235)}{r(238)}\frac{y(235)}{y(238)}=\frac{r(235)}{r(238)}p_{58} \tag{21}$$

where $p_{58}=y(235)/y(238)$ is the constant ratio of their yields, which is numerically equal to the ratio of all of these isotopes ever produced. The remainder ratio converts the production ratio to the surviving ratio. It is known from the terrestrial age that the initial terrestrial uranium at $t=t_\odot$ for the interstellar medium was $^{235}\mathrm{U}/^{238}\mathrm{U}=0.31$. If for sake of argument and comparison one accepts Fowler's (1987) conclusion that the production ratio $p_{58}=1.34\pm0.19$, it follows that the remainder ratio at solar formation must have been $[r(235)/r(238)]_{t_\odot}=0.31/(1.34\pm0.19)=0.23\pm0.03$. This value required at $t=t_\odot$ is shown as a horizontal band in Fig. 1. The curves depict the same ratio $r(235)/r(238)$ at $t=t_\odot$ for several different values of the standard-family infall parameters as a function of the age of the Galaxy, $T_G=t_\odot+4.6$ Gyr. The curves are properties of the infall and evolution model only, in the sense that they do not depend upon the uncertain value of the production ratio. Its value determines in turn the location of the horizontal band through which the remainder ratio must pass. The models shown there are for $k=0$ (no infall), $k=1$, $k=2$, and $k=3$, which are distinguished by the nature of the curve (very heavy, solid, long dash, short dash). Values of $\Delta=0.1$, 4, 8 Gyr are explicitly attached. Each curve has taken an initial metallicity $\alpha=0.1$ for the disc and infall strictly for the sake of example.

There are several things to be said about Fig. 1, all of which strengthen the conclusion that 235/238 is not a useful technique for determining the age of the Galaxy. First, only the $k=0$ solution (no infall) enters the favoured band for $T_G<10$ Gyr, crossing the 'best guess' value 0.23 near 11 Gyr. This $k=0$ model is clearly extreme in the sense that it is well below all of those with significant past infall; moreover, it is (essentially) this model that Fowler (1987) used to conclude that $T_G=10$ Gyr (although not on the basis of this ratio alone). Secondly, even if a model does enter the favoured band it does so with a slope so nearly horizontal that no fix on the age can be claimed. For example the $k=1$, $\Delta=8$ Gyr curve enters the favoured band at $T_G=11.3$ Gyr but is still within it at $T_G=20$ Gyr. Thirdly, the precision used for $p_{58}=1.34\pm0.19$ in Fig. 1 may be optimistic. Thus 235/238 does not usefully constrain the galactic age. It is interesting more because it does constrain the acceptable range of models. For example, the $k=2$, $\Delta=0.1$ model, which has a relatively constant star formation rate, is ruled out unless the production ratio is badly estimated or unless the Galaxy is very old indeed.

Equation (20) is easily evaluated for any value of initial metallicity α, but the lack of determinable age is not changed. Without showing the superfluous figure I will just report that increasing initial metallicity to $\alpha=0.3$ makes each curve noticeably more horizontal while lowering the remainder ratio at the right-hand edge by $\Delta r(235)/r(238)=-0.010$ for each curve. The steepest dependence on galactic age occurs if $\alpha=0$, but not enough so to advocate that exercise.

At this point it is useful to notice a general feature of the remainder expressed in equation (20) – namely, it does not depend on the gas consumption constant ω. Only the product $y\omega$ occurred as a common factor in equation (19), and that product was eliminated in the ratio that formed the remainder. Thus in this family of models the chronological information is completely independent of ω, a result noted by Clayton (1985a). It is for this reason that ω does not appear among the galactic parameters labelling the curves of Fig. 1. However, that independence is not a general mathematical one nor a physically necessary one. It came from defining the family of solutions by $d\theta/dt=f/M_G\equiv k/(t+\Delta)$, independent of ω. The solution to equation (4) expressed by the model leading to equation (9) shows that if $\theta(t)$ is independent of ω, so will the remainder also. In the next section I will use an alternate new family of models in which $\theta(t)$ does depend on ω, because many readers will find it more useful for many purposes.

2.2 EXPONENTIAL INFALL $f_0 e^{-\omega' t}$

The exponential infall in the $k=1$ standard model is a rather special one in two respects: (i) the rate of decline ω is the same as the gas consumption coefficient of equation (3); (ii) the initial infall rate has the value M_{G0}/Δ, which means that it cannot be varied at fixed time independently of $x=(t+\Delta)/\Delta$, although that limitation is removable (Clayton 1986). To make exponential infall more general Clayton (1987a) discovered exact solutions to infall $f(t)=f_0 e^{-\omega' t}$, where $\omega' \neq \omega$. His equations (31) and (32) give respectively the functions $\mu=M_G/M_{\rm Total}$ and $Z(t)$ for stable primary nuclei. He showed

$$\text{gas mass:} \qquad M_G(t)=e^{-\omega t}\left\{M_{G0}+\frac{f_0}{\omega'-\omega}\,[1-e^{-(\omega'-\omega)t}]\right\}. \tag{22}$$

This function also rises to a maximum before its subsequent monotonic decline, the very behaviour called for by Lynden-Bell (1975) to ameliorate the G-dwarf problem. Several examples are shown in Fig. 2. By setting its derivative to zero one finds that at that maximum (located by arrows on Fig. 2)

$$M_G(\max)=M_{G0}\,\frac{\omega_f}{\omega}\,e^{-\omega' t_m} \tag{23}$$

where t_m is the time of maximum and ω_f is a new parameter that measures the initial rate for infall to replace gas mass: $\omega_f \equiv f_0/M_{G0}$. This parameter ω_f measures the strength of the initial infall and ω' measures its now arbitrary rate of decline. This model gives a positive t_m if $\omega_f>\omega$ whereas the model of the last section does so only if $k/\omega>\Delta$. I present Fig. 2 here to facilitate physical

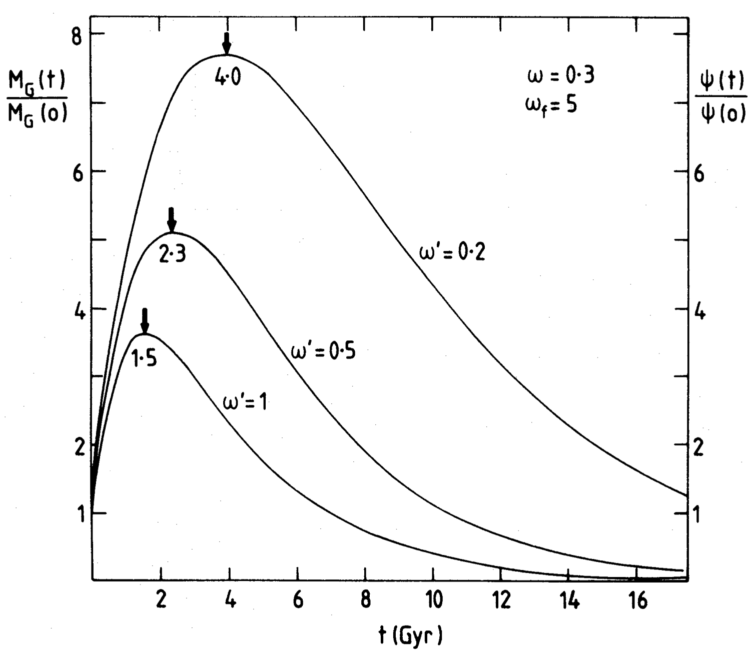

Figure 2. The interstellar gas mass $M_G(t)$ measured in terms of the initial disc mass M_{G0} for the analytic models having exponential infall $f=\omega_f M_{G0} e^{-\omega' t}$. The time of maximum local gas mass is marked by arrows. The sequence shows that increasing ω' decreases the gas-mass growth by decreasing the duration of infall and its total integrated amount $\int f\,dt=\omega_f M_{G0}/\omega'$. These examples each take $\omega_f=5\,{\rm Gyr}^{-1}$ and the gas consumption rate $\omega=0.3\,{\rm Gyr}^{-1}$. Because the models are linear, these curves also measure the star formation rate as indicated by the right-hand ordinate, demonstrating that, for $\omega'=0.2$ for example, the star formation rate at $t=15$ Gyr is only 0.4 times the average past star formation rate despite $\omega=0.3\,{\rm Gyr}^{-1}$. It is the early increase in star formation rate that can solve the G-dwarf problem (Lynden-Bell 1975).

appreciation of this family of analytic models by the reader, because it should become quite useful.

One easily shows that the gas cycle number

$$\text{cycle number:} \qquad \theta(t)=\ln\left\{1+\frac{\omega_f}{\omega'-\omega}\,[1-e^{-(\omega'-\omega)t}]\right\} \tag{24}$$

so that the stable metallicity from equation (9) is for $\lambda=0$

$$\text{stable:} \qquad Z-Z_0=y\omega\,\frac{[1+\omega_f/(\omega'-\omega)]t+[\omega_f/(\omega'-\omega)^2][e^{-(\omega'-\omega)t}-1]}{1+[\omega_f/(\omega'-\omega)][1-e^{-(\omega'-\omega)t}]} \tag{25}$$

a result already established by Clayton (1987a). For the present study we need to generalize it to radioactive nuclei, which is easily accomplished through equation (9):

radioactivity:

$$Z_\lambda-Z_0\,e^{-\lambda t}=y\omega\,\frac{[1+\omega_f/(\omega'-\omega)](1-e^{-\lambda t})/\lambda+\omega_f(\omega'-\omega)^{-1}(\omega'-\omega-\lambda)^{-1}[e^{-(\omega'-\omega)t}-e^{-\lambda t}]}{1+[\omega_f/(\omega'-\omega)][1-e^{-(\omega'-\omega)t}]}. \tag{26}$$

Equation (26) clearly reduces to equation (25) as $\lambda\to 0$. Note also that in contrast to the standard model of the last section, $Z(t)$ depends in an essential way upon ω, which will not disappear in forming the remainder. This dependence persists because the generator of the analytic family $d\theta/dt=f/M_G$ depends upon ω. Nonetheless, as we will see, the physical nature of the conclusions is unchanged. The physical interpretation of the parameters is aided for this family by noting that the total integrated infall exceeds initial disc mass by the simple factor

$$\int_0^\infty f(t)\,dt/M_{G0}=\omega_f/\omega' \tag{27}$$

which has the value 10 for the example $\omega_f=5$, $\omega'=0.5$ shown in Fig. 2, whose gas mass reaches a maximum $5.1\,M_{G0}$ at $t_m=2.4$ Gyr. Total infall tenfold times initial disc mass may seem like a lot to readers imagining our Galaxy today, but notice that much of it occurs early in galactic history. The same infall function today, say $t=15$ Gyr for example, has the value $f(15)/M_{G0}=\omega_f\,e^{-0.5(15)}=0.0028\,\text{Gyr}^{-1}$, and since $M_G(15)=0.275\,M_{G0}$ in this model, the local gas today would be replenished by infall over the rather long time $M_G(15)/f(15)=100$ Gyr. Thus the lack of heavy infall observed today does not rule out moderately strong infall functions, as for example with that set of parameters. Speaking purely from physical intuition, I find it more plausible to think that the disc mass at the solar position increased by a factor of 10 from the time when the inner disc was first established than I do to suppose the local disc was entirely in place from the beginning of the inner Galaxy. But the chronological ages are vastly different for the two cases.

The remainder follows from the ratio of equations (26) to (25). But just as in the standard-model section, it is more useful to first eliminate the yield by demanding $Z(t_\odot)=Z_\odot$ for stable elements and by setting $Z_0=\alpha Z_\odot$. For compactness define the function

$$\Omega(t)\equiv\frac{\omega_f}{\omega'-\omega}\,[1-e^{-(\omega'-\omega)t}] \tag{28}$$

and let $b=\alpha/(1-\alpha)$ and $D=[1+\omega_f/(\omega'-\omega)]$, so that the stable element in equation (25) takes then the form

$$Z(t)=y\omega\left[\frac{b[Dt_\odot-\Omega_\odot/(\omega'-\omega)]}{1+\Omega_\odot}+\frac{Dt-\Omega/(\omega'-\omega)}{1+\Omega}\right] \tag{29}$$

from which $y\omega$ can be eliminated by setting $Z(t_\odot)=Z_\odot$. This gives a generally more useful form analogous to equation (16) for the standard model. Making the same writing simplifications in equation (26) gives

$$Z_\lambda(t)=y\omega\,\frac{b\,e^{-\lambda t}[Dt_\odot-\Omega_\odot/(\omega'-\omega)]}{1+\Omega_\odot}+\frac{D(1-e^{-\lambda t})/\lambda-[\Omega-(D-1)(1-e^{-\lambda t})]/(\omega'-\omega-\lambda)}{1+\Omega} \tag{30}$$

with the remainder $r(t)$ being the ratio of equations (30) to (29). Clearly the dependence on ω remains in the remainders in this family of analytic models. To save space it will not be displayed explicitly.

The remainder ratio $r(235)/r(238)$ at $t=t_\odot$ was then calculated as a function of $t_\odot$ for this family as well, and it is displayed in Fig. 3 as a function of galactic age $T_G=t_\odot+4.6$ to enable a direct comparison with the standard family in Fig. 1. The infall-free limit, in this family $\omega_f=0$, is again displayed as a double-heavy curve, and is identical to the $k=0$ curve in Fig. 1. The other curves

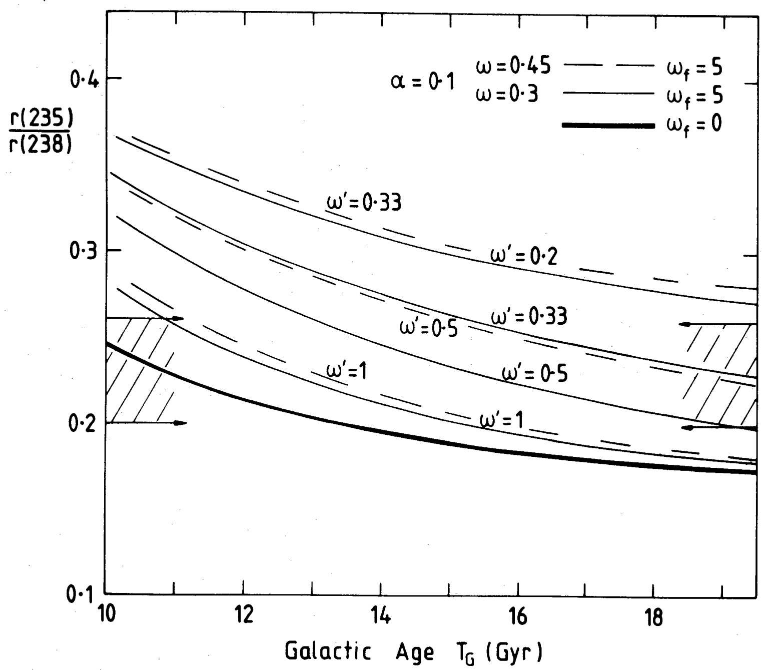

Figure 3. The ratio of the remainders for the uranium isotopes at the time $t=t_\odot$ when the Sun formed as a function of galactic age. Identical to Fig. 1 except that equations (30) and (29) of the family of exponential infall models is used here. Two different gas consumption rates $\omega=0.3$ and $\omega=0.45$ are shown with four values of ω', all for initial infall rate $\omega_f=5$ (all in Gyr^{-1}). The closed-box model $\omega_f=0$, shown doubly heavy, is identical to the $k=0$ curve in Fig. 1. Increasing ω' brings the remainders continuously closer to the closed-box result. For given infall ω_f and ω', increased gas consumption rate ω drives the remainders further from the infall-free relation. The insensitivity to galactic age evident in Fig. 1 is repeated here. Both analytic families display the sensitivity to infall parameters well.

show four choices for the infall decay rate $\omega'=1, 0.5, 0.33$ and 0.2 for two different choices for the gas consumption rate $\omega=0.3$ and 0.45 (all in Gyr^{-1}). The general resemblance to Fig. 1 is obvious and the lessons to be learned are the same. One sees that increasing ω' reduces the integrated infall and hence moves the family closer to the infall-free limit. Increasing gas consumption ω, on the other hand, makes infall more important because it reduces the amount of disc gas to be mixed with the infall. The physical conclusion that 235/238 cannot usefully proscribe the galactic age is reinforced.

The ratio of remainders for the longer-lived actinide pair, $r(^{238}U)/r(^{232}Th)$, at $t=t_\odot$ is shown in Fig. 4 for this family of chemical evolution models. The infall-free $\omega_f=0$ model is as always shown

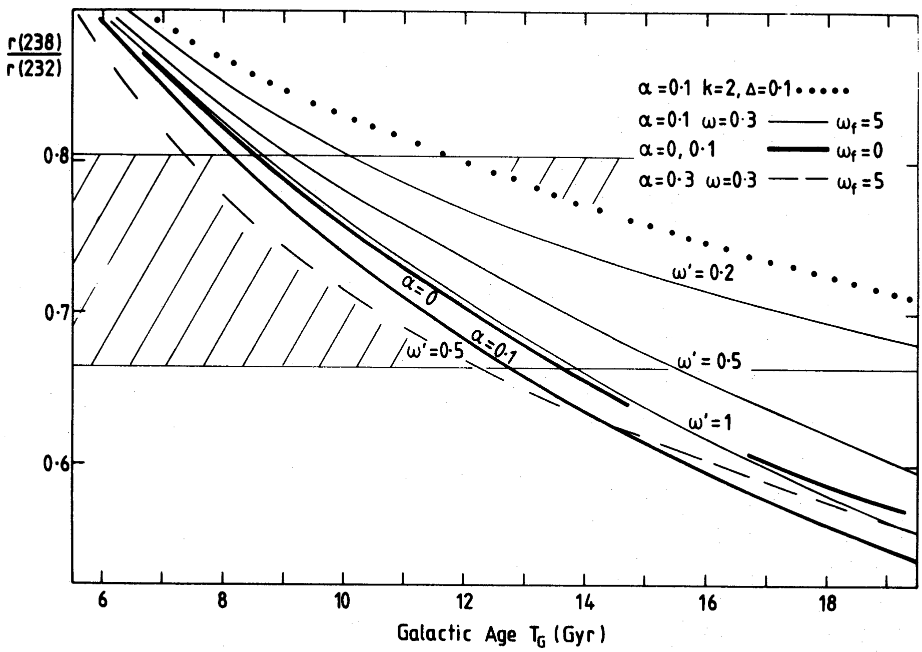

Figure 4. Remainder ratio $r(^{238}\mathrm{U})/r(^{232}\mathrm{Th})$ at $t=t_\odot$ calculated from equation (30) for the exponential infall family, except for one strong-infall member ($k=2$, $\Delta=0.1$) of the standard family (shown dotted). As in Fig. 2, the initial metallicity $\alpha=0.1$ except for one comparison with $\alpha=0.3$ for the $\omega'=0.5$, $\omega=0.3$ case. Infall-free models with $\alpha=0$ and with $\alpha=0.1$ are shown doubly heavy. The horizontal band shows the acceptable range of values for the remainder ratio if the initial $(\mathrm{Th/U})_\odot=2.33\pm0.02$ and if the r-process production ratio $y(232)/y(238)=1.71\pm0.07$. Thus acceptable closed models have $8<T_G<13$; however, models with past infall may be much older. This chronometer is the only one that places any useful upper limit on the value of T_G, and that limit is restricted to independent proof that infall was weaker than in the $\omega_f=5$, $\omega'=0.5$ model, which would limit $T_G<16$ if the abundance and production ratio can also both be trusted.

double heavy, although it is not the most extreme curve shown in Fig. 4. Increasing the initial metallicity to 30 per cent of solar lowers the $\omega'=0.5$ curve to slightly beneath the closed-box curve having $\alpha=0.1$. Taking a large but freely decaying metallicity in the infall is thereby seen to lower somewhat the inferred age from this ratio. The favoured band for $r(238)/r(232)$ at $t_\odot$ comes from evaluating

$$\frac{r(238)}{r(232)}=\left[\frac{Z(\mathrm{U})}{Z(\mathrm{Th})}\right]_\odot\frac{y(232)}{y(238)}=\frac{p_{28}}{2.33\pm0.20}=0.734\pm0.07$$

where the initial solar abundance ratio $^{232}\mathrm{Th}/^{238}\mathrm{U}$ is taken as 2.33 ± 0.20 from Anders & Ebihara (1982) and $p_{28}=1.71\pm0.07$ from Fowler (1987). Perhaps the abundance ratio can be made even more precise in the future; however, that will require not so much better measurements as it will a complete quantitative model of the origin of the solar system in order that one can interpret with confidence the meaning of the abundances in the type C1 carbonaceous meteorites from which the data come. One may note that both the Zr/Hf and Re/Ir abundance ratios are about 13 per cent higher in C2 meteorites than in C1 meteorites, even though both ratios involve refractory pairs of similar chemistry, just as does also the Th/U ratio (Anders & Ebihara 1982), and that this difference is not understood. The meaning of the small 4 per cent error assigned by Fowler to p_{28} is even harder to quantify. The possibility of systematic error in the production ratios has haunted these methods from the beginning. A totally independent calculation by Thielemann, Metzinger & Klapdor (1983) yielded $p_{28}=1.39$, well outside Fowler's judgement of reasonable uncertainty, in which case the remainder ratio at $t_\odot$ above would be reduced to $r(238)/r(232)=0.60$, requiring a much older galaxy. A more recent calculation (Cowan *et al.* 1987) for an explosive helium burning setting, which is probably not the correct environment, produced $p_{28}=1.60$, or a most probable remainder ratio 0.69, near the lower end of the band in Fig. 4. Whatever the correct remainder ratio, increased infall (designated in this family by smaller ω') lengthens the galactic age. It is again worth reminding that Fowler (1987) took the infall-free model with $\alpha=0.17$, essentially the same as but slightly below the double-heavy curve of Fig. 4. One sees that this exercise gives a galactic age $8<T_G<11.5$, just as Fowler claims. The problem with that claim is easily seen to be the systematic astrophysical uncertainty about infall combined with lingering doubts over the production ratio. The curve $\omega'=0.5$ belonging to $\alpha=0.1$ and $\omega=0.3$ extends the upward age limit to $T_G=15.5$ Gyr, even with the band as shown. One model from the standard-model family ($k=2$, $\Delta=0.1$) having heavy infall and a relatively constant star formation rate throughout the time period shown would not leave the band until $T_G=25$ Gyr. To disregard these more elderly fits to the data it is necessary to rule out such models with astrophysical arguments.

The other conclusion to be drawn from these figures is that Fowler's (1987) specification of a young Galaxy rests on his adoption of the extreme model in these figures, namely the one without infall. That this is not what Fowler says (*op. cit.* p. 102) regarding infall to the disc demands some careful clarification, which I now provide.

2.3 AGE-METALLICITY VERSUS AGE SPECTRUM OF SOLAR NUCLEI

The rate of growth of the interstellar metallicity $Z(t)$ is frequently confused with the age spectrum of the elements in solar material. The former reveals itself astronomically through the age–metallicity relationship for stars, whereas it is the latter that enters into cosmochronological calculations based on solar abundances. The equating of concepts is an easy one to fall into, because it matches our physical intuition and is in fact valid for the closed-box model that we commonly hold as our mental picture of these things. However, the two are vastly different, as I will show, and it is their distinction that restricts Fowler's (1987) calculation to the assumption of a closed box even though he describes it as being relevant even in the face of infall of matter on to the disc. But that infall of low-Z material continuously dilutes the oldest nuclei in the interstellar medium so that they are poorly represented in solar material. This is most easily appreciated by setting infall metals $Z_f=0$ and nucleosynthesis $y=0$, in which case equation (6) shows that the initial disc concentration Z_0 of a stable nucleus is diluted according to

$$Z(t)=e^{-\theta(t)}Z_0 \tag{31}$$

showing that the cycle number of equation (7) measures the gradual dilution of initial metallicity. But by the same token, any nuclei synthesized in the earliest epochs will be diluted by the same factor. For a standard-model of order k that dilution $e^{-\theta(t_\odot)}=[\Delta/(t_\odot+\Delta)]^k$, which for a $k=1$ model with $\Delta=1$ Gyr is about 1/10. That is, only 10 per cent or so of the oldest nuclei synthesized in the disc in that model are remembered by the solar material. This is true even though the growth of $Z(t)$ is approximately linear in the standard model if $\Delta \ll t_\odot$. For example, the exact growth (equation 13) of metal concentration in that linear $k=1$ model is $Z(t)=y\omega/2[(t+\Delta)/\Delta-\Delta/(t+\Delta)]$, which is almost exactly linear if $\Delta=1$ Gyr. Therefore it is perfectly logical in a model with infall to have $Z(t)$ growing linearly. However, the nuclei created early are very underrepresented in solar material according to equation (31), so the *age spectrum* of solar system nuclei is far from flat despite the linear growth of $Z(t)$. It is that quirk that restricts Fowler's calculation to the closed model (for linear star formation with constant yield).

We can derive the *age spectrum* of solar system nuclei in these models in the following way (again setting $Z_0=0$, $Z_f=0$ for simplicity). The solution of differential equation (4) becomes for this simple case

$$Z = e^{-\theta(t)} \int_0^t y\omega \, e^{\theta(t')} \, dt' \tag{32}$$

even for time-dependent yield, as is evident on inspection. Consider then, as a mental exercise, that the yield $y(t)$ is non-zero only at $t=t_n$, say, so that over a small time interval Δt_n about t_n it can be written $y(t)=y\omega\Delta t_n\delta(t-t_n)$. When this is inserted in equation (32) there results a contribution ΔZ_n to the solar abundances given by $\Delta Z_n = y\omega\Delta t_n \, e^{-\theta(t_\odot)\,\theta(t_n)}$. All of this contribution ΔZ_n to $Z_\odot$ consists of nuclei having the same age, $t_\odot - t_n$, at solar birth. Letting a sequence of t_n and Δt_n contributions become a continuum, in which the *birthdate of the solar elements* is then represented by τ becomes

$$\frac{\Delta Z_n}{\Delta t_n} \rightarrow \frac{dZ}{d\tau} = y\omega \, e^{-\theta(t_\odot)+\theta(\tau)}. \tag{33}$$

This result is the spectrum of birthdates in the solar material $Z_\odot$. It is also a mere formality to see that the integral over birthdates τ gives $Z_\odot$:

$$\int_0^{t_\odot} \frac{dZ}{d\tau} \, d\tau = e^{-\theta(t_\odot)} \int_0^{t_\odot} y\omega \, e^{\theta(\tau)} \, d\tau$$

which by equation (32) is equal to $Z_\odot$. Again taking one of the analytic models as an example, in the standard family of models described in Section 2.1 the birthdate spectrum is

$$\frac{dZ}{d\tau} = y\omega \, e^{-\theta(t_\odot)+\theta(\tau)} = y\omega \left(\frac{\tau+\Delta}{t_\odot+\Delta}\right)^k. \tag{34}$$

The interpretation is that although the young elements are represented by their full complement $dZ/d\tau = y\omega$, the old elements born near $\tau=0$ are diluted to $dZ/d\tau = y\omega[\Delta/(t_\odot+\Delta)]^k$. Only $k=0$ gives a flat age spectrum. It is this spectrum – equation (34) – that must be used to reckon solar radioactivity, not a flat age spectrum. Finally, the thorough reader may confirm that the integral of $dZ/d\tau$ over all ages at $t=t_\odot$ gives $Z(t_\odot) = y\omega/(k+1)\{(t_\odot+\Delta)/\Delta - [\Delta/(t_\odot+\Delta)]^k\}$, just as in equation (13). Through these exercises one understands that Fowler's (1987) prescription is appropriate only for a closed solar neighbourhood, because it is only for $k=0$ that $Z(t)$ grows linearly and that the age spectrum is flat. And therein lies the problem, because astronomical observations may rule out the closed-box model of the solar neighbourhood. For example, the star formation rate in that model has decreased by a large factor over time (unless one disavows the linear model by insisting that the star formation rate has been relatively constant despite the decrease in gas mass by an order of magnitude, a rather arbitrary prescription also giving older radioactive age); and the number of low-Z dwarfs exceeds those having $Z \approx Z_\odot$ by an unacceptably large factor unless one appeals to a large prompt initial enrichment. The use of the age-spectrum equation (34) also confirms the abundance of radioactive nuclei given in equation (19) if that abundance is instead calculated by the intuitively appealing integral

$$Z_\lambda(t_\odot) = \int_0^{t_\odot} \frac{dZ}{d\tau} \, e^{-\lambda(t_\odot-\tau)} \, d\tau. \tag{35}$$

There is yet one other remark that should be made concerning the initial spike of radioactive metals Z_0 shared by disc and by infall. Exactly because it is shared by the infalling material in the model I have adopted in this work, there is no dilution of Z_0 with time. The mathematical statement of this physical conclusion is evident in the absence of an $e^{-\theta}$ term multiplying Z_0 in equation (9), which describes that model. But one must be wary in physical situations that advocate an initial disc metallicity much larger than that in the infall (or, equivalently, an early infall carrying metallicity that decreases rapidly with time). In such cases (Cox 1985) the initial disc may be super-metal-rich, but that contribution to Z_0 today is only $Z_0\,e^{-\theta(t_\odot)}$. Thus one must not say that an initial spike, of 17 per cent say, in the disc translates to 17 per cent of the solar nuclei having the age of the disc. The physical truth about infall is essential.

Returning to the very important argument of Fowler (1987) that seemed to limit the galactic age, one might suppose that abandoning the linear star formation model could save his argument. Clayton (1985b) already showed that taking a star formation rate quadratic in the gas mass, $\psi(t)\propto M_G(t)^2$, does indeed increase the early nucleosynthesis rate per unit mass of medium, causing the remainder ratio $r(235)/r(238)$ to cross the value 0.23 in Fig. 1 about 3.5 Gyr earlier than in the linear models, thereby indicating a Galaxy that is much younger (see his fig. 4). He showed, furthermore, that the metallicity growth $Z(t)$ remains quasilinear (see his fig. 2). However, he also showed the astrophysical problem with this model, and why it is that recourse to the astronomical constraints is required. For the infall function utilized by Clayton (1985b), the ratio of early star formation to its present value is much bigger in the quadratic model. Specifically he obtained (for that infall rate only) a ratio $\psi(T_G-12)/\psi(T_G)=20$ for the quadratic model but only 6 for the linear model. General astronomical evidence may favour a more constant star formation rate. It is indeed the restriction to moderately declining star formation rates that restricts that avenue to the postulation of a flat age spectrum for solar nuclei. Yokoi *et al.* (1983) also identified this important cosmochronological constraint in a very significant computerized study having a similar astrophysical philosophy to the one I have developed. To achieve a flat age spectrum without increasing the early rate of star formation per unit mass of gas would seem, from equation (32), to require the yield to be time dependent and to fall rapidly enough to compensate for the increase of $e^{\theta(t)}$. This might be best achieved by taking the *r*-process to occur in very massive stars and to assume that the birth fraction of low-mass stars has greatly increased with time, thereby decreasing the yield. Such a pursuit may be astrophysically justifiable, but I will not pursue it here. Rather, I prefer to return to the linear model with constant yield to consider the other cosmochronometers.

3 The Th/Nd ratio in old G-dwarfs

Butcher (1987) has advanced an exciting new cosmochronological data set based on the thorium remainder. He has succeeded in observing today the Th line strength in old G-dwarfs that formed at an estimated past time t_*. Because the thorium remainder was $r(t_*)$ when each star formed, its present remainder in the same stellar surface is

$$r(\text{Th today})=r(t_*)\,e^{-\lambda(T_G-t_*)}. \tag{37}$$

Butcher was able to measure the Th remainder by comparing its line strength with that of the stable element Nd and by assuming that the production ratio $y(\text{Th})/y(\text{Nd})$ is constant in time. In that case the observed line ratio is proportional to the observed r(Th today). He has excited the astronomy world by arguing in this way that the oldest stars are no older than 10 Gyr instead of the 20 Gyr estimated from the photometric ages. His reasoning is that the thorium remainder observed today should be smaller in older stars because the free decay interval (equation 37) has decreased it more than its contemporary decline in the continuously reactivated interstellar gas.

The sense in which Butcher's conclusion is extreme lies in the assumption, apparently backed up by Butcher's (1975) own prior study of that point, that the *r*-process abundances (Th and the *r*-process part of Nd) have the same temporal growth rate as the *s*-process part of Nd. These temporal evolutions should differ substantially if the *r*-process production is primary (i.e. synthesizeable in Pop II stars) whereas the *s*-process production is secondary (i.e. requiring and being proportional to initial metallicity in stars). The latter assumption follows naturally from the argument that the *s*-process has built on iron seed nuclei present in the star from birth, and, moreover, that the neutron source for the *s*-process also derives from the initial stellar metallicity. These tenets are standard in nucleosynthesis theory. The primary nature of the *r*-process is hard to prove at present because *r*-process models of both primary and secondary type exist (e.g. Norman & Schramm 1979). The primary model rests on creating both the neutrons and the seed nuclei in the neutronized heavy-element-rich regions near the mass cut between the ejected matter and the neutron-star matter during a Type II supernova. Hillebrandt, Takahashi & Kudama (1976) and Symbalisty, Schramm & Wilson (1985) have both advanced interesting and plausible models of this type. That the *r*-process is indeed of this type is supported by observational evidence in certain extreme Pop II stars that the *r*-process abundances are very overabundant with respect to *s*-process abundances (Sneden & Parthasarathy 1983; Sneden & Pilachowski 1985) and that throughout Pop II the *r*/Fe ratio remains constant as each grows in step (e.g. Lambert 1987). At the very least this requires the *r*-process to occur in extreme Pop II stars more massive than any primary *s*-process stars, and is also consistent with but does not prove that the *s*-process is secondary. In these calculations I will generalize my earlier argument (Clayton 1987b) in taking the *r*-process as primary and a portion of the *s*-process as secondary; however, I will present a flexible position on this matter by formally allowing the secondary fraction of Nd to be determined by future research. I will not go so far as to suggest that difficult systematic errors in interpreting line ratios in different stars has caused Butcher (1975) to reach a mildly incorrect conclusion on r/s constancy; but his *is* an extreme assumption. Instead I will suggest a possible resolution of that puzzle that could give it theoretical support. My main result will be to support my earlier one showing that the Th/Nd interpretation is more sensitive to that issue than it is to the age of the Galaxy, so that the latter can be addressed only when the former has been completely resolved.

To avoid unnecessary formulae I will utilize only the standard family of models described in Section 2.2. I will take the initial disc metals $\alpha Z_\odot$ to be of primary nuclei only, so that equation (16) describes the primary growth, whereas the secondary part of *s*-nuclei is described by equation (18). If I now require of the chemical evolution that at time $t_\odot$ the secondary *s*-abundance Nd_s is a fraction s of $\mathrm{Nd}_\odot$, whereas the primary abundance is a fraction $(1-s)$ of $\mathrm{Nd}_\odot$, I can form the linear sum indicated schematically as $s\cdot$(equation 18)$+(1-s)\cdot$(equation 16): $\mathrm{Nd}_\odot = s\mathrm{Nd}_\odot + (1-s)\mathrm{Nd}_\odot$. The aforementioned equations are constructed upon the supposition that the seed abundance for the secondary production has temporally evolved as a primary nucleus. The $k=0$ (closed-box) and $k=1$ standard models will be explicitly displayed for the pedagogic value to the reader;

$$k=0: \quad \frac{\mathrm{Nd}(t)}{\mathrm{Nd}_\odot} = (1-s)[\alpha+(1-\alpha)t/t_\odot] + s\,\frac{2\alpha t/t_\odot + (1-\alpha)(t/t_\odot)^2}{1+\alpha}$$

$$k=1: \quad \frac{\mathrm{Nd}(t)}{\mathrm{Nd}_\odot} = (1-s)\left[\alpha + (1-\alpha)\,\frac{x-1/x}{x_\odot - 1/x_\odot}\right]$$

$$+s\,\frac{\frac{1}{2}\alpha(x-1/x) + \frac{1}{3}(1-\alpha)(x^2-3+2/x)/(x_\odot - 1/x_\odot)}{\frac{1}{2}\alpha(x_\odot - 1/x_\odot) + \frac{1}{3}(1-\alpha)(x_\odot^2 - 3 + 2/x_\odot)/(x_\odot - 1/x_\odot)} \tag{37}$$

with higher values of k following the same pattern. This abundance appears in the denominator of the Th/Nd ratio in G-dwarfs if t is identified with t_*, the formation time of each star.

Because it is the Th/Nd ratio being measured it is convenient to form a similar ratio for $\mathrm{Th}/\mathrm{Th}_\odot$

with the aid of equations (19) and (36):

$$\frac{\mathrm{Th}(t_* \to T_\mathrm{G})}{\mathrm{Th}_\odot} = \mathrm{e}^{-\lambda(T_\mathrm{G}-t_\odot)} \frac{[\alpha/(1-\alpha)][\Delta/(k+1)](x_\odot - x_\odot^{-k}) + x^{-k} I_k(t_*, \lambda)}{[\alpha/(1-\alpha)][\Delta/(k+1)](x_\odot - x_\odot^{-k}) + x_\odot^{-k} I_k(t_\odot, \lambda)} \tag{38}$$

where $\lambda = 0.0499\ \mathrm{Gyr}^{-1}$ is the Th decay rate, $\mathrm{Th}(t_* \to T_\mathrm{G})$ is the abundance observed today in a star formed at t_*, and $\mathrm{Th}_\odot$ is, as always in my convention, the *initial* Th abundance in the Sun, related to the value observed today by $\mathrm{Th}_\odot(T_\mathrm{G}) = \mathrm{e}^{-\lambda(T_\mathrm{G}-t_\odot)}\mathrm{Th}_\odot$. The line strength ratio observed is then *proportional* to the normalized ratio $[\mathrm{Th}(t_* \to T_\mathrm{G})/\mathrm{Nd}(t_*)]/(\mathrm{Th}_\odot/\mathrm{Nd}_\odot)$. The quantity thus defined is normalized in such a way that it passes through $\mathrm{e}^{-\lambda(T_\mathrm{G}-t_\odot)} = 0.80$ for other G-dwarfs born at $t_* = t_\odot$ rather than through unity for such stars. This is the quantity that I will display in Figs 5 and 6, but first it is necessary to consider the secondary parameter s.

The decomposition of solar abundances into s-process and r-process portions follows the approach developed by Seeger *et al.* (1965), updated by Käppeler *et al.* (1982), and again by Mathews & Käppeler (1984). The last two of these references have been a part of a continuing Karlsruhe program for defining the s-process more precisely by measuring with accuracy its key cross-sections. By measuring specifically the Nd cross-sections, Mathews & Käppeler (1984) not only provided the numbers needed for this Th/Nd study, but they also cleared up a critical problem for s-process theory that was identified by Clayton (1978) as so acute that he rejected published measurements and calculations of Nd cross-sections in favour of inferred values. With their measurements Mathews & Käppeler resolved the conflict satisfactorily and were able to determine the s-process fraction 0.52 in Solar System material. That is, Nd in the Earth is roughly half s and half r; but it is essential to remember that this is true in the ISM only at $t = t_\odot$. Equations (37) show explicitly that in the class of models displayed, the secondary component rises from zero initially to values greater than solar for stars formed recently. The main question is what fraction of the s-abundance is truly secondary. The simplest case to evaluate is the one used in my preliminary report (Clayton 1987b), the closed ($k=0$) model without initial enrichment ($\alpha=0$) having totally secondary s-process ($s=0.52$), which was utilized because a closed model with $\alpha=0$ was also employed by Butcher (1987) to motivate the meaning of his study, in which case equation (37) gives $\mathrm{Nd}/\mathrm{Nd}_\odot = (1-s)(t/t_\odot) + s(t/t_\odot)^2$, and the ratio of equation (38) to (37) becomes proportional to

$$\frac{\mathrm{Th}(t_* \to T_\mathrm{G})}{\mathrm{Nd}(t_*)} \quad \frac{\mathrm{e}_*^{\lambda t} - 1}{1 - s + s t_*/t_\odot} \qquad (k=0,\ \alpha=0) \tag{39}$$

which fit Butcher's data well, whereas Butcher's (1987) argument omitted the denominator on the belief that s and r processes grow in step. To generalize my results I designate by s' that fraction of the s-process abundance that is secondary (so that $1-s'$ is primary). Then the value of s to be used in equation (37) should be $s = 0.52 s'$.

In Fig. 5 I show how the observed ratio depends on the standard-model parameters in a Galaxy with fixed age $T_\mathrm{G} = 15$ Gyr. These models are freely chosen in the sense that their parameters have not been restricted to achieve a gas mass of 10 per cent or any other astronomical constraint. Most of the curves use $s=0.52$, but closed-model ($k=0$) comparisons for $s=0$, 0.3 and 0.7 are shown to produce a rotating sequence of heavy-dot curves. As mentioned above, all curves pass through 0.80 at $t_* = t_\odot$ ($=10.5$ Gyr for this value of T_G). It is immediately evident that change of the secondary portion s alters the nature of the curves in a way more substantial than the changes between different members of the standard family. For $s=0$ the observed Th/Nd ratio is larger in stars formed recently, which was Butcher's (1987) motivating argument; however, for $s=0.3$ it is almost flat and for $s=0.7$ it is considerably larger in the stars formed early. It is pretty evident then that this method will not provide a true chronological constraint without a precise fix of s.

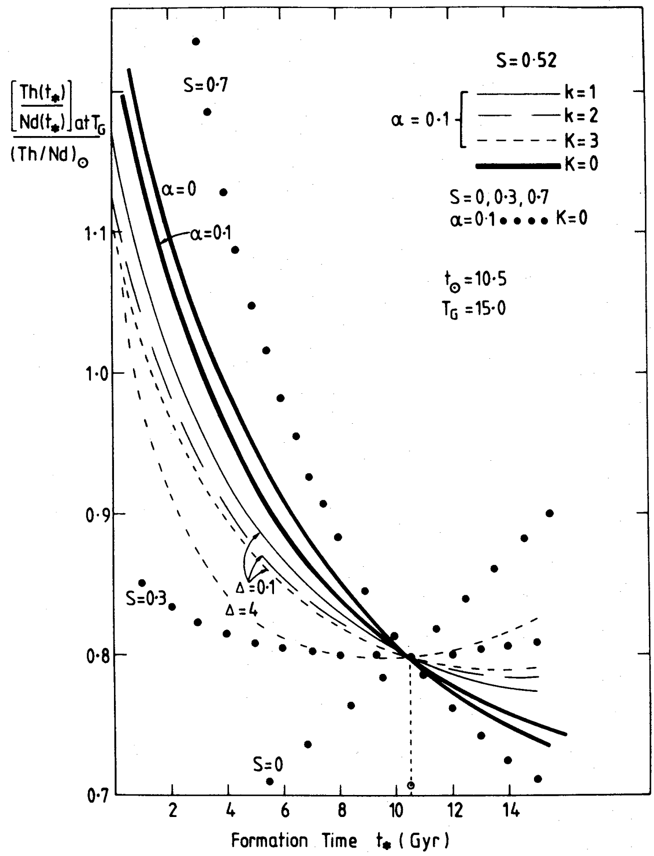

Figure 5. The abundance ratio Th($t_* \rightarrow T_G$)/Nd(t_*) observed today in a star formed at $t=t_*$ as a function of t_* in a Galaxy having a set age $T_G=15$ Gyr. Calculated from the ratio of equation (38) to equation (37), the normalization is defined such that the ratio has the value 0.80 for stars formed at $t_*=t_\odot$. The largest changes in slope belong to different assumptions $s=0$, 0.3, 0.52 and 0.7 (shown as heavy dots) for the secondary fraction of the comparison element (Nd) in the solar composition, showing that parameter to dominate temporal considerations. For the element Nd, $s=0.52$ if s-process has secondary yield. With that value of s several galactic infall models of the standard family are shown along with two closed models. The differences are interesting but small, showing that infall is not the major issue for this chronometer.

Standard-family curves with the single value $s=0.52$ are shown for different infall functions $k=0$, 1, 2 and 3, and for $\Delta=0.1$ and 4 Gyr. There are interesting differences, but it is clear that infall history is not as important as the correct value of s.

Fig. 6 shows Butcher's published data for a Galaxy of age $T_G=20$ Gyr along with three curves: (i) the $\alpha=0, s=0.52$ closed model, which fits the published star ages and line ratios well and which is exactly the curve published in this context by Clayton (1987b); (ii) the $\alpha=0, s=0$ closed model (dashed curve), which is the expectation Butcher (1987) used in his own confrontation with the acceptance of these ages; (iii) a $k=1, \Delta=1$ Gyr standard model with initial metallicity $\alpha=0.1$. This last curve provides a reasonable caricature of other astronomical data (Clayton 1985a), and is seen here to produce an almost exactly flat curve for a Galaxy of this age. Mathews & Schramm (1988) have numerically calculated the near flatness of these curves for galaxies older than 15 Gyr when the s-process is taken to be secondary. In conclusion, therefore, I repeat that the Th/Nd ratio does indeed constitute a chronometer but that its calibration requires very accurate knowledge of the rate of s-process nucleosynthesis, which I have parameterized into secondary (s') and primary ($1-s'$) portions, with $s=0.52s'$.

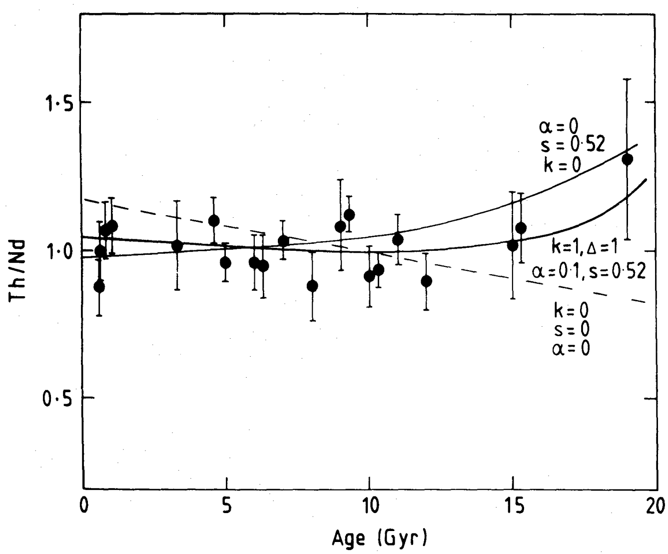

Figure 6. The stellar data for Th/Nd in G dwarfs published by Butcher (1987). The $\alpha=0, s=0.52$ closed model is the one published by Clayton (1987b) to show that the secondary nature of the s-process quite naturally fits these data for old stars in the simplest possible model of chemical evolution, whereas the dashed line is the same galactic model, but with no secondary Nd, advanced by Butcher (1987) as a confrontation with the data. The middle curve is a moderate-infall member of the standard family, showing that the infall slightly flattens an already almost flat curve.

3.1 SIMULATING A PRIMARY *S*-PROCESS

Because the relative temporal growths of s-process and r-process are so important to the Th/Nd cosmochronometer, I consider the possibility that s and r grow together for almost all disc stars, meaning for $Z>0.1\,Z_\odot$. What would this mean?

The data on constant r/Fe in very metal poor stars (e.g. Sneden & Pilachowski 1985; Lambert 1987) seem to me to require a primary r-process, and in massive stars at that. But constancy of the r/s growth would then require that the s-process also be primary. How can that be when the s-process so clearly builds upon initial iron seed? The very early s underabundances may not mean that s is secondary, but only that it occurs in more slowly evolving stars, for example asymptotic giant branch stars, so that the 'primary' s-process yield had not yet occurred when the very first metal poor halo star formed. But if data indicate equal growths over long times, both must be primary.

The key to understanding this possibility may be the following idea. The s-process occurs primarily via the $^{13}C(\alpha,n)$ neutron source rather than the $^{22}Ne(\alpha,n)$ source. This occurs, for example, in AGB stars in a turbulently diffusive intershell region between the hydrogen shell and the ^{12}C-rich ashes of the connective helium zone outside the degenerate carbon core (Sanders 1967; Cowan & Rose 1977). This speculation has such a long history that I will not review it. But what I now point out is that the yield of an s-process nucleus (Zr or Ba) does not fall as the seed metallicity falls because the neutron fluences will be greater, and the greater fluence more than compensates for the diminished seed. The neutron density $n_n=S_n/\Sigma\sigma_A N_A$, where S_n is the rate of the neutron source (say cm^{-3} s^{-1}), σ_A is the (n,γ) cross-section of each neutron absorber of abundance N_A, and the product must be summed. Consider first the more popular ^{22}Ne neutron source. It is secondary, being proportional to initial CNO content, so $S_n \propto Z$. But the neutron absorbers (largely ^{22}Ne itself and its progeny as well as the heavy elements) are also secondary, so $N_A \propto Z$ as well. Thus the metallicity Z cancels, giving n_n independent of Z. But since the Fe seed itself is also proportional to Z, the Ba yield is proportional to Z; i.e. Ba is secondary.

Contrast this with a $^{12}C+p$ mixing source. Take the stellar structure to be independent of Z, as is the intershell ^{12}C content because it is manufactured by the burning within the star. Thus the neutron source S_n is in this case *independent* of initial metallicity Z. But the major absorbers [again $(^{22}Ne+Fe) \propto Z$] are metallicity dependent, and if they dominate the source poisons (^{12}C, ^{13}C, ^{14}N, etc.), as well they should, the neutron density $n_n \propto (Z)^{-1}$. That is, *more metal-poor stars produce larger neutron fluences!*

To see the potential effect of this one need only turn to the first quantitative analysis of the *s*-process. Clayton *et al.* (1961) showed that the Ba yield per initial Fe nucleus ψ_{Ba} (see their equation 37) for a given neutron fluence τ is approximately given by

$$\frac{y(\mathrm{Ba})}{\mathrm{Fe}} \propto \psi_{\mathrm{Ba}} = \frac{\sigma(\mathrm{Ba})N(\mathrm{Ba})}{\mathrm{Fe}(0)} = \lambda \frac{(\lambda\tau)^{m-1}e^{-\lambda\tau}}{\Gamma(m)} \tag{40}$$

where λ and m are parameters that depend upon all prior cross-sections and both are for Ba approximately equal to 20 (see fig. 11 of Clayton *et al.* 1961). It is then elementary to see that doubling the neutron fluence near $\tau \approx 0.5$ ($\times 10^{27}$ n cm^{-2}) results in a 50-fold increase in y(Ba)/Fe, which more than compensates for the factor-of-2 decrease in Fe seed. If the stellar irradiation consisted of that single fluence, low-Z stars would produce more Ba than high-Z stars. However, following Ulrich (1973) we know that these mixing processes lead instead to an exponential distribution of fluences $\varrho(\tau)=e^{-\tau/\tau_0}$. Seeger *et al.* (1965) had already shown that such fluence distributions produce a Ba yield per Fe seed

$$\frac{y(\mathrm{Ba})}{\mathrm{Fe}(0)} \propto (1+1/\lambda\tau_0)^{-m} \tag{41}$$

where, as before, λ and $m \approx 20$. In this case one also confirms that increasing τ_0 from 0.2 to 1.0 (by reducing Z by a factor of about 5) increases the y(Ba) by a factor of 6.5! We now know that the exponential distribution can be done more accurately (Clayton & Ward 1974), but only with a very careful treatment of the neutron competition as a function of initial metallicity. The needed studies will show that n_n cannot go exactly as $1/Z$, because the neutron source has its own absorbers. So although these estimates showing greater Ba yield from lower-Z stars are overly optimistic, they will illustrate the plausibility of maintaining a constant y(Ba) in chemical evolution even though the Ba builds from a secondary (Fe) seed nucleus. So if r/s yields are constant during the growth of metallicity, I suggest that it is due to the effect I describe here. If so it will mark a significant breakthrough in *s*-process theory, requiring, however, many numerical studies of stellar evolution, intershell mixing, and nuclear reaction networks. An equal burden falls on the stellar spectroscopist – to remeasure with the maximum modern sensitivity the Eu/Ba ratio in metal deficient dwarfs.

Since discovering this exciting possibility in chemical evolution, I have undertaken a hurried literature survey and cannot find it addressed. However, the reader will be interested in a very thorough study by Malaney (1986a, b) of the processes under discussion, and particularly of the neutron-liberating reaction networks. Although he does not appear to have noticed the very important consequences for chemical evolution described above, Malaney compares in his table 3 (p. 691) a specific stellar mixing model of Population I metallicity (line 4) with an identical model having ^{22}Ne and Fe, both present as consequences of initial metallicity, decreased by a factor of 10^3 (i.e. essentially absent; line 5). His numerical result in column 7 shows the neutron flux to be increased by a factor of 50 by this reduction! Clearly those iron seeds that are there will be processed rapidly outward, enormously increasing heavy element yields. Nor will the capture path follow the line of stability. In the companion paper, Malaney (1986b) shows in his fig. 8(b) a Zr overabundance of 10^3 after only eight flashes in a model having metallicity 0.1 solar, whereas fig. 7(b) with solar metallicity does not yet even show Zr production after eight flashes. Clearly Malaney has already discovered numerically the nuclear effect that I have here advanced, and it should now assume a role of prominence in the chemical evolution of the Galaxy.

A final remark: all of the above depends upon the ^{13}C production by intershell mixing on the AGB. That must be demonstrated. It must also be demonstrated that the associated neutron source dominates s-process production, at least for $Z<0.5\,Z_\odot$, and that the much celebrated ^{22}Ne neutron source is of secondary importance. An appropriate value for s', and hence s, can be determined from their relative contributions, with the specific $y(\mathrm{Ba}, Z)$ calculated by detailed metallicity dependent comparisons. It makes an exciting prospect.

4 The cosmoradiogenic chronologies

The advantages of the cosmoradiogenic chronologies are substantial. The daughter abundance increases in time rather than decreases. The integration over galactic time places quantitative bedrock under the interpretation of the daughter abundance. The sum of parent and daughter abundance is constrained to have the value the parent would have if it were stable. The relevant production rates are largely measurable, in contrast to the r-process calculations. The half-lives are long enough to be sensitive to the galactic age in its upper range of values. These virtues, dimly perceived at the time, inspired this writer to undertake their definition (Clayton 1964).

In each of the four decay schemes the equations are the same when expressed in terms of the parent's remainder r. At any time in the ISM

parent abundance $= r \times$ (total parent abundance if stable)
daughter abundance $= (1-r) \times$ (total parent abundance if stable).

Taking the ratio (which in practice will be evaluated at $t=t_\odot$ when the solar system formed with its known abundances) one finds

$$\left(\frac{\text{daughter}}{\text{parent}}\right)=\frac{1-r}{r} \tag{42}$$

where the left-hand side is determined at $t=t_\odot$ by measured solar abundances and their decomposition with the aid of nucleosynthesis theory into the cosmoradiogenic component, whereas the right-hand side is a function only of the chronological model of the Galaxy and the decay rate of the nucleus. Thus logically the equation (42) reads

(decomposition of solar abundance) = (chronology model).

What first made this decomposition possible was my preoccupation with the construction of the quantitative details of a theory of the s-process, so that when I noticed that ^{187}Os was too abundant to fit the theory I also understood that the fault lay not in the theory but in a radioactive leak.

The disadvantages of the cosmoradiogenic chronologies have also proved to be substantial. Time has revealed one obstacle after another; but one by one they are yielding to combined experimental and theoretical attack. In what follows I will evaluate the status of these methods and how they fit into the context of the models of chemical evolution described in Section 2.

4.1 COSMORADIOGENIC $^{187}Os_c$

Designating by subscript c the cosmoradiogenic component of the daughter abundance, equation (42) becomes

$$\frac{^{187}\mathrm{Os}_c}{^{187}\mathrm{Re}}=\frac{1-r(187)}{r(187)} \tag{43}$$

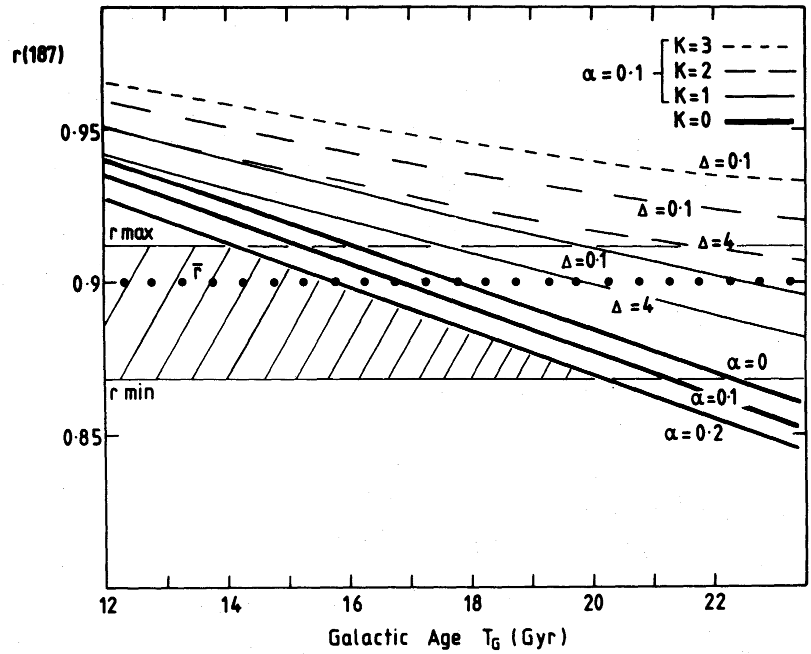

Figure 7. The ^{187}Re remainder at $t=t_{\odot}$ for a wide range of standard-family models as a function of galactic age T_G. Increased infall yields increased galactic age. The horizontal band shows the value of the remainder required by the abundances. If the terrestrial decay rate $\lambda_\beta(187)$ is applicable, the most probable value, $r(187)=0.90$ shown dotted, suggests $T_G=16$–18 Gyr for closed models, but markedly older models with infall. On the other hand, the study by Yokoi *et al.* (1983) seems to indicate a 40 per cent increase in λ_β owing to ionization during stellar residence, in which case the age scale is approximately reduced by $\Delta T_G=-4$ Gyr. Because of the near equality of decay rate, this remainder also applies to ^{87}Rb, another cosmoradiogenic chronometer.

where the remainder for ^{187}Re within the chemical evolution models that I have utilized is given by equation (20) for the standard family and by the ratio of equation (30) to equation (29) for the exponential-infall family. For the ^{187}Re terrestrial decay rate I take $\lambda_\beta(187)=0.0162\,\mathrm{Gyr}^{-1}$ as determined from meteoritic isochrons by Luck, Birck & Allegre (1980). I illustrate the remainder $r(187)$ in Fig. 7 for the standard family only, and in the same line-labelling format employed earlier. For the closed-box ($k=0$) models I show three different values for the initial r-process metallicity α, but for the higher-k members I use only the example $\alpha=0.1$. The curves are uneventful, a slightly diverging envelope sloping linearly (because of the great ^{187} Re lifetime) downward.

To ascertain the meaning one must follow Clayton's (1964) treatment of the left-hand side of equation (43). Because ^{186}Os and ^{187}Os are shielded from r-process production by ^{186}W and by ^{187}Re respectively, only the s-process contributes substantially to their direct production. Because the 'local approximation' of σN equality is probably valid in this mass range to high accuracy (but see Arnould, Takahashi & Yokoi 1984), one writes

$$^{187}\mathrm{Os}_c={}^{187}\mathrm{Os}-{}^{187}\mathrm{Os}_s={}^{187}\mathrm{Os}-{}^{186}\mathrm{Os}\,\sigma(186)/\sigma(187) \qquad (44)$$

where as always each chemical symbol designates the initial solar abundance. With their isochrons Luck & Allegre (1983) determined initial osmium to be $^{187}\mathrm{Os}/^{186}\mathrm{Os}=0.807\pm0.006$. The pursuit of the thermally averaged radiative neutron cross-section ratio called for by Clayton (1964), who guessed 0.4 ± 0.1, has been challenging and eventful. Browne & Berman (1981) measured 0.39 ± 0.03. Following Fowler's astute observation that the (n,γ) cross-section of the first excited state of ^{187}Os at 9.8 keV is also required because it has an even greater population in the star at $kT=30$ keV than the ^{187}Os ground state, Woosley & Fowler (1979) brought the elaborate theoretical tools of the compound-nuclear statistical model to bear on the problem. They calculated that the measured ground-state cross-section should be multiplied by a factor f that they also calculated to be $f=0.82$, but with a range of uncertainty that could reasonably encompass $0.7<f<1.3$. They called attention to inelastic (n, n') scattering measurements on ^{187}Os as a way to learn of the neutron width of the excited state. Winters & Macklin (1982) undertook again the needed measurements, finding that the laboratory ratio of ground-state cross-sections decreases gently from $\sigma(186)/\sigma(187)=0.478\pm0.022$ at $kT=30$ keV to 0.411 ± 0.017 at $kT=10$ keV, and based on their evidence regarding $\sigma(n, n')$ they favoured $f=1$. This much evidence now makes it reasonable to take $f=0.91\pm0.09$. I will continue the practice of taking the 30 keV cross-sections, although the lower temperature could be more appropriate if $^{13}C(\alpha, n)$ is the major source of s-process neutrons. That low temperature would also lower the importance of the ^{187}Os excited state. But for this discussion I take $\sigma(186)/\sigma(187)=0.43\pm0.04$, not significantly different than the recommendation of Winters & Macklin (1982). Inserting these data in equation (44), and taking the abundance ratio $^{187}Re/^{186}Os=3.20$ from Luck *et al.* (1980), allows $r(187)$ to be calculated from equation (43). The most likely value, $\bar{r}=0.90$ is shown as horizontal dots in Fig. 7 within a horizontal band of allowed values.

It is evident that the galactic ages indicated by this reasoning would be large. For the closed models ($k=0$) that I have here argued as being implausible the smallest ages are obtained: $14<T_G<22$ Gyr. Even the mild infall of the $k=1$, $\Delta=4$ Gyr model increases the band to $18<T_G<25$. Thus infall, even early infall, increases the ages inferred from this technique as well. The dilution factor $e^{-\theta(t)}$ severely reduces the importance of the radiogenic $^{187}Os_c$ from the early galactic epochs, thereby diluting one of the advantages of cosmoradiogenic chronologies.

But there are problems also with the ^{187}Re decay rate that still require more attention. Within stars the ion is partially stripped. Its low endpoint energy renders its decay rate potentially sensitive to the state of ionization! I have brought this problem to scientific attention repeatedly: Clayton (1964, most of which was deleted from the published manuscript to shorten it); Clayton (1969), who stressed that $^{187}Re/^{185}Re$ measurements in the solar wind could provide a benchmark that has been influenced by ambient conditions up to severity of the base of the solar convection zone and has integrated over all geologic time; by my graduate student Perrone (1971), whose PhD thesis addressed this topic but was never published, though referred to later by others who used it, because I personally lost confidence in the adequacy of some of his approximations. I none the less encouraged the interest expressed by my Rice University colleague, R. J. Talbot (1973), who used Perrone's rates to evaluate the importance of stellar astration of ^{187}Re, thermally causing a small fraction of it to decay to ^{187}Os within the stellar interiors before they were returned to the interstellar medium, and thereby falsely indicating an excessive age (Clayton 1969). It is necessary to limit such a study to material heated to less than the temperature of neutron liberation, because an s-process-like flux destroys ^{187}Re and harmlessly resets the $^{177}Os/^{186}Os$ s-process ratio. With these ideas Talbot (1973) showed that stellar acceleration of ^{187}Re decay would not be an important factor because the large fraction of interstellar gas cycled through outer layers of massive stars did not spend a sufficient time therein to take advantage of the accelerated rates calculated by Perrone. The situation changed when Takahashi & Yokoi (1983) re-examined the theory of the decay of the ^{187}Re ion and showed that above 10^7 K the non-unique first-forbidden bound-state decay to the first excited state of ^{187}Os becomes energetically available, thereby augmenting the slower first-forbidden-unique ground-state transition studied in this context by Perrone (1971). This important contribution by Takahashi & Yokoi (1983) requires that a very detailed history be given of the material returned from stars, and they collaborated with Arnould on such a project (Yokoi *et al.* 1983), in which they formed a numerical treatment of the chemical evolution of the solar neighbourhood that merits careful reading by students of nuclear cosmochronology. Although the results of their calculation are rather obscurely presented, the net effect of their physical reasoning can be seen by comparing their figs 9 and 10. They describe these figures as presenting the ratio of *s*-process yields $^{187}Os/^{186}Os$ that would be required in order for a given chronological model to be consistent with their treatment of the ^{187}Re decay. I find it more informative to turn this result around by comparing the amounts of cosmoradiogenic $^{187}Os_c$ required for the same $T_G=15$ Gyr model in order to be consistent with those figures. In doing that I find from their results that after 15 Gyr $^{187}Os_c$ is 40 per cent greater in the case with enhanced stellar decay than without it. Since the 15 Gyr models composed are identical, their result is equivalent to stating that $\lambda(187)$ was increased 40 per cent by the stellar acceleration of its decay. That is, the half-life is reduced from 62 to 44 Gyr by their treatment. One approach, therefore, would be to recompute my Fig. 7 with this new half-life; but the effect is easily seen. Since $1-r$ is increased by 40 per cent for r near 0.9, the value of r is decreased by about $\Delta r=-0.04$; i.e. each calculated curve in Fig. 7 can be imagined as lowered by this amount. That has the effect of lowering the ages by roughly 5 Gyr for the closed models ($k=0$), which have the youngest ages, and slightly larger reduction for the models with infall. Another way to see the same result is to increase *every rate* in my time dependent models by 40 per cent, in which case Fig. 7 still applies but the value of $t_\odot$ used in measuring the abscissa must be divided by the factor 1.4; i.e. the value $T_C=18.5$ becomes $T_G=14.5$. This decrease caused Yokoi *et al.* (1983) to conclude that better concordance among models results from including the accelerated decay. My present results cause me to agree, although the uncertainties are still too great to assign any concordant age. However, the proponents of a young Galaxy can take little solace from this uncertainty. The ^{187}Re chronometer cannot be disregarded. The infall still remains a key question, because it makes infall models several Gyr older than the $k=0$ models. Only the closed models allow ages in the 10–12 Gyr range. The $k=1$, $\Delta=1$ Gyr model now indicates $14<T_G<20$ Gyr, with the most probable value $T_G=16$ Gyr. So this trend is in keeping with the U and Th results. And as a final point, I see no reason that the calculation cannot be made even more secure in the future. It is just a lot of hard work, requiring very careful treatment of the stellar interiors and of the log ft value for the non-unique first-forbidden decay of ^{187}Re to $^{187}Os^*$. Very detailed treatment of neutron liberation and capture is needed to delineate that temperature above which neutrons have destroyed the ^{187}Re and $^{187}Os_c$. The reality of intershell mixing in AGB stars leading to a strong ^{13}C neutron source will be important in this regard. And the reduction of ^{187}Os by electron capture must also be included as Arnould (1974) and Arnould *et al.* (1984) argued, even if the ^{187}Re has itself been destroyed by neutrons, which *increases* the true cosmoradiogenic age by reducing the *s*-process yield of ^{187}Os. If analytic linear models are not employed, one must choose specific models and evaluate the full equations of galactic chemical evolution numerically, as Hainebach & Schramm (1977) did for a few specific models and as Yokoi *et al.* (1983) have done. This decay remains one of the best chronometers.

4.2 COSMORADIOGENIC $^{87}Sr_c$

Clayton (1964) pointed out that in principle a ^{87}Rb–^{87}Sr cosmochronometer operates exactly like the Re–Os one. Because the ^{87}Rb half-life is so close to that of ^{187}Re, the curves for the remainder $r(87)$ look almost identical to those for $r(187)$ shown in Fig. 7, so I will not display them here. The difficulty with this cosmochronometer was also pointed out by Clayton (1964) in remarks pessimistic about its eventual utilization: cosmoradiogenic $^{87}Sr_c$ is a much smaller fraction of ^{87}Sr than $^{187}Os_c$ is of ^{187}Os owing to a smaller parent/daughter abundance ratio in the former case, requiring much more accurate cross-sections; and the constancy of σN through the Sr isotopes was not expected to be as good an approximation as it is for Os. Branches in the *s*-process flow through Rb further complicate the latter issue. None the less, following their measurements of capture cross-sections, Beer & Walter (1984) have attempted to perform the subtraction required to determine the amount of $^{87}Sr_c = ^{87}Sr - ^{87}Sr_s$. From their result they estimated $T_G = 13.5 \pm 6$ Gyr. The first utilization is a high achievement, but its chronological relevance must be addressed. Not only is the uncertainty too large to much constrain the true galactic age, but they employed a now traditional caricature of galactic nucleosynthesis that makes the Galaxy appear to be young – the so-called 'exponential models of nucleosynthesis' (Fowler & Hoyle 1960; Clayton 1964; Fowler 1987), which are in reality a postulated exponential distribution of the ages for the Solar System nuclei *with the oldest nuclei enhanced*. That is, they ignore infall and even skew the age spectrum in the other direction, as I discussed in Section 2.3. The models used here would give considerably larger ages than those estimated by Beer & Walter (1984) from the same experimentally determined values for $^{87}Sr_c/^{87}Rb$. They do not themselves explicitly state these values, so I have derived them from their results. The most likely value is $^{87}Sr_c/^{87}Rb = 0.086$, with the range 0.024 to 0.18 being consistent with their estimated 9.5 per cent uncertainty in the cross-section ratio $\sigma(86)/\sigma(87)$. Converting these to remainders for ^{87}Rb yields $0.85 < r(87) < 0.97$, with the most probable value $r(87) = 0.92$. This horizontal band is thus seen to be very comparable to the band for ^{187}Re shown in Fig. 7. The slightly greater lifetime for ^{87}Rb means that the age scale in Fig. 7 should be increased by $\Delta T_G = 1.3$ Gyr for purposes of evaluating the ^{87}Rb remainder. So, for example, the closed ($k=0$) model with no initial enrichment ($\alpha = 0$) can be seen to be in the range $8 < T_G < 25$ Gyr for the ^{87}Rb decay, with the most probably value $T_G = 16$ Gyr. Any past infall lengthens this age estimate in a way illustrated for several models in Fig. 7.

Not only is the inferred age very uncertain, but also very sensitive to the experimentally determined cross-section ratio. Walter & Beer (1985) have now remeasured the crucial ratio to obtain $\sigma(^{86}Sr)/\sigma(^{87}Sr) = 0.740 \pm 0.058$ instead of their previous value 0.68 ± 0.08 (Beer & Walter 1984) and this modest change is sufficient to shift the age to a younger Galaxy. The remainder in Fig. 7 calculated from their newer result allows any value between $1 < r(^{87}Sr) < 0.90$. Thus the age information from this decay has insufficient resolution to be of use, although such attempts to pin it down by even more accurate cross-sections are to be encouraged.

4.3 COSMORADIOGENIC Pb

Equation (42) assumes two representations in the U–Pb system:

$$\frac{^{206}Pb_c}{^{238}U} = \frac{1 - r(238)}{r(238)}, \tag{45a}$$

$$\frac{^{207}Pb_c}{^{235}U} = \frac{1 - r(235)}{r(235)} \tag{45b}$$

with a third expressing $^{208}Pb_c/^{232}Th$ that cannot be utilized in a practical sense. Clayton (1964) introduced their use to cosmochronology and described the decomposition of Pb isotopes along the lines

$$Pb = Pb_s + Pb_r + Pb_c \tag{46}$$

that would be needed to do so. Although equations (45a, b) are independent, Clayton emphasized also that they are constrained by a third relation

$$\frac{y(235)}{y(238)} \left[\frac{r(235)}{r(238)}\right]_{t_\odot} = \left(\frac{^{235}U}{^{238}U}\right)_{t_\odot} = 0.31 \tag{47}$$

that introduces the possibility of a $^{207}Pb/^{206}Pb$ chronology in addition to the two independent ones contained in equations (45a, b). In what follows I will display these relations for the analytic models of chemical evolution that I have introduced for that purpose.

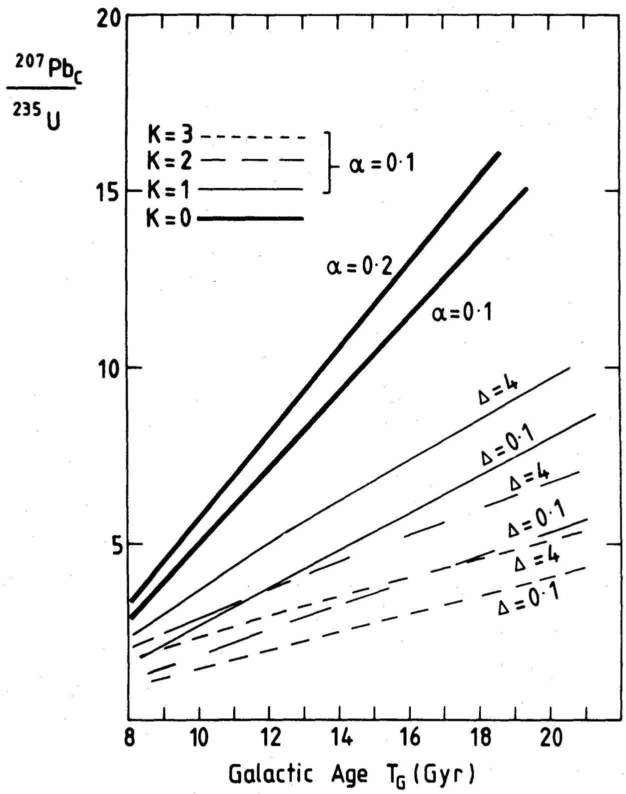

Figure 8. The ratio of cosmoradiogenic $^{207}Pb_c$ to ^{235}U at $t=t_{\odot}$ as a function of T_G. The only credible estimates are those in the text which obtain by one straight subtraction method the values $^{207}Pb_c/^{235}U=21$ and 6.4 for Pb=2.85 and 2.3 respectively. The former value indicates an old Galaxy, but the latter could be as young as 11 Gyr if the solar neighbourhood never experienced metal-poor infall. By a second method, concordant with $^{206}Pb_c$ by virtue of the use of Fig. 9, I obtain $^{207}Pb_c/^{235}U=20$ or 16 for Pb=2.85 or 2.3 respectively, requiring an old Galaxy.

Fig. 8 shows the relation (45b). The analogous figure for equation (45a) is almost identical except for an ordinate scale smaller by a factor of approximately 8. Only the standard family is shown, including the closed-box ($k=0$) models with initial metallicity $\alpha=0.1$ and $\alpha=0.2$ along with the $\Delta=0.1$ and $\Delta=4$ Gyr infall models belonging to $k=1$, 2 and 3. One immediately sees that the closed models carry considerably larger Pb_c/U ratios. This can be immediately understood physically as a result of the dilution of old nuclei in the age spectrum within the Solar System. The initial metallicity $\alpha Z_{\odot}$ carried the largest Pb_c/U ratio into the solar mixture ($\rightarrow\infty$ for $^{207}Pb_c/^{235}U$), but that oldest component along with early disc production is just the part that is most diluted by $e^{-\theta(t)}$. One may notice that in Fig. 8 the curves characterizing infall lie below the closed model, whereas the reverse was true of earlier figures. The difference, of course, is that Pb_c/U increases with galactic time, whereas all remainders decrease with time. That the infall makes the Galaxy look older can be seen by supposing that nuclear physics and nucleosynthesis theory can establish the correct value of $^{207}Pb_c/^{235}U$, say, in which case Fig. 8 demonstrates that the infall models reach that value at larger T_G than do the closed models. Thus the sense of the shift toward older age by a history of metal-poor infall is the same for all chronometers. The fundamental reason is that the infall has reduced the representation of old nuclei in the age spectrum of Solar System nuclei. I will return later to the question of estimating the ratios Pb_c/U, but the reader should first consider once again this reminder: the equations (45a, b) are time-dependent ones, valid at all past times; but what is plotted in Fig. 8 is the value at $t=t_{\odot}$, because the geochemical data is obtained only at $t=t_{\odot}$ (when the Solar System formed).

Now consider formally the last of the Clayton (1964) cosmoradiogenic chronologies,

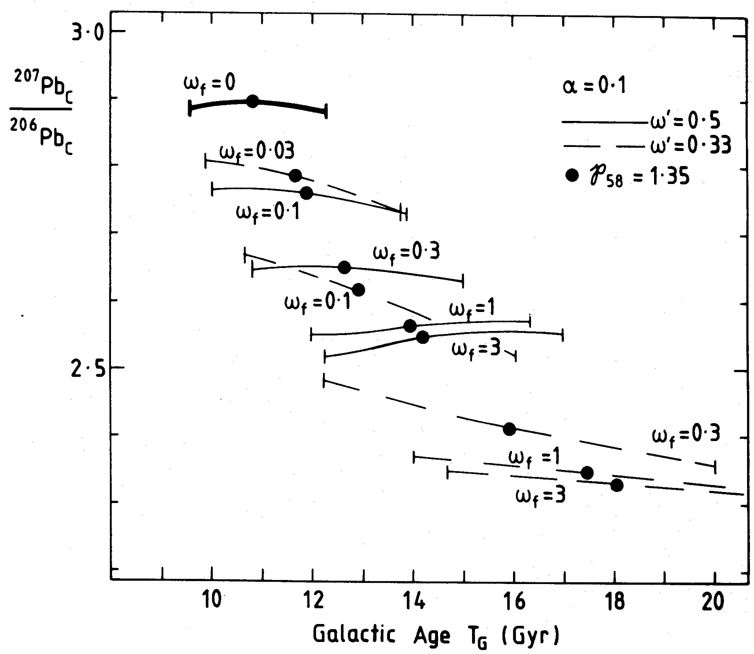

Figure 9. The concordant ratio $^{207}Pb_c/^{206}Pb_c$ for a suite of galactic models having exponential infall. The constraint $^{235}U/^{238}U=0.31$ is built into the calculation. Each model alone gives a fairly constant ratio, but for different ranges of galactic age. The segment of the large unexplicit curve that is consistent with $p_{58}=1.35\pm0.15$ is shown for each model, with the best value shown as a heavy dot. As a family of models, these define a rather smooth inverse relation between $^{207}Pb_c/^{206}Pb_c$ and the galactic age. That concordance is used in the text to derive a value for both cosmoradiogenic abundances and gives $^{207}Pb_c/^{206}Pb_c=2.3$, quite old.

$^{207}Pb_c/^{206}Pb_c$. This ratio, obtainable from the ratio of equation (45b) to (45a) is itself disappointingly flat in time. Even when the constraint of equation (47) is applied, yielding

$$\left[\frac{^{207}Pb_c}{^{206}Pb_c}\right]_{t_\odot}=\frac{1-r_\odot(235)}{1-r_\odot(238)}\,\frac{y(235)}{y(238)}, \tag{48}$$

a rather flat result is still obtained for each galactic model when the yield ratio $y(235)/y(238)=p_{58}$ is chosen for each age T_G to have just that value that gives $(^{235}U/^{238}U)_\odot=0.31$. But the theoretical constraint $p_{58}=1.35\pm0.15$, chosen to be similar to but slightly narrower than Fowler's (1987) recommended value, narrows the acceptable portion of each evolution model to the segments shown in Fig. 9, which is calculated with the exponential-infall family. The large central dot locates $p_{58}=1.35$ within the galactic model designated by $\alpha=0.1$, ω_f and ω' (see Section 2.2); whereas $p_{58}=1.20$ is the left-hand bar and $p_{58}=1.50$ is the right-hand bar. Not only are the loci for each model interesting and different from one another, but there exists a clear trend for the entire family of models, from upper left to lower right (where for clarity the reader should concentrate on a fixed value of p_{58}, say the heavy dots). I will return below to this significant trend as the key to wringing an answer for cosmoradiogenic Pb.

Now turn to abundance constraints that may yield the age. They must be determined from equation (46), requiring nucleosynthesis theory, nuclear data, and natural abundances. I now present a tentative solution to the problem, obtaining an illustrative answer in conclusion.

4.3.1 Pb$_s$

The exponential distribution of neutron fluences $\varrho(\tau)=Ge^{-\tau/\tau_0}$ allows each *s*-process abundance to be calculated exactly (Clayton & Ward 1974; Käppeler *et al.* 1982); however the ongoing determinations of the neutron cross-sections have led to continuous revision of the value of τ_0. But significant for our present purposes is the result that an almost flat σN curve is established between $A \approx 150$ and $A=205$, and that s – only ^{204}Pb must lie on that curve. This argument, with analysis of the branches in neutron current through the Hg–Tl region, has been used by Beer & Macklin (1985) with recent measurements (Horen *et al.* 1984) of $\sigma(204)$ to set the lead abundance at Pb=2.85±0.19 on the usual geochemical scale Si=10^6. The abundance falls between Pb=3.15±0.25 measured in meteorites (Anders & Ebihara 1982) and Pb=2.3±0.5 measured in the solar spectrum (Hauge & Sorli 1973). I will favour 2.85 because I think that argument is the best one; but I note that the age will depend upon this choice. But given that Pb abundance, one concludes for use in equation (46) that ^{206}Pb=0.541, ^{207}Pb=0.587 and ^{208}Pb=1.667.

The next question is the value of $^{206,207}\text{Pb}_s$. The same carefully fitted exponential distribution and branching analysis enabled Beer & Macklin (1985) to determine $^{206}\text{Pb}_s^1=0.290\pm0.022$, $^{207}\text{Pb}_s^1=0.272\pm0.026$ and $^{208}\text{Pb}_s^1=0.809\pm0.204$ with the choice of 30 keV (n, γ) cross-sections (Allen *et al.* 1973; Macklin, Halperin & Winters 1977). Regrettably, the yields are temperature sensitive and will be different if ^{13}C(α, n) is the main neutron source, and the large error on $^{208}\text{Pb}_s^1$ results from the severe difficulty of measuring accurately a cross-section smaller than 10^{-27} cm^2 (Macklin *et al.* 1977). The superscript 1 distinguishes this (first) s-process component from a second one. The need of a second component was discussed by Clayton & Rassbach (1967), on two grounds: (i) the dominance of ^{208}Pb is not adequately accounted for by the exponential distribution; (ii) there should exist a 'strong s-process' of larger fluence that may originate from remixing through more than one pulsating source, or by using some r-process nuclei as seed (doubtful), or by the physical existence of rare sources (~1 per cent of the main source in frequency) that produce much larger fluences. We also produced a major unpublished study (Ward & Clayton 1979) that further elaborated these requirements and that has been useful to cognoscenti of this problem (e.g. Beer & Macklin 1985). These authors argued that a second exponential distribution having a much larger value of τ_0 so that it drives the Pb_s to cycling equilibrium (Clayton & Rassbach 1967) was needed and warranted. It produces the ^{208}Pb-dominant component needed to rationalize the isotopic composition of Pb. The quantitative difficulty is that the large error on $^{208}\text{Pb}_s^1$ appears linearly in the amount of this component needed to account for the remainder of ^{208}Pb. Taking that needed amount to be $^{208}\text{Pb}_s^2=0.69\pm0.25$, Beer & Macklin (1985) calculated $^{206}\text{Pb}_s^2=0.033\pm0.017$ and $^{207}\text{Pb}_s^2=0.044\pm0.023$. I concur with that part of their analysis. Summing these two s-process contributions then yields $^{206}\text{Pb}_s=0.323\pm0.030$ and $^{207}\text{Pb}_s=0.316\pm0.035$ (curiously and circumstantially equal to each other).

What if the Pb abundance is not 2.85? Let Pb=2.85Q, where Q is an abundance parameter with a best value of unity. If one takes the same exponential distribution to fit the σN curve, the values of $^{206,207}\text{Pb}_s^1$ will be unchanged, whereas $^{206,207}\text{Pb}_s^2$ will change owing to a different shortfall of ^{208}Pb. On the other hand, σN for ^{204}Pb would not then lie on the σN curve. I argue instead that taking $Q \neq 1$ must also redefine the appropriate σN level to put ^{204}Pb back on it, in which case both $^{206,207}\text{Pb}_s^{1,2}$ will be proportional to Q. Therefore, I recommend at present that all of the s-process abundances be also taken to be proportional to Q:

$$\text{Pb}=2.85Q, \qquad {}^{206}\text{Pb}_s=(0.323\pm0.030)Q, \qquad {}^{207}\text{Pb}_s=(0.316\pm0.035)Q. \tag{49}$$

4.3.2 Pb_r

Although individual r-process abundances are virtually impossible to calculate, Clayton (1964) argued at some length that the problem is in this case made tractable by the fact that $^{206,207,208}\text{Pb}_r$ are respectively the sums of eight, seven and six transbismuth r-process progenitors. Therefore the individual fluctuations can be expected to equalize in the sum, giving the predictable yield ratio $y_r(206)/y_r(207)=(8/7)\times1.15$ (Clayton 1964), where the extra factor 1.15 was thrown in to account for a weak tendency for even-A yields to exceed those of odd-A by 15 per cent. This odd/even effect is somewhat controversial, and I will discard it below on the basis of experimental evidence in the range $200\leq A\leq205$. (The reader will appreciate that the choices I make here are illustrating the way to a solution rather than to the stronger argument that the answer I obtain is necessarily the correct one.) To illustrate an estimate that is biased as little as possible by theoretical prejudices, we turn again to the decomposition of natural abundances into s and r components (Käppeler *et al.* 1982). With further update of cross-sections measured for Hg, Beer & Macklin (1985) were able to ascertain these r-process residuals in Hg and Tl: $^{199}Hg_r=0.042\pm0.005$, $^{200}Hg_r=0.028\pm0.009$, $^{201}Hg_r=0.025\pm0.004$, $^{202}Hg_r=0.024\pm0.015$, $^{203}Tl_r=0.015\pm0.006$, $^{204}Hg_r=0.021\pm0.002$ and $^{205}Tl_r=0.046\pm0.014$. Interpret the ^{199}Hg result as being large because of its lying on the wing of the $A=191$ r-process peak. Taking the sums of remaining odd-A(3) and even-A(3) abundances yields an average r-yield between $A=200$ and 205 inclusive equal to

$$\bar{N}_r(\text{odd})=0.0287, \qquad \bar{N}_r(\text{even})=0.0243.$$

These values indicate no preference for even A. Because it does not seem generally valid to interpret the slightly larger odd-A abundance as being anything but a statistical fluke, I will tentatively take a flat $N_r=0.0265$ in this region, being the average of the six r abundances known there (no even-A enhancement).

From the remainders $r(232)$ and $r(238)$ calculated in this work it is now possible to assert that the transuranic r-process yields were smaller than those in the $200\leq A\leq205$ region. Because ^{232}Th has six progenitors and ^{238}U has 3.2 (Fowler 1987) it is possible to estimate the average abundance that each progenitor would have had if they were all stable as

$$\bar{N}_r(\text{Th})=\frac{1}{6}\,\frac{^{232}\text{Th}_\odot}{r(232)}, \qquad \bar{N}_r(\text{U})=\frac{1}{3.2}\,\frac{^{238}\text{U}_\odot}{r(238)}.$$

The exact values of the remainders depend upon the detailed model, but no large error is made for this survey to adopt values near $t=12$ in mild infall models, namely $r(232)\approx0.7$ and $r(238)\approx0.4$, giving $\bar{N}_r(\text{Th})\approx0.010$ and $\bar{N}_r(\text{U})\approx0.014$ with the use of the Anders & Ebihara (1982) abundances Th=0.042 and ^{238}U=0.018 for the initial Sun. It seems reasonable therefore to take their average $\bar{N}_r=0.012$ as being the characteristic yield near $A\approx244$ (the centroid of their progenitors). Because this yield is a factor of 2 smaller than between $A=200$ and 205, I take the r-process yield to have declined linearly,

$$\bar{N}_r=0.0265-0.0145\,(A\text{-}204)/40 \tag{50}$$

which gives a linear decline from 0.0265 for the Hg region to 0.012 for average transuranic progenitors. Having chosen that, the sums of eight and of seven progenitors yield $^{206}Pb_r=0.160$ and $^{207}Pb_r=0.150$. For better or for worse, and recognizing in all of this much with which to argue, I take these to be unbiased estimates. I also regard them as independent of the Pb abundance parameter Q.

4.3.3 *Cosmoradiogenic Pb*

For these two key isotopes of Pb equation (46) now reads

$$^{206}Pb={}^{206}Pb_s+{}^{206}Pb_r+{}^{206}Pb_c=0.323Q+0.160+{}^{206}Pb_c=0.541Q,$$

$$^{207}Pb={}^{207}Pb_s+{}^{207}Pb_r+{}^{207}Pb_c=0.316Q+0.150+{}^{207}Pb_c=0.587Q,$$

which can be combined by subtracting (16/15) times the second equation from the first to obtain

$$(16/15)^{207}\text{Pb}_c - {}^{206}\text{Pb}_c = 0.071Q, \tag{51}$$

which ensures that ^{207}Pb$_c$ exceed ^{206}Pb$_c$ by a significant and calculable amount, although here and in what follows I have totally dropped the error propagation in the interest of finding a definite solution. Error propagation is not the best way to understand the uncertainties in such a complicated argument, as I will illustrate below.

If we try to proceed directly from the first set of equations it is easy to see that uncertainty makes a firm conclusion very difficult indeed. Writing the $A=207$ equation, $^{207}\text{Pb}_c=0.271Q-0.150$, one gets formally for the best Pb abundance ($Q=1$) the value $^{207}\text{Pb}_c=0.12$. On Fig. 8 this value would fall at the location $^{207}\text{Pb}_c/^{235}\text{U}=0.12/5.73\times10^{-3}=21$, which lies totally off scale, indicating a Galaxy having $T_G>20$ Gyr even for the closed model. That is an interesting first result favouring an old Galaxy. But notice that if the Pb abundance is 20 per cent smaller (the solar-spectrum value $Q=0.8$) and ^{207}Pb$_r$ is really 20 per cent greater because the yield does not decline as estimated in equation (50), the result would instead be $^{207}\text{Pb}_c/^{235}\text{U}=6.4$. This value falls in Fig. 8 in the realm of a young Galaxy ($T_G\approx11$) for closed models and in the realm $T_G\approx16$ for infall models belonging to the $k=1$ family, etc. The point of that comparison is to me more obvious than formal error propogation; namely, each abundance must be known very well to obtain an age by straight subtraction. This is even more obvious from an attempted result by Beer & Macklin (1985), whose r-process yields I could not agree with, who obtain $^{207}\text{Pb}_c/^{235}\text{U}=39$, which would either indicate a very old Galaxy indeed (*cf.* Fig. 8) or a rather arbitrary model in which the ratio of r-process yield to star formation rate was much larger in the early galactic epochs than during its mid-life (which they unknowingly utilized).

But it is possible to do even better than my subtraction estimate of the previous paragraph. To do so return to Fig. 9 and consider that some specific value of production ratio, say $p_{58}=1.35$, is indeed correct. Fig. 9 then shows that, in rough caricature, the ratio of radiogenic isotopes declines with self-consistent galactic ages encompassing a wide range of infall models approximately as

$$\frac{^{207}\text{Pb}_c}{^{206}\text{Pb}_c} \approx 3.0-0.8\left(\frac{T_G-8}{12}\right). \tag{52}$$

The reader can see that the heavy dots, for example, if fitted by a line, would approximate such a decline. Algebraically it is clear that this condition (which is really just a caricature of the more complete one expressed by Fig. 9) allows equation (51) to be uniquely solved for values of both ^{207}Pb$_c$ and ^{206}Pb$_c$ that stand in an acceptable chronometric ratio. Letting the galactic age parameter $(T_G-8)/12=t'$ generates these results:

$$\frac{^{206}\text{Pb}_c}{^{238}\text{U}} = \frac{3.92Q}{2.2-0.854t'}, \qquad \frac{^{207}\text{Pb}_c}{^{235}\text{U}} = 12.4Q\,\frac{3-0.8t'}{2.2-0.854t'} \tag{53}$$

which may be overlayed conceptually on Fig. 8 and its analog for ^{206}Pb$_c$, because their right-hand sides are also functions of T_G (via t'). These two relations intersect at the allowable age for each model. When this is done with $Q=1$, no solutions are found except for $T_G>20$. Only if the Pb abundance is lowered to the solar value ($Q=0.80$) can a closed model intersection near $T_G=20$ Gyr be found. It is of course concordant in both Pb isotopes.

What has been achieved by this analysis? I took 'the best' unbiased values for the decomposition of Pb and showed that the methods of Clayton (1964) do lead to a unique concordant solution. For the values adopted, that age is old, $T_G\gtrsim20$ Gyr. The bad news is that if all of the uncertainties are introduced, especially the Pb abundance, the ratio and magnitudes of ^{206}Pb$_r$ and

$^{207}Pb_r$, the uranium production ratio p_{58}, and the error limits of the calculation of Pb_s resulting from imperfectly known cross-sections, almost any age can be achieved by certain combinations. *But the method is none the less established.* The errors can continue to be refined. They are similar to and no worse than the errors in the $^{232}Th/^{238}U$ method. This information may one day be regarded as even more convincing than that required for Th/U.

5 Conclusions

This paper has addressed several simultaneous objectives. I now wish to restate them and the contributions made toward these objectives.

Nuclear cosmochronology is not a technique that can stand alone for determining the age of the Galaxy. It must be evaluated within the context of descriptive histories of the chemical evolution of the Galaxy that fit the astronomical facts. In that sense it has now become the province of the astronomer and astrophysicist rather than of the nuclear physicist. I have emphasized this need by displaying nuclear cosmochronology within the context of families of exact analytic solutions of a physical model of the chemical evolution of the solar neighbourhood. I have been motivated by Larson's and Tinsley's research and by my own physical intuition to utilize models in which continuous early accretion of matter has enlarged the total mass in the solar annulus, probably doing so considerably more gradually than the collapse that established the central portion of the disc. Therefore the infall rate $f(t)$ has played a key role in the analytic families that I have presented, and the parameters measuring $f(t)$ are explicitly shown in my formulae and figures. And, indeed, the nuclear ages are very strongly influenced by that infall rate and by the composition of matter in that infalling material. I have used throughout a model in which the metallicity of the infall is equal to the initial metallicity of the disc ($\alpha Z_\odot$), with radioactive concentrations that are exponentially decaying. But I warn the reader that different chronological results may be obtained by a vastly different composition within the infalling material.

My next objective, in keeping with the first, is to render nuclear cosmochronology more accessible to the non-nuclear astrophysicist, who must clarify it with astronomical arguments. Thus the equations and figures that I have presented represent logical relationships of chemical evolution rather than nuclear knowledge. I emphasize the logical concept 'remainder', the ratio of a radioactive abundance in the interstellar gas to the abundance it would have had were it stable. The remainders are functions only of the global features of the chemical evolution of the Galaxy, and do not depend upon nuclear arguments (except for the decay rates and the primary versus secondary question). Therefore the equations and figures are fixed for future use, and will not change as nuclear data changes. The nuclear arguments enter only in trying to specify the appropriate areas of intersection of the curves with relevant nuclear quantities. Thus I emphasize the relationship of the observable nuclear quantities to families of analytic models whose parameters can be chosen by the astronomer in a way that attempts to come to grips with the large astronomical and astrophysical lore concerning the growth and chemical evolution of the solar neighbourhood. The astronomer can confirm my solutions to the equations of chemical evolution and can understand the relations plotted and can relate them to his own studies (e.g. age–metallicity, stellar counts versus metallicity, galactic growth dynamics, etc.) and thereby make profound contributions in an important arena that has been incorrectly regarded as the province of the nuclear astrophysicist.

The next objective of this work was of more immediate relevance to cosmological science. The nuclear age of our Galaxy has been rather emphatically set at $T_G \simeq 10$ Gyr by two important and influential works (Butcher 1987; Fowler 1987). This age datum, as both emphasized, is of overriding importance (matched only by the correct values of the Hubble constant and of the global mass density) to the determination of the nature of the Universe. Therefore I evaluated the existing nuclear data from seven nuclear chronometric techniques to place most probable values on the figures of this paper. In all cases the nuclear techniques are also compatible with an older Galaxy, say $12<T_G<20$. I therefore contend that the conclusions of Butcher and of Fowler cannot be drawn solely from the arguments advanced by them. The weakness in Fowler's calculation lies in its neglect of infall, even modest and early infall, and perhaps in overoptimistic evaluation of the ^{235,238}U, ^{232}Th production ratios. I have gone to some lengths to distinguish between the rate of growth of interstellar metallicity and the age spectrum of Solar System nuclei in order to clarify the chronological importance of that distinction. The weakness in Butcher's argument, on the other hand, lies not in the more modest role that infall plays in it but in the question of simultaneous growth of *s*-process and *r*-process parts of Nd. In the traditional theoretical picture the secondary nature of the *s*-process makes its growth lag that of *r*-process nuclei in a way that very much alters the chronological interpretation. I displayed this dependence in terms of the fraction *s* of Nd that actually is secondary. I also addressed the alternative possibility that Butcher has correctly inferred an exactly simultaneous growth rate for *r* and *s* nuclei from his earlier study (Butcher 1975), and to that end I have presented what I believe to be an original solution to the problem of *s*-process abundances growing as rapidly as those of primary nuclei. If my interpretation is correct, the ^{13}C(α, n) neutron source has been the important one, rather than ^{22}Ne(α, n), for *s*-process nucleosynthesis, calling for many significant theoretical studies of the mixing mechanism and the metallicity dependence of its yields.

Taking all of the nuclear chronological techniques together, I would say that $12<T_G<20$ Gyr is the most unbiased interpretation of the nuclear data alone. Other arguments may, of course, give other answers. Only the ^{238}U/^{232}Th technique suggests a smaller age, and that only if the infall history can be eliminated and if the abundance ratio and production ratio can be more accurately limited than at present. The ^{235}U/^{238}U ratio is not very useful for determining galactic age, except for its important constraint on cosmoradiogenic Pb. The cosmoradiogenic chronologies favour an older Galaxy but still have large uncertainties. The best hope for the future? Continued refinement of the cosmoradiogenic chronometers in conjunction with improved certainty of the correct Th/U abundance ratio (i.e. understanding meteoritics) and of the ^{232}Th/^{238}U production ratio in the *r*-process. Equally important will be improvements in observational and interpretational precision of the Th/Nd ratio and the *r*/*s* ratio versus galactic time. The most important astrophysical prerequisite to radioactive dating is ascertainment of the infall history of the solar neighbourhood, as Tinsley (1977) discovered.

These many uncertainties, vastly and excitingly reduced during the last decade, still plague nuclear cosmochronology even within the simplest idealized model of chemical evolution of the solar neighbourhood: i.e. linear star formation, a well-mixed solar annulus, infall that is continuous and with constant composition. When we consider the much wider range of astrophysical possibilities (e.g. episodic infall with varying composition, incomplete mixing, variation in the ratio of *r*-process yields to one another and to the rate of star formation, metal-enhanced star formation, etc.) it is clear that much remains to be done before the galactic age will be known in this way.

Acknowledgments

I dedicate this paper to those determined leaders in neutron physics, R. L. Macklin, H. Beer and F. Käppeler, who have patiently striven to make the cosmoradiogenic chronologies a reality. I thank Professors Arnold Wolfendale and Richard Ellis of the Physics Department, Durham University, for supporting so enthusiastically my sabbatical leave there in 1987 and for many pointed discussions of the topic of this paper. St Mary's College and the Society of Fellows of Durham University also supported this work by a Research Fellowship. Bernard Pagel, the referee, made many helpful comments to improve this paper, for which I am grateful. This research was also supported in part by the Robert A. Welch Foundation and in part by NASA.

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