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SOLAR MODELS OF LOW NEUTRINO-COUNTING RATE: THE DEPLETED MAXWELLIAN TAIL

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ABSTRACT

We present evolutionary sequences for the Sun which confirm that the ^{37}Cl counting rate will be much reduced if the high-energy tail of the Maxwellian distribution of relative energies is progressively depleted. If the depletion is characterized by exp $[-\delta(E/kT)^2]$, the counting rate falls below 1 SNU for $\delta \geq 10^{-2}$. Observational tests of these ideas are discussed.

Subject headings: interiors, solar — neutrinos — nuclear reactions

I. INTRODUCTION

The evolutionary sequences for the Sun that we report here have been calculated under the assumption that the relative energies of ions are not exactly Maxwellian. Our results confirm in more detail Clayton's (1974) conclusion that neutrino counting rates below 1 SNU[1] can result from a progressive depletion of the high-energy tail of the distribution function of relative energies.

We cannot argue that there should be a deficiency in the number of energetic pairs, because we cannot calculate a physical cause. Clayton (1974) suspected that the problem lies in the realm of many-body physics, i.e., that the long-range Coulomb interaction conspires in some unknown way to quench relative energies above about $15kT$. In that case the quenching will be a smooth and continuous function, and Clayton suggested a correction factor to the distribution function of the form

$$\exp f(E) \approx \exp(\beta_0 - \beta_1 E - \beta_2 E^2 - \cdots) . \qquad (1)$$

On the other hand, Kocharov (1972) has suggested that inelastic collisions may somehow deplete the density of high-energy ions. Bound K-shell electrons of Fe will, for example, be collisionally invisible except for ion energies greater than about 7 keV, which is about $6kT$ near the solar center. Removal of high-energy ions by some selective mechanism can also drag down the number of ions at energies approaching that threshold, since the usual balanced migration upward and downward in energy may be interrupted. If such a "demon" can exist, its effect can probably also be cast in the form of equation (1), which we will adopt explicitly for purposes of testing its effect in solar models. When constructing the models, we ignore the first-order term $-\beta_1 E$ in the Taylor expansion of $f(E)$ for the reasons described by Clayton (1974).

Although we are describing this effect as "depletion of the Maxwellian tail," it should be pointed out that other types of modification of the distribution function could also lower the SNU rate. Clayton (1974) showed that the condition $d^2f/dE^2 < 0$ in the range of energies between $E_0(p, p)$ and $E_0(p, {}^7\mathrm{Be})$ characterizes those distributions that produce a lowered neutrino flux, and he investigated at that time (unpublished) the function $f = \alpha(E/kT)^{1/2}$, which has a negative second derivative. It lowers the SNU rate despite *increasing* the Maxwellian tail, a result it achieves by increasing the *p-p* power at several kT at the expense of fewer low-energy pairs. This effect may, in the long run, seem more reasonable since some many-body effects—namely, plasma effects involving large numbers of particles—seem more likely to influence the density of low-energy pairs. In particular, a physical mechanism that increases the number of pairs at several kT relative to the number of low-energy pairs lowers the SNU rate. One requires in this case an energizer to transport large numbers of low-energy particles to the few-kT region. Nonetheless, for the purposes of this study in solar structure, we will use equation (1), which is most easily interpreted as a depletion of the high-energy tail.

In § II we present a revised evaluation of thermonuclear reaction rates and of the pressure for a distribution function modified by the factor given in equation (1). In § III we present the effect of these results on solar models calculated with a simple Henyey code. In § IV we discuss the long-range possibility of using the Sun as a laboratory for measuring departures of the two-particle distribution from the Maxwellian.

[1] 1 solar neutrino unit (SNU) = 10^{-36} solar neutrino captures per second per target atom.

II. THERMONUCLEAR REACTION RATES AND PRESSURE

The thermonuclear reaction rate can be written (Clayton 1968) as

$$r_{12} = N_1 N_2 \int_0^\infty \sigma v n(E) dE\,, \tag{2}$$

where all quantities have their usual meanings. The distribution $n(E)$ is usually taken to be Maxwellian in the center-of-mass energy E of a pair of particles. A non-Maxwellian distribution may be written

$$n'(E) = n(E) \exp [f(E)]\,, \tag{3}$$

where $f(E)$ must be such as to conserve the particle density;

$$\int n'(E) dE = N = \int n(E) \exp [f(E)] dE\,. \tag{4}$$

Thus when $f(E)$ is approximated by

$$f(E) \approx \beta_0 - \beta_2 E^2 + \cdots\,, \tag{5}$$

the parameter β_0 is a known function of β_2. A related parameter $\delta \equiv \beta_2 (kT)^2$ is a dimensionless measure of the deviation of the distribution from Maxwellian. From equation (4) the required value of β_0 is given by

$$\exp \beta_0 = 1 + \tfrac{15}{4}\delta - 30\delta^2 + \cdots\,. \tag{6}$$

When the cross section is written in the usual form (Clayton 1968)

$$\sigma(E) = \frac{S(E)}{E} \exp (-bE^{-1/2})\,, \tag{7}$$

the reaction rate integral becomes

$$r_{12}' = N_1 N_2 \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty \exp \left[\beta_0 - \delta \left(\frac{E}{kT}\right)^2 - \frac{E}{kT} - bE^{-1/2}\right] dE\,. \tag{8}$$

Evaluation by the method of steepest descents reveals that the integrand has its maximum at the most effective energy;

$$E_0' = E_0 \left(1 + 2\delta \frac{E_0'}{kT}\right)^{-2/3}\,, \tag{9}$$

where E_0 is the usual expression for the most effective energy (Clayton 1968, p. 302), and the reaction rate becomes

$$r_{12}' = r_{12} \left(1 + \frac{15}{4}\delta - \frac{7}{3}\delta \frac{E_0}{kT} + \cdots\right) \exp (-\Delta_{12})\,, \tag{10}$$

where r_{12} is the usual rate and where

$$\Delta_{12} = 3 \frac{E_0}{kT} \left[\left(1 + \frac{5}{3}\delta \frac{E_0'}{kT}\right)\left(1 + 2\delta \frac{E_0'}{kT}\right)^{-2/3} - 1\right]. \tag{11}$$

The pressure is also modified by alteration of the distribution function. The pressure for a nonrelativistic nondegenerate gas is

$$P_g = \tfrac{2}{3} \int_0^\infty E n(E) dE \tag{12}$$

so that reduction of $n(E)$ at the highest energies will somewhat reduce the pressure. The difficulty for the stellar models is that because the basic proposition is ad hoc, it does not indicate whether the altered distribution function is appropriate for the ions only or for both ions and electrons, nor does it guarantee that the laboratory-frame distribution function for the ions is altered in the same manner as was the distribution function for the relative energies. However, if all distribution functions are altered by the same factor, we find

$$P_g' = NkT(1 - 5\delta + 46\delta^2 - \cdots)\,. \tag{13}$$

For the solar-neutrino problem, values of δ near 10^{-2} will be of interest, suggesting pressure errors of up to 5 percent if the electrons are similarly affected but only about 2 percent if only the ions are so affected. In the construction of the solar models, this uncertainty is masked by the basic composition uncertainty within the Sun—namely, the number of particles per gram. We find that the solar neutrino flux is almost insensitive to the influence of δ on the pressure because one need only take a slightly different initial composition in order that the models pass through $L = L_\odot$ at $t = t_\odot$. The models actually reported were computed for the usual $P_g = NkT = \rho RT/\mu$.

III. EVOLUTION OF SOLAR MODELS

A series of evolutionary sequences for 1 $M_\odot$ stars from the zero-age main sequence to $L = L_\odot$ have been calculated using a Henyey-type stellar evolution code with a grid of 70 mass zones (Newman 1975). The equation of state used was the perfect gas law plus radiation pressure, and the opacity law was the fit of Iben and Ehrman (1962) to the Keller and Meyerott opacities. Since we were interested primarily in interior conditions, fitting to a grid of atmospheres was not done; instead, photospheric boundary conditions were applied (Cox and Giuli 1968). The opacity law was cut off as it approached the turnover at the H and He ionization zones, in order that the envelope could conveniently be treated by the same method as the interior. As a result our models are radiative throughout, and of course are not to be taken seriously for conditions in the outer envelope. We confirm, however, that the structure of the interior is but little affected by the errors in the atmosphere, and that the characteristics of our "standard Sun" (the model with $L = L_\odot$ at $t \approx 4.5 \times 10^9$ yr calculated with no deviation from the Maxwellian distribution) differs but slightly from the models of other workers who have used more realistic atmospheres (see, for example, Bahcall, Bahcall, and Ulrich 1969).

We treated the nuclear reactions in some detail in order to trace the changes of neutrino fluxes with changes in δ. The exact nuclear rates used for such a comparative study are not of special importance; however, for purposes of comparison we used the same nuclear rates used by Bahcall and Ulrich (1971). The formulation of the *p-p* chains was as described in Clayton (1968), with ^{3}He considered to be always in equilibrium. Rates for the CNO cycle were computed only for the reactions $^{12}C(p, \gamma)^{13}N$, $^{14}N(p, \gamma)^{15}O$, and $^{16}O(p, \gamma)^{17}F$.

The calculations were begun at $t = 0$ with a model in hydrostatic equilibrium having a uniform composition with ^{12}C, ^{14}N, and ^{16}O initially in the abundance ratio suggested by Cameron (1973). Thus the time ($\sim 10^8$ yr) required for contraction to the main sequence is not included in the ages reported in Table 1 and Figure 1. The evolutionary sequences were calculated for several different compositions and several different values of the depletion parameter δ. Our aim throughout was not to construct the best possible models for the Sun, but rather to obtain a realistic measure of the dependence of the several neutrino fluxes on δ.

Table 1 presents the behavior of several quantities of interest as a function of δ at constant age. The solar luminosity was attained at approximately 4.5×10^9 years for a standard model having Maxwellian distributions ($\delta = 0$) and having initial composition $X = 0.737$ and $Z = 0.0149$. This model results in a counting rate of 5.32 SNU (neglecting about 0.3 SNU from the *pep* reaction, which we chose to ignore for this study). This standard model is very similar to others in the literature. If the tail deviates from Maxwellian according to $\delta = 0.01$, then $X = 0.7466$ is required (holding Z/X constant) to reach the present solar luminosity near the same time; moreover, the neutrino counting rate is reduced to 0.63 SNU (again neglecting *pep*). Thus a change in δ by 0.01 is compensated by a change in X of 0.010 insofar as age versus luminosity is concerned; however, the non-*pep* SNUs have been reduced by a factor of about 8. Most of this reduction has come at the expense of the ^{8}B neutrino flux, which at $\delta = 0.01$ actually contributes even less than ^{7}Be neutrinos to the total counting rate.

The effect of δ on evolution is indicated in the panels of Figure 1. Nonzero δ tends to make a star *more* luminous, with the result that the solar luminosity is reached sooner. This effect comes about because δ *reduces* the rate of energy generation at a given temperature; the solar center therefore contracts to higher temperature to generate sufficient power. The higher central temperature gives a larger temperature gradient and hence a larger luminosity.

Despite the increase of central temperature at $L = L_\odot$, the SNUs decrease with δ. Our proposed explanation of

TABLE 1

CHARACTERISTICS OF THREE SOLAR MODELS WITH $L = L_\odot$ AT 4.5×10^9 YEARS

X	0.737	0.742	0.7466
δ	0	0.005	0.010
T_c (10^6 K)	14.88	15.09	15.40
ρ_c (g cm^{-3})	136.1	145.6	156.6
$\phi(pp)$ (10^{10} cm^{-2} s^{-1})	6.13	6.34	6.43
$\phi(^7Be)$ (10^9 cm^{-2} s^{-1})	3.55	1.85	1.09
$\phi(^8B)$ (10^6 cm^{-2} s^{-1})	3.03	0.67	0.21
$\phi(^{13}N)$ (10^8 cm^{-2} s^{-1})	2.65	1.17	0.96
$\phi(^{15}O)$ (10^8 cm^{-2} s^{-1})	1.81	0.30	0.07
^{37}Cl rate (SNU*)	5.32	1.49	0.63

* The counting rate given does not include the *pep* neutrinos. They were not explicitly calculated, but they increase each entry by about 0.3 SNU.

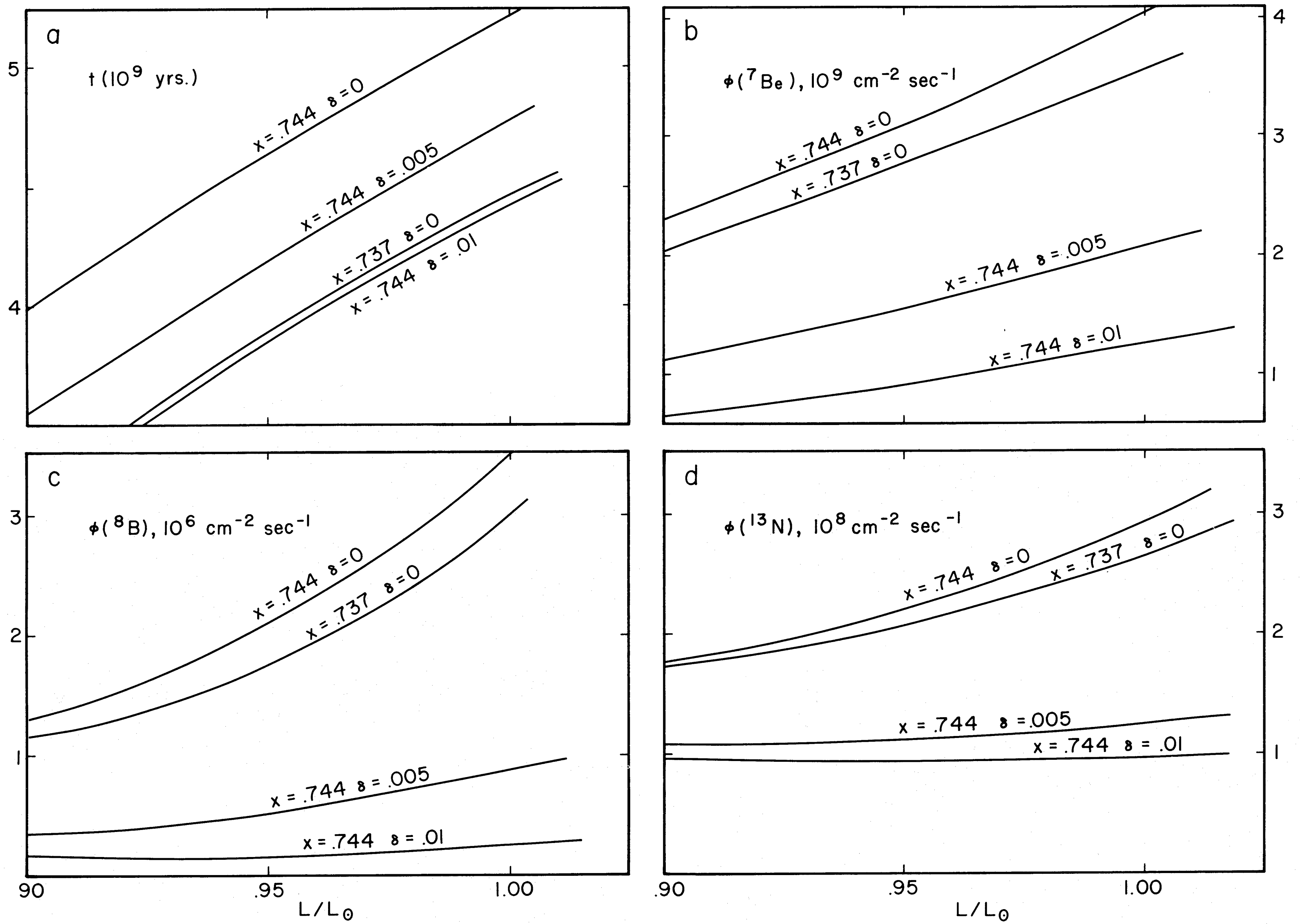

FIG. 1.—(*a*) Time from the zero-age main sequence versus luminosity. Models with nonzero values of the depletion parameter δ evolve in much the same way as those with unperturbed Maxwellian distribution functions. The effect of changing δ here resembles the effect of changes in composition. (*b*) ^{7}Be neutrino flux versus luminosity. As δ is increased the number of PPII and PPIII completions is decreased. The resultant decrease in neutrinos from the ^{7}Be electron capture is reflected by an increase in the flux of "invisible" *p-p* neutrinos. (*c*) ^{8}B neutrino flux versus luminosity. The effect of δ on the high-energy ^{8}B neutrinos is even more dramatic than for ^{7}Be. For $\delta = 0.01$ they are down by more than an order of magnitude near $L_\odot$. (*d*) ^{13}N neutrino flux versus luminosity. The depletion parameter δ does not depress CNO neutrinos as drastically as one might anticipate owing to the activity of the shell where CN equilibrium is being established (see Fig. 3).

the solar neutrino experiment differs from others in this regard, for most other explanations are in fact attempts at *lowering* the central temperature of the present Sun (Ulrich 1974; see, however, Iben 1969 for a counterexample). The flux $\phi(pp)$ increases with δ because a higher percentage of the completions occurs via PPI; i.e. $\phi(pp)$ increases at the expense of $\phi(^7\mathrm{Be})$, $\phi(^8\mathrm{B})$, and $\phi(\mathrm{CNO})$.

IV. MEASURING δ WITH THE SUN

Traditionally astronomy has detected new laws of physics by use of the cosmic laboratory, i.e., by taking advantage of the fact that nature performs experiments that we cannot. In principle, the Sun could measure many-body properties of a dense Coulomb plasma. How could neutrino astronomy measure δ in the Sun? Our work suggests two ways which we will call *neutrino spectroscopy* and *neutrino photography*. These may become practical with future advances in neutrino technology.

a) *Neutrino Spectroscopy*

The special property of δ is the manner in which it shifts the relative yield from different neutrino sources. We show this explicitly in Figure 2 in the form of ratios of fluxes. If neutrinos from the Sun can be positively detected, and if spectroscopy enables their sources to be identified, a test will clearly be possible. If there does exist a deviation from the Maxwellian distribution, and if that deviation can be characterized as we have done it here with the parameter δ, there should only be one value of δ that correctly characterizes the ratios of the separate fluxes. Inspection of Figure 2 shows that the test is very sensitive if the measurements can be made.

b) *Neutrino Photography*

The time dependence and emission structure of the $^{13}\mathrm{N}$ neutrinos is quite interesting. Whereas the other fluxes increase monotonically with time due to the increasing central temperature, $\phi(^{13}\mathrm{N})$ drops precipitously early in the evolution as the primordial $^{12}\mathrm{C}$ is converted to $^{14}\mathrm{N}$ during the establishment of CN equilibrium. Only then does it begin the slow rise characteristic of the other fluxes.

The radial dependence of $^{13}\mathrm{N}$ emissivity is equally interesting. It decreases away from the center due to the temperature gradient and the associated slowdown of the CN cycle. Some distance out, however, $\phi(^{13}\mathrm{N})$ begins to rise again as one encounters the shell region where primordial $^{12}\mathrm{C}$ is still being converted to $^{14}\mathrm{N}$. This emission structure is shown in Figure 3 for three separate values of δ. We note that insofar as $^{13}\mathrm{N}$ is concerned, the usual symbol for the Sun (⊙) is an appropriate one. The outer ring source is not weak, and its size is independent of δ. Of the total $^{13}\mathrm{N}$ neutrinos, the ring contributes 40 percent at $\delta = 0$, 68 percent at $\delta = 0.005$, and 88 percent at $\delta = 0.01$. The central $^{13}\mathrm{N}$ source is the varying one, being practically extinguished at $\delta = 0.01$. The effect of δ on

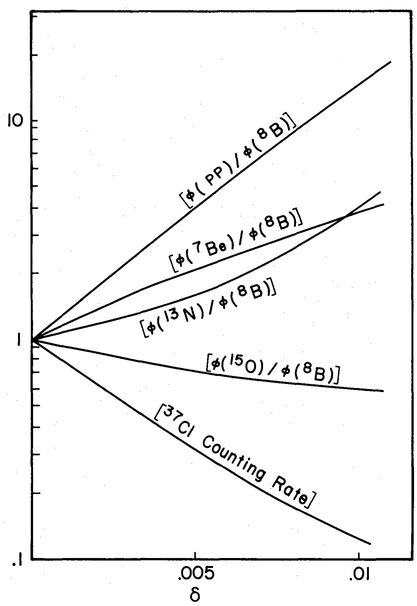

FIG. 2.—The dependence at constant composition ($X = 0.744$, $Z = 0.015$) of several quantities on the depletion parameter δ. Each quantity is normalized to unity at $\delta = 0$. The ratios of neutrino fluxes from different sources indicate the possibilities for measuring δ through neutrino spectroscopy. The predicted counting rate of the $^{37}\mathrm{Cl}$ experiment (normalized) neglects the contribution of *pep*, which is largely independent of δ.

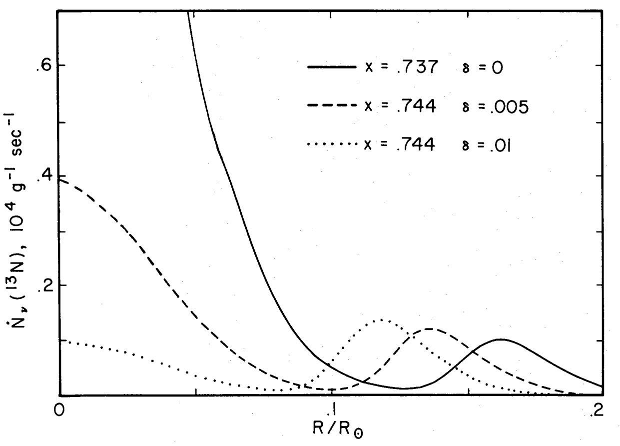

FIG. 3.—Rate of production of ^{13}N neutrinos per unit mass as a function of radius. As δ is increased, the central source is weakened and the shell source moves inward.

the shell source is not to change its strength but to move it inward toward higher temperatures and hence smaller radii. Figure 3 suggests, then, a measurement of δ via neutrino photography of the Sun.

V. CONCLUSIONS

We have shown that a deviation from the Maxwellian distribution of relative energies for energies in excess of $10kT$ can produce marked reduction in the counting rate of the ^{37}Cl experiment (SNUs), while producing only minimal changes in the usual solar models. Those changes that do occur can be compensated for by small changes in the initial helium concentration; such compensations are routinely made in seeking models of one solar mass which reach the present solar luminosity at the solar age. The mechanism does, however, leave its own signature in the energy spectrum of solar neutrinos and in their place of origin. In principle, this mechanism is subject to observational test when neutrino astronomy is sufficiently developed.

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