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D C. Marinescu<br>Clemson University, dcm@clemson.edu

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# Electron-Dephasing Time in a Two-Dimensional Spin-Polarized System with Rashba Spin-Orbit Interaction 

D. C. Marinescu<br>Department of Physics and Astronomy, Clemson University, Clemson, South Carolina 29634, USA

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#### Abstract

We calculate the dephasing time $\tau_{\phi}(B)$ of an electron in a two-dimensional system with a Rashba spinorbit interaction, spin-polarized by an arbitrarily large magnetic field parallel to the layer. $\tau_{\phi}(B)$ is estimated from the logarithmic corrections to the conductivity within a perturbative approach that assumes weak, isotropic disorder scattering. For any value of the magnetic field, the dephasing rate changes with respect to its unpolarized-state value by a universal function whose parameter is $2 E_{Z} / E_{\mathrm{SOI}}$ ( $E_{Z}$ is the Zeeman energy, while $E_{\text {SOI }}$ is the spin-orbit interaction), confirming the experimental report published in Phys. Rev. Lett. 94, 186805 (2005). In the high-field limit, when $2 E_{Z} \gg E_{\text {SOI }}$, the dephasing rate saturates and reaches asymptotically to a value equal to half the spin-relaxation rate.


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In two-dimensional electron gases (2DEG) with spinorbit interaction (SOI), the skew-scattering of the electron spin leaves a trademark fingerprint on the quantum corrections to the conductivity in the form of a positive contribution, the antilocalization term. This results from the SOI mediated coupling of trajectories of electrons with opposite momenta and opposite spins, a configuration aptly named a singlet Cooperon, in the impurity-averaged diffusion equation [1]. The phase difference in the interfering paths of the two electrons is measured by the dephasing time $\tau_{\phi}$.

A magnetic field of intensity $B$ applied parallel to the 2D layer diminishes the spin-scattering effect of SOI by aligning the spins parallel to its direction. In this situation, $\tau_{\phi}(B)$ is expected to depend on two relevant parameters: the Zeeman splitting, proportional with the effective gyromagnetic factor $\gamma, E_{Z}=2 \gamma B$ and the spin-orbit energy, expressed in terms of the spin-relaxation time $\tau_{s}(0)$ [2], $E_{\mathrm{SOI}}=\hbar / \tau_{s}(0)$.

A theory of this effect, developed at low magnetic fields by considering the Zeeman interaction as a perturbation on the electron diffusion equation [1], finds that, for a SOI with linear (Rashba [3]) momentum coupling, $\tau_{\phi}(B)$ is proportional to $\left(E_{Z} / E_{\mathrm{SOI}}\right)^{2}$ [4]. This behavior has been confirmed by several separate experiments [5,6].

The linear $\left(E_{Z} / E_{\mathrm{SOI}}\right)^{2}$ dependence of $\tau_{\phi}(B)$ inferred from the low field estimates has been challenged recently by data obtained by Meijer et al. [7] for intense magnetic fields. When $E_{Z} / E_{\text {SOI }} \gg 1$ the dephasing time saturates, becomes independent of such system parameters as the electron density, Rashba splitting, or the elastic scattering time, and is represented by a universal function of $\left(E_{Z} / E_{\text {SOI }}\right)^{2}$.

Intrigued by these experimental findings, we calculate the dephasing time and study its variation with $E_{Z}$ from a general theory of the localization effects in a spinpolarized 2DEG with Rashba spin-orbit coupling. The first
important result of this Letter is that within the weak, isotropic scattering approximation, $\tau_{\phi}(B)$ is, indeed, a universal function of $\left(2 E_{Z} / E_{\mathrm{SOI}}\right)^{2}$. This dependence, rather than the $\left(E_{Z} / E_{\text {SOI }}\right)^{2}$ parametrization discussed in the experiment, might be explained by the argument that the additional dephasing introduced by the longitudinal magnetic field in the trajectories of the opposite spin electrons that form the singlet Cooperon is such that it preserves the relative orientation of the two spins, making $2 E_{Z}$ (corresponding to two spin flips) the appropriate energy scale. The second important result is that in the high-field limit, the calculated relative change of the dephasing rate in respect to its unpolarized value tends asymptotically to $1 / 2 \tau_{s}(0)$. Of course, at low fields we recover the dephasing rate obtained in Ref. [4].

The object of this study is a 2 DEG , spin-polarized by a magnetic field parallel to the layer, $\vec{B}=B \hat{z}$. The magnetic field determines an imbalance between the number of spins parallel to the field, $n_{\uparrow}$, and those opposite to the field, $n_{\downarrow}$. The degree of spin polarization, $\zeta=\left(n_{\uparrow}-n_{\downarrow}\right) /\left(n_{\uparrow}+n_{\downarrow}\right)$, varies continuously from -1 to 1 as a function of $B$ and is considered a parameter of the problem. The direction of the magnetic field, in the plane of the gas, corresponds to the quantization axis of the electron spin, $\hat{z}$ ( $\hat{x}$ is in the plane, while $\hat{y}$ is perpendicular on the plane). The noninteracting electron energies, in the absence of the spin-orbit interaction, reflect the Zeeman splitting and are written, for an electron of momentum $\vec{p}$, spin $\sigma$, and effective mass $m$ (considered to be spin independent), as $\boldsymbol{\epsilon}_{\vec{p}, \sigma}=p^{2} / 2 m+$ $\gamma \sigma B$, where $\sigma=1$ for a spin parallel to the field and $\sigma=$ -1 for the opposite.

In the presence of the Rashba spin-orbit coupling, the single-electron Hamiltonian is

$$
\begin{equation*}
H_{\mathrm{SOI}}=\frac{\vec{p}^{2}}{2 m}-\frac{\tilde{\lambda}}{\hbar} \vec{\sigma} \cdot(\vec{p} \times \hat{y})+\gamma \vec{\sigma} \cdot \vec{B}, \tag{1}
\end{equation*}
$$

with $\tilde{\lambda}$ the coupling constant of the Rashba interaction, and $\vec{\sigma}$ the Pauli spin operator.

To calculate the dephasing time, we generalize the wellknown theory of localization [8] for a spin-polarized system. By considering the spin polarization as a preexisting condition of the problem, rather than a perturbation in comparison with SOI, as in Ref. [4], we allow an unconstrained relationship between $E_{Z}$ and $E_{\text {SOI }}$ to occur. The logarithmic corrections to the conductivity are generated by the poles of a quantum diffusion equation, or, in an equivalent representation [9], by the eigenvalues of the Cooperon equation. The propagator (or Cooperon) represents the convolution in the momentum space of two single-particle Green functions, averaged over the impurity configuration. To construct the propagator, we start by first obtaining an expression for the single-electron Green function in a spin-polarized electron system with Rashba interaction.

In the absence of SOI, the single-particle Green function in the 2 DEG is represented by a diagonal $2 \times 2$ matrix in spin space:

$$
G^{R, A}(\vec{p}, \omega)=\left(\begin{array}{cc}
G_{\uparrow}^{R, A}(\vec{p}, \omega) & 0  \tag{2}\\
0 & G_{\downarrow}^{R, A}(\vec{p}, \omega)
\end{array}\right),
$$

where $G_{\sigma}^{R, A}(\vec{p}, \omega)=\left(\omega-\hbar^{-1} \epsilon_{\vec{p} \sigma} \pm \frac{i}{2 \tau_{0}}\right)^{-1}$ are the spindependent retarded $(R)$ (with + ) and advanced ( $A$ ) (with - ) components. The impurity scattering is assumed to be isotropic, characterized by a rate $\tau_{0}=2 \pi N_{0} u^{2}$, with $N_{0}=$ $m / 2 \pi \hbar^{2}$ the single spin density of states at the Fermi surface, and $u^{2}$ the mean-square impurity potential, considered spin independent.
The nonaveraged propagator, $P^{0}(\vec{q}, \Omega)$ defined by

$$
\begin{equation*}
P^{0}(\vec{q}, \Omega)=\int d \omega \sum_{\vec{p}} G^{R}(\vec{p}, \omega) G^{A}(-\vec{p}+\vec{q}, \omega+\Omega) \tag{3}
\end{equation*}
$$

involves two Green functions associated with two different spin- $1 / 2$ particles. Its matrix representation occurs in the $S_{1} \otimes S_{2}$ space, where the Pauli spin operators and the identity that act as a basis are, respectively, $\left\{\hat{I}_{1}, \hat{\sigma}_{x, y, z}\right\}$ and $\left\{\hat{I}_{2}, \hat{\rho}_{x, y, z}\right\}$. In this description, $P^{0}(\vec{q}, \Omega)$ becomes:

$$
\begin{align*}
P^{0}(\vec{q}, \Omega)= & \sum_{\sigma}\left[P_{\sigma, \sigma}^{0}(\vec{q}, \Omega) \frac{\hat{I}_{1}+\sigma \hat{\sigma}_{z}}{2} \otimes \frac{\hat{I}_{2}+\sigma \hat{\rho}_{z}}{2}\right. \\
& \left.+P_{\sigma,-\sigma}^{0}(\vec{q}, \Omega) \frac{\hat{I}_{1}+\sigma \hat{\sigma}_{z}}{2} \otimes \frac{\hat{I}_{2}-\sigma \hat{\rho}_{z}}{2}\right] . \tag{4}
\end{align*}
$$

The coefficients $P_{\sigma, \sigma^{\prime}}^{0}(\vec{q}, \Omega)$ are calculated from Eq. (3) for the corresponding pairs of spins in the perturbative approach of the weak scattering approximation, by considering $1 / \tau_{0} \gg$ much larger than all the other frequencies involved:

$$
\begin{gather*}
P_{\sigma \sigma}^{0}=2 \pi N_{0} \tau_{0}\left[1+i \Omega \tau_{0}-D_{\sigma} q^{2} \tau_{0}\right],  \tag{5}\\
P_{\sigma,-\sigma}^{0}=2 \pi N_{0} \tau_{0}\left[1+i(\Omega+2 \sigma \gamma B / \hbar) \tau_{0}-D_{\sigma} q^{2} \tau_{0}\right] . \tag{6}
\end{gather*}
$$

$D_{\sigma}=D_{0}(1+\sigma \zeta)$ are the spin-dependent diffusion coefficients, which in the case of the unpolarized gas become equal to the diffusion coefficient in 2D, $D_{0}=v_{F}^{2} \tau_{0} / 2$, where $v_{F}$ is the Fermi energy.

The introduction of SOI changes substantially this picture because the spin direction is no longer parallel to the applied magnetic field, but rather undergoes a process of randomization induced by the coupling to the orbital motion. A single-electron Green function that incorporates both interactions can be obtained by using the exact eigenfunctions and the corresponding eigenvalues of $H_{\text {SOI }}$. Because of the spin mixing, however, this algorithm is laden with serious mathematical difficulties, especially where the impurity average is concerned.

In a different approach, adopted here, we exploit the linear coupling of the electron momentum to the spin in the Rashba term. This coupling allows the introduction of a spin-dependent vector potential $m \tilde{\lambda}(\vec{\sigma} \times \hat{y}) / \hbar$ that is being used as a generator of a non-Abelian unitary transformation in the spin space to produce the Green's function of the spin-polarized electrons in the presence of the spin-orbit interaction. Such an approximation has been discussed before in connection with a magnetic vector potential [8], with the Aharonov-Casher effect [10], and with SOI in parallel quantum dots [11]. Consequently, we define

$$
\begin{equation*}
G_{\vec{p}, \omega}^{R, A}\left(\vec{r}, \vec{r}^{\prime}\right)=e^{-i \lambda(\vec{\sigma} \times \hat{y}) \cdot \Delta \vec{r}} G^{R, A}(p, \omega) \tag{7}
\end{equation*}
$$

as the single-particle Green function in the presence of SOI, with $\lambda=\tilde{\lambda} m / \hbar$ a reduced Rashba coupling constant. On account of SOI, the new Green function $G$ is not diagonal. Accordingly, the nonaveraged propagator $\mathcal{P}^{0}(\vec{q}, \Omega)$, defined by Eq. (3), becomes a 16 term sum in the vector space $S_{1} \otimes S_{2}$, that reflects the skew-scattering of the electron spins.

Performing the statistical average over the impurity configuration requires the propagator $\mathcal{P}(\vec{q}, \Omega)$ to satisfy:

$$
\begin{equation*}
\mathcal{P}(\vec{q}, \Omega)=u^{2}+u^{2} \mathcal{P}^{0}(q, \Omega) \mathcal{P}(\vec{q}, \Omega) . \tag{8}
\end{equation*}
$$

An elegant solution to Eq. (8) is obtained in a basis of the $S_{1} \otimes S_{2}$ vector space in which the kernel $\mathcal{P}^{0}(\vec{q}, \Omega)$ assumes a diagonal form. The vectors $\Psi_{i}(\vec{r})$ of this basis are found by solving

$$
\begin{equation*}
\int d \vec{r}^{\prime} \mathcal{P}^{0}\left(\vec{r}, \vec{r}^{\prime}\right) \Psi_{i}\left(\vec{r}^{\prime}\right)=\Lambda_{i} \Psi_{i}(\vec{r}), \tag{9}
\end{equation*}
$$

for the corresponding eigenvalues $\Lambda_{i}$. In this representation, $\mathcal{P}^{0}\left(\vec{r}, \vec{r}^{\prime}\right)=\sum_{i} \Lambda_{i}(\vec{q}, \Omega) \Psi_{i}(\vec{r}) \Psi_{i}^{*}\left(\vec{r}^{\prime}\right)$ leading to an immediate solution to Eq. (8):

$$
\begin{equation*}
\mathcal{P}\left(\vec{r}, \vec{r}^{\prime}\right)=\sum_{i} \frac{u^{2}}{1-u^{2} \Lambda_{i}(\vec{q}, \Omega)} \Psi_{i}(\vec{r}) \Psi_{i}^{*}\left(\vec{r}^{\prime}\right) . \tag{10}
\end{equation*}
$$

The logarithmic corrections to the conductivity are then given by the poles of $\mathcal{P}(\vec{q}, \Omega)$, estimated in the limit of small $\vec{q}: \lim _{\vec{q} \rightarrow 0}\left(1-u^{2} \Lambda_{i}\right)$ [8]. The problem is centered,
therefore, on solving the eigenfunction-eigenvalue equation of the nonaveraged propagator.

Equation (9) is linearized by expanding $P_{\sigma \sigma^{\prime}}^{0}$ and $\Psi_{i}\left(\overrightarrow{r^{\prime}}\right)$ up to second order around $\vec{r}$ [8]. The final result involves the Fourier transforms of the propagators in the absence of SOI, $P_{\sigma \sigma^{\prime}}^{0}$, defined in Eq. (6), and their derivatives with respect to $q$. Simultaneously, the eigenfunction $\Psi_{i}\left(\overrightarrow{r^{\prime}}\right)$ is replaced by $\Psi_{i}\left(\vec{r}^{\prime}\right)=\Psi_{i}(\vec{r})+\nabla \Psi_{i}(\vec{r}) \Delta \vec{r}+\frac{1}{2} \nabla^{2} \Psi_{i}(\vec{r}) \times$ $(\Delta \vec{r})^{2} . \Psi_{i}(\vec{r})$ is a spinor in $S_{1} \otimes S_{2}$ whose general form is a linear combination of eigenvectors of $S_{z i}$, in $S_{1} \otimes S_{2}$, $\mid \xi>_{i}$ for spin-up and $\mid \eta>_{i}$ for spin-down $(i=1,2)$ : $\Psi(\vec{r})=e^{\vec{q} \cdot \vec{r}}\left(a\left|\xi>_{1}\right| \xi>_{2}+b\left|\xi>_{1}\right| \eta>_{2}+c \mid \eta>_{1}\right.$
$\left|\xi>_{2}+d\right| \eta>_{1} \mid \eta>_{2}$ ), where $a, b, c, d$ are numerical coefficients. With this test solution, the characteristic equation for the eigenvalues of the propagator is obtained to be

$$
\left|\begin{array}{cccc}
Z_{1 \uparrow} & q_{z} & q_{z} & -\lambda  \tag{11}\\
q_{z} & Z_{0 \uparrow} & -\lambda & q_{z} \\
q_{z} & -\lambda & Z_{0 \downarrow} & q_{z} \\
-\lambda & q_{z} & q_{z} & Z_{1 \downarrow}
\end{array}\right|=0,
$$

where the diagonal coefficients that incorporate the eigenvalues $\Lambda$ are $Z_{1 \sigma}=\left[1+i \Omega \tau_{0}-D_{\sigma} \tau_{0}\left(q^{2}-4 \lambda q_{x}-\right.\right.$ $\left.\left.6 \lambda^{2}\right)-\Lambda\right] / 2 \lambda D_{\sigma} \tau_{0} \quad$ and $\quad Z_{0 \sigma}=[1+i(\Omega+$ $\left.2 \sigma \gamma B / \hbar) \tau_{0}-D_{\sigma} \tau_{0}\left(q^{2}-2 \lambda^{2}\right)-\Lambda\right] / 2 \lambda D_{\sigma} \tau_{0} . q_{x}$ and $q_{z}$ are the corresponding components of $\vec{q}$.

The straightforward evaluation of the determinant leads to a quartic equation in $\Lambda$ (imbedded in $Z_{i \sigma}$ ):

$$
\begin{align*}
\left(Z_{0 \uparrow} Z_{0 \downarrow}-\lambda^{2}\right)\left(Z_{1 \uparrow} Z_{1 \downarrow}-\lambda^{2}\right)= & q_{z}^{2}\left(Z_{1 \uparrow}+Z_{1 \downarrow}+2 \lambda\right) \\
& \times\left(Z_{0 \uparrow}+Z_{0 \downarrow}+2 \lambda\right) \tag{12}
\end{align*}
$$

Even though, in principle, this equation can be solved for any values of $q$, for the problem at hand suffices to obtain its solutions for $q=0$. It is also instructive to investigate the solutions of this equation in the absence of the mag-
netic field. In the relevant limit $\vec{q} \rightarrow 0$, we obtain $Z_{1 \pm}=$ $\pm \lambda$ and $Z_{0 \pm}= \pm \lambda$, which generate the following values for the logarithmic corrections to the conductivity:

$$
\lim _{q \rightarrow 0} \frac{1-u^{2} \Lambda_{i}}{\tau_{0}}=\left\{\begin{array}{l}
-i \Omega+8 \lambda^{2} D_{0}  \tag{13}\\
-i \Omega+4 \lambda^{2} D_{0} \\
-i \Omega+4 \lambda^{2} D_{0} \\
-i \Omega
\end{array}\right.
$$

By using the definitions of the diffusion coefficient and of the reduced Rashba interaction, we identify $4 D_{0} \lambda^{2}=$ $2 \tilde{\lambda}^{2} k_{F}^{2} \tau_{0}$ ( $k_{F}$ is the Fermi momentum) as the Dyakonov spin-relaxation rate, $1 / \tau_{s}(0)$ [2]. This is associated with the SOI induced rate of change of the in-plane spin components: $d S_{i} / d t=-S_{i} / \tau_{i i},(i=x, z)$, where $\tau_{i i}=\tau_{s}(0)$. The rate of change of the perpendicular spin component, $d S_{y} / d t$, is described by $\tau_{y y}=\tau(0) / 2$. The imaginary frequency is replaced by the dephasing rate, $\tau_{\phi}^{-1}(0)$. With these substitutions, we readily regain the traditional form of the conductivity poles expressed in terms of the spinrelaxation times as derived in Ref. [1]. One can also show that the eigenfunction $\Psi(r)$ associated with the eigenvalue $\tau_{\phi}^{-1}(0)$ corresponds to the pairing of two electrons of opposite spins and momenta, i.e., the singlet Cooperon, and generates the antilocalization correction [1].

Inspired by this analysis, we rewrite Eq. (12) in terms of a new set of unknowns, $u_{i, \sigma}=Z_{i, \sigma}+\lambda$, such that

$$
\begin{equation*}
\left[\frac{u_{1 \uparrow} u_{1 \downarrow}}{u_{1 \uparrow}+u_{1 \downarrow}}-\lambda\right]\left[\frac{u_{0 \uparrow} u_{0 \downarrow}}{u_{0 \uparrow}+u_{0 \downarrow}}-\lambda\right]=q_{z}^{2} . \tag{14}
\end{equation*}
$$

In this form, the limit $q \rightarrow 0$ can be taken and the resulting two uncoupled quadratic equations can be solved independently. With $\mu_{1 \sigma}=2 \pi N_{0} \tau_{0}\left[1+i \Omega \tau_{0}-4 \lambda^{2} D_{\sigma} \tau_{0}\right]$ and $\mu_{0 \sigma}=2 \pi N_{0}\left[1+i(\Omega+2 \gamma \sigma B / \hbar) \tau_{0}-2 \lambda^{2} D_{0} \tau_{0}\right]$, the solutions are:

$$
\begin{equation*}
\Lambda_{i, \pm}=\frac{1}{2}\left[\mu_{i \uparrow}+\mu_{i \downarrow}-2 \lambda^{2}\left(D_{\uparrow}+D_{\downarrow}\right) \tau_{0} \pm \sqrt{\left[\left(\mu_{i \uparrow}-\mu_{i \downarrow}\right)-2 \lambda^{2} \tau_{0}^{2}\left(D_{\uparrow}-D_{\downarrow}\right)\right]^{2}+16 \lambda^{4} \tau_{0}^{4} D_{\uparrow} D_{\downarrow}}\right] \tag{15}
\end{equation*}
$$

A quick inspection shows that $1-\left.u^{2} \Lambda_{1, \pm}\right|_{\vec{q}=0} / \tau_{0}$ and $1-\left.u^{2} \Lambda_{0,-}\right|_{\vec{q}=0} / \tau_{0}$ correspond to the first three lines of Eq. (13). Their dependence on the magnetic field is producing a negligible effect on the logarithmic corrections to the conductivity compared with the unpolarized case.

The opposite is true, however, about the effect of the magnetic field on the solution that describes the weak antilocalization,

$$
\begin{equation*}
\lim _{q \rightarrow 0} \frac{1-u^{2} \Lambda_{0,+}}{\tau_{0}}=-i \Omega+2 D_{0} \lambda^{2}-2 D_{0} \lambda^{2} \sqrt{1+2 \zeta\left(\frac{i \gamma B}{\hbar D_{0} \tau_{0} \lambda^{2}}\right)-\left(\frac{\gamma B}{\hbar D_{0} \lambda^{2} \tau_{0}}\right)^{2}} \tag{16}
\end{equation*}
$$

In analogy with the unpolarized case, described by the last line of Eq. (13), a dephasing time $\tau_{\phi}(B)$ is defined as a measure of the antilocalization correction in the presence of a magnetic field:

$$
\begin{equation*}
\frac{1}{\tau_{\phi}(B)}=\frac{1}{\tau_{\phi}(0)}+2 D_{0} \lambda^{2} \mathfrak{R} e\left[1-\sqrt{1+2 \zeta\left(\frac{i \gamma B}{\hbar D_{0} \tau_{0} \lambda^{2}}\right)+\left(\frac{i \gamma B}{\hbar D_{0} \tau_{0} \lambda^{2}}\right)^{2}}\right] \tag{17}
\end{equation*}
$$

( $\mathfrak{R} e$ designates the real part of the expression.)
Following the notations of Ref. [2], we introduce the variation of the dephasing rate from the unpolarized case, $\Gamma_{\phi}^{s}(B)=$ $\left[\tau_{\phi}(B)\right]^{-1}-\left[\tau_{\phi}(0)\right]^{-1}$ and recognize that $\left(\gamma B / \hbar D_{0} \tau_{0} \lambda^{2}\right)^{2}=\left\{4 \gamma B /\left[\hbar / \tau_{s}(0)\right]\right\}^{2}=\left(2 E_{Z} / E_{\text {SOI }}\right)^{2}$. Moreover, since on account of the large electron density and the low effective mass the spin polarization is very small, in the following


FIG. 1 (color online). The calculated dephasing time from Eq. (18) is compared with the experimental data of Meijer et al. [7] for $\zeta=0$.
considerations $\zeta$ is set to 0 . Thus, by extracting the real part of Eq. (17) we obtain that the change of the dephasing time induced by the longitudinal field is simply:

$$
\begin{equation*}
\tau_{s}(0) \Gamma_{\phi}^{s}(B)=\frac{1}{2}\left\{1-\sqrt{\frac{1-\left(\frac{2 E_{\mathrm{Z}}}{E_{\mathrm{SOI}}}\right)^{2}+\left|1-\left(\frac{2 E_{\mathrm{Z}}}{E_{\mathrm{SOI}}}\right)^{2}\right|}{2}}\right\} . \tag{18}
\end{equation*}
$$

We compare the relative dephasing rate give by Eq. (18) with the experimental data in Fig. 1.

The dephasing mechanism suggested by Eq. (18) can be understood by considering that in the presence of the longitudinal magnetic field the down-spin component of the singlet Cooperon is forced to undergo a spin flip. This action is counterbalanced by the spin-orbit coupling which will redirect the spin in some other direction. For the singlet Cooperon to continue to exist and thus contribute to the antilocalization, the dephasing of the original downspin electron trajectory has to be such that, as a result of the spin-orbit coupling, the electron spin reverts to the down position in respect to the direction of the field. The two consecutive spin flips imply the $2 E_{Z}$ energy dependence.

At weak fields, when $\left(2 E_{Z} / E_{\text {SOI }}\right)^{2} \ll 1$, from Eq. (18), we obtain

$$
\begin{equation*}
\tau_{s}(0) \Gamma_{\phi}^{s}(B)=\left(\frac{E_{Z}}{E_{\mathrm{SOI}}}\right)^{2} \tag{19}
\end{equation*}
$$

This is equivalent with $\tau_{\phi}^{-1}(B)-\tau_{\phi}^{-1}(0)=2 \gamma^{2} B^{2} /\left(k_{F}^{2} \tau_{0}^{2}\right)$, the result of Ref. [4]. For $\left(2 E_{Z} / E_{\text {SOI }}\right)^{2} \simeq 1$, Eq. (18) becomes:

$$
\begin{equation*}
\tau_{s}(0) \Gamma_{\phi}^{s}(B)=\frac{1}{2} \frac{\left(\frac{2 E_{Z}}{E_{\mathrm{soI}}}\right)^{2}}{1+\sqrt{1-\left(\frac{2 E_{\mathrm{Z}}}{E_{\mathrm{soI}}}\right)^{2}}} \tag{20}
\end{equation*}
$$

establishing the onset of the saturated behavior, that continues through the high-field limit, when the dephasing rate
is close to $1 / 2 \tau_{s}(0)$ as observed experimentally [7]. In this regime, at a given intensity of the spin-orbit coupling $E_{\mathrm{SOI}}$, there is a maximum value of the longitudinal magnetic field determined by $2 E_{Z \max }=E_{\mathrm{SOI}}$ for which the modifications induced in the direction of one of the spins in the singlet Cooperon can be compensated by the skewscattering produced by SOI, such that the singlet pairing is preserved. Increasing the intensity of the magnetic field beyond this limit leads to the disappearance of the antilocalization correction as the singlet Cooperon configuration is not realized anymore, exactly as observed experimentally [7]. The dephasing of the two trajectories remains constant since it is established solely by the spin-orbit coupling at a value equal to half the spin-relaxation rate along the $z$ direction, $\left[2 \tau_{s}(0)\right]^{-1}, 2 \tau_{s}(0)$ being the amount of time in which the original down-spin flips and then realigns itself with the $\hat{z}$ axis. In the ideal 2 D system modeled here, no surface effects have been considered. Consequently, this calculation does not reproduce the slow slope of the curve registered experimentally [7].

We conclude, therefore, that a general theory of localization effects in spin-polarized 2DEGs explains the saturated universal dependence on $\left(2 E_{Z} / E_{\text {SOI }}\right)^{2}$ of the electrondephasing time, matching the data of Meijer et al. [7].

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