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Tunneling between dissimilar quantum wells: A probe of the energy-dependent quasiparticle lifetime

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Tunneling between two narrow quantum wells with different effective masses is proposed as a probe of the quasiparticle inelastic lifetime at finite excitation energy. Conservation of energy and of \vec{k} , the momentum parallel to the interface, allows the tunneling conductance to be large only if the crossing of the two energy bands [$E_1(\vec{k}) = E_2(\vec{k}) + eV$] at an applied voltage V occurs between the two Fermi levels. The abruptness of the change in tunneling current as this crossing passes through one of the Fermi levels can be used to investigate the lifetimes of the quasiparticle states involved. Results based on the random phase approximation are used as an illustration.

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I. INTRODUCTION

Measurements of the tunneling current that appears between two identical quantum wells under bias V allowed Murphy *et al.*¹ to determine directly the electron inelastic lifetime from the width of the peaks in the differential conductance dI/dV . In the system employed in the experiment (two identical high-mobility GaAs/Al_xGa_{1-x}As quantum wells) the energy and momentum conserving interwell tunneling occurs only when the edges of the minibands in the two wells cross, i.e., when the externally applied electric potential is zero. In this situation the electronic states available for tunneling are those in the vicinity of the Fermi surface, whose broadening under the effect of the Coulomb scattering determines the width of the conductance peak.

In this paper we explore the possibility of tunneling between two quantum wells of different effective electron masses m_1^* and m_2^* . A suitable heterostructure for experimental analysis of this proposal involves two high-mobility quantum wells between which tunneling occurs with conservation of electron momentum and energy. This property has been verified with great accuracy in the case of GaAs/Al_xGa_{1-x}As heterostructures by measuring the tunneling conductance in magnetic fields parallel to the two-dimensional (2D) layers.² The same system serves as prime candidate for the following investigation.

When $m_1^* \neq m_2^*$, the tunneling current is nonzero for an entire range of values of the applied voltage V corresponding to a superposition of occupied states in the first well and empty states in the second well that have the same energy and momentum. The tunneling conductance peaks will occur when eV is equal to a fraction of the Fermi energy, as determined by the ratio of the two masses, $\alpha = m_1^*/m_2^*$. Because the states involved in the tunneling can be quite far from the Fermi energy, the broadening of the conductance peaks will

serve as a test of any theory of electron relaxation at higher energies, where in addition to decay into electron-hole excitations, it is expected that the decay into plasma modes gives an important contribution.

The electronic inelastic lifetime τ_{QE} has been the subject of numerous theoretical papers.³⁻¹² It is well established that within the random phase approximation, the decay into particle-hole excitations generates a relaxation rate proportional to $\Delta^2 \ln \Delta$ when $k_B T \ll \Delta \ll \mu$ and to $T^2 \ln T$ when $\Delta \ll k_B T \ll \mu$. Here Δ is the excitation energy above the Fermi level, while $k_B T$ is the thermal energy. The exact proportionality coefficient at finite temperatures, first estimated in Ref. 10, was a recent subject of debate^{11,12} when the tunneling experiments¹ revealed a numerical discrepancy with the original calculation.

Following the general method outlined in Ref. 10 we estimate the broadening of the tunneling conductance peaks at finite temperatures, by considering both the decay into electron-hole excitations and the decay into plasma modes. These estimates will be useful by comparing them with the data obtained in the possible experiment previously described.

II. TUNNELING CONDUCTANCE

The system under investigation consists of two narrow quantum wells separated by a distance d . In each well, the energy spectrum of the two-dimensional electron gas (2DEG) is described in parabolic band approximation by $E_i(\vec{k}) = \hbar^2 k^2 / 2m_i^* + E_{i0}$, with $i=1,2$, the layer index, and E_{i0} the bottom of the conduction subband in each of the wells. The occupation number of each electronic state corresponds to the Fermi statistics, $n(E) = \{1 + e^{[E - \mu]/k_B T}\}^{-1}$, where μ is the chemical potential.

The tunneling processes between two electronic states,

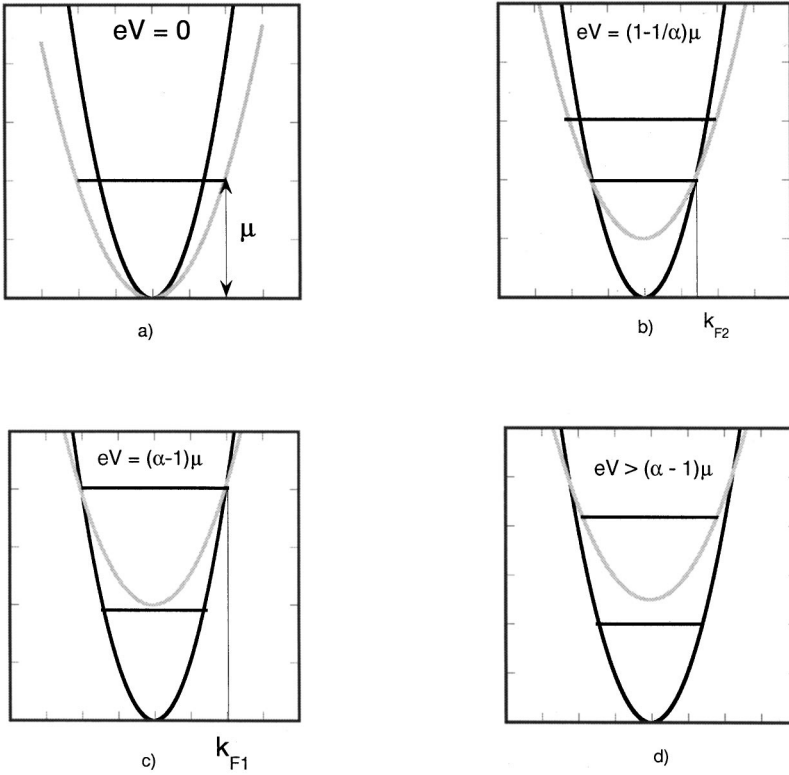


FIG. 1. (a) The one-electron energy in the parabolic approximation in the two wells of electron effective masses m_1^* and m_2^* , $m_1^* > m_2^*$. The same Fermi energy and the same energy origin is assumed in both wells. (b) The tunneling current is abruptly turned on at k_{F2} , when the energy of the heavy electrons (m_1^*) becomes equal to the Fermi energy E_{F2} in the “light” layer. (c) The energy and momentum conserving tunneling occurs for values of the applied electric voltage in the interval $[(1-\alpha^{-1})\mu, (\alpha-1)\mu]$. The upper value corresponds to the two parabolas crossing at the Fermi momentum of the heavy electrons at $k=k_{F1}$. (d) For larger values of the applied voltage, there are no states with the same energy and momentum and the tunneling current is zero.

$\vec{k}, E_1(\vec{k})$ in the first well, and $\vec{p}, E_2(\vec{p})$ in the second well, subject to an electric potential difference V , gives rise to an electric current I expressed in terms of the tunneling probability $|T_{\vec{k}\vec{p}}^-|^2$ and the spectral density of states in each layer, $A^{(i)}[E, E_i(\vec{k})]$, as¹³

$$I = 2e \sum_{\vec{k}, \vec{p}} |T_{\vec{k}\vec{p}}^-|^2 \int_{-\infty}^{\infty} dE A^{(1)}[E, E_1(\vec{k})] \times A^{(2)}[E + eV, E_2(\vec{p})] [n(E) - n(E + eV)]. \quad (1)$$

In Eq. (1) both spin orientations are accounted for by the factor of 2 that multiplies the sum over states. In the absence of any phonon scattering, the momentum of the initial and final tunneling states is conserved and the tunneling matrix element can be written as $|T_{\vec{k}\vec{p}}^-| = T \delta_{\vec{k}\vec{p}}^-$. ($\delta_{\vec{k}\vec{p}}^-$ is the Kronecker symbol.) In the following considerations we will assume that the quasiparticle energies involved are still small enough, such that the tunneling matrix element $T_{\vec{k}\vec{p}}$ can be considered a constant independent of energy.

For wells with equal electron effective masses, the energy and momentum conserving tunneling is realized only at $eV = |E_{20} - E_{10}|$, since application of a finite electric potential will shift the energy subbands in one well relative to the other. In the bilayer system we consider, the ratio of the two effective masses is $\alpha = m_1^*/m_2^* \geq 1$. In equilibrium, the chemical potential is constant throughout the sample as shown in Fig. 1(a) for the case in which $E_{10} = E_{20}$ (which we will assume for the sake of simplicity throughout the rest of this work).

Following this discussion, in Eq. (1), $E_2(k) = \alpha E_1(k)$, and the tunneling current integral is rewritten:

$$I = 2eN_1N_2T^2 \int_{-\infty}^{\infty} dE_1 \int_{-\infty}^{\infty} dE A^{(1)}[E, E_1] \times A^{(2)}[E + eV, \alpha E_1] [n(E) - n(E + eV)]. \quad (2)$$

The spectral function $A(E, \vec{k})$ is related to the imaginary part of the retarded Green function, $A(E, \vec{k}) = -2\text{Im}\mathcal{G}_{\text{ret}}(E, \vec{k})$. Near the quasiparticle peak, the spectral function can be approximated by a Lorentzian,

$$A(E, \vec{k}) = \frac{1}{2\pi} \frac{\Gamma[E(\vec{k}) - \mu, \vec{k}]}{[E - E(\vec{k}) + \mu]^2 + \frac{\{\Gamma[E(\vec{k}) - \mu, \vec{k}]\}^2}{4}}, \quad (3)$$

where $\Gamma[E(\vec{k}) - \mu, \vec{k}] \equiv -2\text{Im}\Sigma_{\text{ret}}[E(\vec{k}) - \mu, \vec{k}]$ is a slowly varying function of energy. This behavior allows one to consider it constant inside the tunneling current integral. In the case of a noninteracting, disorder-free, 2DEG, $\Gamma[E(\vec{k}) - \mu, \vec{k}] \rightarrow 0$ and $A(E, \vec{k}) = -\delta[E - E(\vec{k}) + \mu]$.

The tunneling current turns on abruptly when the applied voltage suffices to raise the energy of the heavy electrons (m_1^*), such that there are occupied states located exactly at the Fermi surface of the light layer ($\mu = \hbar^2 k_{F2}^2 / 2m_2^*$), as described in Fig. 1(b). The conservation of momentum and energy leads to $\hbar^2 k_{F2}^2 / 2m_2^* = \hbar^2 k_{F2}^2 / 2m_1^* + eV$ and the corresponding value of $eV = (1 - \alpha^{-1})\mu$. At values of eV for which the intersection of the two parabolas lies between the two Fermi levels, the current remains approximately constant. For these states, $\hbar^2 k^2 / 2m_2^* = \hbar^2 k^2 / 2m_1^* + eV$. The electrons that tunnel from occupied states in layer 1 into empty states in layer 2, acquire an excitation energy with respect to

the Fermi level, $\Delta_e = \hbar^2 k^2 / 2m_2^* - \mu = \alpha eV / (\alpha - 1) - \mu$, while the holes left behind have an excitation energy of $\Delta_h = \hbar^2 k^2 / 2m_1^* - \mu = eV / (\alpha - 1)$. The current is turned off when the crossing point corresponds to the Fermi momentum of the heavy electron layer ($\mu = \hbar^2 k_{F1}^2 / 2m_1^*$) as in Fig. 1(c). Then, the voltage satisfies $\hbar^2 k_{F1}^2 / 2m_2^* = \hbar^2 k_{F1}^2 / 2m_1^* + eV$, with $eV = (\alpha - 1)\mu$. When eV becomes larger than this critical value as shown in Fig. 1(d), the tunneling current vanishes.

In the simplest approximation—that of a noninteracting 2DEG—by inserting the corresponding δ function dispersion in Eq. (1), the tunneling current is obtained:

$$I = \frac{2eN_1N_2T^2}{\alpha - 1} \left[n\left(\frac{eV}{\alpha - 1}\right) - n\left(\frac{\alpha eV}{\alpha - 1}\right) \right], \quad (4)$$

where N_1 and N_2 are the corresponding two-dimensional densities of states at the Fermi surface, $N_i = m_i^* / \pi \hbar^2$. The limit $\alpha \rightarrow 1$ of Eq. (4) reproduces the result of the tunneling current in identical quantum wells; i.e., tunneling occurs only at $eV = 0$.¹¹ For finite α , the tunneling will occur for all values of the electric potential V for which the difference between the Fermi functions is nonzero. At very low temperatures, when the Fermi distribution can be approximated by a step function, this interval is $[(1 - \alpha^{-1})\mu, (\alpha - 1)\mu]$. When the electron-electron interaction is considered the width of the transition broadens, the broadening being determined by the finite electron relaxation time. The corresponding differential tunneling conductance $G = dI/dV$ is, from Eq. (4),

$$G = \frac{2e^2N_1N_2T^2}{\alpha - 1} \{ \delta[eV - (1 - \alpha^{-1})\mu] - \delta[eV - (\alpha - 1)\mu] \}. \quad (5)$$

The differential conductance peaks occur at those values of the applied voltage where the tunneling current turns on and off. Of course, when $\alpha = 1$, the only peak is centered around $V = 0$. Equation (5) indicates that the first conductance peak, at $eV = \mu(1 - \alpha^{-1})$, corresponds to electrons tunneling from layer 1 to layer 2 at $\Delta_e = 0$ and to holes tunneling from layer 2 to layer 1 at $\Delta_h = -\mu(1 - \alpha^{-1})$. The second conductance peak occurs at $eV = (\alpha - 1)\mu$ and corresponds to the $1 \rightarrow 2$ tunneling of the electrons at $\Delta_e = \mu(\alpha - 1)$ and with the $2 \rightarrow 1$ tunneling of holes at $\Delta_h = 0$.

When a finite spectral dispersion, Eq. (3), is considered in Eq. (2) the expression of the tunneling current becomes

$$I = \frac{eN_1N_2T^2}{2\pi^2} \int_{-\infty}^{\infty} dE_1 \int_{-\infty}^{\infty} dE \times \frac{\Gamma_1\Gamma_2[n(E) - n(E + eV)]}{[(E - E_1 + \mu)^2 + \Gamma_1^2/4][\{E + eV - \alpha E_1 + \mu\}^2 + \Gamma_2^2/4]}. \quad (6)$$

The integral after E_1 , the convolution of the two Lorentzians, generates a new Lorentzian function $(\alpha\Gamma_1 + \Gamma_2)(2\pi)^{-1/2} / \{[(\alpha - 1)E - \mu - eV]^2 + (\alpha\Gamma_1^2 + \Gamma_2^2)/2\}$ whose insertion in Eq. (2) leads to

$$I = \frac{eN_1N_2T^2}{\pi} \int_{-\infty}^{\infty} dE \frac{\alpha\Gamma_1 + \Gamma_2}{[(\alpha - 1)E - \mu - eV]^2 + (\alpha\Gamma_1^2 + \Gamma_2^2)/2} \times [n(E) - n(E + eV)]. \quad (7)$$

With the change of variable under the integral, $x = (\alpha - 1)E - eV$, we obtain

$$I = \frac{eN_1N_2T^2}{\pi(\alpha - 1)} \int_{-\infty}^{\infty} dx \frac{\alpha\Gamma_1 + \Gamma_2}{x^2 + (\alpha\Gamma_1^2 + \Gamma_2^2)/2} \times \left[n\left(\frac{x + eV}{\alpha - 1}\right) - n\left(\frac{x + \alpha eV}{\alpha - 1}\right) \right]. \quad (8)$$

The conductance $G = dI/dV$ is obtained by differentiating with respect to V in Eq. (8),

$$G = \frac{e^2N_1N_2T^2}{(\alpha - 1)} \left\{ \frac{\alpha\Gamma_1 + \Gamma_2}{[eV - (1 - \alpha^{-1})\mu]^2 + (\alpha\Gamma_1^2 + \Gamma_2^2)/2} - \frac{\alpha\Gamma_1 + \Gamma_2}{[eV - (\alpha - 1)\mu]^2 + (\alpha\Gamma_1^2 + \Gamma_2^2)/2} \right\}, \quad (9)$$

where it was considered that the derivative of the Fermi function behaves like a δ function. The widths of the two conductance peaks are determined by $\sqrt{2(\alpha\Gamma_1^2 + \Gamma_2^2)}$. We note that, since Γ_1 and Γ_2 are weak functions of energy, their values in Eq. (9) are those corresponding to the excitation energies at each of the two peaks, respectively. In each layer, the relationship of Γ to the quasielectron and quasihole inelastic lifetimes τ_{QE} and τ_{QH} is

$$\Gamma[E(\vec{k}) - \mu, \vec{k}] = \hbar\tau_{QE}^{-1} + \hbar\tau_{QH}^{-1}, \quad (10)$$

where $n[E(\vec{k})]\tau_{QE}^{-1} = \{1 - n[E(\vec{k})]\}\tau_{QH}^{-1}$.

Since the width of the conductance peaks can be determined experimentally, by comparison, one can verify the accuracy of the theoretical estimates of the various relaxation times as well as the validity of Eq. (10).

III. FINITE ELECTRONIC LIFETIME

As one of the most important problems in condensed matter theory, the electron inelastic lifetime has been studied in both two- and three-dimensional systems for many years.⁴⁻¹² For the problem at hand we will analyze the decay processes of an excited quasielectron that has tunneled from layer 1 into layer 2. The general formalism adopted here is based on the Fermi golden rule, which leads to a probability of decay as a result of the intra- and interlayer interactions with another quasiparticle equal to

$$\frac{1}{\tau_{2QE}(k)} = \frac{2\pi}{\hbar} \sum_{\vec{p}, \vec{q}, \sigma', j=1}^2 n_{\vec{p}, \sigma'} (1 - n_{\vec{p}-\vec{q}, \sigma', j}) (1 - n_{\vec{k}+\vec{q}, \sigma, i}) \times |V_{2j}(\vec{k}, \vec{q})|^2 \delta[E_2(\vec{k}+\vec{q}) + E_j(\vec{p}-\vec{q}) - E_j(\vec{p}) - E_2(\vec{p})], \quad (11)$$

where $|V_{ij}(\vec{p}, \vec{q})|$ is the matrix element of the dynamically screened electron-electron interaction. Its expression can be easily inferred from general electrostatic considerations. When a test charge ρ is embedded in layer 2, the screened potential experienced by an electron in the same layer in the random phase approximation (RPA) reflects self-consistently the effect of the charge and that of the concomitant induced electron density fluctuations in both layers:

$$V_{22} = v(q) [\rho + \Delta n_2 + F \Delta n_1], \quad (12)$$

where $v(q) = 2\pi e^2/q$ is the Fourier transform of the intralayer Coulomb interaction. The interlayer interaction is smaller by a form factor $F(q) = e^{-qd}$. The same test charge determines a screened electron potential in layer 1 equal to

$$V_{21} = v(q) [F(\rho + \Delta n_2) + \Delta n_1]. \quad (13)$$

The induced fluctuations are proportional to the screened electron potentials through polarization functions of the 2D electron gas, χ^0 .¹⁴ Therefore, $\Delta n_i = \chi_i^0 V_{ii}$. Substituting these into Eqs. (12) and (13) leads to a system of equations for V_{22} and V_{21} that can be solved with the following results:

$$V_{22} = v(q) \rho \frac{[1 - v(q)(1 - F^2)\chi_2^0]}{D}, \quad (14)$$

$$V_{21} = v(q) \rho \frac{F}{D}. \quad (15)$$

D is the determinant of the system, given by

$$D = 1 - v\chi_2^0 - v\chi_1^0 + v^2\chi_1^0\chi_2^0(1 - F^2). \quad (16)$$

In the absence of any electronic spin effects, in the RPA, V_{22} and V_{21} are also equal to the effective electron-electron interactions in the corresponding layers.

By making use of the fluctuation dissipation theorem¹⁵ or by direct computation, the sum over \vec{k} and σ' in Eq. (11) can be calculated in terms of the imaginary part of the electron gas polarization, $\chi^0(q, \omega)$.¹⁴ Therefore, one can write ($j = 1, 2$)

$$\sum_{\vec{p}, \sigma'} n_{\vec{p}, \sigma'} (1 - n_{\vec{p}-\vec{q}, \sigma'}) \delta[E_j(\vec{p}-\vec{q}) - E_j(\vec{p}) - \hbar\omega] = - \frac{\hbar \text{Im} \chi^0(\vec{q}, \omega)}{\pi S (1 - e^{-\hbar\omega/k_B T})}, \quad (17)$$

where S is the total surface area of the gas. Here $\hbar\omega = E(\vec{k}-\vec{q}) - E(\vec{k}) = E(\vec{p}) - E(\vec{p}+\vec{q})$, on account of conservation of energy.

Upon the insertion of Eq. (17) into Eq. (11) and integration over all the available transition energies $\hbar\omega$, the relaxation rate becomes

$$\frac{1}{\tau_{2QE}(k)} = \frac{-1}{\hbar S} \sum_{\vec{q}, j=1}^2 \int_{-\infty}^{\infty} d(\hbar\omega) |V_{2j}|^2 \times \frac{1 - n[E_2(\vec{k}) - \hbar\omega]}{1 - e^{-\hbar\omega/k_B T}} \text{Im} \chi_j^0(\vec{q}, \omega) \times \delta\{\hbar\omega - [E_2(\vec{k}) - E_2(\vec{k}+\vec{q})]\}. \quad (18)$$

We introduce $\epsilon_2(\vec{q}, \omega)$ as the dielectric function of the electrons in layer 2, given by

$$\epsilon_2(\vec{q}, \omega) = 1 - v\chi_2^0 - \frac{\chi_1^0 v F^2}{1 - \chi_1^0(1 - F^2)} \quad (19)$$

and write

$$\sum_{j=1}^2 |V_{2j}|^2 \text{Im} \chi_j^0(\vec{q}, \omega) = v \text{Im} \frac{1}{\epsilon_2(\vec{q}, \omega)}. \quad (20)$$

The summation over \vec{q} in Eq. (18) can be transformed into an integral in the usual fashion, $\sum_{\vec{q}} \rightarrow S(2\pi)^{-2} \int_0^\infty q dq \int_0^{2\pi} d\phi_q$. The result of the angular integration is

$$\int_0^{2\pi} d\phi_q \delta\{\hbar\omega - [E_2(\vec{k}) - E_2(\vec{k}+\vec{q})]\} = \begin{cases} \frac{2}{\sqrt{\left(\frac{\hbar^2 q k}{m_2^*}\right)^2 - \left(\hbar\omega + \frac{\hbar^2 q^2}{2m_2^*}\right)^2}}, & \text{for } \left| \hbar\omega + \frac{\hbar^2 q^2}{2m_2^*} \right| \leq \frac{\hbar^2 q k}{m_2^*}, \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

The consecutive steps exposed in Eqs. (19)–(21) lead to the following general equation for the relaxation time of an excited quasielectron (QE) in layer 2:

$$\frac{1}{\tau_{2QE}(k)} = - \frac{2e^2}{\pi \hbar} \int_{-\infty}^{\infty} d(\hbar\omega) \int_{q_{min}}^{q_{max}} dq \frac{\{1 - n[E_2(\vec{k}) - \hbar\omega]\}}{1 - e^{-\hbar\omega/k_B T}} \frac{\text{Im} \frac{1}{\epsilon_2(\vec{q}, \omega)}}{\sqrt{\left(\frac{\hbar^2 q k}{m_2^*}\right)^2 - \left(\hbar\omega + \frac{\hbar^2 q^2}{2m_2^*}\right)^2}}, \quad (22)$$

with the constraint of Eq. (21) which limits the range of q and $\hbar\omega$ integrals. Equation (22) is quite general, applying equally well for two and three dimensions, for all possible decay processes. The particulars for each case are generated by $\text{Im}[1/\epsilon_2(\vec{q}, \omega)]$.

IV. DECAY PROCESSES

The imaginary part of the dielectric function acquires specific values depending on the type of process involved. In the RPA, $\text{Im}[1/\epsilon_2(\vec{q}, \omega)]$ can be divided as

$$\text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right] = \text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right]_{e-h} + \text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right]_{pl}. \quad (23)$$

A. Electron-hole decay

At finite but low temperatures, such that $\Delta \leq k_B T \leq \mu$, the electron-hole decay occurs for small values of q and ω . For a given ω , the wave vector q spans an interval limited by the solutions of Eq. (21):

$$q_{\pm} = k \left(1 \pm \sqrt{1 - \frac{\hbar\omega}{\Delta + \mu}} \right). \quad (24)$$

Excitation of electron-hole pairs takes place in both layers, as reflected by $\text{Im}[1/\epsilon_2(\vec{q}, \omega)]$:

$$\begin{aligned} \text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right]_{e-h} &= \frac{|1 - \chi_1^0 v(1 - F^2)|^2}{|D|^2} \\ &\times \left[\text{Im}\chi_2^0 + \frac{F^2}{|1 - \chi_1^0 v(1 - F^2)|^2} \text{Im}\chi_1^0 \right]. \end{aligned} \quad (25)$$

Since the probability of excitation of electron-hole pairs in layer 1 is diminished by a factor $F^2(q)$, the main contribution comes from electron-hole excitations in the same layer in which the excited quasiparticle exists. A quick inspection of the integral over q in Eq. (22) indicates that for the same layer des-excitation processes, an increased weight carry those that occur for values of q and ω that cancel the square root. Following Fig. 2, we conclude that the regions of the (q, ω) plane involved are those at the limit of the electron-hole continuum, associated with small frequency and small wave vector $q \sim 1$ (forward scattering) and large wave vector $q \sim 2k_{F2}$ (backscattering). In the first instance, $(q/2k_{F2}) \ll (m_2^* \hbar\omega / qk_{F2})$, and the imaginary part of the dielectric function, Eq. (19), can be approximated by¹⁰

$$\text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right]_{e-h} = -\frac{\hbar\omega}{2e^2 k_{F2}} \sqrt{1 - \left(\frac{m_2^* \omega}{\hbar q k_{F2}}\right)^2}. \quad (26)$$

In the opposite limit, when $q/2k_{F2} \sim 1$, the imaginary part of the dielectric function becomes

$$\text{Im}\left[\frac{1}{\epsilon_2(\vec{q}, \omega)}\right]_{e-h} = -\frac{\hbar\omega}{2e^2 k_{F2}} \frac{1}{\sqrt{1 - \left(\frac{q}{2k_{F2}}\right)^2}}. \quad (27)$$

Upon the insertion of Eqs. (26) and (27) into Eq. (22), the integral after q can be performed with the same result in both cases, leading to

$$\begin{aligned} \frac{1}{\tau_{2QE}(k)} \Big|_{e-h} &= \frac{2m_2^*}{\pi \hbar^2 k_{F2}^2} \int_{-\infty}^{\infty} d(\hbar\omega) \left(\omega \ln \left| \frac{4\mu}{\hbar\omega} \right| \right) \\ &\times \frac{\{1 - n[E(\vec{k}) - \hbar\omega]\}}{1 - e^{-\hbar\omega/k_B T}}. \end{aligned} \quad (28)$$

When the excitation energy measured with respect to the Fermi surface, $\Delta = \hbar^2 k^2 / 2m_2^* - \mu$, is introduced in Eq. (28) and the change of variable, $y = \hbar\omega / k_B T$, is performed, the relaxation rate becomes

$$\frac{\hbar}{\mu \tau_{2QE}(\Delta)} \Big|_{e-h} = \frac{1}{\pi} \left(\frac{k_B T}{\mu} \right)^2 \int_{-\infty}^{\infty} dy \frac{y \ln \left| \frac{4\mu}{k_B T y} \right| e^y e^{\Delta/k_B T}}{(e^y + e^{\Delta/k_B T})(e^y - 1)}. \quad (29)$$

At finite temperatures, the excitation energy Δ involved in the electron-hole decay process is much smaller than $k_B T$, much lower than the Fermi energy. The most important contribution to the integral comes from points near the origin, where the $(1 - e^{-y})$ becomes almost zero. Within logarithmic accuracy, in the vicinity of $y=0$, only $y \ln|4\mu/k_B T|$ can be retained inside the integral with the result

$$\begin{aligned} \frac{\hbar}{\mu \tau_{2QE}(\Delta)} \Big|_{e-h} &= -\frac{1}{\pi} \left(\frac{k_B T}{\mu} \right)^2 \ln \left(\frac{k_B T}{4\mu} \right) \\ &\times \int_{-\infty}^{\infty} dy \frac{y e^y e^{\Delta/k_B T}}{(e^y + e^{\Delta/k_B T})(e^y - 1)}. \end{aligned} \quad (30)$$

The integral over y can be integrated exactly with the result¹⁶

$$\frac{\hbar}{\mu \tau_{2QE}(\Delta)} \Big|_{e-h} = -\frac{1}{2\pi} \left(\frac{k_B T}{\mu} \right)^2 \ln \left(\frac{k_B T}{\mu} \right) \frac{\pi^2 + (\Delta/k_B T)^2}{1 + e^{-\Delta/k_B T}}. \quad (31)$$

At the Fermi surface, when $\Delta=0$, τ_{2QE} becomes

$$\frac{\hbar}{\mu \tau_{2QE}(0)} \Big|_{e-h} = -\frac{\pi}{4} \left(\frac{k_B T}{\mu} \right)^2 \ln \left(\frac{k_B T}{\mu} \right). \quad (32)$$

In the limit $\Delta \ll k_B T$, Eq. (32) generates a finite width of the conductivity peak equal to

$$\Gamma_{2e-h} = \frac{2\hbar}{\tau_{2QE}|_{e-h}} = -\mu \frac{\pi}{2} \left(\frac{k_B T}{\mu} \right)^2 \ln \left(\frac{k_B T}{\mu} \right), \quad (33)$$

a result also obtained in Ref. 11.

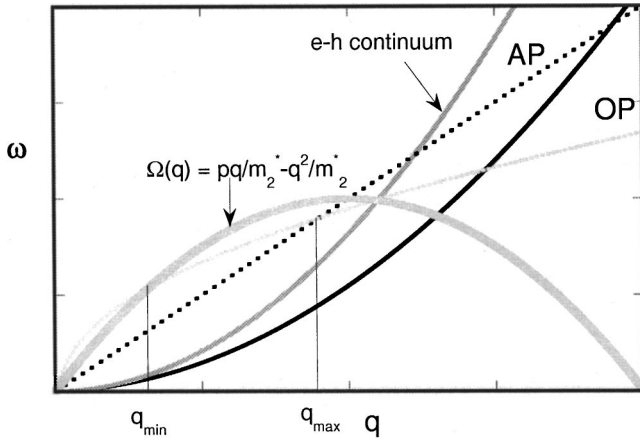


FIG. 2. The $(\omega-q)$ plane of the bilayer system. Electron pairs can be excited in the electron-hole continuum common to both layers or only in the electron-hole continuum of layer 2. The optical (OP) and acoustical (AP) plasmon branches are represented outside the electron-hole continuum. The available decay states are limited by the solid angle integration in q space, described by the solid thick line.

For an arbitrary excitation energy, one should use the result of the numerical evaluation of Eq. (29) in Eq. (10) to determine Γ . Figure 3 shows $\tau_{QE}|_{e-h}$ for $\Delta=0$, obtained both by direct numerical integration and by logarithmic approximation, and for $\Delta=0.5\mu$ and $\Delta=\mu$.

B. Decay into plasma modes

The plasma modes are obtained at frequencies which cancel the real part of the dielectric function, Eq. (19), when simultaneously its imaginary part is also zero. When the distance between the wells is large, each layer exhibits independent plasma oscillations whose small wave vector limit is approximated by $\omega_{pi}^2 \approx 2\pi n_i e^2 q / m_i^*$.¹⁴ The contribution of the single-layer plasmon decay is identical to that studied in the case of a single 2DEG and the results obtained in Ref. 17 are expected to apply here.

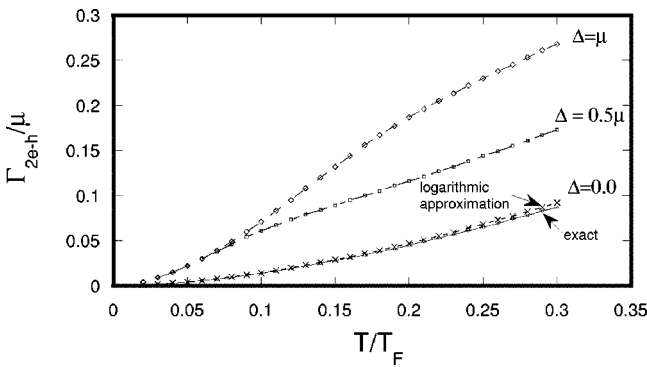


FIG. 3. The relaxation rate for the electron-hole pair decay is presented as a function of temperature at $\Delta=0$, $\Delta=0.5\mu$ and $\Delta=\mu$. For $\Delta=0$, we show by comparison the results of the exact numerical integration and the result of the logarithmic approximation.

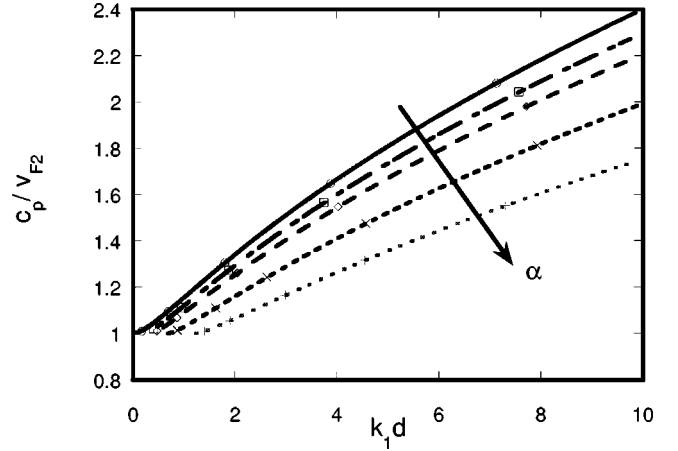


FIG. 4. The group velocity of the acoustic plasma modes of the bilayer system is calculated numerically for values of $m_1^*/m_2^* = \alpha$ ranging from 1.0 to 2.0 as a function of $k_1 d$, the product of the Thomas-Fermi wave vector in the first layer ($k_1 = 2e^2 m_1^*/\hbar^2$) and the interlayer distance.

When the distance between layers is small and the form factor F becomes larger, the interlayer interaction couples the plasmon modes in the two layers. The result is the existence of two orthogonal plasma oscillations that correspond to the linear superposition of the in-phase and out-of-phase density fluctuations.^{18,19} The salient features of this phenomenon can be extracted by investigating the zeros of the dielectric function of the bilayer system, Eq. (19). The condition $\text{Re}\epsilon_2=0$, along with $\text{Im}\chi_{1,2}^0=0$, leads to the following equation:

$$1 - v(\chi_1^0 + \chi_2^0) + v^2 \chi_1^0 \chi_2^0 (1 - F^2) = 0. \quad (34)$$

In the small- q limit, it is expected that the frequency of the symmetric mode satisfies $\omega_+ \gg qv_F$. In this case, in Eq. (34) the asymptotic expressions of the polarization functions can be used: $\chi_i v = \omega_{ip}^2 / \omega^2$. This mode, often called an optical plasmon (OP), has a dispersion law similar to a single-electron layer, $\omega_+ = \sqrt{\gamma_+} q$, and represents the excitation frequency of the in-phase density fluctuations in the two layers. The constant γ_+ is readily obtained to be

$$\gamma_+ = \frac{2\pi n_1 e^2}{m_1^*} + \frac{2\pi n_2 e^2}{m_2^*} = \frac{4\pi n_1 e^2}{m_1^*}, \quad (35)$$

where the same Fermi level was considered in the two wells.

1. Decay into acoustic plasmons

The out-of-phase density mode is an acoustic plasmon (AP), $\omega_- = c_p q$. Following Ref. 19, we determine numerically the group velocity for different values of α , as shown in Fig. 4. The undamped propagation of the AP mode outside of the electron-hole continuum is realized when c_p is larger than $v_{F2} = \max(v_{F1}, v_{F2})$ as indicated by analysis of the $\omega-q$ plane, Fig. 2. Experimentally, this situation can always be achieved when the distance between the layers is made larger than a critical value.¹⁹

The contribution of the plasmon modes to the relaxation processes is strongly limited by three factors. First, AP modes exist outside the electron-hole continuum when

$$\omega(q) < \frac{\hbar k_{F2} q}{m_2^*} + \frac{\hbar q^2}{2m_2^*}. \quad (36)$$

Here, the right-hand side corresponds to $\min[\hbar k_{F1} q/m_1^* + \hbar q^2/2m_1^*; \hbar k_{F2} q/m_2^* + \hbar q^2/2m_2^*]$. Second, the solid angle integration in q space imposes, for the layer in which the excited quasiparticle resides (layer 2 in our problem),

$$\left| \omega(q) + \frac{\hbar q^2}{2m_2^*} \right| < \frac{\hbar p q}{m_2^*}. \quad (37)$$

At $T=0$ K a third condition is imposed by the maximum excitation energy available, Δ ; hence, $\omega(q) < \Delta$. Satisfying Eqs. (36) and (37) simultaneously at finite temperatures limits the integration after q in Eq. (22) to a finite interval $[0, q_{max}]$, as seen in Fig. 2. Here q_{max} is determined from a simple analysis to be dependent on the energy of the excited quasiparticle, $\Delta = \hbar k^2/2m_2^* - \mu$:

$$q_{max} = \begin{cases} 0, & \text{for } \Delta < \mu \left[\left(\frac{c_p}{v_{F2}} \right)^2 - 1 \right], \\ 2k_{F2} \left(\sqrt{\frac{\Delta}{\mu} + 1} - \frac{c_p}{v_{F2}} \right), & \text{for } \mu \left[\left(\frac{c_p}{v_{F2}} \right)^2 - 1 \right] < \Delta < 4\mu \frac{c_p}{v_{F2}} \left(\frac{c_p}{v_{F2}} - 1 \right), \\ 2k_{F2} \left(\frac{c_p}{v_{F2}} - 1 \right), & \text{for } \Delta > 4\mu \frac{c_p}{v_{F2}} \left(\frac{c_p}{v_{F2}} - 1 \right). \end{cases} \quad (38)$$

The threshold for decay into AP modes is defined by a critical momentum value $k_{c1} = m_2^* c_p / \hbar$ and a corresponding critical excitation energy

$$\Delta_{c1} = \mu \left[(c_p/v_{F2})^2 - 1 \right]. \quad (39)$$

In the problem at hand, the maximum excitation energy for the quasidelectron tunneling in layer 2 is $\Delta_e = \mu(\alpha - 1)$. Therefore, for the AP mode excitation to be possible, $\Delta_e > \Delta_{c1}$, leading to

$$\alpha > (c_p/v_{F2})^2. \quad (40)$$

At the same time, the maximum quasihole energy $\Delta_h = \mu(1 - \alpha^{-1})$ suffices to excite AP modes in layer 1 if

$$2 > \frac{1}{\alpha} + \alpha \left(\frac{c_p}{v_{F2}} \right)^2. \quad (41)$$

Since $c_p/v_{F2} > 1$, this equation does not have solutions. Therefore, the AP modes will be excited only in layer 2 by the QE decay. Within the RPA, at plasma frequency, the imaginary part of the inverse dielectric function can be approximated by

$$\text{Im} \left[\frac{1}{\epsilon(\vec{q}, \omega)} \right]_{pl} = - \frac{\pi}{\left| \frac{\partial \text{Re} \epsilon(\vec{q}, \omega)}{\partial \omega} \right|} \delta(\omega - \omega_+(q)). \quad (42)$$

For acoustic plasmons it can be shown that

$$\frac{\partial \text{Re} \epsilon}{\partial \omega} = \frac{k_2}{\hbar q^2 c_p} \frac{1}{\sqrt{1 - (v_{F2}/c_p)^2}}, \quad (43)$$

where $k_2 = 2e^2 m_2^* / \hbar^2$ is the Thomas-Fermi screening wavelength in the second layer. The relaxation time integral follows from Eq. (22):

$$\begin{aligned} \frac{1}{\tau_{2QE}(\Delta)} \Big|_{AP} &= \frac{\hbar c_p}{m_2^*} \sqrt{1 - (v_{F2}/c_p)^2} \int_0^{q_{max}} dq \\ &\times \frac{\{1 - n[E(\vec{k}) - \hbar c_p q]\}}{1 - e^{-\hbar c_p q/k_B T}} \\ &\times \frac{q}{\sqrt{\frac{2}{m_2^*} (\Delta + \mu) - \left(c_p + \frac{\hbar q}{2m_2^*} \right)^2}}. \end{aligned} \quad (44)$$

A change of variable inside the integral to $x = q/k_{F2}$ leads to

$$\begin{aligned} \frac{\hbar}{\mu \tau_{2QE}(\Delta)} \Big|_{AP} &= 2 \sqrt{(c_p/v_{F2})^2 - 1} \int_0^{q_{max}/k_{F2}} dx \\ &\times \frac{\{1 - n[E(\vec{k}) - 2\mu(c_p/v_{F2})x]\}}{1 - e^{-2\mu(c_p/v_{F2})x/k_B T}} \\ &\times \frac{x}{\sqrt{(1 + \Delta/\mu) - (c_p/v_{F2} + x)^2}}. \end{aligned} \quad (45)$$

Analytic results can be obtained at very low temperatures when $k_B T \leq \Delta \leq \mu$ and the Fermi factors under the integral are equal to 1. By using for the upper limit of the integral

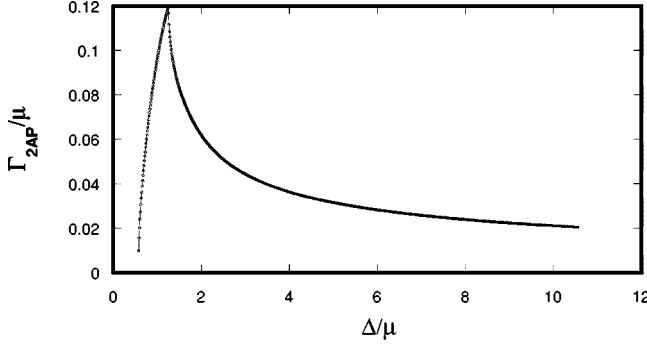


FIG. 5. The decay rate of an excited QE into acoustic plasmons as a function of the excitation energy is calculated for $\alpha=2$ at $T=0.2T_F$ for a sample with electronic density $n_2=1.6 \times 10^{11} \text{ cm}^{-2}$, interlayer distance $d=200 \text{ \AA}$, and effective electron mass $m_2^*=0.1m_e$.

after q the corresponding values given in Eq. (38) we obtain for $\Delta \in \{\mu[(c_p/v_{F2})^2 - 1]; 4\mu(c_p/v_{F2})(c_p/v_{F2} - 1)\}$

$$\frac{\hbar}{\mu\tau_{2QE}(\Delta)} \Big|_{AP} = \frac{8}{A} \frac{c_p}{v_{F2}} \left[\sqrt{\frac{\Delta}{\mu} + 1 - \left(\frac{c_p}{v_{F2}}\right)^2} - \frac{c_p}{v_{F2}} \arcsin \sqrt{1 - \frac{(c_p/v_{F2})^2}{\Delta/\mu + 1}} \right] \quad (46)$$

and for $\Delta > 4\mu(c_p/v_{F2})(c_p/v_{F2} - 1)$

$$\begin{aligned} \frac{\hbar}{\mu\tau_{2QE}(\Delta)} \Big|_{AP} = & \frac{8}{A} \frac{c_p}{v_{F2}} \left\{ \sqrt{\frac{\Delta}{\mu} + 1 - \left(\frac{c_p}{v_{F2}}\right)^2} \right. \\ & - \sqrt{\frac{\Delta}{\mu} + 1 - \left(\frac{2c_p}{v_{F2}} - 1\right)^2} - \frac{c_p}{v_{F2}} \\ & \times \arcsin \left[\frac{2c_p/v_{F2} - 1}{\sqrt{\Delta/\mu + 1}} \sqrt{1 - \frac{(c_p/v_{F2})^2}{\Delta/\mu + 1}} \right] \\ & \left. - \frac{c_p/v_{F2}}{\sqrt{\Delta/\mu + 1}} \sqrt{1 - \frac{(2c_p/v_{F2} - 1)^2}{\Delta/\mu + 1}} \right\}. \end{aligned} \quad (47)$$

The relaxation rate of a quasielectron due to the decay into acoustic plasmons, $\Gamma_{2AP} = \hbar/\{\tau_{2QE}[1 - n(\Delta + \mu)]\}$, at $T=0.2T_F$ (T_F is the Fermi temperature) is presented in Fig. 5 as a function of Δ . Here Γ_{AP} increases with Δ up to the point where the energy of the acoustic plasmon is equal to the maximum energy transfer for a given momentum k of the quasiparticle. When this point is reached, the upper limit of the integral becomes constant, equal to the value imposed by the intersection with the electron-hole continuum and Γ_{AP} decreases as $1/\sqrt{\Delta}$. The temperature dependence of Γ_{AP} when $\Delta = \mu$ appears in Fig. 6.

2. Decay into optical plasmons

The relaxation processes involving OP's are possible only when the quasiparticle excitation energy $\hbar^2 p^2/2m_2^* - \mu$ is larger than a critical value Δ_{c2} determined by the lowest

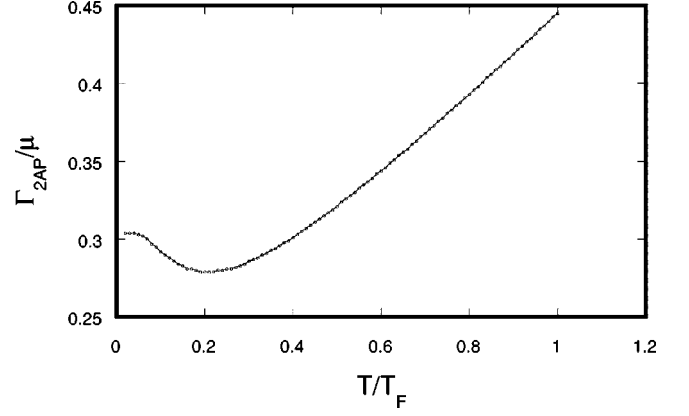


FIG. 6. The temperature dependence of the rate of decay into acoustic plasmons is plotted for $\Delta = \mu$. Here $\alpha=2.0$ and $c_p/v_{F2} = 1.25$.

value of the quasiparticle momentum p_{c2} for which Eq. (37) is satisfied, exactly as in the case of a single-2D-electron layer analyzed in Refs. 10 and 17. Here p_{c2} and q^* , the corresponding momentum transfer, are found by imposing that the OP curve $\omega_+ = \sqrt{\gamma + q}$ and $\Omega(q) = \hbar pq/m_2^* - \hbar q^2/2m_2^*$ admit a common tangent in the (ω, q) plane. This condition leads to $q^* = (k_2 k_{F2}^2)^{1/3}$ and $p_c = 3q^*/2$, conducing to a critical excitation energy

$$\Delta_{c2} = \mu \left[\frac{9}{4} \left(\frac{k_2}{k_{F2}} \right)^{2/3} - 1 \right]. \quad (48)$$

In the experiment we propose, the maximum QE energy is fixed by the band structure alignment in the two wells at $(\alpha - 1)\mu$. For the OP plasmon decay to occur, it is then necessary that

$$\alpha > \frac{9}{4} \left(\frac{k_2}{k_{F2}} \right)^{2/3}. \quad (49)$$

The imaginary part of the inverse dielectric function can be directly estimated, with the result

$$\text{Im} \left[\frac{1}{\epsilon_2} \right] = -\pi \omega_+ \delta(\omega - \omega_+). \quad (50)$$

By using this value of $\text{Im}[1/\epsilon_2]$ in Eq. (22), in the vicinity of the critical excitation energy, for a quasiparticle of momentum k , the relaxation time can be approximated by

$$\begin{aligned} \frac{\hbar}{\mu\tau_{2QE}(k)} \Big|_{OP} = & \frac{4\sqrt{2}m_2^*e^2}{\hbar\mu} \sqrt{\frac{\gamma_+}{3(k+p_{c2})}} \\ & \times \frac{1 - n[E_2(k) - \hbar\omega_+(q^*)]}{1 - e^{-\hbar\omega_+(q^*)/k_B T}}. \end{aligned} \quad (51)$$

A similar result was obtained in Ref. 17.

V. DISCUSSION

The aim of the experiment proposed in this paper is to verify the correctness of the theoretical predictions for the

quasiparticle relaxation lifetime when decay into plasmonic modes is an important mechanism. For this purpose, appropriately high excitation energies should be reached, as established by Eqs. (40) and (48). A numerical solution to Eq. (40) can be obtained from Fig. 3, where we find, for example, that for $\alpha=2$ one can select $c_p/v_{F2}=1.25$. To illustrate our results, we assume generic sample parameters: carrier density $N=1.6\times 10^{11}\text{ cm}^{-2}$, interlayer distance $d=200\text{ \AA}$, and effective electron mass $m_2^*=0.1m_e$. We note that for this sample, the values obtained for the Fermi vector, $k_F=\sqrt{2\pi N}=10^8\text{ m}^{-1}$, and for the Thomas-Fermi wavelength, $k_2=3.7\times 10^9\text{ m}^{-1}$, would require $\alpha=25$ for the optical plasmons to be excited, according to Eq. (48). Therefore, the excitation of the optical plasmons will not be possible in this experiment.

The width of the conductance peaks, Eq. (9), is given by $\sqrt{2(\alpha\Gamma_1^2+\Gamma_2^2)}$, where Γ_1 and Γ_2 are to be considered functions of the excitation energy of the corresponding peak. At the first peak, $\Delta_e=0$ for the electron in layer 2 and $\Delta_h=-\mu(1-\alpha^{-1})$ for the hole in layer 1 allow only the excitation of electron-hole pairs in both layers. At the second peak, where $\Delta_e=(\alpha-1)\mu$ and $\Delta_h=0$, AP modes are excited along with electron-hole pairs in layer 2. Employing Eqs.

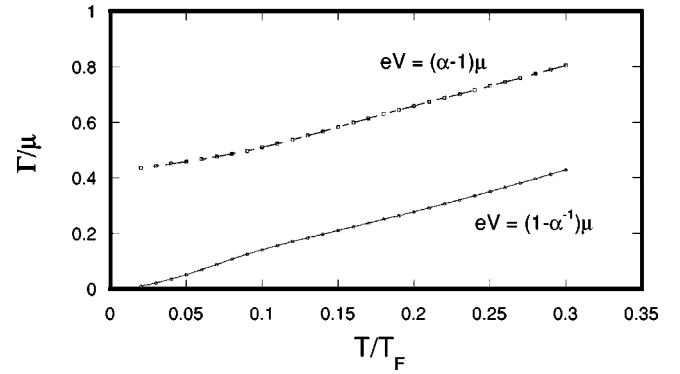


FIG. 7. The finite tunneling widths of the two conductivity peaks are plotted as a function of temperature for the case $\alpha=2.0$. At $eV=(1-\alpha^{-1})\mu$, the width $\sqrt{2(\alpha\Gamma_1^2+\Gamma_2^2)}$ is determined by the relaxation into electron-hole pairs in both layers. At $eV=(\alpha-1)\mu$, the relaxation into AP modes that occurs only in layer 2 makes a finite contribution.

(29) and (10) we plot the temperature dependence of the conductivity peaks in Fig. 7. Assuming that the decay into electron-hole pairs is a mechanism well understood, relevant information can be extracted about the temperature dependence of the decay into AP modes.

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