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The coupling of spin waves with charge- and spin-density waves is shown to be induced by a spin-dependent interaction in a quantum well, which is spin polarized by a dc magnetic field at an angle θ to the symmetry axis. The mixing of the plasmonic and magnonic modes, which occurs for both intra- and intersubband transitions, depends on the coupling constant of the spin-spin interaction, the tilt angle θ, and the initial spin polarization ζ. [S0163-1829(99)08247-8]

An intense experimental effort employing infrared absorption and inelastic light-scattering techniques has been focused on probing the excitation spectrum of an electron gas confined to a quasi-two-dimensional semiconductor heterostructure (2DEG).[1,2] The observed absorption peaks indicate that linearly independent charge-density waves (CDW's), spin-density waves (SDW's), and spin waves (SW's) propagate at frequencies corresponding to the poles of the appropriate response functions. The first two excitations are generated by fluctuations in the local density of particles with a given spin, while the spin waves are associated with spin-flip processes.

When a dc magnetic field is applied perpendicular to the 2D layer, the energy levels within each subband are determined by the ratio of the Zeeman splitting $2\gamma^*B$ and the cyclotron energy $\hbar\omega_c=\hbar eB/m^*c$ (γ^* is the effective g value and m^* is the effective mass). If $2\gamma^*B\ll\hbar\omega_c$ (as in GaAs heterostructures), each subband is characterized by a sequence of Landau levels, each of which displays a small spin splitting.[3] In the opposite limit, $2\gamma^*B\gg\hbar\omega_c$ (as can occur for dilute magnetic semiconductor structures), the main splitting within each subband is into widely separated spin-up and spin-down components. The cyclotron energy gives rise to small splittings of the spin subbands into Landau-level ladders. The SW's are not coupled to the CDW and the SDW when the magnetic field is normal to the 2D layer (considered here as the x-z plane).

The shift of the collective excitation frequency from the single-particle transition is a consequence of the many-body interaction. Theoretical investigations of the elementary spectrum of a 2D electron gas in a normal magnetic field have shown that the existence of the CDW and intersubband SDW is a consequence of the mean electrostatic field, the random-phase approximation (RPA) of the Coulomb repulsion.[4] Intrasubband SDW and both intra- and intersubband SW's appear only when a spin-dependent interaction is included.[5] The former is negligible in GaAs where γ^* is very small but can give observable effects in dilute magnetic semiconductors, where the effective gyromagnetic factor is enhanced by the electron interaction with the magnetic ions to values up to a hundred times its band value.

In this paper, we consider the applied dc magnetic field $\vec{B}$ at an angle θ to the symmetry axis of the dilute magnetic semiconductor well and show that the tilt of the magnetic field produces the coupling of the spin- and charge-density waves with the spin waves. The excitation frequencies for the collective modes are derived within the RPA approximation by using the equation of motion for the one-electron distribution function. Analytic results are obtained for small values of the inclination angle θ. The dependence of the collective-mode frequencies on the equilibrium polarization and θ are studied in the long-wavelength limit.

The electron state in a quantum well of width L is a plane wave of wave vector $\vec{k}$ (in the $\hat{x}$-$\hat{z}$ plane) modulated by an envelope function that reflects the $\hat{y}$-axis confinement. The associated field operator is

$$\psi_\alpha(\vec{r},y,\vec{s})=\sum_{\vec{k}\sigma} a_{\alpha;\vec{k}\sigma}e^{i\vec{k}\cdot\vec{r}}\xi_\alpha(y)\chi_\sigma(\vec{s}), \tag{1}$$

with $\xi_\alpha(y)=\sqrt{2/L}\sin\alpha\pi/L$ ($\alpha=1,2,\ldots$), the subband α wave function. The spinor $\chi_\sigma(\vec{s})$ corresponds to a projection σ of the spin $\vec{s}$ along the $\hat{z}$ axis. If $c^\dagger_{k\sigma}$ and $c_{k\sigma}$ are the creation and destruction operators for an electron with spin projection σ along the direction of the dc magnetic field $\hat{u}=\hat{y}\sin\theta+\hat{z}\cos\theta$, the equilibrium ground-state average $\langle c^\dagger_{k\sigma}c_{k\sigma}\rangle=n_{k\sigma}$ is the number of electrons with spin σ parallel to $\hat{u}$ per unit area. Elementary quantum mechanics determines that

$$a_{k\sigma}=\cos\frac{\theta}{2}c_{k\sigma}+i\sin\frac{\theta}{2}c_{k\bar{\sigma}}. \tag{2}$$

The spin polarization of the electron gas is just $\zeta=(n_\uparrow-n_\downarrow)/(n_\uparrow+n_\downarrow)$. The magnetization,

$$\vec{m}=-\gamma\sum_{\vec{s}}\sum_\alpha\int_0^L dy\int d^2r\,\psi^\dagger_\alpha(\vec{r},y,\vec{s})(\vec{\sigma}\cdot\vec{B})\psi_\alpha(\vec{r},y,\vec{s}),$$

has nonzero components on $\hat{y}$ and $\hat{z}$ axes proportional to the corresponding components of the dc magnetic field.

The equilibrium Hamiltonian of an electron gas in a dc magnetic field is

$$H_0=\sum_{\vec{k},\sigma,\alpha}\left(\frac{\hbar^2k^2}{2m^*}+\gamma^*B_z\,\mathrm{sgn}(\sigma)\right)a^\dagger_{\alpha;\vec{k}\sigma}a_{\alpha;\vec{k}\sigma}$$
$$+\sum_{\alpha;\vec{k}}[\gamma^*B^-a^\dagger_{\alpha;\vec{k}\uparrow}a_{\alpha;\vec{k}\downarrow}+\gamma^*B^+a^\dagger_{\alpha;\vec{k}\downarrow}a_{\alpha;\vec{k}\uparrow}]$$
$$+\frac{1}{2}\sum_{\alpha,\beta,\alpha',\beta'}\sum_{\vec{k},\vec{k}',\vec{q}',\sigma,\sigma'}[v^{\alpha'\beta'}_{\alpha\beta}(\vec{q}')$$
$$-4\pi(\gamma^*)^2\,\mathrm{sgn}\,\sigma\,\mathrm{sgn}\,\sigma']a^\dagger_{\alpha';\vec{k}-\vec{q}'/2,\sigma}$$
$$\times a^\dagger_{\beta';\vec{k}'+\vec{q}'/2,\sigma'}a_{\beta;\vec{k}'-\vec{q}'/2,\sigma'}a_{\alpha;\vec{k}+\vec{q}'/2,\sigma}$$
$$+8\pi(\gamma^*)^2\sum_{\alpha,\beta,\alpha',\beta'}\sum_{\vec{k},\vec{k}',\vec{q}'}a^\dagger_{\alpha';\vec{k}-\vec{q}'/2,\uparrow}$$
$$\times a^\dagger_{\beta';\vec{k}'+\vec{q}'/2,\downarrow}a_{\beta;\vec{k}'-\vec{q}'/2,\uparrow}a_{\alpha;\vec{k}+\vec{q}/2,\downarrow}\,, \tag{3}$$

where the signum function $\mathrm{sgn}(\sigma)$ is 1 for a spin up and -1 for a spin down. The many-body interaction term in Eq. (3) includes the Coulomb repulsion $v^{\alpha'\beta'}_{\alpha\beta}(\vec{q})$ and the effect of a self-consistent magnetization. $v^{\alpha'\beta'}_{\alpha\beta}(\vec{q})$ is the product of the Fourier transform of the 2D Coulomb interaction $v(\vec{q})=2\pi\mathrm{e}^2/q$ and the form factor, which describes the $\hat{y}$-axis overlap of the electronic wave functions:

$$v^{\alpha'\beta'}_{\alpha\beta}(\vec{q})=v(\vec{q})\int_0^L dy\int_0^L dy'\,\zeta_{\alpha'}(y)\zeta_{\beta'}(y)$$
$$\times e^{-q|y-y'|}\zeta_\beta(y')\zeta_\alpha(y). \tag{4}$$

A small electromagnetic perturbation [an electric potential $\varphi(\vec{r},y,t)\sim\varphi(\vec{q},y)e^{i(\omega t-\vec{q}\cdot\vec{r})}$ and a magnetic induction of arbitrary orientation $\vec{b}(\vec{r},y,t)\sim\vec{b}(\vec{q},y)\mathrm{e}^{i(\omega t-\vec{q}\cdot\vec{r}]}$ modifies the local energy of the electrons and changes their distribution functions. This generates density fluctuations and spin-flip processes, which have a time and position variation imposed by the perturbation. In a Fourier-transform representation, these are functions of the wave vector $\vec{q}$, and frequency ω. The interaction Hamiltonian is

$$H_{\mathrm{int}}=\sum_{\alpha,\beta}\sum_{\vec{k},\vec{q}}\int_0^L dy\left\{\sum_\sigma[-e\varphi(-\vec{q},y)\right.$$
$$+\gamma b_z(-\vec{q},y)\mathrm{sgn}(\sigma)]a^\dagger_{\beta;\vec{k}-\vec{q}/2,\sigma}a_{\alpha;\vec{k}+\vec{q}/2,\sigma}$$
$$+\gamma[b^+(-\vec{q},y)a^\dagger_{\beta;\vec{k}-\vec{q}/2,\downarrow}a_{\alpha;\vec{k}+\vec{q}/2,\uparrow}$$
$$\left.+b^-(-\vec{q},y)a^\dagger_{\beta;\vec{k}-\vec{q}/2,\uparrow}a_{\alpha;\vec{k}+\vec{q}/2,\downarrow}]\right\}, \tag{5}$$

where a uniform notation for the external fields was used: $\varphi=\int_0^L dy\,\xi_\alpha(y)\varphi(\vec{q},\omega,y)\xi_\beta(y)$. The $\vec{q}$ and ω dependence is implicitly understood.

There are three types of induced fluctuations. The field components that leave the spin state unchanged, φ and b_z, determine variations in the electron density whose spin remains parallel to the $\hat{z}$ axis. The transverse components of the magnetic induction, $b^\pm=b_x\pm ib_y$, drive spin-flip processes such that the spin projection is in the $\hat{x}$-$\hat{y}$ plane. Furthermore, the collective modes can involve excitations in the same subband, $\alpha=\beta$, or in a different one, $\alpha\neq\beta$.

The frequency- and wave-vector-dependent density fluctuations $\Delta n_{\alpha\sigma;\beta\sigma'}(\vec{q},\omega)$ correspond to a transition of an electron from a state $(\alpha,\vec{k}-\vec{q}/2,\sigma)$ to a state $(\beta,\vec{k}+\vec{q}/2,\sigma')$, by exchanging momentum $\vec{q}$ and energy $\hbar\omega$ with the external perturbation. In a linear-response approximation, this process is averaged over the ground state of the nonperturbed system. The frequency- and wave-vector-dependent fluctuation $\Delta n_{\alpha\sigma;\beta\sigma}(\vec{q},\omega)$ is

$$\Delta n_{\alpha\sigma;\beta\sigma}(\vec{q},\omega)=\int_0^L dy\,\xi_\alpha(y)\xi_\beta(y)$$
$$\times\sum_{\vec{k}}[\langle a^\dagger_{\beta;\vec{k}-\vec{q}/2,\sigma}(\omega)a_{\alpha;\vec{k}+\vec{q}/2,\sigma}(\omega)\rangle$$
$$-\langle a^\dagger_{\alpha;\vec{k}\sigma}a_{\alpha;\vec{k}\sigma}\rangle\delta_{\alpha,\beta}\delta_{\vec{q},0}$$
$$\times\langle a^\dagger_{\alpha;\vec{k}\sigma}a_{\alpha;\vec{k}\sigma}\rangle\delta_{\alpha,\beta}\delta_{\vec{q},0}]. \tag{6}$$

Analogously, a spin-flip process, associated with the Pauli raising and lowering operators σ^+ and σ^-, is described by

$$\Delta n_{\alpha\sigma;\beta\bar{\sigma}}(\vec{q},\omega)=2\int_0^L dy\,\xi_\alpha(y)\xi_\beta(y)$$
$$\times\sum_{\vec{k}}[\langle a^\dagger_{\beta;\vec{k}-\vec{q}/2,\bar{\sigma}}(\omega)a_{\alpha;\vec{k}+\vec{q}/2,\sigma}(\omega)\rangle$$
$$-\langle a^\dagger_{\alpha;\vec{k}\bar{\sigma}}a_{\alpha;\vec{k}\sigma}\rangle\delta_{\alpha,\beta}\delta_{\vec{q},0}]. \tag{7}$$

The frequency dependence of the creation and destruction operators is determined by the usual equation of motion, which involves their commutator with the total Hamiltonian.[6]

When the commutators are estimated, the fully interacting equations for the density fluctuations are obtained. However, this infinite chain of coupled equations cannot be solved analytically if some approximations are not performed. For simplicity, we choose to analyze only transitions between two given minibands, α and β. In this case, the Coulomb interaction matrix element, Eq. (4), becomes

$$v^{\alpha\beta}(q)=\frac{2\pi\mathrm{e}^2}{q}\left\{\frac{2}{qL}+\frac{1}{(qL)^2+(2\pi\alpha)^2}\delta_{\alpha,\beta}\right.$$
$$\left.-\frac{2}{(qL)^2}\frac{(2\pi\alpha)(2\pi\beta)}{[(qL)^2+(2\pi\alpha)^2][(qL)^2+(2\pi\alpha)^2]}\right\}. \tag{8}$$

Another simplification occurs for values of θ which assure that γ^*B is much smaller than all the energies involved in the problem. The coupling between oscillations corresponding to different wave vectors generated by the static components of the applied magnetic field are neglected under the assumption that $\sin\theta \ll 1$.

To show how a magnetic interaction intermediates the coupling between the density and spin fluctuations, we revert to the simplest approximation, that of a self-consistent magnetic field. We introduce the generalized polarization coefficients of the electron gas $P^{\alpha\beta}_{\sigma\sigma'}$ to be

$$P^{\alpha\beta}_{\sigma\sigma'} = \sum_{\vec{k}} \frac{\left(\cos^2\frac{\theta}{2} n^{\beta}_{\vec{k}-\vec{q}/2,\sigma} + \sin^2\frac{\theta}{2} n^{\beta}_{\vec{k}-\vec{q}/2,\overline{\sigma'}}\right) - \left(\cos^2\frac{\theta}{2} n^{\alpha}_{\vec{k}+\vec{q}/2,\sigma} + \sin^2\frac{\theta}{2} n^{\alpha}_{\vec{k}+\vec{q}/2,\overline{\sigma'}}\right)}{\hbar\omega - (\epsilon_{\beta;\vec{k}+\vec{q}/2,\sigma'} - \epsilon_{\alpha;\vec{k}-\vec{q},\sigma})}. \tag{9}$$

In a mean-field theory, the linear-response approximation determines that the density fluctuations, $\Delta n^{\alpha\beta} = \Delta n^{\alpha\beta}_{\uparrow} + \Delta n^{\alpha\beta}_{\downarrow}$, and the spin-density fluctuations, $\Delta s^{\alpha\beta} = \Delta n^{\alpha\beta}_{\uparrow} - \Delta n^{\alpha\beta}_{\downarrow}$, satisfy

$$\begin{aligned}\Delta n^{\alpha\beta} = {} & P^{\alpha\beta}(-e\varphi + v^{\alpha\beta}\Delta n^{\alpha\beta}) + \gamma^*\zeta \cos\theta P^{\alpha\beta} \\ & \times (b_z - 4\pi\gamma^*\Delta s^{\alpha\beta}_z) + i\gamma^*\zeta\frac{\sin\theta}{2}P^{\alpha\beta} \\ & \times[b^+ - 4\pi\gamma^*\Delta n^{\alpha\beta}_+] - i\gamma^*\zeta\frac{\sin\theta}{2}P^{\alpha\beta} \\ & \times[b^- - 4\pi\gamma^*\Delta n^{\alpha\beta}_-],\end{aligned} \tag{10}$$

$$\begin{aligned}\Delta s^{\alpha\beta} = {} & \zeta P^{\alpha\beta}(-e\varphi + v^{\alpha\beta}\Delta n^{\alpha\beta}) + \gamma^* P^{\alpha\beta} \\ & \times(b_z - 4\pi\gamma^*\Delta s^{\alpha\beta}_z)\cos\theta.\end{aligned} \tag{11}$$

In the same approximation, the spin-flip fluctuations are given by

$$\begin{aligned}\Delta n^{\alpha\beta}_+ = {} & \gamma^*\Pi^{\alpha\beta}_{\uparrow\downarrow}[b^+ - 4\pi\gamma^*\Delta n^{\alpha\beta}_+] \\ & + i\zeta\sin\theta P[-e\varphi + v_{\alpha\beta}\Delta n^{\alpha\beta}],\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta n^{\alpha\beta}_- = {} & \gamma^*\Pi^{\alpha\beta}_{\downarrow\uparrow}[b^- - 4\pi\gamma^*\Delta n^{\alpha\beta}_-] + i\zeta\sin\theta P \\ & \times[-e\varphi + v_{\alpha\beta}\Delta n^{\alpha\beta}].\end{aligned} \tag{13}$$

Equations (11) and (13) form a self-consistent system of equations, which describes the oscillations generated by the applied electromagnetic perturbation in the quantum well. The excitation frequencies of the collective modes are determined by requiring that the homogeneous system obtained in the absence of the outside field has a nontrivial solution. The coupling among density fluctuations of electrons with different wave vectors $\vec{k}$ mediated by the transverse components of the dc magnetic field, $B^{\pm} = \pm iB\sin\theta$, makes it very difficult to obtain a solution for an arbitrary tilt angle.

The excitation frequency of the various collective modes is a solution of the secular equation,

$$\begin{aligned}& \{(1 - P^{\alpha\beta}v^{\alpha\beta})[1 + 4\pi(\gamma^*)^2P^{\alpha\beta}] + 4\pi(\gamma^*)^2(P^{\alpha\beta})^2 \\ & \quad \times v^{\alpha\beta}\zeta^2\cos^2\theta\}[1 + 8\pi(\gamma^*)^2P^{\alpha\beta}_{\uparrow\downarrow}][1 + 8\pi(\gamma^*)^2\Pi_{\downarrow\uparrow}] \\ & \quad + 2\zeta^2\pi(\gamma^*)^2\{P^{\alpha\beta}_{\downarrow\uparrow}[1 + 8\pi(\gamma^*)^2P^{\alpha\beta}_{\uparrow\downarrow}] \\ & \quad + P^{\alpha\beta}_{\uparrow\downarrow}[1 + 8\pi(\gamma^*)^2P^{\alpha\beta}_{\downarrow\uparrow}]\}(P^{\alpha\beta}\tilde{v})^2\sin^2\theta = 0.\end{aligned} \tag{14}$$

The last term of Eq. (14) describes the coupling between the density fluctuations and the spin fluctuations. It exists only if the 2D electron gas is spin polarized under a tilt angle θ. The strength of this hybrid mode is determined by the coupling constant of the spin-spin interaction, which in the approximation we used is just the self-consistent magnetization. The loss of axial symmetry, which occurs when the dc magnetic field is tilted at an angle θ, is at the origin of this effect. The excitations generated by fluctuations in the density of the electrons with spin parallel to the $\hat{z}$ axis are expected to occur at frequencies much larger than the Zeeman energy $2\gamma^*B$, where the spin-wave modes begin. By neglecting $P_{\sigma\bar{\sigma}}$ in Eq. (14), a quadratic equation in $(P^{\alpha\beta}v^{\alpha\beta})$ is obtained and solved for both intra- and intersubband modes.

The intrasubband excitations occur within the same energy subband. For the lowest energy subband ($\alpha=1$), $Pv(q)$, from Eqs. (9), is simply equal to ω_p^2/ω^2, where $\omega_p = 2\pi ne^2/m^*$ is the plasma frequency for a 2D electron system. These results are equivalent to the long-wavelength limit of the noninteracting electron-gas polarization functions, characteristic of plasma oscillations. In this situation, Eq. (14) has just one valid solution for ω:

$$\omega^2 = \omega_p\left[1 - \kappa\zeta^2\left(1 - \frac{\sin^2\theta}{2}\right)\right], \tag{15}$$

where $\kappa = 4\pi(\gamma^*)^2/v(q)$. The excitation is the charge-density wave, associated with the symmetric combination of density fluctuations in the up- and down-spin electrons. It is excited at the plasma frequency in 2D corrected by a term that reflects the coupling with the spin modes. Spin-flip processes about the initial polarization direction generate a nonzero contribution to the $\hat{z}$-axis spin dynamics, proportional to $4\pi(\gamma^*)^2n\zeta^2\sin^2\theta/m^*$ as one can determine by comparing Eq. (15) with its value in the case $\theta=0$.

As an example of the intersubband collective modes, we calculate the excitation frequencies between the ground state ($\alpha=1$) and the first excited level ($\beta=2$). For simplicity, we will assume that the electron density in the β subband is much smaller than the electron density in subband α. The intersubband modes start at excitation frequencies comparable to the single-particle transition, $\Omega = \epsilon_2 - \epsilon_1$. In this case, $P^{\alpha\beta}v^{\alpha\beta}$ is given by

$$Pv(q) = \frac{2nv(q)(\hbar\Omega)^2}{(\hbar\omega)^2 - (\hbar\Omega)^2}. \tag{16}$$

There are two collective modes. One corresponds to the spin-symmetric density fluctuation, charge-density wave, and it begins at

$$\hbar\omega_{\mathrm{CDW}}=\hbar\Omega\left[1+\frac{n v(q)}{(\hbar\Omega)}(1-\kappa\zeta^2)\right]+8\pi(\gamma^*)^2 n\zeta^2\frac{\sin^2\theta}{2}. \tag{17}$$

The lower-frequency mode corresponds to antisymmetric spin oscillations, that form a spin-density wave. The excitation frequency is

$$\hbar\omega_{\mathrm{SDW}}=\hbar\Omega\left[1-\frac{4\pi(\gamma^2)^2 n}{(\hbar\Omega)}(1-\zeta^2)\right]-4\pi(\gamma^*)^2\frac{\sin^2}{2}. \tag{18}$$

A gap opens up in the intrasubband excitation spectrum, which is a result of an initial spin polarization and of the inclination angle. This effect was also observed in the case of the inter-Landau-level transitions in GaAs structures.[7]

The same analysis can be performed in the case of the excitation frequencies of the spin-flip processes. In a first-order approximation in $(4\pi\gamma^*)$, we obtain that intrasubband down-up excitations, that generate the spin waves, occur at

$$\hbar\omega^+=2\gamma^*B+8\pi(\gamma^*)^2\zeta n+\frac{\hbar^2q^2}{2m^*n\zeta}+8\pi(\gamma^*)^2\left(\frac{m^*\omega_p}{2\pi\hbar}\right)^2 n\zeta\frac{\sin^2\theta}{4}. \tag{19}$$

The up-down excitation mode starts at a frequency $\omega^- =\omega^+(-\zeta)$. The SW dispersion law is odd in ζ since the spin-flipping process depends on the direction of the magnetic field. Of course, the CDW and the SDW, which are density modes determined only by the magnitude of $\vec{B}$, are functions of ζ^2.

Intersubband spin-flip transitions are also analyzed for transitions between the subbands $\alpha=1$ and $\beta=2$. In this situation, the generalized polarization functions, Eq. (9) become

$$P_{\downarrow\uparrow}v=\frac{1}{2}\left[\frac{n(1+\zeta\cos\theta)}{\hbar\omega-\hbar\Omega^+}-\frac{n(1-\zeta\cos\theta)}{\hbar\omega+\hbar\Omega^-}\right], \tag{20}$$

$$P_{\uparrow\downarrow}v=\frac{1}{2}\left[\frac{n(1-\zeta\cos\theta)}{\hbar\omega-\hbar\Omega^-}-\frac{n(1+\zeta\cos\theta)}{\hbar\omega+\hbar\Omega^+}\right], \tag{21}$$

where $\Omega^+=\epsilon_\uparrow^\beta-\epsilon_\downarrow^\alpha$ and $\Omega^-=\epsilon_\downarrow^\beta-\epsilon_\uparrow^\alpha$ are the single-particle spin-flip transitions. By solving the secular equation, Eq. (14), we obtain the excitation frequencies for the collective modes of the interacting system,

$$\hbar\omega^+=\hbar\Omega^+-4\pi(\gamma^*)^2 n(1+\zeta\cos\theta)\times\left[1+\left(\frac{nm^*v(q)}{2\pi\hbar^2}\right)\zeta\frac{\sin^2\theta}{2}\right], \tag{22}$$

$$\hbar\omega^-=\hbar\Omega^+-4\pi(\gamma^*)^2 n(1-\zeta\cos\theta)\times\left[1+\left(\frac{nm^*v(q)}{2\pi\hbar^2}\right)\zeta\frac{\sin^2\theta}{2}\right]. \tag{23}$$

The influence of the plasma modes on these excitation frequencies is mediated by the term proportional to $\sin^2\theta$, which also includes the wave-vector-dependent Coulomb interaction matrix element.

We have shown that in an asymmetrically spin-polarized quantum well a weak self-consistent magnetic perturbation generates a coupling between the plasmonic modes and the spin waves, dependent on the anisotropy angle θ and on the degree of initial polarization, ζ. This coupling occurs for both intra- and intersubband excitations. The general algorithm described in this paper, can be extended to include any other type of self-consistent spin-dependent interaction. In this sense, the results we obtained should be interpreted as qualitative, rather than quantitative. Of great interest, of course, will be the inclusion of the spin-dependent short-range Coulomb effects, which are expected to dominate the self-consistent magnetization, even in the case of dilute magnetic semiconductors.

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