12-15-1997

Composite Fermions and the Half-Filled State

D C. Marinescu
Clemson University, dcm@clemson.edu

J J. Quinn
University of Tennessee

P Sitko
University of Tennessee

Composite fermions and the half-filled state

D. C. Marinescu
Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

J. J. Quinn
Department of Physics, University of Tennessee, Knoxville, Tennessee 37906

P. Sitko
Department of Physics, University of Tennessee, Knoxville, Tennessee 37906
and Institute of Physics, Technical University of Wraclaw, 50-370 Wraclaw, Poland

K. S. Yi
Department of Physics, Pusan National University, Pusan 609-735, Korea
(Received 25 June 1997)

Under appropriate conditions electron-hole symmetry should apply to a partially filled Landau level of a two-dimensional electron gas. This suggests that the application of Jain's composite fermion (CF) picture to either electrons or holes should lead to equivalent results. Surprisingly, for a system of N_e electrons on a Haldane sphere, this is not true for three values of the Landau level degeneracy $2S+1$. When $N_e-1\leqslant S\leqslant N_e$, the sum of the electron filling factor ν and the hole filling factor μ, as determined from Jain's picture, is smaller than unity. Because of this, use of the relation $\nu=1-\mu$ can lead to "twin" or "alias" states having different values of ν for the same N_e and $2S+1$. One example is the "half-filled" state. It is determined by requiring the effective (mean-field) flux $2S^*$ "seen" by one CF to vanish. Different results are obtained when $S_e^*=S_e-(N_e-1)$ and $S_h^*=S-(N_h-1)$ are set equal to zero. The same problem arises in the CF hierarchy picture when the number of quasielectrons n_{QE} is related to the effective flux $2S^*$ by $2(n_{\rm QE}-1)\leqslant 2S^*\leqslant 2n_{\rm QE}$. [S0163-1829(97)00647-4]

I. INTRODUCTION

A strongly correlated two-dimensional electron gas, subjected to a magnetic field B, behaves like an incompressible quantum fluid generating fractional quantum Hall states when the ratio of the particle density n to the magnetic flux density, expressed in flux quanta, is a simple fraction with an odd denominator $\nu=n\phi_0/B$ ($\phi_0=hc/e$, the flux quantum). This property of the interacting electron system can be explained in terms of the composite fermion (CF) picture, proposed by Jain[1] to describe the sequence of Laughlin states with $\nu=1/3,2/5,3/7,\ldots$. The composite fermion transformation, as defined by Jain, attaches to each electron an even number $2p$ of flux quanta oriented opposite to the applied magnetic field, such that the effective mean field flux per particle is $\nu^{*-1}=\nu^{-1}-2p$. Fractional fillings correspond then to integer quantum Hall states of the weakly interacting CF system, described by ν^*. This theoretical approach was extended by Halperin, Lee, and Read[2] to the properties of the compressible $\nu=1/2$ state, the accumulation point of the odd-denominator sequences. For an infinite number of particles in the electron gas, it has been established that the half-filled state occurs when the effective magnetic field "seen" by a composite fermion is zero.

Important insight into the nature of the fractional filling states has been obtained by studying a system of N_e electrons on a sphere of radius R that contains at its center a magnetic monopole of strength $2S\phi_0$ generating a radial field $B=(\hbar c/e)SR^{-2}$.[3] The single electron energy $\xi=(\hbar\omega_c/2S)[l(l+1)-S^2]$ depends on the angular momentum $l=S+n$ ($n=0,1,2,\ldots$), but not on its z component. Therefore, the nth Landau level (or nth angular momentum shell) has a degeneracy $g_n=2S+(2n+1)$.

When the composite fermion transformation is applied, the effective monopole strength S^* experienced by one CF electron is

$$2S_e^*=2S-2p(N_e-1). \tag{1}$$

For simplicity, henceforth we consider $p=1$. The single CF states are quantized in the same way as the electron states if S_e^* replaces S and ω_c^* replaces ω_c. If N_e composite fermions fill exactly an integer number ν^* of shells, then

$$\nu=\frac{\nu^*}{1+2\nu^*}, \tag{2}$$

where $\nu^*=\pm1,\pm2,\pm3,\ldots$ describes the condensed states in the principal Jain sequence. This leads to a unique relationship between the value of S and N_e at which a particular filling occurs. At $S_e^*=0$, the effective magnetic field seen by a CF is zero. By analogy to the infinite system, this criterion has been used to define a half-filled Landau level.[4]

For magnetic fields large enough that $\hbar\omega_c\gg e^2/l$, all but the lowest electron Landau level can be neglected. One can then invoke particle-hole symmetry and apply the composite fermion transformation to holes instead of electrons. Obviously,

$$N_e+N_h=2S+1. \quad (3)$$

The effective monopole strength seen by a CF hole is

$$2S_h^*=2S-2(N_h-1). \quad (4)$$

The requirement that the CF holes fill an integer number μ^* of shells gives a fractional filling

$$\mu=\frac{\mu^*}{1+2\mu^*}, \quad (5)$$

where $\mu^*=\pm 1, \pm 2, \ldots$ describes the CF ''hole'' filling of the condensed liquid states. The analog of the half-filled state in this case is obtained for $S_h^*=0$. The half-filled state should be obtained for the same values of the monopole flux in the two representations. However, $S_e^*=0$ when $2S=2(N_e-1)$, whereas $S_h^*=0$ when $2S=2N_e$. In other words, for the same value of N_e the half-filled state occurs for an electron CF at values of $2S$ different from that for a CF hole.

In the light of the above considerations, one could think of three possible descriptions of the half-filled Landau level (or the half-filled shell): $2N_e = 2N_h=2S+1$, which occurs only when $2S$ is odd (because N_e and N_h are integers) and corresponds to having half as many electrons as there are single-particle states $2S+1$; $2S_e^*=0$, which occurs at $2S=2(N_e-1)$ or at $2S=2N_h$ and corresponds to CF electrons seeing an effective magnetic field $B_e^*=0$; and $2S_h^*=0$, which occurs at $2S=2(N_h-1)$ or at $2S=N_e$ and corresponds to an effective magnetic field B_h^* seen by the CF holes equal to zero. In this paper we show how this dilemma leads to ''twin'' or ''alias'' states[6] and we discuss the appropriate description of the half-filled Landau level for systems with small number of particles.

II. COMPOSITE FERMION TRANSFORMATION AND ELECTRON-HOLE SYMMETRY

The states of the Jain sequence occur when $2S_e$ is such that an integral number of shells is filled with CF electrons. If n shells are filled, then the number of electrons satisfies

$$\sum_i^n g_i=N_e. \quad (6)$$

When the summation is performed this is written

$$(2|S_e^*|+n)n=N_e. \quad (7)$$

If we identify n with the magnitude of ν^*, the inverse of the effective flux seen by a CF electron, and impose the condition that ν^* have the same sign as S_e^*, then $\nu^*=\nu_+^*$ if $S>(N_e-1)$, $\nu^*=\nu_-^*$ if $S<(N_e-1)$, and ν^* is undefined if $S=N_e-1$. Here $\nu_\pm^*$ are given by

$$\nu_\pm^*=-S_e^*\pm[(S_e^*)^2+N_e]^{1/2}. \quad (8)$$

The same analysis applied to CF holes filling an integral number of shells gives $\mu^*=\mu_+^*$ if $S<N_e$, $\mu^*=\mu_-^*$ if $S>N_e$, and μ^* is undefined if $S=N_e$. Here $\mu_\pm^*=-S_h^*\pm[(S_h^*)^2+N_h]^{1/2}$. From their definition, $S_e^*+S_h^*=1$. This implies that $(S_e^*)^2+N_e=(S_h^*)^2+N_h$ and

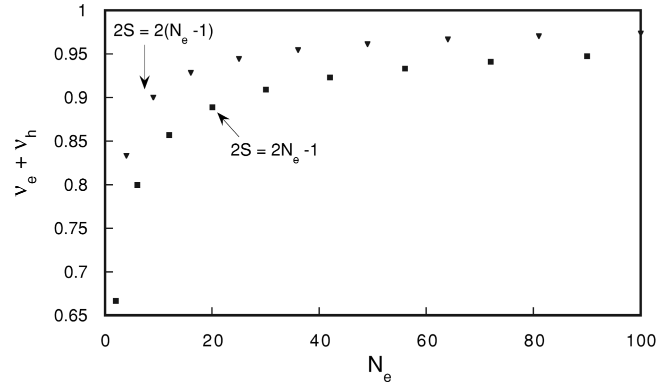

FIG. 1. Plot of $\nu_e+\nu_h$ as determined from the Jain construction for $2S=2(N_e-1)$ and $2S=2N_e-1$.

$\nu_+^*+\mu_-^*=\nu_-^*+\mu_+^*=-(S_e^*+S_h^*)=-1$. From this one can easily demonstrate that for values of S outside the range $N_e-1\leqslant S\leqslant N_e$, the sum of ν and μ [as defined by Eqs. (2) and (5), respectively] is equal to unity. For the three values in the range $N_e-1\leqslant S\leqslant N_e$, the sum $\nu+\mu$ is always less than unity.

In Fig. 1 we plot $\nu+\mu$ vs N_e for states in the principal Jain sequence that have $S=N_e-1$ and $S=N_e-1/2$. It has been common to assume that $\nu+\mu=1$ for all values of $2S$. We have just demonstrated that for finite-size systems in a spherical surface this is not true if S is in the range $N_e-1<S<N_e$. We believe that use of $\nu+\mu=1$ in situations where it is not valid and defining values of ν^* (or μ^*) when $S_e^*=0$ (or $S_h^*=0$) leads to twin or alias states discussed by other authors.[5,6] The electron-hole symmetry can be employed to resolve alias states obtained for $(N_e,2S)=(n^2,2n^2-2)$. In these cases, $|\nu^*|=n$ and two fractional fillings, both corresponding to incompressible states, can be envisioned, $\nu=\nu^*/(2p\nu^*\pm 1)$, depending on the sign chosen for ν^*. The $N_h=n^2-1$, CF holes, however, experience a nonzero effective field $S_h^*=1$ and occupy exactly $\mu^*=n-1$ shells. From Eq. (5), μ is $(n-1)/(2n-1)$. ν is then simply determined, as $1-\mu$, leading to the only possible electron fractional filling $\nu=n/(2n-1)$.

The fractional fillings of states belonging to the principal Jain sequence for an $N_e=12$ electron system calculated for values of $2S$ going from 11 to 33 are given in Table I. Equally good candidates for the ''half-filled'' Landau level are the states occurring at $2S=22$ when $S_e^*=0$ and at $2S=24$ when $S_h^*=0$. The former is represented as being half filled with electrons, whereas the latter is half filled with holes. The CF electron state with $2S=22$ can be thought of as containing three quasielectrons of the $\nu=3/5$ state (or of the $\nu=3/7$) state. Since each quasielectron has angular momentum $l_{\mathrm{QE}}=3$, the allowed multiplets of the mean-field ground state have $L=0\oplus 2\oplus 3\oplus 4\oplus 6$ and the ground state is highly degenerate. This is expected for a half-filled state.[4] For $2S=23$, both descriptions give a 3/7-filled state, since this state is less than half filled for both CF electrons and CF holes. It has $\nu^*=\mu^*=3$. Notice that for $2S<22$ and $2S>24$, $\nu^*+\mu^*=-1$ and $\nu+\mu=1$.

However, whenever N_e is the square of an integer $N_e=n^2=2^2,3^2,\ldots$, the value $S_e^*=0$ occurs at an integral CF filling $|\nu^*|=n$. This means that even though $S_e^*=0$ and

TABLE I. Values of $2S_e^*$, the effective flux seen by a composite fermion electron, and $2S_h^*$, that seen by a CF hole, are given for various values of $2S$. g_{ei} (g_{hi}) are the degeneracies of the ith electron (hole) CF level. ν_e^* and ν_h^* are the effective integral CF fillings and $\nu_e = \nu_e^*/(1+2\nu_e^*)$ and $\nu_h = \nu_h^*/(1+2\nu_h^*)$. $N_e = 12$.

$2S$	$2S_e^*$	g_{e1}	g_{e2}	g_{e3}	ν^*	ν	N_h	$2S_h^*$	g_{h1}	g_{h2}	g_{h3}	μ^*	μ
11	−11	12			−1	1	0						
18	−4	5	7		−2	2/3	7	6	7			1	1/3
21	−1	2	4	6	−3	3/5	10	3	4	6		2	2/5
22	0	1				1/2		11	2	2			
23	1	2	4	6	3	3/7	12	1	2	4	6	3	3/7
24	2	3					13	0	1				1/2
26	4	5	7		2	2/5	15	−2	3	5	7	−3	3/5
33	11	12			1	1/3	22	−9	10	12		−2	2/3

the effective magnetic field $\omega_c^* = 0$, there is a gap in the spectrum produced by the kinetic energy $(\hbar^2/2mR^2)n(n+1)$. The ground state is nondegenerate and it does not appear to have the properties expected for a half-filled state. Thus the cancellation of the effective field experienced by a CF is a rather necessary but not sufficient condition for a compressible state. This argument is supported by the results of exact numerical diagonalization of the Hamiltonian for a system $(N_e, 2S) = (9,16)$, which show that the ground state of total angular momentum $L = 0$ is clearly separated from the first excited states. Furthermore, the energy spectrum is identical, up to an overall constant, to that of $(N_e, 2S) = (8,16)$. In this latter case, $2S_e^* = 2$ and $\nu = 2/5$. It seems that the description of $(N_e, 2S) = (9,16)$ as a half-filled state using as an argument just the cancellation of the effective field is ill advised. Some care must be exercised to be sure that the candidate half-filled state has a highly degenerate ground state.

III. SUMMARY

In this paper we have demonstrated that due to finite-size effects $\nu + \mu$ as defined by Jain's CF picture is not equal to unity for values of $2S$ in the range $N_e - 1 \leqslant S \leqslant N_e$. We believe that inappropriate use of $\nu + \mu = 1$ leads to twin or alias states. In addition, when $2S_e^* = 0$ (or $2S_h^* = 0$) a half-filled state does not necessarily result since the actual energy spectrum can be identified with that of a well-defined condensed state of the Jain sequence. In the CF hierarchy picture[7] in which flux tubes are attached to quasiparticle excitations (CF in an otherwise empty shell) to give new mean-field composite fermions, this same ambiguity arises. Whenever the effective flux value $2S^*$ at a given value of the hierarchy is related to the number of quasielectrons by $2(n_{QE} - 1) \leqslant 2S^* \leqslant 2n_{QE}$, attaching flux quanta to quasiholes instead of quasielectrons gives fillings ν_i and μ_i that do not sum to unity.

ACKNOWLEDGMENTS

We are grateful for the support in completing this work provided by Lockheed Martin Energy Research and the Department of Energy. P.S. would also like to acknowledge support by the KBN Grant No. PB 674/P03/96/10. K. S. Y. acknowledges support by the BSRI-97-2412 program of the Ministry of Education, Korea.

[1] J. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[2] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[3] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1993).
[4] E. Rezayi and N. Read, Phys. Rev. Lett. **72**, 900 (1994).
[5] J. K. Jain, Phys. Rev. Lett. **73**, 1051 (1994).
[6] Claudius Gross and A. H. MacDonald, Phys. Rev. B **42**, 9514 (1990).
[7] P. Sitko, K. S. Yi, and J. J. Quinn (unpublished).