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Exchange and Correlation Corrections to the Response Functions of a Spin-Polarized Electron Gas

D C. Marinescu
Clemson University, dcm@clemson.edu

J J. Quinn
University of Tennessee

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Exchange and correlation corrections to the response functions of a spin-polarized electron gas

D. C. Marinescu
Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

J. J. Quinn
Department of Physics, University of Tennessee, Knoxville, Tennessee 37966

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We analyze the spin and charge responses induced in a spin-polarized electron gas by a weak electromagnetic field. The coupled spin-charge response is derived from the equation of motion of the particle distribution function in the presence of the perturbation. To obtain the correct frequency and the wave-vector dependence we introduce the spin-dependent local-field factors, $G_\sigma^{\pm}=G_\sigma^x \pm G_\sigma^c$, which give the exchange (x) and correlation (c) corrections to the random phase approximation. For an arbitrary degree of polarization of the electron gas, we derive the exact analytical expressions for $G_\sigma^{\pm}(\vec{q},\omega)$ in the limit of high frequency or large wave vectors. The results for $\vec{q}\to\infty$ are expressed in terms of the two-particle correlation function, $g(\vec{r})$ at $r=0$. [S0163-1829(97)04628-6]

I. INTRODUCTION

In the study of the response functions of an electron gas with a positive background, the concept of a local-field correction originates in the difference between the effective potential experienced by an electron and the mean field value. Exchange and correlation effects associated with the Coulomb repulsion, which determine a decrease in the number of particles surrounding a given electron are responsible for this deviation. The exchange-correlation hole modifies the interaction between a probe electron and the rest of the electron gas, leading to a potential variation. A self-consistent approximation first proposed by Kukkonen and Overhauser[1] can be generalized to[2]

$$\Delta\Phi_\sigma(\vec{q},\omega)=v(\vec{q})[1-G_\sigma^{+}(\vec{q},\omega)]\Delta n(\vec{q},\omega) - G^{-}(\vec{q},\omega)v(\vec{q})\vec{\sigma}\cdot\Delta\vec{s}(\vec{q},\omega), \tag{1}$$

where $v(\vec{q})=e^2/4\pi q^2$ is the Fourier transform of the Coulomb interaction. Equation (1) defines the local-field corrections as the coupling functions of an electron with spin σ to density and spin fluctuations, $\Delta n(\vec{q},\omega)$ and $\Delta\vec{s}(\vec{q},\omega)$, respectively. The spin-symmetric character of Δn indicates that G_σ^{+} is the sum of the parallel and antiparallel spin effects, whereas the spin-antisymmetry of $\Delta\vec{s}$ imposes that G_σ^{-} is the difference. If we introduce G_σ^x for the exchange, $G_{\sigma\sigma}^c$ for the same-spin correlation, and $G_{\sigma\bar{\sigma}}^c$ for opposite-spin correlation, we can write

$$G_\sigma^{+}(\vec{q},\omega)=G_\sigma^x(\vec{q},\omega)+G_{\sigma\sigma}^c(\vec{q},\omega)+G_{\sigma\bar{\sigma}}^c(\vec{q},\omega), \tag{2}$$

$$G_\sigma^{-}(\vec{q},\omega)=G_\sigma^x(\vec{q},\omega)+G_{\sigma\sigma}^c(\vec{q},\omega)-G_{\sigma\bar{\sigma}}^c(\vec{q},\omega). \tag{3}$$

In the spin-polarized electron gas a static magnetic field induces an equilibrium imbalance in the number of electrons of opposite spins. As a consequence, the response to a weak electromagnetic perturbation consists of coupled charge and spin fluctuations, characterized by appropriate susceptibility functions: pure electric, χ_{ee}, pure magnetic (longitudinal and transversal), χ_{mm}, and coupled magnetic-electric, χ_{em} and χ_{me}. The investigation of these functions has been done just recently.[3] In this case, the exchange-correlation interactions are functions of the initial direction of the electron spin and the local-field corrections depend parametrically on the degree of polarization of the system.

Because of the difficulty involved in approximating many-body interactions, limited knowledge has been gained about the exact expressions of the local-field corrections. For an unpolarized electron gas, it has been established from the compressibility relation that at small $\vec{q}$, $G_\sigma^{\pm}(\vec{q},\omega)$ varies quadratically.[4] Using the equation-of-motion method, Niklasson[5] and Zhou and Overhauser[6] derived the limits for large wave vectors:

$$\lim_{q\to\infty} G^{+}(\vec{q},\omega)=\tfrac{2}{3}[1-g(0)], \tag{4}$$

$$\lim_{q\to\infty} G^{-}(\vec{q},\omega)=\tfrac{1}{3}[4g(0)-1]. \tag{5}$$

Here $g(0)$ is the two-particle correlation function at the origin. In this paper we obtain the equivalent asymptotes for the local-field corrections in a spin-polarized electron gas.

II. MANY-BODY CORRECTIONS IN THE EFFECTIVE FIELD APPROXIMATION

An electron gas—N electrons with a uniform positive background, confined in the volume ν—is spin polarized by a static magnetic field, $\vec{B}=B\hat{z}$, such that in equilibrium there are $N_\uparrow$ electrons with spins parallel to the field, and $N_\downarrow$ electrons with spins of opposite orientation. Any degree of polarization of the system, $\zeta=(N_\uparrow-N_\downarrow)/N$, can be obtained by adjusting the value of $\vec{B}$. The external perturbation consists of an electric potential, $\phi(\vec{r},t)$, and a magnetic field, $\vec{b}(\vec{r},t)$, whose direction is specified by the unit vector $\hat{u}=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$. Assuming a sinusoidal time and position dependence of the fields, based on Eq. (1), a self-consistent approximation for the Fourier component of the effective one-particle perturbing Hamiltonian is

$$H_\sigma(\vec{q},\omega)=\gamma\vec{b}(\vec{q},\omega)\cdot\vec{\sigma}-e\phi(\vec{q},\omega)+v(\vec{q})[(1-G_\sigma^+)\Delta n(\vec{q},\omega)-G_\sigma^-\vec{\sigma}\cdot\Delta\vec{s}(\vec{q},\omega)],\tag{6}$$

where γ is the effective Bohr magneton. The exchange-correlation effects are considered through the local-field factors, G_σ^+ and G_σ^-, defined by Eqs. (2)–(3). $\vec{\sigma}$ is the usual Pauli operator for the electron spin. In Eq. (6) we have neglected the magnetic spin-spin interaction, much smaller than the Coulomb repulsion. Introducing the raising and lowering spin operators, σ_+ and σ_-, respectively, the product $\vec{b}\cdot\vec{\sigma}$ can be written as

$$\vec{b}\cdot\vec{\sigma}=\tfrac{1}{2}b_+\sigma_-+\tfrac{1}{2}b_-\sigma_++b_z\sigma_z.\tag{7}$$

In this form it is easy to see that $b_+=b_x+ib_y$ causes electrons to flip spins from up to down, while $b_-=b_x-ib_y$ has the opposite effect. These processes generate the transverse (in respect to the direction of initial polarization) spin response. The electric field and b_z preserve the initial spin state and induce fluctuations in the number of electrons whose spin remains parallel to the direction of the static field. As a result of the spin-response anisotropy, the associated local-field correction, G_σ^-, is going to be direction dependent: $G_{L,\sigma}^-$ for longitudinal variations and $G_{T,\sigma}^-$ for the transverse ones.

The effective potential experienced by an electron of spin-projection σ along the $\hat{z}$ axis is then, from Eqs. (6) and (7),

$$\Phi_\sigma(\vec{q},\omega)=\gamma b_z(\vec{q},\omega)\mathrm{sgn}(\sigma)-e\phi(\vec{q},\omega)+v(\vec{q})[(1-G_\sigma^+)\Delta n(\vec{q},\omega)-\mathrm{sgn}(\sigma)G_{L,\sigma}^-\Delta\vec{s}(\vec{q},\omega)],\tag{8}$$

where sgn(σ) is equal to 1 (or -1) when the spin is parallel (or antiparallel) to the field.

Two complementary equations are written for the effective potentials experienced by electrons which flip spin,

$$\Phi_+(\vec{q},\omega)=\gamma b_+(\vec{q},\omega)-v(\vec{q})G_{T,\uparrow}^-\Delta s_+(\vec{q},\omega),\tag{9}$$

$$\Phi_-(\vec{q},\omega)=\gamma b_-(\vec{q},\omega)-v(\vec{q})G_{T,\downarrow}^-\Delta s_-(\vec{q},\omega).\tag{10}$$

The spin index of the transverse local-field corrections is chosen to correspond to the initial direction of the spin. Observe that for an arbitrary degree of polarization, $|\zeta|\leqslant1$, $G_\sigma^-(\zeta)=G_{\bar{\sigma}}^-(-\zeta)$.

The first order perturbation theory shows that the induced fluctuations are linearly related to the effective potentials by

$$\Delta n_\sigma(\vec{q},\omega)=\Pi_{\sigma\sigma}\Phi_\sigma(\vec{q},\omega),\tag{11}$$

$$\Delta n_+(\vec{q},\omega)=\Pi_{\uparrow\downarrow}\Phi_+(\vec{q},\omega),\tag{12}$$

$$\Delta n_-(\vec{q},\omega)=\Pi_{\downarrow\uparrow}\Phi_-(\vec{q},\omega).\tag{13}$$

We elect to include all the many-body effects in the local-field corrections, $G_\sigma(\vec{q},\omega)$ and, consequently, write the coefficient of proportionality, $\Pi_{\sigma\sigma'}$, as the noninteracting electron response function:

$$\Pi_{\sigma\sigma'}(\vec{q},\omega)=\frac{1}{\nu}\sum_{\vec{k}}\frac{n_{\vec{k}-\vec{q}/2,\sigma'}-n_{\vec{k}+\vec{q}/2,\sigma}}{\hbar\omega-(\epsilon_{\vec{k}+\vec{q}/2,\sigma}-\epsilon_{\vec{k}-\vec{q}/2,\sigma'})},\tag{14}$$

where $\epsilon_{\vec{k},\sigma}=\hbar^2k^2/2m-\gamma\mathrm{sgn}(\sigma)B$ is the equilibrium energy in the static magnetic field for an electron with statistical distribution $n_{\vec{k},\sigma}$, momentum $\hbar\vec{k}$, and spin projection σ along the $\hat{z}$ axis. [An alternate definition of $\Pi_{\sigma\sigma'}$ which includes many-body corrections to the one electron energies is discussed by Sturm.[8]]

When written for up and down spins, Eq. (11) leads to a system of coupled equations which can be solved for $\Delta n_\uparrow$ and $\Delta n_\downarrow$ in terms of the external perturbation. We express these results as charge density, $\Delta\rho=-e(\Delta n_\uparrow+\Delta n_\downarrow)$, and longitudinal magnetization, $\Delta m_z=-\gamma(\Delta n_\uparrow-\Delta n_\downarrow)$, fluctuations. The coefficients of the linear system which relates the induced response to the perturbing fields are the susceptibility functions. Therefore,

$$\Delta\rho=\chi_{ee}\phi+\chi_{em}b_z,$$

$$\Delta m_z=\chi_{me}\phi+\chi_{mm}b_z.\tag{15}$$

Because of the initial spin polarization, the response consists of coupled charge and spin fluctuations described by

$$\chi_{ee}=\frac{e^2}{D}[\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow}+2\Pi_{\uparrow\uparrow}\Pi_{\downarrow\downarrow}v(\vec{q})(G_{L,\uparrow}^-+G_{L,\downarrow}^-)],\tag{16}$$

$$\chi_{em}=-\frac{e\gamma}{D}[\Pi_{\uparrow\uparrow}-\Pi_{\downarrow\downarrow}+v(\vec{q})\Pi_{\uparrow\uparrow}\Pi_{\downarrow\downarrow}(G_{L,\downarrow}^--G_{L,\uparrow}^-)],\tag{17}$$

$$\chi_{me}=\frac{e\gamma}{D}[\Pi_{\uparrow\uparrow}-\Pi_{\downarrow\downarrow}+2v(\vec{q})\Pi_{\uparrow\uparrow}\Pi_{\downarrow\downarrow}(G_\downarrow^+-G_\uparrow^+)],\tag{18}$$

$$\chi_{mm}=-\frac{\gamma^2}{D}[\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow}-v(\vec{q})\Pi_{\uparrow\uparrow}\Pi_{\downarrow\downarrow}(2-G_\downarrow^+-G_\uparrow^+)].\tag{19}$$

D is the determinant of the system:

$$D=\tfrac{1}{2}[1-2v(\vec{q})\Pi_{\uparrow\uparrow}(1-G_\uparrow^+)][1+2v(\vec{q})\Pi_{\downarrow\downarrow}G_{L,\downarrow}^-]+\tfrac{1}{2}[1-2v(\vec{q})\Pi_{\downarrow\downarrow}(1-G_\downarrow^+)][1+2v(\vec{q})\Pi_{\uparrow\uparrow}G_{L,\downarrow}^-].$$

Here, the wave vector and frequency dependence of the local-field corrections was implicitly assumed. The transverse magnetic fluctuations are obtained from Eqs. (12) and (13):

$$\Delta m_+ = -\gamma \Delta n_+ = \chi^+_{mm} b_+ ,$$

$$\Delta m_- = -\gamma \Delta n_- = \chi^-_{mm} b_- , \tag{20}$$

along with the corresponding susceptibilities:

$$\chi^+_{mm} = -\frac{2\gamma^2 \Pi_{\uparrow\downarrow}}{1+2v(\vec{q})\Pi_{\uparrow\downarrow} G^-_{T,\uparrow}}, \tag{21}$$

$$\chi^-_{mm} = -\frac{2\gamma^2 \Pi_{\downarrow\uparrow}}{1+2v(\vec{q})\Pi_{\downarrow\uparrow} G^-_{T,\downarrow}}. \tag{22}$$

These results have been derived by Yi and Quinn[3] and can be used as alternative definitions for the many-body local-field corrections, $G^{\pm}_{\sigma}$, longitudinal or transverse. A correct wave-vector and frequency dependence of the susceptibility functions can be obtained only when the right behavior of the many-body corrections is known.

III. MICROSCOPIC DERIVATION OF THE MANY-BODY CORRECTIONS

We proceed by extending the equation-of-motion method, first proposed by Niklasson,[5] to the physical system described in the previous section. The unperturbed Hamiltonian of the system, H_0, is

$$H_0 = \sum_{\vec{k},\sigma} \epsilon_{\vec{k},\sigma} c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma} + \frac{1}{2\nu} \sum_{\vec{q}} v(q) \sum_{\vec{k},\sigma} \sum_{\vec{k}',\sigma'} c^\dagger_{\vec{k}-\vec{q}/2,\sigma} c^\dagger_{\vec{k}'+\vec{q}/2,\sigma'} \times c_{\vec{k}-\vec{q}/2,\sigma'} c_{\vec{k}+\vec{q}/2,\sigma} , \tag{23}$$

with $\epsilon_{\vec{k},\sigma} = \hbar^2/2m - \gamma B \operatorname{sgn}(\sigma)$, the energy of an electron with spin σ in the static magnetic field. Operators $c^\dagger_{\vec{k},\sigma}$ and $c_{\vec{k},\sigma}$ create and annihilate an electron of momentum $\hbar\vec{k}$ and spin projection σ *along an arbitrary axis* $\hat{u}$. In particular, we choose $\hat{u}$ to be along the perturbing magnetic field, such that the electronic spinors are eigenfunctions of $\vec{\sigma}\cdot\vec{b}$. In this representation, the Hamiltonian of the perturbation is just

$$H_1(t) = \sum_{\vec{q}} \sum_{\vec{k},\sigma} [\gamma b(-\vec{q},t)\operatorname{sgn}(\sigma) - e\phi(-\vec{q},t)] c^\dagger_{\vec{k}-\vec{q}/2,\sigma} c_{\vec{k}+\vec{q}/2,\sigma} . \tag{24}$$

The orthogonal transformation to creation or annihilation operators for electrons with spins projected along the $\hat{z}$ axis is realized by

$$c_{\vec{k},\uparrow} = a_{\vec{k},\uparrow} \cos\frac{\theta}{2} + a_{\vec{k},\downarrow} \sin\frac{\theta}{2} e^{i\phi}, \tag{25}$$

$$c_{\vec{k},\downarrow} = a_{\vec{k},\uparrow} \sin\frac{\theta}{2} - a_{\vec{k},\downarrow} \cos\frac{\theta}{2} e^{i\phi}. \tag{26}$$

Since the kinetic energy and the Coulomb interaction terms do not depend on the axis on which the spin is projected, we can immediately write, in terms of the a's operators,

$$H_0 = \sum_{\vec{k}} \epsilon_{\vec{k},\sigma} a^\dagger_{\vec{k},\sigma} a_{\vec{k},\sigma} + \frac{1}{2\nu} \sum_{\vec{q}} v(q) \sum_{\vec{k},\sigma} \sum_{\vec{k}',\sigma'} a^\dagger_{\vec{k}-\vec{q}/2,\sigma} a^\dagger_{\vec{k}'+\vec{q}/2,\sigma'} \times a_{\vec{k}-\vec{q}/2,\sigma'} a_{\vec{k}+\vec{q}/2,\sigma} . \tag{27}$$

Substituting Eqs. (25) and (26) into (24), the interaction Hamiltonian becomes

$$H_1(t) = \gamma \sum_{\vec{q},\vec{k}} \Big\{ b_+(-\vec{q},t) a^\dagger_{\vec{k}-\vec{q}/2,\uparrow} a_{\vec{k}+\vec{q}/2,\downarrow} + b_-(-\vec{q},t) a^\dagger_{\vec{k}-\vec{q}/2,\downarrow} a_{\vec{k}+\vec{q}/2,\uparrow} + \sum_{\sigma} [b(-\vec{q},t)\operatorname{sgn}(\sigma) - e\phi(-\vec{q},t)] a^\dagger_{\vec{k}-\vec{q}/2,\sigma} a_{\vec{k}+\vec{q}/2,\sigma} \Big\} . \tag{28}$$

The first two terms describe electronic spin-flip processes driven by the transverse components of the magnetic field, b_+ and b_-, while the third one gives the coupled density and spin fluctuations of the electrons whose initial spin state remains unchanged under the perturbation.

A. The longitudinal response

The electric potential and the $\hat{z}$ component of the perturbing magnetic field induce fluctuations in the number of electrons of a given spin without changing their spin state. The dynamical deviations from equilibrium are described by a Wigner distribution function:

$$f^{(1)}_{\vec{k},\sigma}(\vec{q},t) = \langle a^\dagger_{\vec{k}-\vec{q}/2,\sigma}(t) a_{\vec{k}+\vec{q}/2,\sigma}(t) \rangle, \tag{29}$$

such that the induced electron density is

$$\Delta n_\sigma(\vec{q},t) = \sum_{\vec{k}} \Delta f^{(1)}_{\vec{k},\sigma}(\vec{q},t). \tag{30}$$

$\Delta f^{(1)}_{\vec{k},\sigma}$ is the perturbation produced by the external field $\Delta f^{(1)}_{\vec{k},\sigma}(\vec{q},t) = f^{(1)}_{\vec{k},\sigma}(\vec{q},t) - \delta_{\vec{q},0} n_{\vec{k},\sigma}$. The time-dependent behavior of the Wigner distribution is determined by the equation of motion:

$$i\hbar \frac{\partial}{\partial t} \langle a^\dagger_{\vec{k}-\vec{q}/2,\sigma}(t) a_{\vec{k}+\vec{q}/2,\sigma}(t) \rangle = \langle [a^\dagger_{\vec{k}-\vec{q}/2,\sigma}(t) a_{\vec{k}+\vec{q}/2,\sigma}(t), H_0 + H_1(t)] \rangle. \tag{31}$$

Taking the Fourier transform one obtains

$$[\hbar\omega-(\epsilon_{\vec{k}+\vec{q}/2,\sigma}-\epsilon_{\vec{k}-\vec{q}/2,\sigma})]\Delta f^{(1)}_{\vec{k},\sigma}(\vec{q},\omega)$$
$$=\frac{1}{\nu}(n_{\vec{k}-\vec{q}/2,\sigma}-n_{\vec{k}+\vec{q}/2,\sigma})\{\gamma b_z(\vec{q},\omega)\mathrm{sgn}(\sigma)$$
$$-e\phi(\vec{q},\omega)+v(\vec{q})[\Delta n_\uparrow(\vec{q},\omega)+\Delta n_\downarrow(\vec{q},\omega)]\}$$
$$+\frac{1}{\nu}\sum_{\vec{q}'}v(\vec{q}')\sum_{\vec{k}',\sigma'}[\Delta f^{(2)}_{\vec{k}-\vec{q}'/2,\sigma;\vec{k}',\sigma'}(\vec{q}-\vec{q}',\vec{q}';t)$$
$$-\Delta f^{(2)}_{\vec{k}+\vec{q}'/2,\sigma;\vec{k}',\sigma'}(\vec{q}-\vec{q}',\vec{q}';t)], \tag{32}$$

where

$$\Delta f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';t)$$
$$=\langle a^\dagger_{\vec{k}-\vec{q}/2,\sigma}(t)a^\dagger_{\vec{k}'-\vec{q}'/2,\sigma'}(t)a_{\vec{k}'+\vec{q}'/2,\sigma'}(t)a_{\vec{k}+\vec{q}/2,\sigma}\rangle$$
$$-\langle a^\dagger_{\vec{k}-\vec{q}/2,\sigma}(t)a_{\vec{k}+\vec{q}/2,\sigma}(t)\rangle\langle a^\dagger_{\vec{k}'-\vec{q}'/2,\sigma'}(t)a_{\vec{k}'+\vec{q}'/2,\sigma'}\rangle$$
$$-\delta_{\vec{q}+\vec{q}',0}\,f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q}) \tag{33}$$

is the perturbed part of the two-particle distribution function. At equilibrium, this function is

$$f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q})=\langle 0|a^\dagger_{\vec{k}-\vec{q}/2,\sigma}a^\dagger_{\vec{k}'+\vec{q}/2,\sigma'}a_{\vec{k}'+\vec{q}/2,\sigma'}a_{\vec{k}+\vec{q}/2,\sigma}|0\rangle$$
$$-\langle 0|a^\dagger_{\vec{k}-\vec{q}/2,\sigma}a_{\vec{k}+\vec{q}/2,\sigma}|0\rangle$$
$$\times\langle 0|a^\dagger_{\vec{k}-\vec{q}/2,\sigma'}a_{\vec{k}'+\vec{q}/2,\sigma'}|0\rangle. \tag{34}$$

Therefore, the induced density fluctuations for electrons of a given spin σ can be obtained from Eqs. (30) and (32):

$$\Delta n_\sigma(\vec{q},\omega)=\Pi_{\sigma\sigma}[-e\phi(\vec{q},\omega)+\gamma\mathrm{sgn}(\sigma)b_z(\vec{q},\omega)$$
$$+v(\vec{q})(\Delta n_\uparrow+\Delta n_\downarrow)]+\frac{1}{\nu}\sum_{\vec{q}'}v(q')$$
$$\times\sum_{\vec{k},\sigma}\sum_{\vec{k}',\sigma'}\frac{1}{\hbar\omega-(\epsilon_{\vec{k}+\vec{q}/2,\sigma}-\epsilon_{\vec{k}-\vec{q}/2,\sigma})}$$
$$\times[\Delta f^{(2)}_{\vec{k}-\vec{q}'/2,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}',\omega)$$
$$-\Delta f^{(2)}_{\vec{k}+\vec{q}'/2,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}',\omega)]. \tag{35}$$

Again, $\Pi_{\sigma\sigma}$ is the polarization function for spin σ electrons in the absence of the interaction, defined by Eq. (14). Since the fluctuation of the electron density for a given spin depends self-consistently on the induced densities of electrons of both spins, the dielectric and magnetic responses are coupled.

Equation (35) does not give a straightforward solution for Δn_σ since the behavior of $\Delta f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}',\omega)$ is not known. The two-particle fluctuations are determined by the commutator of two-body distribution function with the Hamiltonian. They satisfy the following equation:

$$\left[\hbar\omega-\frac{\hbar^2\vec{k}\cdot\vec{q}}{m}-\frac{\hbar^2\vec{k}\cdot\vec{q}'}{m}\right]\Delta f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)$$
$$=\frac{1}{\nu}[f^{(2)}_{\vec{k}-(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}')$$
$$-f^{(2)}_{\vec{k}+(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}')]$$
$$\times\Psi_\sigma(\vec{q}+\vec{q}',\omega)+\frac{1}{\nu}[f^{(2)}_{\vec{k}'-(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q})$$
$$-f^{(2)}_{\vec{k}'+(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q})]\Psi_{\sigma'}(\vec{q}+\vec{q}',\omega)$$
$$+\overline{F}^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)+\overline{F}^{(m)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)$$
$$+\overline{F}^{(m)}_{\vec{k}',\sigma';\vec{k},\sigma}(\vec{q}',\vec{q};\omega). \tag{36}$$

Ψ_σ is the external perturbing field experienced by an electron. For a spinor projected along the $\hat{z}$ axis, $\Psi_\sigma=-e\phi+\gamma b_z\mathrm{sgn}(\sigma)$.

In this expression, the first two terms on the right-hand side originate in the interaction of one electron with the external field, in the presence of another electron. We have followed Niklasson[5] and used $\overline{F}^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}$ to describe the mutual interaction between the two electrons in the presence of all the other members of the gas. $\overline{F}^{(m)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)$ and $\overline{F}^{(m)}_{\vec{k}',\sigma';\vec{k},\sigma}(\vec{q},\vec{q}';\omega)$ represent the interaction of one electron with the rest of the particles, in the presence of the other electron. Three-particle correlations are involved in all these terms.

B. The transverse response

Under the influence of the transverse components of the magnetic perturbation, $b_\pm(\vec{q},\omega)$, some of the electrons change their initial spin state: $b_+(\vec{q},\omega)$ determines transitions from up to down, while $b_-(\vec{q},\omega)$ has an opposite effect. The appropriate Wigner distribution to describe the induced magnetization can be derived from the following considerations. The particle number operator for electrons with spin projection σ *along* the $\hat{x}$ *axis* is

$$\eta^x_{\vec{k},\sigma}(\vec{q},t)=c^{x\dagger}_{\vec{k}-\vec{q}/2,\sigma}(t)c^x_{\vec{k}+\vec{q}/2,\sigma}(t), \tag{37}$$

where the operator $c^x_{\vec{k},\sigma}$ and its conjugate are given by Eq. (26) particularized for the $\hat{x}$ axis $(\theta=90°,\phi=0°)$. After the substitutions are performed, one can write immediately

$$\eta^x_{\vec{k},\uparrow}(\vec{q},t)-\eta^x_{\vec{k},\downarrow}(\vec{q},t)=a^+_{\vec{k}-\vec{q}/2,\uparrow}(t)a_{\vec{k}+\vec{q}/2,\downarrow}(t)$$
$$+a^+_{\vec{k}-\vec{q}/2,\downarrow}(t)a_{\vec{k}+\vec{q}/2,\uparrow}(t), \tag{38}$$

and analogously for the $\hat{y}$ axis $(\theta=90°,\phi=90°)$,

$$\eta^y_{\vec{k},\uparrow}(\vec{q},t)-\eta^y_{\vec{k},\downarrow}(\vec{q},t)=i[a^\dagger_{\vec{k}-\vec{q}/2,\uparrow}(t)a_{\vec{k}+\vec{q}/2,\downarrow}(t)$$
$$-a^\dagger_{\vec{k}-\vec{q}/2,\downarrow}(t)a_{\vec{k}+\vec{q}/2,\uparrow}(t)]. \tag{39}$$

It is convenient to form the linear combinations, $\Delta\eta^\pm=\Delta\eta^+\pm i\Delta\eta^-$,

$$\Delta\eta^{+}(\vec{q},t)=2a^{\dagger}_{\vec{k}-\vec{q}/2,\downarrow}(t)a_{\vec{k}+\vec{q}/2,\uparrow}(t), \tag{40}$$

$$\Delta\eta^{-}(\vec{q},t)=2a^{\dagger}_{\vec{k}-\vec{q}/2,\uparrow}(t)a_{\vec{k}+\vec{q}/2,\downarrow}(t). \tag{41}$$

Clearly, the relevant time-dependent Wigner distributions for the spin-flip processes induced by b_+ and b_-, respectively, are

$$f^{+}_{\vec{k}}(\vec{q},t)=2\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\downarrow}(t)a_{\vec{k}+\vec{q}/2,\uparrow}(t)\rangle,$$

$$f^{-}_{\vec{k}}=2\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\uparrow}(t)a_{\vec{k}+\vec{q}/2,\downarrow}(t)\rangle. \tag{42}$$

The corresponding Fourier components of the transverse magnetic fluctuations can be obtained by summing over $\vec{k}$:

$$\Delta m^{+}(\vec{q},\omega)=-\gamma\sum_{\vec{k}}\Delta f^{+}_{\vec{k}}(\vec{q},\omega), \tag{43}$$

$$\Delta m^{-}(\vec{q},\omega)=-\gamma\sum_{\vec{k}}\Delta f^{-}_{\vec{k}}(\vec{q},\omega). \tag{44}$$

Following the method outlined in the previous section, we derive the time variation of $\Delta m^{+}(\vec{q},\omega)$:

$$\begin{aligned}\Delta m^{+}(\vec{q},\omega)=&-2\gamma\Pi_{\uparrow\downarrow}b_{+}(\vec{q},\omega)-\frac{2\gamma}{\nu}\sum_{\vec{q}'}v(\vec{q}\,')\sum_{\vec{k}}\sum_{\vec{k}',\sigma'}\frac{1}{\hbar\omega-(\epsilon_{\vec{k}+\vec{q}/2,\uparrow}-\epsilon_{\vec{k}-\vec{q}/2,\downarrow})}\\&\times[\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\downarrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}+\vec{q}\,'-\vec{q}/2,\uparrow}(\omega)\rangle\\&-\langle a^{\dagger}_{\vec{k}-\vec{q}\,'+\vec{q}/2\downarrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}+\vec{q}/2,\uparrow}(\omega)\rangle].\end{aligned} \tag{45}$$

The four-operator averages included in Eq. (45) describe electrons interacting through the Coulomb interaction, while at the same time one of them changes its spin state. They are simply the two-particle fluctuations considered for electron spinors projected *along the* $\hat{x}$ *and* $\hat{y}$ *axis*. Employing the set of transformations given by Eqs. (25) and (26), one can show that

$$\begin{aligned}&\sum_{\sigma'}\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\downarrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}+\vec{q}/2,\uparrow}(\omega)\rangle\\&=\sum_{\sigma,\sigma'}[\mathrm{sgn}(\sigma)\langle c^{x\dagger}_{\vec{k}-\vec{q}/2,\sigma}(\omega)c^{x\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)c^{x}_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)c^{x}_{\vec{k}+\vec{q}/2,\sigma}(\omega)\rangle\\&\quad-i\,\mathrm{sgn}(\sigma)\langle c^{y\dagger}_{\vec{k}-\vec{q}/2,\sigma}(\omega)c^{y\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)c^{y}_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)c^{y}_{\vec{k}+\vec{q}/2,\sigma}(\omega)\rangle]\\&=\frac{1}{2}\sum_{\sigma,\sigma'}\{[\mathrm{sgn}(\sigma)\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)]_{x}-i[\mathrm{sgn}(\sigma)\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)]_{y}\}.\end{aligned} \tag{46}$$

$[\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)]_x$ (or $[\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)]_y$) is the perturbation of the distribution function of two particles whose spins are projected along $\hat{x}$, (or $\hat{y}$) direction, respectively.

An identical analysis can be performed for $\Delta m^{-}(\vec{q},\omega)$, with the result

$$\begin{aligned}\Delta m^{-}(\vec{q},\omega)=&-2\Pi_{\downarrow\uparrow}\gamma b_{-}(\vec{q},\omega)-\frac{2\gamma}{\nu}\sum_{\vec{q}'}v(\vec{q}\,')\sum_{\vec{k},\vec{k}',\sigma'}\frac{1}{\hbar\omega-(\epsilon_{\vec{k}+\vec{q}/2,\downarrow}-\epsilon_{\vec{k}-\vec{q}/2,\uparrow})}\\&\times[\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\uparrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}-\vec{q}\,'+\vec{q}/2,\downarrow}(\omega)\rangle\\&-\langle a^{\dagger}_{\vec{k}+\vec{q}\,'-\vec{q}/2,\uparrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}+\vec{q}/2,\downarrow}(\omega)\rangle],\end{aligned} \tag{47}$$

where from (25) and (26)

$$\begin{aligned}&\sum_{\sigma'}\langle a^{\dagger}_{\vec{k}-\vec{q}/2,\downarrow}(\omega)a^{\dagger}_{\vec{k}'-\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}'+\vec{q}\,'/2,\sigma'}(\omega)a_{\vec{k}+\vec{q}/2,\uparrow}(\omega)\rangle\\&=\frac{1}{2}\sum_{\sigma,\sigma'}\{[\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)\mathrm{sgn}(\sigma)]_{x}+i[\Delta f^{(2)}_{k,\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}\,';\omega)\mathrm{sgn}(\sigma)]_{y}\}.\end{aligned} \tag{48}$$

The frequency dependence of the pair distribution functions for transverse spin states is determined by equations of motion analogous to Eq. (36). In these cases, $\Psi_\sigma = \mathrm{sgn}(\sigma) b_\pm$. The corresponding equations for the four-operator averages are finally

$$\left[\hbar\omega - \frac{\hbar^2 \vec{k}\cdot\vec{q}}{m} - \frac{\hbar^2 \vec{k}'\cdot\vec{q}'}{m}\right]\sum_{\sigma'} \langle a^\dagger_{\vec{k}-\vec{q}/2,\downarrow} a^\dagger_{\vec{k}'+\vec{q}'/2,\sigma'} a_{\vec{k}'-\vec{q}'/2,\sigma'} a_{\vec{k}+\vec{q}/2,\uparrow}\rangle$$

$$= \frac{1}{2\nu}\sum_{\sigma,\sigma'} [f^{(2)}_{\vec{k}-(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}') - f^{(2)}_{\vec{k}+(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}')]\gamma b_+(\vec{q}+\vec{q}',\omega)$$

$$+ \frac{1}{2\nu}\sum_{\sigma,\sigma'} [f^{(2)}_{\vec{k}'-(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q}) - f^{(2)}_{\vec{k}'+(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q})]\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma')\gamma b_+(\vec{q}+\vec{q}',\omega)$$

$$+ \sum_{\sigma,\sigma'} [\overline{F}^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}',\omega) + \overline{F}^{(m)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega) + \overline{F}^{(m)}_{\vec{k}',\sigma';\vec{k},\sigma}(\vec{q},\vec{q}';\omega)]. \tag{49}$$

Also,

$$\left[\hbar\omega - \frac{\hbar^2 \vec{k}\cdot\vec{q}}{m} - \frac{\hbar^2 \vec{k}'\cdot\vec{q}'}{m}\right]\sum_{\sigma'} \langle a^\dagger_{\vec{k}-\vec{q}/2,\uparrow} a^\dagger_{\vec{k}'+\vec{q}'/2,\sigma'} a_{\vec{k}'-\vec{q}'/2,\sigma'} a_{\vec{k}+\vec{q}/2,\downarrow}\rangle$$

$$= \frac{1}{2\nu}\sum_{\sigma,\sigma'} [f^{(2)}_{\vec{k}-(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}') - f^{(2)}_{\vec{k}+(\vec{q}+\vec{q}')/2,\sigma;\vec{k}',\sigma'}(-\vec{q}')]\gamma b_-(\vec{q}+\vec{q}';\omega)$$

$$+ \frac{1}{2\nu}\sum_{\sigma,\sigma'} [f^{(2)}_{\vec{k}'-(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q}) - f^{(2)}_{\vec{k}'+(\vec{q}+\vec{q}')/2,\sigma';\vec{k},\sigma}(-\vec{q})]\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma')\gamma b_-(\vec{q}+\vec{q}';\omega)$$

$$+ \sum_{\sigma,\sigma'} [\overline{F}^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega) + \overline{F}^{(m)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega) + + \overline{F}^{(m)}_{\vec{k}',\sigma';\vec{k},\sigma}(\vec{q},\vec{q}';\omega)]. \tag{50}$$

As before, the first terms on the right-hand side of the above equations describe the interaction between an electron and the external field in the presence of the other electron (as a result of the external field one electron flips spin, while the ''spectator'' does not). Since the equilibrium values of the two-particle distribution functions do not depend on the particular axis of the spin projection, we have employed the following equality:

$$[f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q})]_{x,(y)} = f^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q}), \tag{51}$$

where $\overline{F}^{(2)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)$ represents the mutual interaction between the two electrons in the presence of the rest of the electron gas. $F^{(m)}_{\vec{k},\sigma;\vec{k}',\sigma'}(\vec{q},\vec{q}';\omega)$ and $\overline{F}^{(m)}_{\vec{k}',\sigma';\vec{k},\sigma}(\vec{q},\vec{q}';\omega)$ are a consequence of the many-body effects and correspond to the interaction between one electron and the rest of the gas in the presence of the other electron. These terms contain the perturbed parts of the three-particle distribution functions.

IV. THE LIMIT OF LARGE $\vec{Q}$ OR ω

The iterative procedure involved by the equation-of-motion method can be continued indefinitely: the time evolution of the n-body distribution function is dependent not only on the external fields, but also on the $(n+1)$-particle correlations. This chain can be terminated in the limit of large wave vectors or high frequency, when the Coulomb interaction between electrons becomes negligible with respect to the external perturbations. We consider that the outside electromagnetic field is strong enough, so only two-particle correlations are significant. Consequently, we neglect all the terms denoted by $\overline{F}_{\vec{k},\sigma,\vec{k}',\sigma'}$ in Eqs. (36), (49), and (50). Furthermore, we assume that $\hbar\omega$ and $\hbar^2 q^2/2m$ are much larger than the Zeeman splitting in the static magnetic field, γB. With these assumptions, the simplified equations for the two-particle distribution functions are substituted in Eqs. (35), (43), and (44). Keeping the leading terms in $\hbar\omega$ and $\hbar^2 q^2/2m$, we obtain

$$\Delta n_\sigma = \Pi_{\sigma\sigma}[\gamma b_z(\vec{q},\omega)\mathrm{sgn}(\sigma) - e\phi(\vec{q},\omega) + v(\vec{q})(\Delta n_\uparrow + \Delta n_\downarrow)] + \gamma\Gamma_{1\sigma} b_z(\vec{q},\omega) - e\Gamma_{2\sigma}\phi(\vec{q},\omega). \tag{52}$$

$\Gamma_{1\sigma}$ and $\Gamma_{2\sigma}$ represent the exchange and correlation contribution to the response, obtained from the equation of motion of the two-particle distribution function:

$$\Gamma_{1\sigma_0}=\frac{1}{2\nu^2}v(q)\left[\frac{\hbar^2q^2/m}{(\hbar\omega)^2-(\hbar^2q^2/2m)^2}\right]^2\left\{\sum_{\vec{q}\,'}\alpha(\vec{q},\omega)\left(\frac{\vec{q}\cdot\vec{q}\,'}{q^2}\right)^2\frac{v(\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}[1+\mathrm{sgn}(\sigma_0)\mathrm{sgn}(\sigma)]f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right.$$
$$\left.-\left[\frac{\vec{q}\cdot(\vec{q}+\vec{q}\,')}{q^2}\right]^2\frac{v(\vec{q}+\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}\mathrm{sgn}(\sigma')[\mathrm{sgn}(\sigma)+\mathrm{sgn}(\sigma_0)]f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right\}, \tag{53}$$

$$\Gamma_{2\sigma_0}=-\frac{1}{2\nu^2}v(\vec{q})\left[\frac{\hbar^2q^2/m}{(\hbar\omega)^2-(\hbar^2q^2/2m)^2}\right]^2\left\{\sum_{\vec{q}\,'}\alpha(\vec{q},\omega)\left(\frac{\vec{q}\cdot\vec{q}\,'}{q^2}\right)^2\frac{v(\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}[1+\mathrm{sgn}(\sigma_0)\mathrm{sgn}(\sigma)]f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right.$$
$$\left.-\left[\frac{\vec{q}\cdot(\vec{q}+\vec{q}\,')}{q^2}\right]^2\frac{v(\vec{q}+\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}[1+\mathrm{sgn}(\sigma_0)\mathrm{sgn}(\sigma)]f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right\}, \tag{54}$$

with

$$\alpha(\vec{q},\omega)=\frac{1}{2}\left[\left(\frac{\hbar\omega+\hbar^2q^2/2m}{\hbar\omega-\hbar^2q^2/2m}\right)^2+\left(\frac{\hbar\omega-\hbar^2q^2/2m}{\hbar\omega+\hbar^2q^2/2m}\right)^2\right]. \tag{55}$$

In an analogous way, in the limit of large wave vectors or high frequencies, the transverse magnetic fluctuations are found to be

$$\Delta m_+=2\Pi_{\uparrow\downarrow}\gamma b_+(\vec{q},\omega)+2\gamma b_+(\vec{q},\omega)\Gamma_T(\vec{q},\omega), \tag{56}$$

$$\Delta m_-=2\Pi_{\downarrow\uparrow}\gamma b_-(\vec{q},\omega)+2\gamma b_-(\vec{q},\omega)\Gamma_T(\vec{q},\omega), \tag{57}$$

where $\Gamma_T(\vec{q},\omega)$ is expressed by

$$\Gamma_T(\vec{q},\omega)=-\frac{1}{2\nu^2}v(\vec{q})\left[\frac{\hbar^2q^2/2m}{(\hbar\omega)^2-(\hbar^2q^2/2m)^2}\right]^2\sum_{\vec{q}\,'}\left\{\alpha(\vec{q},\omega)\left[\frac{\vec{q}\cdot\vec{q}\,'}{q^2}\right]^2\frac{v(\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right.$$
$$\left.-\left[\frac{\vec{q}\cdot(\vec{q}+\vec{q}\,')}{q^2}\right]^2\frac{v(\vec{q}+\vec{q}\,')}{v(\vec{q})}\sum_{\vec{k},\sigma}\sum_{\vec{k}\,',\sigma'}\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma')f^{(2)}_{\vec{k},\sigma;\vec{k}\,',\sigma'}(\vec{q}\,')\right\}. \tag{58}$$

The system of equations obtained from Eq. (35) written for up and down spins, gives the charge and longitudinal spin responses. The corresponding susceptibilities functions are

$$\chi_{ee}(\vec{q},\omega)=e^2\frac{\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow}+\Gamma_{1\uparrow}+\Gamma_{2\downarrow}}{1-v(\vec{q})(\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow})}, \tag{59}$$

$$\chi_{em}(\vec{q},\omega)=-e\gamma\frac{\Pi_{\uparrow\uparrow}-\Pi_{\downarrow\downarrow}+\Gamma_{2\uparrow}-\Gamma_{2\downarrow}}{1-v(\vec{q})(\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow})}, \tag{60}$$

$$\chi_{me}(\vec{q},\omega)=e\gamma\frac{\Pi_{\uparrow\uparrow}-\Pi_{\downarrow\downarrow}+\Gamma_{1\uparrow}-\Gamma_{2\downarrow}}{1-v(\vec{q})(\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow})}, \tag{61}$$

$$\chi_{mm}(\vec{q},\omega)=-\gamma^2\frac{\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow}-4\Pi_{\uparrow\uparrow}\Pi_{\downarrow\downarrow}+\Gamma_{2\uparrow}[1-2\Pi_{\uparrow\uparrow}v(\vec{q})]+\Gamma_{2\downarrow}[1-2\Pi_{\downarrow\downarrow}v(\vec{q})]}{1-v(\vec{q})(\Pi_{\uparrow\uparrow}+\Pi_{\downarrow\downarrow})}. \tag{62}$$

The transverse susceptibilities are derived from (56) and (57):

$$\chi^+_{mm}=-2\gamma^2(\Pi_{\uparrow\downarrow}+\Gamma_T), \tag{63}$$

$$\chi^-_{mm}=-2\gamma^2(\Pi_{\downarrow\uparrow}+\Gamma_T). \tag{64}$$

The response functions have been previously obtained in Eqs. (16)–(22) using the local-field approximation. Thus, by comparison, we can derive the exact expressions of the many-body corrections for large $\vec{q}$ or high frequency.

In the same limit, the polarization functions of the free electrons, Eq. (14), become

$$\Pi_{\sigma_0,\sigma_0}=[1+\mathrm{sgn}(\sigma_0)\zeta]\Pi_0, \tag{65}$$

$$\Pi_{\sigma_0,\overline{\sigma_0}}=\left[1-\mathrm{sgn}(\sigma_0)\zeta\frac{\hbar\omega}{\epsilon_q}\right]\Pi_0, \tag{66}$$

with $\epsilon_q=\hbar^2q^2/2m$. Π_0 is the asymptotic value of the same spin free-electron response function for $\xi=0$:

$$\Pi_0=\left(\frac{N}{2\nu}\right)\frac{\hbar^2q^2/2m}{(\hbar\omega)^2-(\hbar^2q^2/2m)^2}. \tag{67}$$

With this substitution, the microscopic expression for the transverse local-field corrections, $G^-_{T,\uparrow}$, is obtained from Eqs. (21), (58), and (63):

$$G^-_{T,\uparrow}(\vec q,\omega)=\frac{1}{N^2(1-\zeta\hbar\omega/\epsilon_q)^2}\sum_{\vec q'}\left\{\alpha(\vec q,\omega)\left(\frac{\vec q\cdot\vec q'}{q^2}\right)^2\frac{v(\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right.$$
$$\left.-\left[\frac{\vec q\cdot(\vec q+\vec q')}{q^2}\right]^2\frac{v(\vec q+\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma')f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right\}. \tag{68}$$

The complementary result, $G^-_{T,\downarrow}$, is a consequence of the symmetry relation, $G^-_{T,\downarrow}(\zeta)=G^-_{T,\uparrow}(-\zeta)$, satisfied for any degree of polarization, $|\zeta|\leqslant 1$.

The comparison of the systems of equations, (16)–(19), and (59)–(62), leads, after long but simple mathematical manipulations, to the expressions of the local-field corrections associated with density and longitudinal spin fluctuations. Therefore, for an electron of spin projection σ_0,

$$G^-_{L,\sigma_0}=\frac{[1-\zeta\mathrm{sgn}(\sigma_0)]}{N^2(1-\zeta^2)^2}\sum_{\vec q'}\left\{\alpha(\vec q,\omega)\left(\frac{\vec q\cdot\vec q'}{q^2}\right)^2\frac{v(\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}[\mathrm{sgn}(\sigma_0)+\mathrm{sgn}(\sigma)][\mathrm{sgn}(\sigma)-\zeta]f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right.$$
$$\left.-\left[\frac{\vec q\cdot(\vec q+\vec q')}{q^2}\right]^2\frac{v(\vec q+\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}[\mathrm{sgn}(\sigma_0)+\mathrm{sgn}(\sigma)][\mathrm{sgn}(\sigma')-\zeta]f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right\}, \tag{69}$$

$$G^+_{\sigma_0}=\frac{[1+\zeta\mathrm{sgn}(\sigma_0)]}{N^2(1-\zeta^2)^2}\sum_{\vec q'}\left\{\alpha(\vec q,\omega)\left(\frac{\vec q\cdot\vec q'}{q^2}\right)^2\frac{v(\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}[1+\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma_0)][1-\zeta\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma_0)]f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right.$$
$$\left.-\left[\frac{\vec q\cdot(\vec q+\vec q')}{q^2}\right]^2\frac{v(\vec q+\vec q')}{v(\vec q)}\sum_{\vec k,\sigma}\sum_{\vec k',\sigma'}[1+\mathrm{sgn}(\sigma)\mathrm{sgn}(\sigma_0)][1-\zeta\mathrm{sgn}(\sigma')]f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q')\right\}. \tag{70}$$

As before, $G^{\pm}_{L,\uparrow}(\zeta)=G^{\pm}_{L,\downarrow}(-\zeta)$. These expressions are exact in the large wave-vector or high frequency limit. It is remarkable that the frequency and wave-vector dependence of the longitudinal and transverse local-field corrections are different as a result of the initial polarization of the system. For $\hbar\omega=\epsilon_q/\zeta$, the many-body effect for the transverse response is divergent, leading to a negligible susceptibility. Thus, the magnitude of the response can be controlled, by varying ζ.

The equilibrium two-particle distribution function is related to the pair-correlation function through

$$\sum_{\vec k}\sum_{\vec k'}f^{(2)}_{\vec k,\sigma;\vec k',\sigma'}(\vec q)=N_\sigma N_{\sigma'}\int d\vec r e^{-i\vec q\cdot\vec r}[2g_{\sigma\sigma'}(\vec r)-1]. \tag{71}$$

The pair-correlation function, $g_{\sigma\sigma'}(\vec r)$ represents the probability of finding an electron of spin σ' at a distance $\vec r$ from a particular electron of spin σ located at the origin.

The asymptotic expressions of the many-body corrections in the limit $\vec q\to\infty$ are obtained by making use of the following mathematical identity:

$$\frac{1}{\nu}\sum_{\vec q'}\left(\frac{\vec q\cdot\vec q'}{q^2q'^2}\right)e^{-i\vec q'\cdot\vec r}=\frac{1}{3}\delta(\vec r). \tag{72}$$

When Eq. (71) is introduced in Eqs. (68), (69), and (70), using the above mathematical identity, the long-wave limits of the local-field factors are expressed in terms of $g_{\sigma\sigma'}(0)$:

$$\lim_{\vec q\to\infty}G^+_{\uparrow}(\vec q,\omega)=\frac{1}{3(1+\zeta)}[(2+3\zeta)-(2+4\zeta)g_{\uparrow\downarrow}(0)], \tag{73}$$

$$\lim_{\vec q\to\infty}G^-_{T,\uparrow}(\vec q,\omega)=\frac{1}{3}\{2(1-\zeta^2)[g_{\uparrow\downarrow}(0)+g_{\downarrow\uparrow}(0)]+3\zeta^2-1\}, \tag{74}$$

$$\lim_{\vec{q}\to\infty} G^{-}_{L,\uparrow}(\vec{q},\omega)=\frac{1}{3(1+\zeta)}[(2\zeta+4)g_{\uparrow\downarrow}(0)-1]. \quad (75)$$

Of course, by the virtue of the exclusion principle, $g_{\uparrow\uparrow}(0)=g_{\downarrow\downarrow}(0)=0$. The $\vec{q}\to\infty$ limit maintains the anisotropy of the local-field corrections associated with the spin response. When $\zeta=0$, the anisotropy disappears and we recover the results obtained by Niklasson[5] and Zhu and Overhauser[6] for G^{+} and G^{-}, respectively:

$$\lim_{\zeta\to 0}\lim_{\vec{q}\to\infty} G^{+}=\tfrac{2}{3}[1-g(0)], \quad (76)$$

$$\lim_{\zeta\to 0}\lim_{\vec{q}\to\infty} G^{-}=\tfrac{1}{3}[4g(0)-1]. \quad (77)$$

The values of the pair-correlation functions at the origin, $g_{\uparrow\downarrow}(0)$ and $g_{\downarrow\uparrow}(0)$, have to be calculated self-consistently. Following a simple model proposed by Zhu and Overhauser,[2] we derive in the Appendix their expressions as functions of the degree of polarization, ζ, and obtain [Eq. (A11)]

$$g_{\sigma\bar{\sigma}}(0)=\frac{32[1-\zeta\,\mathrm{sgn}(\sigma)]}{(8+3r_s)^2}, \quad (78)$$

where r_s measures in Bohr radii the distance between two electrons.

V. CONCLUSIONS

Local-field corrections, $G^{\pm}(\vec{q},\omega)$, were introduced as approximations for the many-body interactions in the self-consistent, one-particle Hamiltonian, Eq. (6). In the equation-of-motion method, the time variation of the particle density at large wave vectors or high frequency leads to an analytic expression for $G^{\pm}(\vec{q},\omega)$. This result is exact since it is a consequence of an external constraint and does not require additional assumptions about the multiple-particle correlations. The asymptotic values can be used as physical limits for any extrapolation of the correction factors at arbitrary $\vec{q}$ and ω.

These results should be tested in experiments. A spin-polarized electron gas can be created by a static magnetic field applied to a dilute magnetic superconductor quantum well embedded in a modulation-doped nonmagnetic host. In this system, a weak electromagnetic field can induce coupled spin and charge fluctuations, the resonances of the response being associated with intra- and inter-subband excitations. The analysis of the infrared absorption spectrum should allow then a comparison with theoretical determinations.

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APPENDIX: THE TWO-PARTICLE CORRELATION FUNCTION AT THE ORIGIN

The two-particle correlation function, $g_{\sigma\sigma'}(r)$ is defined as the probability of finding an electron of spin σ' at a distance $\vec{r}$ from the electron of spin σ located at the origin, $\vec{r}=0$. For fermions, the value of this function at $\vec{r}=0$ is determined by the particles of opposite spin to the one chosen as a reference, in agreement with the Pauli principle. In a spin-polarized electron gas, the number of up and down spins are different. Consequently, the values of $g_{\sigma\sigma'}(0)$ are different when the reference spin is up or down. The corresponding pair correlation values are $g_{\uparrow\downarrow}(0)$ and $g_{\downarrow\uparrow}(0)$, respectively.

To obtain $g_{\sigma\bar{\sigma}}(0)$ for an electron of a given spin σ in a spin-polarized electron gas we follow a simple calculation proposed by Overhauser.[7] A pair of electrons with opposite spins forms a singlet state whose wave function is

$$\Psi(\vec{r})=\Phi(\vec{r})\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2-|\downarrow\rangle_1|\uparrow\rangle_2), \quad (A1)$$

where $\Phi(\vec{r})$ is the spatial wave function, while $|\uparrow\rangle$ and $|\downarrow\rangle$ are the spin eigenfunctions. $\vec{r}$ is the coordinate of the relative motion. Since the interaction between electrons is spherically symmetric, $\Phi(\vec{r})\sim R(r)$. The Schrödinger equation satisfied by the radial wave function, $R(r)$, is

$$-\frac{\hbar^2}{m}\left(\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}\right)+V(r)R=ER. \quad (A2)$$

$V(r)$ is the effective potential, and m is the mass of the electron. E is considered to be equal to two free-electron energies, $E=\hbar^2k^2/m$, with k the momentum of the relative motion. In the absence of the interaction, the solution is

$$R(r)=\frac{\sqrt{2}}{kr}\sin kr. \quad (A3)$$

The probability that both electrons are localized at $r=0$ is then $|R(0)|^2=2$. Assume now that the spin of one electron in the pair is specified σ. The weight of the term $|\sigma\rangle_1|\overline{\sigma}\rangle_2$ in the spin eigenfunction is 1/2. Furthermore, the chance of encountering an electron of spin state $\overline{\sigma}$ is $[1-\mathrm{sgn}(\sigma)\zeta]/2$. Thus, the particle density at the same position with the electron of spin σ is reduced to

$$g_{\sigma\bar{\sigma}}(0)=\frac{1-\mathrm{sgn}(\sigma)\zeta}{2}. \quad (A4)$$

When the Coulomb interaction is considered, because of the repulsion, the density around a particular electron decreases. To calculate this decrement, an expression for the screened Coulomb repulsion, $V(r)$, is needed. In the Overhauser model, $V(r)$ is approximated with the potential of a sphere uniformly filled with screening charge density (outside the sphere the screening is zero). The total screening charge inside the sphere is e, while the radius a is just the separation between two electrons. In this approximation,

$$V(r)=\begin{cases}\frac{e^2}{r}\left(\frac{a}{r}+\frac{1}{2}\frac{r^2}{a^2}-\frac{3}{2}\right), & r\leqslant a\\ 0, & r\geqslant a,\end{cases} \tag{A5}$$

where $a=(3/4\pi n)^3$.

With this choice of potential, we solve the Schrödinger equation. The change of variable, $u(r)=R(r)/r$, leads to

$$-\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2}\right)+V(r)u=Eu. \tag{A6}$$

The solution, $u(r)$, and its derivative, du/dr, have to be continuous on both sides of $r=a$. We introduce the dimensionless variables: $s=r/a$, $q=ka$, $r_s=a/a_B$, with $a_B=\hbar^2/me^2$ the Bohr radius, and write for $s\leqslant 1$,

$$\frac{d^2u}{ds^2}+\left[q^2-r_s\left(\frac{1}{s}+s^2-\frac{3}{2}\right)\right]u=0. \tag{A7}$$

Outside the sphere, in the absence of any potential, $u(s)$ satisfies $u''+q^2u=0$. An exact solution for $s\geqslant 1$ is $u(s)=\sin(qs-\phi)$ (ϕ is a phase shift). The amplitude and slope of the solution $u(s)$ have to be the same for $s=1$. The dominant behavior of u is in the vicinity of the origin, at $s=0$. Then we can approximate u with $u(s)=\beta s$, where β is the slope at the origin. Hence, from Eq. (A7), we obtain immediately

$$u'(s)\approx\beta r_s\left(s+\frac{s^4}{8}-\frac{3}{4}s^2\right)+\beta. \tag{A8}$$

The slope of $u(s)$ at $s=1$ is then

$$u'(1)=\beta[1+\tfrac{3}{8}r_s], \tag{A9}$$

which has to be equal to the slope obtained for $s\geqslant 1$, $u'(s)=1$. Thus,

$$\beta=\frac{1}{1+\frac{3}{8}r_s}. \tag{A10}$$

The reduction in the pair-correlation function at the origin caused by the Coulomb repulsion is then

$$g_{\sigma\bar{\sigma}}(0)=\frac{32[1-\mathrm{sgn}(\sigma)\zeta]}{(8+3r_s)^2}. \tag{A11}$$

[1] C. A. Kukkonen and A. W. Overhauser, Phys. Rev. B **20**, 550 (1979).
[2] Xiaodong Zhu and A. W. Overhauser, Phys. Rev. B **33**, 925 (1986).
[3] K. S. Yi and J. J. Quinn, Phys. Rev. B **54**, 13 398 (1996).
[4] R. Dupree and D. J. W. Geldart, Solid State Commun. **9**, 147 (1971).
[5] Goran Niklasson, Phys. Rev. B **10**, 3052 (1974).
[6] Xiaodong Zhu and A. W. Overhauser, Phys. Rev. B **30**, 3158 (1984).
[7] Kurt Sturm, Phys. Rev. B **52**, 8028 (1995).
[8] A. W. Overhauser, Can. J. Phys. **73**, 683 (1995).