Pak. j. eng. technol. sci.
Volume 4, No 1, 2014, 28-40
ISSN: 2222-9930 print
ISSN: 2224-2333 online

Application of Stochastic Volatility Model to KSE-100

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Received on: May20, 2013; Accepted on: Aug2, 2013

ABSTRACT

The paper examines the implementation of stochastic volatility (SV) model to the data of Karachi Stock Exchange 100 index during the period of January 2007 to December 2011. The Stochastic Volatility model is compared with the GARCH (1,1) model for forecasting volatility. The stochastic volatility model is basically a parametric approach to observe volatility that includes two noise terms, tends to capture volatility better than GARCH (1,1) model. Thus this exercise demonstrates the capability of stochastic volatility model to forecast volatility more efficiently for emerging markets such as KSE.

1. INTRODUCTION

In finance, we generally encounter trade-off between returns and risks. Thus identifying and gauging risks is an essential task for financial decision making. Volatility is considered a primary tool to figure out risk. Mathematically, volatility is a conditional second moment for a random variable which depends on other random variable(s). Let Y_t is a vector which evolves over time. It implies that Y_t would have observations $[y_1, y_2, \ldots, y_n]$. Suppose Y is dependent upon some other vectors $X_{1t}, X_{2t}, \ldots, X_{mt}$ – all are indexed with time. In this case the conditional distribution of Y_t can be depicted by $[Y_t | X_{1t}, X_{2t}, \ldots, X_{mt}]$ and the variance of the above distribution would be referred to volatility.

$$\text{Volatility} = \text{Var}\,[Y_t | X_{1t}, X_{2t}, \ldots, X_{mt}] \qquad (1)$$

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With the development of Autoregressive Conditional Hetroscedastic Model (ARCH) by Engle (1982), the class of conditional hetroscedastic models has become instrumental to observe volatility. The ARCH model, at the first lag, that is ARCH (1) is based upon following equations

$$y_t = \sigma_t \varepsilon_t \quad (2)$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 \quad (3)$$

Bollerslev (1986) further generalized the ARCH model into Generalized Autoregressive Conditional Hetroscedastic(GARCH) model. The GARCH (p,q) model, resembles closely with the ARMA (p,q) model and it rectifies the problem of infinite lag autoregressive model that can frequently be observed in ARCH models. The GARCH models for the minimum order, as proposed by Bollerslev (1986), that is GARCH (1,1), can be presented by the following equations

$$y_t = \sigma_t \varepsilon_t \quad (4)$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta y_{t-2}^2 \quad (5)$$

Since the emergence of GARCH models its variants such as EGARCH, TGARCH, NGARCH and GJR-GARCH have been developed. But to date, the most frequently applied model remains GARCH (p,q) model with p=1 and q=1 (Bollersley (2006) in his original work used GARCH (1,1) model).

Apart from conditional hetroscedastic models, some other models have also been developed to observe volatility. Stochastic volatility models come into the category of such models which are not based on conditional hetroscedasticity in previous time epochs, rather it takes into account the arrival of information in any particular market or economy.

The market which is considered in this paper is Karachi Stock Exchange and its all famous KSE-100 has been used as a barometer of its movement. It is common among practitioners to calculate the log returns on the indices of the major equity market of any economy as a proxy for the rate of returns on the economy and to extend the inferences obtained from equity indices analysis to the whole economy. Here we are doing a somewhat similar exercise. In the last decade, so many research works have emerged about the KSE as a result of sharp growth in its market capitalization.

Equally important interest has been taken by researchers in the investigation of market crashes that occurred in this era. The phenomenon of sharp declines has proved to be the foundation of studying volatility at KSE. One recent work about volatility of KSE-100 index has been done by Rafique and Rehman (2011)- who have applied GARCH (1,1) model on KSE data of varying frequencies (daily, monthly and annually) and compared the effect of change in frequency on KSE-100 persistency.

The objective of our study is to apply the stochastic volatility (SV) model on the KSE data and then to compare the volatility modeling of SV model and GARCH (1,1) model and to ascertain which model depicts volatility in a better way. Several researches including Yu (2002) and Krichene (2003) have come to the conclusion that the markets which fall in the category of 'Emerging Markets'- the stochastic volatility models depict their volatilities in a better way. The organization of this paper is as follows. The next section reviews some basics of stochastic volatility. The third section describes the methodology which we applied here in this paper. The fourth section shows the results of our research. And the last section is dedicated to the conclusion of our results.

2.THE STOCHASTIC VOLATILITY MODEL

The idea of Stochastic Volatility was given by Taylor (1986). The volatility in GARCH models and in almost all its variants depends upon the volatility present in previous time epochs. Therefore the different variance series in those models depend upon the variances of previous time periods. Stochastic volatility model gives us the independence from the values of previous time periods-consequently the information arrival became the only determinant of volatility in SV models. SV models are frequently applied for making projections in option pricing but apart from it, these models can also be applied to model variations in the stocks.

The cornerstone for constructing stochastic volatility model stems from the Black-Scholes model for estimating volatility. Black-Scholes (1973)in their seminal paper presented their model for pricing contingent claims. In the Black-Scholes model the returns on asset are assumed to follow a geometric Brownian motion.This geometric Brownian motion translates into the normality of log returns on the asset.

However, various empirical evidences prove contrary to this fact due to higher tails and higher peaks of distribution of log returns. This phenomenon indicates the varying variances of the distribution of log returns. The Black-Scholes model can be computed by the following equation:

$$C(S,K,T,t,r,) = S.N(d_1) - K.N(d_2) \quad (6)$$

Where

S	is the current amount of the underlying asset
K	is the strike price
T	is the maturity time
T	is the current time
R	is the risk free rate
s	is the volatility of S, and
N(.)	is the cumulative normal distribution

Also

$$d_1 = \frac{\log S - \log K + \left(r + 0.5\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad (7)$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \quad (8)$$

The Black-Scholes model which was mentioned above does not consider the time varying volatility (s). Conditional Heteroscedastic models allow us to capture the volatility which evolvesover time. The GARCH (p,q) models can be extended to the stochastic volatility models as follows

$$y_t = \sigma_t \varepsilon_t \quad (9)$$

$$\Delta\sigma_t^2 = \omega + \alpha y_{t-1}^2 + (\beta - 1)\sigma_{t-1}^2 + \sigma_\eta \eta_t \quad (10)$$

Equation 9 is referred as an observation equation of the SV model, whereas equation 10 is referred as transition equation of the SV model.In equation 10, all the parameters of stochastic volatility model are mentioned. For computational purposes, generally, all the parameters are supposed to belong to vector **q**, that is {w,a,b,s_h} Î**q.** The SV model actually depicts the flow of information arriving in any dynamic system [Anderson (1996)]. One of the unique features of SV models is that it uses two innovation terms for error. These terms are e_t which follows

Both the disturbances (e_t and h_t) may or may not be independent. Preminger and Hafner (2006) suggested that the inclusion of an additional error term, makes SV model more flexible as compared to other volatility models. If the disturbances are not independent, or in other words, correlated to each other, then it implies that stock price movements are negatively correlated with the volatility. This phenomenon of correlation of disturbances is referred by Black(1976) as leverage effect. Thus, in the case of rise in volatility, the debt to equity ratio will rise for making investments and consequently the investments will become more risky. In SV model, h_t which is the volatility of volatility, used to depict us the leverage effect.Although SV models provide better modeling facility for returns, but estimation of its parameters is an uphill task. The complexity in estimation of its parameters is amidst the intractability of log-likelihood estimates of SV parameters. The log-likelihood estimates are difficult to obtain, owing to the fact that SV models are used to compute two error terms simultaneously i.e.e_t, the error term innovating in the process andh_t, which is the volatility of volatility. Several algorithms such asJacquier, Polson and Rossi (1994), Heston(1993) and Mills (2008) have been developed to estimate its parameters.

1. METHODOLOGY

As the KSE data is in the form of time series, therefore the utmost part of thisresearch is to check the stationarity of the time series. Establishment of stationarity ensures that causal analysis is not going to yield spurious results. We have applied the frequently used Augmented Dickey Fuller (ADF) test for this purpose. The basic parameter of the ADF test isr, whose value, if turns out to be 1, then we cannot reject the null hypothesis of non stationarity or we can say,that the problem of unit root has emerged in the series.

The ADF test can be performed through different forms such as, with intercept, without intercept and with inclusion of trend factor. However here we are applying the ADF test with intercept and without trend. Following is the equation of ADF test in this form

$$y_t = \beta_t + (1+\rho)y_{t-1} + \varepsilon_t \quad (11)$$

After testing the stationarity, we have first applied the conventional GARCH (1,1) model on the log returns of KSE-100. One of the special interests we have here is to check the persistency of our model. The persistency can be checked in case of GARCH (1,1) model by considering whether the sum of parameters, a and b is less than or equal to 1 or not. If their sum exceeds 1, it means that the model is not persistent and hence not adequate for the particular data.

Then we turn to the central objective of our research that is, the application of SV model to the KSE data. For the purpose of applying SV model, we have followed the algorithm of Mills(2008). The technique of Mills (2008) is based on Kalman filtering. Barndoff-Nielsen and Shephard (2002) described KalmanFilter as eligible for capturing non Gaussian dynamics of volatility and also provides consistent and asymptotically normal set of estimators. As our model is in a dynamic system, the state in SV model keeps changing. Thus we can model the log returns of stock indices via the following differential equation

$$d(Log(P)) = \mu dt + \sigma(t)dt \quad (12)$$

Where P is the price of stock and t indicates the innovation of our time. If we discretize the above equation then x_t can be taken as $Dlog(P_t)$, then the above equation would become

$$x_t = \mu + \sigma_t \varepsilon_t \quad (13)$$

That implies that

$$V(x_t|\sigma_t) = V(\mu + \sigma_t \varepsilon_t) \quad (14)$$

Mills(2008) found that the distribution of the volatility of log returns is lognormally distributed due to the fact that $\log(e_t^2)$ follows a logarthimicc[2] distribution. The expected value of $\log e_t^2$ $E(\log e_t^2)$ comes out to be -1.27 and variance $V(\log e_t^2)$ will be equal to 4.93. This can enable us to estimate the state equation. Using Kalman filters we getBest Linear Unbiased Estimates (BLUE) for volatility vector h_t which depends on the observation equation y_{t-1}.

After applying both GARCH(1,1)model and SV model, mentioned in equations (5) and (10) respectively, we compared both of them to ascertain which model is more suitable for modeling KSE returns. For the purpose of comparison we used two different techniques here, so that we can make our inferences with more certainty; first we compared the root mean square errors (RMSE) values for both models and then compared these models via Asymptotic Relative Efficiency (ARE) of variance of median over variance of mean. Root Mean Square Error can simply be obtained by taking square root of Mean Square Error. For the purpose of getting ARE of median over mean, a bootstrapping approach is applied on the derived realizations from GARCH(1,1) and SV models. The larger values of ARE here means that distribution is getting heavy tailed and hence giving better results. These comparative studies would finally complete our research.

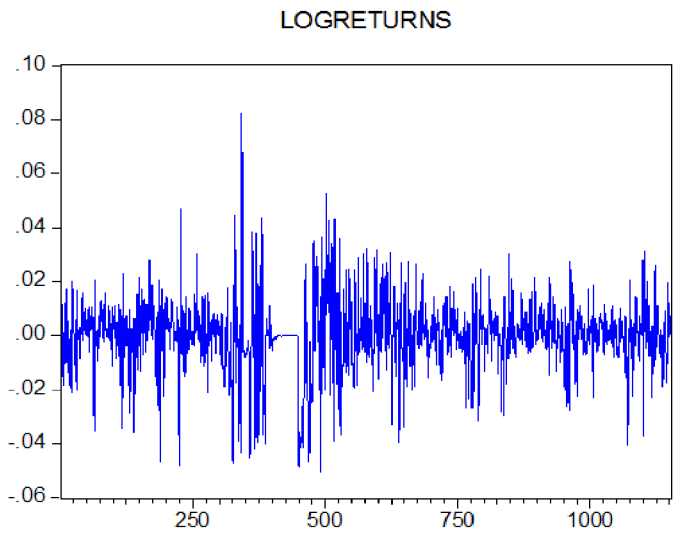
RESULTS

In this paper we have first calculated the descriptive statistics for the log returns on KSE 100 indices as these descriptives are helpful in explaining what actually is going on in our data. The following table represents these descriptives

Table 5.1

Descriptive Measures	Values
Mean	-1.53e-07
Median	0.000327
Standard Deviation	0.014139
Skewness	-0.292146
Kurtosis	5.746020
JarqueBera	378.6655

The descriptive statistics describe some important characteristics of our data. Firstly, we can see that the value of the median is much higher as compared to mean which is a depicter of fat tail phenomena in our series. The value of skewness is also slightly deviating from the standard normal's skewness. Also the value of kurtosis is also higher than that of standard normal. This fat tail and higher kurtosis is a typical characteristic of financial time series. The higher value of Jarque-Bera (which leads to the rejection of null hypothesis of normality) is a natural outcome of those characteristics which we obtained earlier. The figure below shows the plot for the log return series.



Now we will consider the stationarity of the log returns on indices. Though the plot of the log returns series is depicting stationary pattern. However, the conventional ADF test is used for checking the stationary in the series. We have performed this test at level.

The following table shows its value.

Table 5.2

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-27.52967	0.0000

It is obvious from Table 5.2 that ADF test statistic is highly significant. Therefore we can say that the given series is stationary.We then applied the GARCH(1,1) on KSE returns to model the volatility from this conventional model. The following table shows the parameter values of GARCH(1,1)

Table 5.3

Parameter	Value	Sig.
ω	8.34e-06	0.0000
??	0.173724	0.0000
??	0.789145	0.0000

We can see that the value of a+b<1, it means that our obtained parameters are persistent. These results show similar properties as those reported in the study of Rafique and Rehman (2011). Our results also show verify the volatility clustering, that is prevalence of cycles of higher or lower volatility for longer time periods, which can be observed via GARCH(1,1) model.

Now we come to the application of Stochastic Volatility model to the returns of KSE-100 indices. The SV model is implemented here via Kalman filtering. The results of this model are shown in Table 5.4.

Table 5.4

Parameter	Value	Sig.
ω	-0.145507	0.6981
??	0.9399140	0.0000
??	-8.135254	0.0000
σ_η	0.0850600	0.0301
SV	82.26765	0.0000

From Table 5.4, it can be observed that most of the values of stochastic volatility parameter vector **q**come out to be significant. The only insignificant parameter is w. It means that in the long run variance of the model will be close to 0.The value of a implies long memory of the volatility process. However the value of $|\beta| > 1$, which implies that the generated volatility process is not stationary. But negative sign here informs us about the presence of asymmetric affect, that is negative developments would have larger impact on KSE-100 index as compared to positive ones, consequently the leverage will get high in the market.While $s_h$which is the volatility of the process is also significant which confirms the suitability of SV model for given data. Further, the test statistic for transition equation (which is latent) is significant at the highest level.

This shows that SV model provides good approximation for the volatility of an emerging market, such as KSE. Its value is depicting the magnitude of affect that any shock will yield on KSE-100 index. For the purpose of comparison between GARCH(1,1) and SV model, we applied two approaches: the root mean square error (RMSE) and the asymptotic relative efficiency (ARE) of median over mean. Following table shows its results.

Table 5.5

	RMSE	**ARE**
GARCH (1,1)	0.017796	0.161491
SV	0.017118	1.245120

As the values of RMSE are very close to zero in both models, therefore it hints that both the models are quite accurate for our data. But as the value of RMSE is slightly small in SV model, it means that the SV modeling technique for the KSE100 data is more appropriate. In ARE (median over mean) however, larger values show better approximation of higher kurtosis and heavier tails. As the value of ARE is much higher in SV models as compared to GARCH (1,1) model, it means that according to ARE the SV model is more suitable for our data. The results of our comparison between GARCH(1,1) and SV model are quite similar to the results of Yu (2002) and Racicot and Theoret (2010) who found that SV model provides superior estimates than GARCH (1,1) model. However these results are contrary to the results of Preminger and Hafner (2006). The comparison finally completes our analysis.

4. CONCLUSION

This research is an attempt to apply SV models on KSE-100 data. The SV model proved to be better and more suitable for volatility existing in the returns of KSE-100. Thus our research demonstrates the capability of SV model for finding out the volatility on returns of KSE. The research has deep implications for investment analysts and portfolio managers, who make their investments at Karachi Stock Exchange, as they can assess the volatility here via simulation based SV model apart from ARCH type models.

One of the reasons for the success of SV model could be the fact pointed out by Yu(2002), which holds mostly due to lack of regularization and emerging nature of markets. These effects in the market tend to follow random movements, consequently the simulation based method is better for modeling the market movements.

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