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An Analysis of Coincidence between KSE-100 and S&P 500 Index using Spectral Approach

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Abstract

When dealing with time series data, particularly of higher frequency, we are often interested in figuring out periods which are of vital importance. Here in this research, the returns on KSE-100 and S&P 500 index are taken on daily basis from September 2001 to June 2013. As thousands of data points (due to high frequency) are considered, it is impossible for us to figure out any pattern in series, unless suitable filtering is applied on them. For this purpose, a power spectrum will be made by means of a fast fourier transform. This will yield us the events that has influenced KSE-100 index considerably in post 9/11 scenario.

1.0. Introduction:

Since its inceptions, Informational Efficiency in financial markets has been a subject of immense investigation for financial analysts and practitioners. The concept of informational efficiency was originally floated by Fama (1970). Informational Effeciency in any market implies that how rapidly the market is responding to different developments. These developments range from macroeconomic developments to political developments and to the developments that are taking place in other financial markets.

This informational efficiency has given birth to all famous efficient market hypothesis (EMH). EMH is basically an empirical approach to investigate the informational efficiency. Thus, most of the studies that have investigated informational efficiency in financial markets are basically carried out under the umbrella of EMH. In the last quarter century, EMH remained central idea of several studies. These studies have generally deduced hybrid sort of conclusions. EMH is further classified into three categories- which are termed as 'forms'. These forms include 'weak form'- based on the historical data, semi strong form- based on the historical data and all publicly available information and strong form- based on all publicly and privately available information. The first amongst these three forms also forms the basis for most of the technical analysis.

But informational efficiency in any market can be analyzed from different perspectives. One perspective may be the analysis of impacts of macroeconomic and political developments that are domestically affecting any financial market. Another perspective may be the analysis of spillovers on any market from other markets of the same economy. Similarly, the analysis of developments, taking place in financial markets of other economies on local market is also an important study in EMH based analysis. The later mentioned analysis will be studied here.

There are several econometric approaches to analyze the impact of shocks that are evolving in financial markets of different economies on the local stock market. Most of the approaches are based on time domain analysis of local and international stock markets. Such analysis involves econometric tools of cointegration, correlation and causation. Here in this paper, frequency domain analysis is used to investigate the causal relationships between the US and Pakistan's stock markets.

In this study, daily data for US and Pakistan's stock markets has been taken from the post 9/11 period. For depicting the stock market activity in Pakistan and US stock markets, the log returns on KSE-100 index and S&P 500 index are used. The data set which has been used in this study is of high frequency due to the fact that we are using daily data of closing market indices. Altogether, the study uses 2897 value for both the indices on the days of synchronous trading in both markets.

2.0.   Literature Review and Theoretical Framework

The purpose of the study is to assess the impacts of developments that are taking place in the international stock markets on the Karachi Stock Exchange. For modeling the impact of international stock market, Standard and Poor 500 index has been used. Thus, here it is actually intended to verify the informational efficiency of KSE-100 index to the international stock markets.

As indicated in the previous section, a causal relationship is needed to be developed here for testing the EMH. Many different approaches can be deployed for developing a causal relationship. In time series analysis, it is common to divide the analytical tools in to two categories, time domain analysis and frequency domain analysis. The later set of techniques is collectively termed as spectral analysis. In spectral analysis, instead of fitting any polynomial to regress the time series, we consider the spectrum of the frequencies of some time series and then we analyze them further to figure out hidden periodicities that are prevailing in time series.

When we are dealing with such a data set as we have in this study, in which we have large number of observations, it is important to filter the times series to figure out hidden periodicities, as the graphical presentation of such a time series is simply insufficient to model its pattern. In this study we are using cross spectrum to analyze the comovement in both the series. Several other studies have also used the spectral methods.

Smith, K. (2000) used co spectrum method to investigate the lead lag relationships between different international stock markets. He argued, that increased link in the spectrum of two stock markets is an indication that the returns in both financial markets are interlinked. Therefore, if any investor simultaneously invests in such markets which are interlinked, the benefits of diversification will get reduced.

McCullough (1995) analyzed the impacts of changes in turnover - that is, volume of total shares exchanged on the stock prices using smoothed periodogram. He argued that smoothed peroidogram is an unbiased and consistent estimator of frequencies. He also argued that time and frequency domain methods are complementary to each other and they should yield similar results, but in practice, the results which are obtained from both perspectives are different from each other. The results obtained by him also showed that NYSE; the stock market which he analyzed is inefficient, that is price change is not responding to the changes taking place in the volume of shares.

McCullough (1997) again analyzed the EMH from a different approach. In this study, he had taken the returns on prices of shares and the transaction data that is the trading volume. In this study, he argued that lagged correlations used to exist in stock market. He suggested that increased filtering in such data would yield in the loss of correlative properties of time series. For these reasons, he carried out cross spectral analysis. Here he was able to verify that there exists a contemporous relationship between stock market returns and volume change. On the basis of the obtained results from the cross spectral analysis, he suggested that simultaneous equation models such as ILS and 2SLS can be appropriate.

Rocha and Souza (2012) analyzed relationship between financial development and economic growth of six countries, Brazil, India, France, Japan, United States and South Korea for a fifty year period that is 1960 to 2010. For depicting financial growth, the authors used stock market activity. They carried out their analysis in frequency domain. For that, they have used the technique of Breitung and

Candelon (2006). The main purpose of their study was to explore possible asymmetries related to the timings of the trade. As the countries, which were under study here belongs to different time regions; hence the stock market activity does not take place on the same time. They conclude that direction of causality depends upon the period of the cycle.

Bollerslev and Wright (2001) proposed a spectral approach to analyze the volatility. They argued that it's an established fact that volatility in any time series depends upon past values (lagged values) and it can be model by using either parametric time domain methods or frequency domain methods. The authors have used daily exchange rate data of US$ to Deutsche Mark for a ten year period. They have estimated their volatilities using GARCH (p,q) models of different orders. Like many others, they also found empirical results that GARCH (1,1) is most suitable. After they estimated the volatility model, they extend their analysis to find a mutual relationship using a time domain method and frequency domain method. They concluded that both approaches yield similar conclusions, however the frequency domain approach is better in the context that they are less parametric.

Gradojevic and Dobardzic (2013) assessed the flow of information across European Markets of Serbia, Croatia, Slovenia and Germany. The authors argued, that increased linkage between any two stock markets is an indication that benefits of diversification cannot be obtained if any investor pumps his funds into such two markets simultaneously. This study also used the methodology of Breitung and Candelon (2006); where VAR framework was built in frequency domain.

The study concludes that market linkages have been increased across EU markets and now they are more exposed to spillovers among them and the benefits of diversification can only be obtained if investments are been made across sectors.

1.0.   Methodology

Periodogram is treated as one of the most eminent tool of spectral analysis. The idea behind constructing the periodogram is to figure out hidden periodicities that are prevailing in any time series. Thus by constructing periodograms, we are actually retrieving information about frequencies that are embedded in data. It has long been a practice among quantitative analysts to fit a polynomial containing sine and cosine functions on a time series to figure out periodicities in it. This approach of capturing hidden periodicities is termed as estimation of harmonics.

Since fourier series is a combination of sine and cosine functions, hence it has a pivotal role in finding these hidden periodicities. Let {Yt} be a random variable which is generated through an stochastic process and the process has generated a sample of 'n' observations such that sample size n= 2k+1. In that case

$$A_0 = \bar{Y} \qquad\qquad \text{Eq. (3.1)}$$

$$A_j = \frac{2}{n} \sum_{t=1}^{n} Y_t \cos\left(\frac{2\pi t j}{n}\right) \qquad\qquad \text{Eq. (3.2)}$$

$$B_j = \frac{2}{n} \sum_{t=1}^{n} Y_t \sin\left(\frac{2\pi t j}{n}\right) \qquad\qquad \text{Eq. (3.3)}$$

In this case the periodogram 'I' of the function at frequency j/n will be given by the following equation

$$I\left(\frac{j}{n}\right) = \frac{n}{2}\left(A_j^2 + B_j^2\right) \qquad\qquad \text{Eq. (3.4)}$$

We may express the Eq. (4) in the form of complex numbers as follows

$$I\left(\omega_j\right) = \frac{1}{2\pi n} \left|\sum_{t=1}^{n} y_t e^{it\omega}\right|^2 \qquad \text{Eq. (3.5)}$$

Where ω_j represents the frequency at which the periodogram is displaying values. At any point, the height of periodogram shows the strength of sine and cosine pairs at various frequencies. At this point, we will like to discuss another concept; tapering the process of decreasing magnitude of data on both sides and converge it towards data mean. The range of doing tapering is from 0 to 0.5. Beyond this range, we witness the phenomena of aliasing, which occurs when other frequencies show similar pattern to the frequencies that are lying in this range.

When we are having a large set of data, such as that has been taken here, it often becomes necessary to smooth the periodogram. Otherwise our results will become inconsistent, as indicated by Smith (2000). By smoothing the periodogram, we used to induce the bias in the periodogram, but it is necessary to do smoothing, otherwise we cannot be able to deduce conclusions from the periodogram. The concept of periodogram smoothing was given by Daniell (1946). If an 'm' point filter is applied on the periodogram, where 'm' is an odd number, then the smoothed periodogram can be represented by the following equation.

$$\widehat{f(\pi)} = \frac{\left[I(\pi) + 2\sum_{j=1}^{m} I(\pi - \frac{2\pi j}{n})\right]}{m} \qquad \text{Eq.(3.6)}$$

In this analysis, we are considering two time series, so in order to observe the spillover impact, it is suitable to make a cross spectrum of the two series. The cross spectrum can be estimated by the following equation.

$$I_{xy}\,(\omega) \;=\; \frac{1}{2\pi n}\,\left[\left(\sum_{t=1}^{n} x_t\, e^{it\omega}\right)\left(\sum_{t=1}^{n} y_t\, e^{it\omega}\right)\right]\qquad\text{Eq.(3.7)}$$

The estimation of spectrum will ultimately complete our analysis.

4.0.   Results

Table 4.1 showed the descriptive statistics of the two markets- KSE-100 and S&P 500 indices. The descriptive statistics has been calculated on the daily log returns for both the markets. For the overall period, we can see that mean returns in both markets remain close to zero. The overall variance in both he series also remains constant. Skewness also in both markets is close to zero, which is hinting that net asymmetries are less prevalent in both markets and both markets are efficient. However, in comparison to US equity market, KSE seems to be negatively skewed, which implies that here negative developments in the previous time period have more impact. In both the series higher kurtosis is present. S&P 500 has a higher value of kurtosis than KSE-100 index. It implies that in S&P 500, fat tail phenomena is more apparent as compared to KSE-100 index and more extreme events have occurred there in S&P 500 index.

Table 4. 1

	KSE-100	S&P 500
Mean	0.0004417187	-5.813654e-05
Variance	3.915855e-05	3.321027e-05
Skewness	-0.3054113	0.1939423
Kurtosis	6.059893	11.6364
Jarque Bera	1175.222	9021.476

Table 4.2 shows the results of ADF and PP test statistic, which is carried out to check that any series is stationary or not. The test statistics of both reveal that the log returns on both the markets are stationary at level. This implies that returns on any particular day in both markets is not dependent on any past or historical information

but they are actually responding to the day to day developments. It means that both markets can be termed as efficient in accordance to weak form EMH. The results for KSE-100 were in contrast with the work of Akber and Muhammad (2013) who concluded that KSE-100 cannot be treated as fully efficient market in accordance to weak form EMH.

Table 4. 2

	KSE-100	S&P-500
ADF Test Statistic	-12.0054,	-14.0688
PP Test Statistic	-2779.539	-2987.22

After determining that both series are stationary, we go for establishing a causal relationship between them. For this purpose methodology of Smith (2000) is used to make a cross spectrum for both the series. For this purpose, here a smoothed periodogram has been used for both the KSE-100 and S&P 500. As the frequency of the data is very large, therefore a higher order 'm' value- that is order of the filter was required so that we can figure out hidden periodicities, though the higher order would induce an extra amount of bias in it, but applying a higher filter is necessary when we are dealing with such an enormous data. Apart from applying higher filter, full tapering has also been done with both periodograms in order to avoid any leakages. Figure 4.1 shows the co spectrum for both the series.

Figure 4. 1
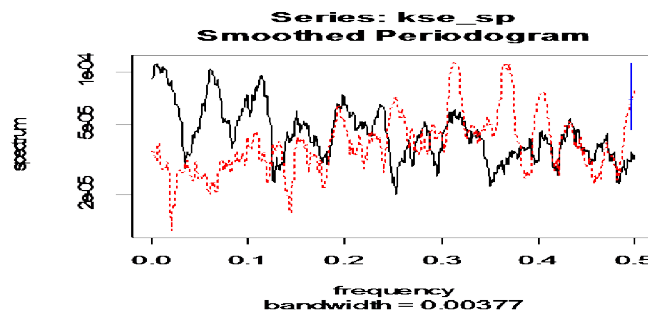
In the above cospectrum, the black line is representing the periodogram of KSE-100 index while the line in red depicts the smoothed periodogram for S&P 500. In the above spectrum we can observe that the peak in KSE-100 is occurring at a very low frequency, almost at $\omega=0.01$, it means that the period of the data is 100 days. Whereas, due to high frequency ($\omega=0.366$) t he period in S&P 500 index is 2.73 days or roughly 3 days.

It is obvious in the cospectrum that in frequency domain, both the series are almost moving in opposite directions. At low frequencies, we can see that KSE is approximately ahead by phase than S&P but at higher frequencies the situation is reverse. Hence we can say on the basis of this empirical evidence that we do not find impact of any development occurring in any market, influencing the other market. These results are in contrast with the work of Iqbal (2012) who argued a relationship between KSE and international stock market.

1.0.   Conclusion

S&P 500 is one of the largest equity markets of the world, whereas KSE is an emerging market and is many times less than S&P in terms of market capitalization. Thus, investing at both places is almost structurally different. However, it is common among investment and financial analysts to diverse their portfolio by investing in different markets. But benefits of diversification can only be obtained when there is no or weak relationship between any two markets. In this context, we can say that international investors, who have exposed themselves in S&P 500 index can get the benefits of diversification if they invest in the equity market of Pakistan as it has a weak relationship with US stock market.

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