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Cognitive Demands and Second-Language Learners:

A Framework for Analyzing Mathematics Instructional Contexts

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Abstract

The issues involved in teaching English language learners mathematics while they are learning English pose many challenges for mathematics teachers and highlight the need to focus on language processing issues related to teaching mathematical content. Two realistic-type problems from high-stakes tests are used to illustrate the complex interactions between culture, language, and mathematical learning. The analyses focus on aspects of the problems that potentially increase cognitive demands for second-language learners. An analytical framework is presented that is designed to enable mathematics teachers to identify critical elements in problems and the learning environment that contribute to increased cognitive demands for ESL students. The framework is proposed as a cycle of teacher reflection that would extend a constructivist model of teaching to include broader linguistic, cultural, and cognitive processing issues of mathematics teaching as well as enable teachers to develop more accurate mental models of student learning.

Cognitive Demands and Second-Language Learners:

A Framework for Analyzing Mathematics Instructional Contexts

English as a second language (ESL) professionals have discussed content ESL pedagogy since the 1980s (Chamot & O'Malley, 1986; Cuevas, 1984). Low mathematics scores on standardized tests for students who speak English as a second or additional language suggest, however, that greater attention to how mathematics content is taught to ESL students is needed (Holmes & Duron, 2000; MacDonald, 2004). Part of the explanation for the low scores is that content teachers in the United States, in general, have not been prepared to provide appropriate instructional programs for English language learners in their classes. "More than 40 percent of all teachers in the nation report that they taught students who are limited in their English proficiency, yet only 12 percent of those teachers had eight or more hours of training in how to teach those students" (Nieto, 2004, p. 219). Given the current emphasis on standards and high-stakes testing in the United States, as well as the growing trend to enact laws that mandate the mainstreaming of English language learners after one year of ESL instruction (Nieto, 2004; Education Commission on the States, 2004), mathematics teachers can no longer wait for ESL students to learn English before they teach them mathematics content.

The issues involved in teaching ESL students mathematics while they are learning English pose many challenges for mathematics teachers. Not only must they identify the linguistic demands of the instructional context and plan instructional activities to teach natural language and the formal mathematics language of textbooks used in classrooms (Dale & Cuevas, 1992), they must also adjust instruction to accommodate the cultural, socio-economic, and linguistic changes occurring in the ESL student population. Countries of origin of new immigrants entering the United States have changed dramatically since the mid 1990s. Before then, newcomers were predominantly from Asia and Latin America (Mace-Matluck, Alexander-Kasparik, & Queen, 1998). Since 1995, however, "larger numbers of refugees have arrived in the United States from Africa, eastern Asia, eastern Europe, and Russia" (Freeman, Freeman, & Mecuri, 2003, p. ix). The new immigrants come from various social and educational backgrounds. Some come with age-appropriate or grade-level mathematics education and are able to use their knowledge to transition to mathematics instruction in English. Increasing numbers of students, however, are from countries at war, or they have fled conditions of oppression and poverty. Some have spent years in refugee camps, have had little or no schooling, or may be illiterate in their native languages. Others come with life experiences and cultural backgrounds that differ greatly from that of their U.S peers.

In addition to the socio-economic, linguistic, and cultural diversity among the new immigrants, an increasing number of students born in the United States are entering school without any knowledge of English and with limited language development in their first language. Schulte (2002) found, for example, that in Montgomery and Fairfax counties in Virginia 35% of the students in ESL classes were born in the United States. These children often live in poverty in isolated language communities. A third group of students that Freeman, Freeman, and Mecuri (2003) refer to as "long-term English language learners" has increased dramatically. Such learners are older students, either immigrant or native born, who have lived in and attended school in the United States but who have not "developed high levels of literacy in either their first language or English" (p. x). Teachers in regular academic classrooms often feel unprepared to meet the diverse needs of these students, especially at the upper elementary and secondary levels where their general pedagogy coursework may not have provided them with strategies to use to support ESL students' English language development (speaking, listening, reading, and writing) while they are learning content-area academic knowledge and skills (Gándara, Maxwell-Jolly, & Driscoll, 2005).

The issues and questions related to teaching a changing population of English language learners (National Coalition of Advocates for Students, 1988) are complex (Freeman, Freeman, & Mecuri, 2003). Yet, attempts to educate these students have not measurably changed; “The vast majority of students still spend most or part of their day in monolingual English classrooms” (Nieto, 2000, p. 190). Once these students leave ESL or bilingual programs, they receive little or no instructional support in furthering their English language development. Nevertheless, it is increasingly expected (and in many states is required by law) that they will learn and achieve at grade-level standards if they are to graduate from high school. In mathematics, the tests used to measure whether or not students meet grade-level standards often involve word problems set in contexts considered to reflect experiences and knowledge readily available to the students. The fact that knowledge required to understand the contexts of word problems is not readily available to all students is illustrated in the following example about a problem involving baseball.

The second author worked with a preservice elementary school teacher who was a second-language learner and who had failed a required university mathematics course several times. The author’s task in a series of meetings with the student was to determine her understanding of the mathematics required to solve problems from exams used in the course she had failed. One problem solution involved the use of the Pythagorean Theorem to determine the distance that a catcher needed to throw the ball in order to throw out a runner at second base. The student was asked to talk aloud as she read and solved the problem. The student told the author that she did not know anything about the game of baseball. She was familiar with the shape of the baseball diamond that was described in the problem but did not know where the catcher stood. After relevant aspects of the game were explained, the student identified the need to use the Pythagorean Theorem, which she named and stated without prompting before applying it correctly to the problem situation. Thus, although the student knew the relevant mathematics to solve the problem, she was not able to create a mental model (Johnson-Laird, 1986) that accurately depicted the problem situation without direct instruction that helped her understand the context of the problem. In effect, brief and contextualized direct instruction filled in the gaps in her "baseball" schema (Sweller & Low, 1992) that then allowed her to identify the type of problem and access information in long-term memory that she needed to use to solve it (Pass, Renkl, & Sweller, 2003).

Problems and other mathematical materials are often written using implicit assumptions about the "typical" student who would use such materials at a particular developmental level or grade. For ESL students, these assumptions may be incorrect. These assumptions, in effect, increase what Pass, Renkl, and Sweller (2003) term "extraneous cognitive load" and require students to "search for referents in an explanation" (p. 2). When teachers identify and make explicit the factors that increase cognitive demands, they can then assess students' understandings in those areas and provide appropriate support in instruction, as in the above example.

Aspects of Task Demands From Realistic Test Items

The baseball example above illustrates how assumptions about prior experience creates extraneous cognitive load and increases the cognitive challenges for second-language learners, especially when solving mathematics problems on their own. To illustrate further the complexity of the cognitive demands that realistic type problems pose for second language learners, we analyze and discuss two mathematics problems from a high-stakes test, making explicit the cognitive processes involved in successfully completing the problems. Work done by Paas, Renkl, and Sweller (2003) and Lamon (2003) provided insights about cognitive load that we have considered in looking at the issues for second-language learners and mathematical problem solving. The amount of information and the number of relevant elements that students need to hold in their working memory in order to solve a problem were considered as we examined the cognitive demands made on students by the language and structure of word problems.

Because a limited amount of information can be stored and processed in working memory, a problem solver must be able to efficiently recall "domain-specific knowledge structures" or information in long-term memory that is relevant to the problem situation, hold it in working memory, and simultaneously work on the solution to the problem (Kalyuga, Ayres, Chandler, & Sweller, 2003). While working on a problem, a problem solver also must hold in memory the strategy being applied and keep in mind the interconnections between different parts of the problem; "The extent to which relevant elements interact is a critical feature" (Paas, Renkl, & Sweller, 2003, p. 1). If the number of combined elements of information is more than the capacity of working memory, cognitive overload results; that is cognitive overload occurs when the task demands on working memory "exceed the available cognitive capacity" (Paas, Touvinen, Tabbers, & Van Gerven, 2003, p. 64).

Problem 1[1]: The Laundry Problem

> Sandy's family does its laundry at a coin-operated laundromat. It costs $1.25 per load to use the washing machines and 25¢ per load to use the dryers for 10 minutes. Sandy's family has 5 loads of laundry to do and each load will need to be in a dryer for 30 minutes. Which expression will give Sandy's family the total cost of doing these loads of laundry?

A. (\$1.25 + \$0.25) × 3 × 5

B. [\$1.25 + (3 × \$0.25)] × 5

C. [(3 × \$1.25) + \$0.25] × 5

D. 3 × (\$1.25 + \$0.25) × 5

Reflections on the solution process for Problem 1. The second and third authors ("A" and "B", respectively below) reflected in detail on what was significant about the solution process for this problem.

> (A) This problem has a lot of numbers and relationships between the numbers that I need to understand. The multiple-choice options contain complex mathematical phrases that I must interpret. Also, I need to hold the goal of finding the total cost of doing all of the loads of laundry in my memory space and match that against four multiple choice options. I considered reading through the choices for the answer; but because the multiple choice options appear complex to me, I made a strategic decision to solve or partially solve the problem and then choose my answer rather than trying to read all of the choices and match them with the complex reading situation in the problem statement. This decision allowed me to ignore the information in the multiple choice options until I had simplified the information in the problem statement so that my solution could be compared in a single step with the information in each multiple-choice option.
>
> To provide some memory space in which to think about the problem situation, I recognized, as I read through the problem for the first time, that I needed to do some writing in order to chunk or link some of the information. I also knew that I can help my cognition by reading the problem several times, picking up some information each time

to build my understanding of the situation. So – as I read the problem a second time – I made some notes.

	Wash	Dry
1 load	\$1.25	25¢ for 10 min
		30 min
5 loads		

Figure 1. Notes for Organizing and Supporting Cognition

The notes helped me to sort and organize the information in the problem and keep the goal in mind. From my notes, I could chunk information appropriately as I thought about the problem. For example, I noticed that information in the third sentence had to be put together with information in the second sentence in order to figure out how much it would cost to wash and dry a load of laundry. My notes put these related numbers into close proximity of one another. The numbers in the problem are quite familiar to me so understanding the number relationships came quickly (automatically). The symbols and story situation invoked knowledge of dollars and cents that I know how to interpret and add, subtract, multiply and divide if necessary. As I read the problem I invoked a laundromat schema from my memory in which I recalled the order in which I wash and dry clothes and how laundromats require money to run the washer and dryers. Because the numbers are familiar, the operations are familiar and I have a laundromat schema to help me order the information, I was able to recognize quickly the relationships between numbers in the problem.

In Step 1 of my solution, I mentally compared 10 minutes and 30 minutes and realized that I need to put 3 quarters in the dryer for each load. That comparison came "automatically" because I know that 10 × 3 = 30. The second step also came quickly. I know that I need 3 quarters to dry a load of laundry and that is \$.75. It costs \$1.25 to wash a load so I mentally added \$.75 to that to get \$2.00 to wash and dry one load of laundry. There were 5 loads so I still needed to multiply that by 5. I now had enough information to look at the possible choices so I started to examine each choice. Choice B describes what I did so I marked it and moved on to the next problem.

(B) The *'\$1.25 per load'* signaled to me that this was a rate problem, so if I focused on getting the units in place the numbers would take care of themselves (a reduction in cognitive load). The answer called for dollars so I needed to multiply dollars/load by loads, a straightforward calculation. But the drying part was a little less easy. It was essentially a hidden rate: 25¢ per load per 10 minutes. So to get dollars I needed to multiply by 10-minute units and then by loads. I could do this in my head, recognizing right away that there are three 10-minute units in 30 minutes. But I wrote the total answer down anyway because the form of the options for answers looked a bit confusing, and I know I could easily make a simple error in translation.

Life experiences, language, cognitive processes, and knowledge of and the ability to apply mathematical content all interact in the solution process (Cai, Jakabcsin, & Lane, 1996; Kastberg, d'Ambrosio, McDermott, & Saada, 2005; van Merriënboer & Sweller, 2005; Silver, Shapiro, & Deutsch, 1993). We can think of the problem-solving process as starting with the establishment of a problem space in which the cognitive work on the problem is completed as illustrated in Figure 2.

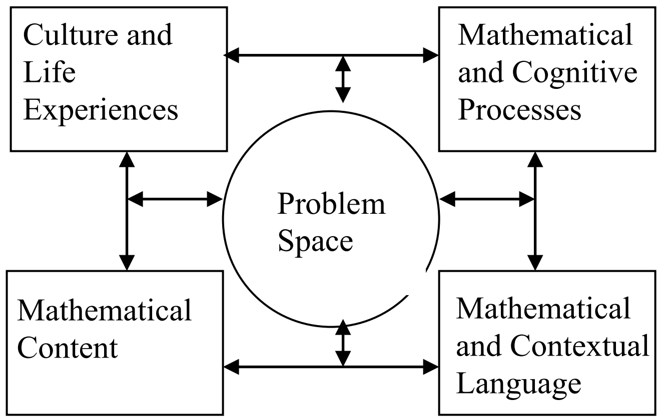

Figure 2: The problem solution process

A problem solver begins with an observation of the complexity of the cognitive demands of interpreting and understanding the problem and the testing situation as the problem is read for the first time. Past experiences with testing situations interact with the awareness of cognitive processes and mathematical language structures to influence a decision to control the cognitive load. In effect, this decision begins the problem-solving process in the upper right box of Figure 2 in interaction with the lower right and upper left boxes. Other cognitive strategies then emerge in interaction with life experiences related to laundromats as notes are made involving the numbers. The numerical content, or intrinsic demands of the problem, is processed automatically as life experiences and understanding of language are used to provide the foundation or schema needed to understand what to do with the mathematical content.

If one focuses solely on the arithmetic operations involved, this problem seems reasonable for seventh graders because it involves the addition and multiplication of simple decimal numbers. However, only half (50.1%) of more than 70,000 students who took the test were able to manage the cognitive demands of this problem with sufficient mathematical understanding to select the correct answer (Choice B). About one third (33.9%) of the students linked the information within the second sentence (Choices A and D) instead of linking information from the second sentence with information from the third sentence. Selection of Choices A and D may result from a failure to recognize that this problem involves a complex rate (25 cents/load/10 minutes), or a failure to use cognitive strategies to control demands on cognition. Prior experience doing laundry in a laundromat would greatly enhance a seventh grader's ability to recognize that rates are involved in this problem and to interpret a complex rate. Thus, students' lack of prior experience with laundromats could be expected to impact their success rate. This observation impels us to ask what is really being tested by this problem.

Linguistic challenges. The language used in Problem 1 compounds the cognitive demands for many students, and particularly for second-language learners. The use of the pronoun *it* in the sentence, "It costs $1.25 per load to use the washing machines and 25¢ per load to use the dryers for 10 minutes", requires a level of interpretation that may not be apparent to ESL students. They may think that "It," which begins the sentence, refers to the previous noun, namely *laundromat*, instead of recognizing that the sentence is referring to the *cost of doing laundry*. The pronoun *it* is not used as an explicit referent for a noun previously mentioned in an earlier phrase. Rather, *it* is used to refer to an unstated noun, the cost imposed by the laundromat, and has to be inferred from the context. Such inferences might require little mental processing by a fluent English reader, but for less fluent readers and especially ESL students, the processing demands are greater and require space in working memory that needs to be devoted to the mathematics of the problem.

Compound sentences also increase cognitive demands. The compounding in the second sentence, "and 25¢ per load", links the "25¢ per load" with the washing machine load until the end of the sentence, at which point prior experience with laundromats signals the reader to interpret the sentence differently. Thus, the language structure places heavy demands on prior experience that a seventh grader is unlikely to have. Additionally, ESL speakers may have interpretation problems because of the differing interpretations of verbs; for example, Spanish speakers may interpret the words *to do* as *make*. For the second-language learner, further difficulties may arise because of the knowledge assumptions inherent in partial use of the language. For example, a student may be familiar with *wash clothes* in English but not with the word *laundry*, and students whose families do laundry at home may not understand the word *laundromat*. Both *load* and *expression* have multiple meanings that may not be familiar to students. *Load* is usually first learned as a verb and when second-language learners read the word *expression*, they may, for example, think about what someone's face looks like. Also, for many ESL students, English first names are not easily recognized. When the first author asked a Southeast Asian eighth-grade student to work a similar problem, the first question he asked was: "What is a Sandy?" He was not familiar with common first names in English. Although these terms can be explained reasonably well in a classroom setting, on a standardized test such language may result in students answering questions incorrectly. They may know and understand the mathematics concepts and processes needed to solve the problem, but the unfamiliar language may interfere with the accurate assessment of the students' mathematical knowledge (Mohan, 1992).

Problem 2[2]: The Soccer Problem

> Jorge's town has a soccer league. He helped sign up players for the different divisions. The table below shows the total number of players who signed up this year.

Division	Number of Players	Sign-up Fee
A (16-18 years old)	49	$20 per player
B (13-15 years old)	54	$20 per player
C (10-12 years old)	67	$30 per player

Jorge needs to figure out the total amount of money from sign-up fees. Which of the following expressions will give him the correct amount?

A. (49 + 54) × $20 + (67 × $30)

B. (49 × $30) + (54 × $20) + (67 × $30)

C. (49 + 54 × $20) + (67 × $30)

D. (49 + 54 + 67) × ($20 + $30)

Reflection on the solution process for Problem 2. As in Problem 1, Problem 2 has a goal of finding the total amount of money without actually finding a simple numerical answer. Rather, the answer is one of several complex multiple-choice options that show possible ways of attempting to arrive at the total. Although, the table visually chunks some of the information by placing the parts in close proximity, there are two ways to complete a computation to determine the total. One way would be to complete three multiplications and then add the three results: (49 × 20) + (54 × 20) + (67× 30). The second way is to group together the first two "Numbers of Players" in the table because the sign-up fees are both the same: (49 + 54) × 20. Then, the result is added to (67 × 30).

If the problem solver computes the answer and then looks at the multiple choice options, the process used may not be an available option. Solving the problem, looking for the solution, recognizing (a different symbolic representation of their solution) that it is not there and then redoing the problem requires seventh graders to have a level of confidence in their own problem-solving abilities that may be unreasonable to expect and increases the length of time to complete the solution. To recognize that there are two ways to compute the solution and hold their two distinct representations in working memory while comparing their discrete elements to the discrete elements of equally complex phrases in the multiple-choice options would cognitively overload most people. Either way, the testing structure increases the complexity of the solution process for this problem introducing what van Merriënboer and Sweller (2005) call extraneous information that increases the complexity of cognitive load that must be processed.

A strategic test-taking approach that might be used to control cognitive demands for this problem would be to start with the first multiple-choice option and determine whether or not it fits the problem situation. This strategy stands in stark contrast to the strategy used in Problem 1. The contrast illustrates that the productive use of problem-solving strategies is dependent upon the problem situation (Lesh, Lester, & Hjalmarson, 2003). It also underscores the importance of classroom discussions about different cognitive strategies and when to apply them as emphasized by Lesh and Zawojewski (in press).

Linguistic challenges. This problem has several attributes that may increase cognitive demands for students, and especially ESL students, by introducing linguistic information that must be processed in working memory and that is extraneous to the mathematical problem (van Merriënboer & Sweller, 2005; Paas, Renkl, & Sweller, 2003). *Division*, *sign-up fee*, *per*, and *league* are words that ESL students may not have seen before or seen only in a mathematical sense, as for example, the word *division*. ESL students often become intently focused on language and can spend a lot of time trying to figure out a word's meaning. They might, for example, initially try to figure out how division applies to this problem situation, thus losing valuable testing time. Test writers can use various strategies to help students reduce the cognitive demands: *Division* could be changed to *Age Group*; *per* could be changed to *for each*; *sign-up fee* might be changed to *cost*; and *league* might be avoided by saying "Jorge's town has many soccer teams and he helps sign up players for the teams." If students are not familiar with reading a table, they may not understand the relationships between the numbers and the labels. Just adding an "s" to the word *division* may reduce cognitive demands for English language learners because the "s" explicitly indicates plural and signals a need to look for multiple groups.

A Transition to Teaching

The above analysis of two problems from a state assessment test illustrates the need for mathematics teachers to consider ways in which assumptions about prior knowledge and life experiences implicit in the language and situations used to contextualize problems may not match students' experience, especially for ESL students (Campbell, 1995; Lamon, 2003). There is *prima facie* evidence that test writers are not as linguistically or culturally aware of the difficulties that particular wording and phrasing in word problems cause students, especially those taking the tests in a second or additional language. Underestimation of students' actual capacities with respect to mathematics may occur as a result of the way in which students use everyday knowledge to answer mathematics test items that use "realistic" contexts to frame the test question (Cooper & Dunne, 1998; Lamon, 2003). Additionally, we have illustrated how underestimation of students' actual capacities with respect to mathematics may occur as a result of second-language learners' lack of familiarity with the context of a problem.

Like the experts who reflected on the problems above, students initially use text to figure out what they need to do mathematically, making assumptions about appropriate interpretations. Wyndhamn and Säljö, (1999) illustrated how students' cognitive activity, while working in small groups, was discursive in nature and followed the descriptions of the interactions of text and student reasoning identified by Laborde (1990). In Wyndhamn and Säljö's study, students began with reading the text and interpreting the problem. After comparing answers, they tested arguments using both intra-linguistic and extra-linguistic information. When the text and mathematics involved did not produce new arguments for resolving discrepancies, students turned to other life experiences in order to structure arguments for the validity of their answers.

Work with urban middle-school youth in the QUASAR[3] Project (Silver, Smith, & Nelson, 1995; Silver & Stein, 1996) pointed to the need to adapt instruction to the needs of diverse learners. This five-year project documented the importance of focusing on multiple representations and multiple strategies within the features of communication and collaboration in classroom instruction. They also identified the need to include language development in the mathematics curriculum. Language infused instruction in combination with appropriate mathematical tasks was critical to supporting meaningful learning for all students, especially when "intended to provoke students to engage in conceptual understanding, reasoning, or problem solving" (p. 483). This instruction was especially important for students learning English as a second language while concurrently learning mathematics in classrooms where instruction was entirely in English. In two three-year longitudinal studies conducted within the larger QUASAR project, ethnically and linguistically diverse students benefited as much from such instruction as did their native English-speaking counterparts.

Discussion of a Model of Mathematics Teaching

The problem analyses and research discussed above illustrate how the interaction between language and mathematics is compounded by the diversity of students' background knowledge and life experiences. Although we could focus our attention on the writing of test items in ways that would make the contexts more readily accessible to all students, we have chosen to focus on how to prepare teachers for classrooms with students with diverse life experiences. The task of supporting language development while teaching mathematics is doable, but the strategies that teachers use in developing the instructional program may need to be significantly modified from the traditional approaches to teaching mathematics. For teachers to plan appropriate instruction they need to be able to identify elements of the interaction between mathematics and language and the ways in which the elements affect the cognitive demands on second-language learners as they comprehend and do mathematics. Instruction then can address the critical elements of the interaction through discussion of background, linguistic or content related knowledge that the student will need in order to learn a concept or solve a problem. Many of the factors we discuss for ESL students also affect the problem-solving success of at-risk students. Teachers who have focused on strategies for helping second-language learners have been pleasantly surprised that other students have also benefited from those strategies (Adler, 1999).

The analytical and interactive process required of teachers was illustrated by Simon's (1995) constructivist model of mathematics teaching derived from his reflections on, and analysis of, his own practice with university preservice teachers. His model evolved as he observed his students from a constructivist perspective within the socially situated context in which he was teaching. Through his interactions with students and reflections on his instructional activities, he continually reconstructed his understanding to better align his pedagogy with the reality of how students understood and processed mathematical situations. His model of constructivist teaching suggests a "cyclical interrelationship of aspects of teacher knowledge, thinking, decision making, and activity" (p. 135). Inherent in this relationship is what Simon called "the critical tension between the teacher's goals with regard to student learning and his responsibility to be sensitive and responsive to the mathematical thinking of the students" (p. 114). Lamon (2003) suggested that this responsiveness includes several critical factors: 1) the ability to "take into account the different knowledge structures with which students come into instruction;" 2) the ability to "build a model of the students' conceptual structures and track changes;" and 3) the ability to "elicit and manage diverse interpretations and ways of thinking" (p. 436).

A reader might be puzzled that we apparently "mix" theories in this paper, perhaps even to the point of seeming to be motivated by an idealism that overlooks practical considerations. It is generally conceded that attempts to base teaching upon constructivist principles of learning (Simon, 1995) are at odds with cognitive load theory (Sweller, 1988) that we used as a framework to analyze the cognitive demands of mathematical problems on second language learners. What, in essence, characterizes constructivist theories of learning? There are, perhaps, as many answers as there are theoreticians. Yet Steffe (in conversation with the third author) once summed up what was for him an essential feature of any constructivist theory of learning: the notion of reflective abstraction. Without reflective abstraction as a motive in learning, constructivist theory might as well be about pure discovery learning, which it is not. So if we take reflective abstraction as a basic and essential piece of a constructivist theory of learning, in what way is this compatible with cognitive load theory? The concept that makes the connection a little clearer is that of memory.

Cognitive load is concerned fundamentally with working memory: the aspects of memory, long and short term that we bring to bear in problem settings. Reflective abstraction is concerned fundamentally with re-enacting situations in mind, bringing to bear long-term memories as we re-think a problem solution. From this perspective, therefore, constructivist theory and cognitive load theory are quite compatible, each dealing with different aspects of a students' use of memory. Without attention to issues of working memory, students are doomed to suffer inefficient and unproductive problem-solving techniques, a message that cognitive load theory has made very clearly. Also, without a stimulus to abstract through reflection on the operations used in the solution of a problem, a student risks never seeing a more inclusive picture. If we want our students to be efficient problem solvers, who learn general principles from their problem-solving experiences, then we need to pay attention to what both cognitive load theory and constructivist theory tell us about student learning – and how that might influence our teaching.

Although we are in general agreement with Simon's model of mathematics teaching (Simon, 1995), we argue that, at a time when high-stakes testing dramatically affects students' sense of achievement and self-worth, his model needs to be extended. What is missing is the explicit inclusion and discussion of language, culture, and prior experiences as they play out in the kindergarten through grade 12 classrooms (ages 5 to 18). Such a modification is needed to fit our mental representations of the classrooms in which there are growing numbers of culturally and linguistically diverse students and where novice or pre-service teachers and in-service teachers find themselves teaching mathematics.

Additionally, Simon (1995) did not discuss or show how pre-service teachers could apply the model. It would have been extremely powerful to demonstrate how the model could be used to help pre-service teachers *become* mathematics teachers capable of reflecting on their own students' mathematical constructions and of adapting their own instruction to meet their students' perceptions and developmental levels in the way that Simon reflected on his own teaching. Unlike Simon, pre-service teachers, in general, have not yet acquired the mathematical or pedagogical sophistication to perceive, attend to, and interpret all of the critical factors in their classrooms. Pre- and in-service teachers learning to work with ESL students in mathematics classes will benefit from a framework that directs their attention to elements in the classroom to which they otherwise would not attend. As they learn to attend to these elements, they develop an initial internal structure that will guide decision-making and instructional planning.

Berger and Luckman (1973) referred to the building of mental models that guide the perceptual processes used to perceive, attend to, and interpret elements in our environment as the structures of the life world. Recent work in mathematics education (Lesh & Doerr, 2003; Lesh & Zawojewski, in press) has examined the critical role these models play in mathematics teaching and learning. As teachers acquire teaching experience, they transform and enrich their internal mental models of mathematics teaching. They also develop models of the "students' conceptual structures" (Lamon, 2003, p. 436) and of changes in student learning. The guidance provided by the initial framework presented below serves to support the development of teachers' ability to monitor and reflect upon their own teaching and check the accuracy of their mental models of mathematics knowledge and their students' learning. The framework provides teachers with questions they can ask themselves in order to enhance their ability to perceive elements in the instructional environment that they otherwise might not have seen and then to use those elements in instructional planning.

Development of a Framework for Integrating Culture and Language

The analytical framework that we propose can be thought of as a cycle of reflection for planning and implementing instruction. Such a cycle of reflection might be activated when teachers establish the social norms of the mathematics classroom (Wertsch & Toma, 1995), and when they select and pose mathematical situations to their students. Our framework focuses attention on elements critical to the teacher's construction of a representation of second-language learners' cognitive processing of mathematics concepts being learned in a second language. It is not a prescriptive methodology for teaching mathematics. Instead, it provides a structure for developing the ability to analyze student experience and prior knowledge in coordination with assumptions inherent in mathematics instruction and materials. Teachers can use that analysis to build more accurate "models of the realities of those with whom they interact" (Steffe & D'Ambrosio, 1995, p. 146).

The development of the framework drew on symbolic interaction theory (Blumer, 1969) and compatible aspects of phenomenology (Schutz, 1967; Berger & Luckman, 1973). We employ what Wertsch (1991) has referred to as a "sociocultural approach to mediated action." Wertsch and Toma (1995) explained that a "fundamental claim of this approach is that mental functioning is assumed to be inherently situated with regard to cultural, historical, and institutional contexts" (p. 159). We propose that reflection on culture, language, and socially situated prior experiences, in addition to reflection on mathematical content and students' cognitive processes and understandings, be incorporated into models of mathematics teaching.

Figure 3 demonstrates the complexity of the three critical components of mathematics pedagogy and their interaction: (a) the classroom as a socially constructed learning environment mediated through teacher-student interaction; (b) the teacher's perceptions of the students and classroom environment mediated by the interaction of experience and knowledge, language, culture, and mathematics; and (c) the students' perceptions of the classroom environment mediated through the interaction of language, culture, mathematics, prior experiences and knowledge. These three components interact to shape the teacher's and students' expectations about what it means to learn and teach mathematics (Campbell, 1995). Based on their expectations, students attend to elements in the instructional context that they perceive to be important and subsequently build models of mathematical concepts based on their “interpretation of the problem-solving situation” (Lesh & Doerr, 2003, p. 9). The teacher’s perceptions of students’ learning are mediated by the teacher’s prior experiences and contexts in which the teacher learned and developed mathematical understandings, as well as earlier teaching experiences. When the teacher and students share many of the same kinds of prior experiences and the teacher’s understandings of mathematics concepts are well developed, the teacher’s perceptions of the students’ models of mathematical concepts can be reasonably accurate. However, when the teacher and students do not share the same language, previous experiences, or culturally based assumptions about what it means to teach and learn mathematics, the teacher’s resulting mental model of student learning may have little to do with the students’ actual understanding and construction of mathematical concepts and processes. The ability to design and plan instruction with a high level of congruence between the instructional intent, classroom organization, and instructional delivery has positive academic consequences for ESL students (Tikunoff, 1985; Roy & Rousseau, 2005).

Figure 3 visually represents the elements of the instructional process and their interaction to which teachers need to attend as they plan mathematics instruction. We have drawn on elements of this representation to develop a framework to help teachers identify factors that influence the varied ways ESL students may perceive and process information presented in a mathematics classroom. In our work as teacher educators, we discovered that teachers often are not aware of how these factors are embedded in assumptions behind curriculum materials and the mathematics pedagogy they use in their instruction.

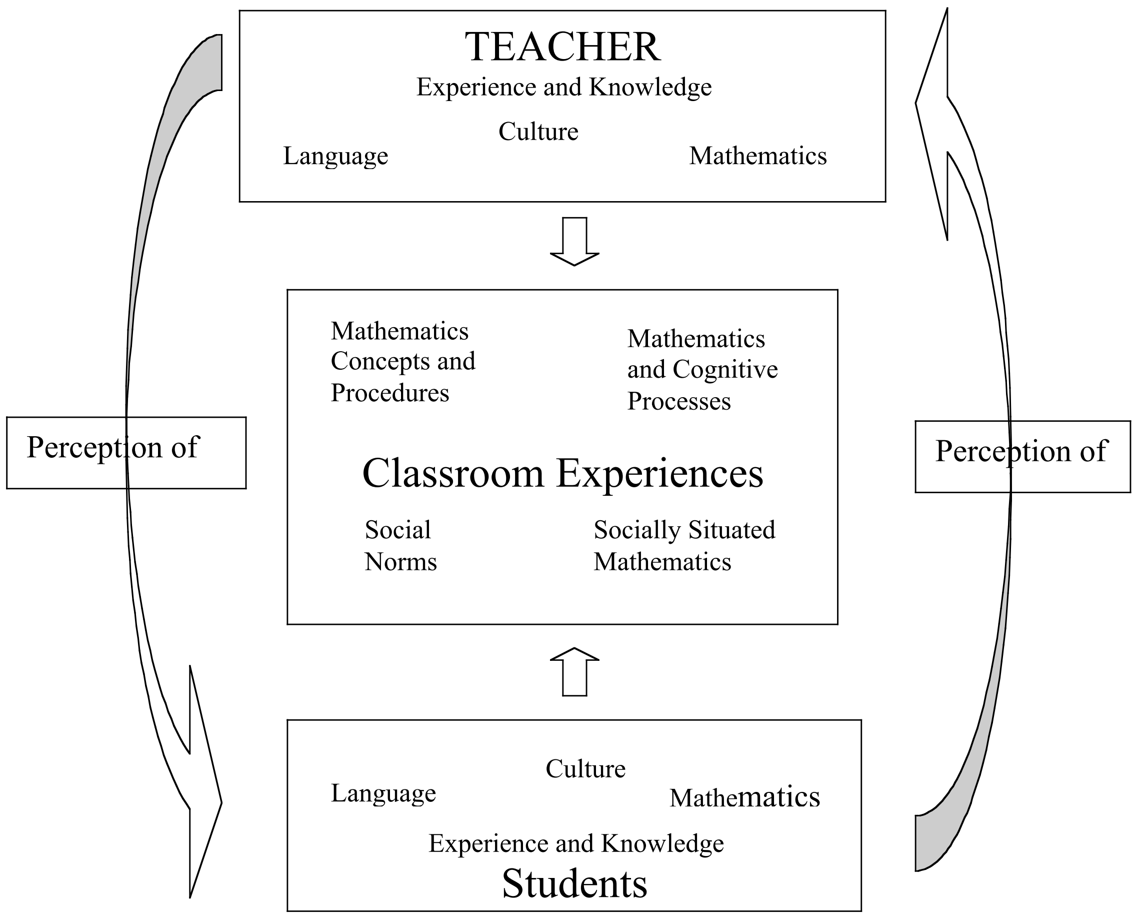

Figure 3: Model of Interactions in Mathematics Teaching and Learning

These factors influence the decisions that teachers make about the mathematical situations they pose, the materials they select, and the structure and management of classroom activities. This lack of awareness results, in part, from the nature of mathematics as a social and cultural construct. That is, the experiential base on which the concepts are formed is dependent upon the social and physical environment where they are learned and are shaped by the language used to express them. Most teachers in public schools in the United States, for example, are monolingual English speakers who received their mathematics coursework in schools, colleges, and universities in the United States. Their understanding of what mathematics entails and what it means to "teach" mathematics is shaped by those experiences. As graduates of the same educational system in which they teach, these teachers may not have been exposed to opportunities allowing or enabling them to reflect on the culture-based assumptions inherent in their methods, modes of presentation, and materials. Furthermore, they may not have reflected on the ways in which language (specifically English in the United States) and assumptions about prior experiences are used to explain and develop conceptual understandings of mathematics.

Historically, the discussion of teaching English as a second language through mathematics began in the early 1980s (Cuevas, 1984) in the United States. Key concepts influencing content ESL pedagogy at that time were those of cognitive academic language and common underlying proficiency articulated by Cummins (1979, 1986). These concepts became the foundation for programs that prepare ESL teachers and for research in effective teaching of English language learners. Cummins hypothesized that if students understood academic concepts in a discipline and had the language to express those concepts in their first language, the concepts and language would serve as a bridge or as common underlying proficiencies for the ESL students to use to learn the English needed to express those underlying concepts.

As content ESL programs developed in the mid-1980s, teacher educators began to apply Cummins theories to research and to the development of teacher preparation programs. The origin of our framework is in the research by Cuevas (1984) and Chamot and O'Malley (1986), who studied the practical application of Cummins' theories applied to mathematics teaching of ESL students. Cuevas was one of the first to examine the relationship between language and mathematical content. He developed a format to teach educators how to analyze a mathematics lesson and how to plan instruction to include content and language objectives and activities when working with ESL students. This approach to content lesson planning is still commonly used when training teachers in content ESL pedagogy (Echevarria, Vogt, & Short, 2004).

One of the limitations of a language-content approach to lesson and materials analysis is that the language analysis tends to be at the surface level. Features such as grammar, word meaning, and syntax are often the focus of analysis, as well as the format in which teachers present information and ask questions. When content is the focus, the models for teaching mathematical concepts, as presented in mathematics textbooks, are often derived from the experts who wrote the textbooks rather than from a conceptualization of how the students process and understand the concepts. In fieldwork with public school content-ESL teachers working with Southeast Asian refugees, Campbell (1992) found that references to popular culture and experiences assumed to be universal for each grade level were embedded within textbooks and only served to confuse ESL students. She concluded that more than language and content needed to be considered in order for teachers to develop a constructivist pedagogy that enabled them to attend to and reflect on critical elements of instructional materials and activities presented in mathematics classes where English was the only language of instruction.

Campbell's (1992) work with teachers in sheltered English classes documented the importance of analyzing the relationship between language, life experiences, culture and instructional content. In such classes, students from a variety of language groups are taught English through a focus on teaching content using ESL strategies. Her work had implications for content teacher preparation, leading to the development of an initial framework that was used in teacher education courses. Although the framework was a useful instrument for analyzing the relationships between language, culture, and instructional content, it was limited. Collaboration between the authors led to the development of the framework articulated in this paper, which focuses more specifically on mathematics and integrates research from mathematics education and ESL-Bilingual education.

The framework presented in this paper evolved from work in teacher education courses for elementary and secondary classroom teachers and pre-service teachers with degrees in content areas who are obtaining a credential as graduate students. The elements of the framework have been successfully taught to students preparing to be mathematics teachers. Their use of this model for instructional planning was documented in their lesson plans.

A Framework for Planning and Reflecting on Mathematics Instruction

In this section, we discuss the framework developed in our teacher education courses. The framework is designed to help teachers more accurately attend to elements in the instructional context, including the natural language of instruction, academic content of mathematics (mathematical language, concepts and processes), cognitive processes, cultural references, and prior experiences related to problem contexts. Attention to these elements, which are difficult for many teachers to recognize, is important because they affect the level of cognitive demands placed on students when solving mathematics problems. Teachers who attend to them develop a more accurate representation of students' knowledge and abilities (Campbell, 1992; Lamon, 2003).

The framework has four components: (a) academic content; (b) mathematical and cognitive processes; (c) mathematical and contextual language; and (d) cultural/life experiences. It provides a structure aimed at enabling teachers to examine the source of cognitive demands for second-language students who are developing their understanding of mathematics concepts and their ability to understand word problems. Their analysis then provides information to use in planning instruction that is more appropriate. The following questions and explanations for each component of the framework focus attention on aspects of the students' prior knowledge that will influence cognitive demands.

Component A: Academic Content

- How experienced are students with mathematics concepts and procedures?

If students must struggle to recall concepts and procedures that they have only been exposed to recently, the cognitive demands will be higher than if they have used the concepts and procedures for a sufficient period of time to develop their ability to automatically process domain-specific schemas (Pass, Renkl, & Sweller, 2003). If, for example, a problem for a fourth grader involves addition of fractions, the problem is likely to have greater cognitive demands than a problem that involves addition of whole numbers less than 100. Likewise, a problem that involves sevenths and ninths will generally have greater cognitive demands than a problem that involves halves and fourths.

- How experienced are the students with concepts from other content areas such as science and social studies that are required?

While those who write mathematics problems should ensure that connections between mathematics and other content areas are supported in other curricula, educators cannot assume that this additional work has been done for commercially prepared materials. When materials are based on inappropriate assumptions regarding prior academic preparation, increased cognitive demands results and will thwart the goal of making sense of the mathematics in the problem situation.

Component B: Mathematical and Cognitive Processes

- What mathematical processes are needed and how experienced are the students at using them?

The National Council of Teachers of Mathematics (2000) identified five process standards for acquiring and using mathematical content: (a) problem solving, (b) reasoning and proof, (c) communication, (d) connections, and (e) representations. Each of these process standards involves a complex knowledge base that is used when solving a mathematics problem. Further development of the framework might include the delineation of cycles of reflection for each of the mathematical processes; however, cycles of reflection that integrate these processes may be more appropriate and additional or different perspectives on processes may be needed. Lesh and Yoon (2004) suggested that "the development of ideas occurs in the presence of diversity, selection, reproduction, and communication" (p. 226) and may involve multiple concept strands that interact and simultaneously develop. Lesh and Yoon used a models and modeling perspective to describe processes that contribute to idea development. As students engage in model development, they "not only need to use existing constructs and conceptual systems, but they also often need to modify or extend them by integrating, differentiating, revising, or reorganizing their initial mathematical interpretations" (p. 210). As they build models, students function like mathematicians by quantifying, dimensionalizing, coordinatizing, and systematizing in order to interpret situations mathematically (Lesh & Doerr, 2003).

♦ What cognitive processing skills are needed?

Cognitive processing skills do not develop automatically. Their development requires a program of intentional instruction that includes well-designed worked examples and instructional guidance (Rieber & Parmley, 1995; Kalyuga, Ayres, Chandler, & Sweller, 2003; van Merriënboer & Sweller, 2005) and incorporates "scaffolding," a process in which strategic control of learning is gradually transferred from experts to novices (Vygotsky, 1980; van Merriënboer, Kirschner, & Kester, 2003). Cognitive load researchers (Renkl & Atkinson, 2003) have referred to this process as a “fading procedure” in which “examples are presented before learners are expected to engage in problem solving, or alternatively, examples are interspersed with the to-be-solved problems” (p. 15). The goal of instruction becomes one of enabling students to take control over their own learning through the practice and development of increasingly complex processes modeled by the teacher in activities and demonstrations, and in the texts and materials (Schoenfeld, 1985). Becoming aware of and controlling our cognition as we solve problems falls under a more general category referred to as metacognition in psychological literature (Lesh & Zawojewski, in press; Schoenfeld, 1992; Wilson & Clark, 2004). Metacognitive and cognitive processes can be taught (Bransford, Brown, & Cocking, 1999; Chamot & O’Malley, 1994; Echevarria, Vogt, & Short 2004; Garofalo & Lester, 1985) and can be introduced during the discussion of problem solutions. Cognitive strategies help students to organize and classify their perceptions of the environment. Such strategies are used to plan, monitor, and evaluate learning (Chamot & O'Malley, 1994). Care must be taken, however, to develop students’ understanding that metacognitive strategies are not recipes to be followed but tools to be used in appropriate problem situations and during different phases of the problem solution process (Lesh & Zawojewski, in press).

Common strategies used in science and mathematics include posing questions, planning, predicting, perceiving relationships, drawing conclusions, formulating and evaluating hypotheses and checking the accuracy of work. Information acquired from using these strategies is as important as the end solution or findings. For example, ESL students’ ability to evaluate accuracy, to identify what they don’t know or where they have made a mistake in calculations, to see relationships, and to modify hypotheses is an important indicator of student learning and comprehension (Tikunoff, 1985; Lesh & Zawojewski, in press).

Students often have difficulty transferring concepts learned in one problem to another problem or a new situation, thus making every problem unique. One strategy that addresses this issue involves reduction of goal specificity. The student must focus on understanding the problem situation instead of focusing on the goal of the problem as defined by the question. Reduction of goal specificity in the early stages of teaching and learning a new principle reduces cognitive load (Owen & Sweller, 1985; van Merriënboer, Kirschner, & Sweller, 2003; Sweller, 1988). When goal specificity is reduced, cognitive processing space can be used for schema acquisition instead of means-end analysis, which involves closing the gap between the problem goal and where the problem solver is in the problem-solving process. Means-end analysis increases cognitive load because, in order to close the gap, the problem solver must

> simultaneously consider the problem goal, the current problem state, relations of the goal state to the current problem state, relations of this relation to the allowable operators, and in addition the maintenance of any sub-goal stack that has been constructed. It is therefore not surprising that novices, with a poor grasp of these many elements, have difficulty learning while using means-ends analysis (Sweller, 1988, p. 284).

The use of reduction of goal specificity may require a change in perception and attitude about what it means to teach and learn mathematical problem solving in schools that currently focus on having students work toward answering the question.

Component C: Mathematical and Contextual Language

- Do the students' prior experiences include the development of mathematical language and the development of the reflective and command functions of natural language in the learning of mathematics?

Mathematics teachers' focus of attention has shifted during the past two decades from an exclusive focus on mathematics content to listening to how students think about and process that content. Although mathematics content is as important as ever, we have begun to acknowledge that teaching mathematics needs to encompass more than just mathematical content. Boero, Douek, and Ferrari (2002), for example, suggested that natural language serves important roles in mathematics learning. Natural language functions as "a mediator between mental processes, specific symbolic expressions, and logical organizations in mathematical activities" (p. 243) and as a mediator between experience and the development of concepts. It is a tool for managing specific mathematical languages (e.g., algebraic) and is a means for developing metalinguistic awareness, which is important in making transitions between languages (e.g., between algebraic and geometric). Additionally, natural language aids in finding counter examples and in developing arguments of validity. Boero et al. recommended that mathematics teachers mediate classroom instruction, building on students' individual productions and on cultural models in order to help them attain a level of sophistication in the use of natural language that will support their learning of advanced mathematics. According to Boero et al., "teachers must have a strong commitment to increasing students' development of linguistic competencies by way of producing, comparing, and discussing conjectures, proofs, and solutions for mathematical problems" (p. 242).

Moschkovich (1999) emphasized the need for ESL students to experience the development of mathematical content and argumentation practices, as well as to experience building vocabulary. The students must be encouraged to participate in mathematical discussions that focus on justifying thinking and interpreting meaning.

- Does the language used in the problem statement or instruction correspond to the level of English language development of ESL students?

If students understand the words *wash clothes* and the problem statement refers to *laundry*, some support needs to be given to help students develop strategies to bridge the language gap and prevent cognitive blockages. Teachers need to be attentive to these issues when they write and develop test questions and discuss problems in class. In both cases, language needs to be used that supports students in their development of contextually related words. That is, if students understand the words *wash clothes* but do not understand the word *laundry*, the problem statement needs to be structured in such a way that students are able to infer meaning for the word *laundry* using the context of the problem.

A related consideration to note when working with ESL students is that the language developed in the ESL curriculum may not correlate with the grade-level curriculum in mathematics. Students may be learning language commonly taught at one to three levels below the grade-level language used in teaching mathematics. For example, students may be learning their number names in the ESL curriculum, when in the math class they are beginning algebra. Thus, mathematics teachers need to communicate with ESL teachers to coordinate the language development of their shared students.

Further considerations need to include dialectal variations in the language of mathematics (Hirigoyen, 1997) from the language groups represented in the classroom. The ways that numbers are represented and named vary across language and cultural groups. The algorithms and notations used as well as symbols for geometry may vary. Dialectal variations may distort measurement of student performance because students' lack of the mathematical dialect used in the classroom may be perceived as a lack of understanding of mathematics.

- Are there words that have specialized meanings in mathematics that have different meanings in natural language?

Second-language learners “must contend with multiple language variables all at the same time” (Khisty, 1995, p. 283). Khisty concluded that the nature of the language used to communicate mathematical ideas “needs to be brought to the students’ attention and its structure needs to be taught, along with the rest of mathematics, in order for students to develop sufficient control in its use as a way of communicating mathematically” (p. 283).

If, for example, a student has learned the natural language meaning of a word such as *table,* but not the mathematical meaning of the same word, for example, *times table*, confusion is likely. Likewise, if the mathematical meaning of the a word such as *division* has been learned but not the natural language meaning and the problem statement applies the natural language meaning, the student may be confused and attempt to divide. One way that teachers can support ESL students’ comprehension is to create glossaries that identify terms with multiple meanings, including the glossaries with mathematical materials they distribute to students.

Component D: Cultural/Life Experiences

- What knowledge of cultural or life experiences is needed to understand the problem statement?

ESL students often bring to the classroom life experiences in which “they have had to solve problems, communicate, and reason, but in ways that are not generally found in math textbooks” (de Abreu, 2002; Buchanan & Helman, 1993). When textbook authors contextualize mathematics in word problems, they often use situations or contexts that are associated with the popular culture or everyday life common to growing up in the United States. ESL students, however, may not have shared in these or even similar experiences. Children raised in other countries may not understand how to play baseball or ride a bus, or they may not be familiar with the values and names of coins, objects or characters from popular movies and literature distributed in the United States. When such information is used to contextualize problem situations, they often confuse or frustrate ESL students who have no frame of reference from which to interpret the situations. Instead of working to solve the problem, they focus their attention on trying to understand the context.

- What connections need to be made between the mathematics of the classroom and student experience?

Migrant ESL students born and raised in the United States, for example, may have life experiences apart from their peers who participate daily in American popular culture. These students may have had experiences related to harvesting food crops, which pay in terms of the amount harvested. They might be able to discuss, in mathematical terms, the amounts of produce harvested and dollars earned, but be unable to relate that knowledge to problems that frame the same problem situations in terms of an hourly wage. In this instance, the teacher would need to help the students make those connections.

Concluding Remarks

A strong argument can be made that students learning English as a second language should have the opportunity to learn mathematics in classrooms in which they can negotiate meaning as the dialogue moves freely between their primary language and English (e.g., Khisty, 1995). However, the language groups represented by the students in many classrooms have become increasingly diverse. In Washington State alone, more than 200 languages are represented in the public schools. “Despite encouraging evidence that learning communities can be successfully created for linguistically diverse student populations, teachers face special challenges in creating safe and supportive environments in these situations” (Silver, Smith, &

Nelson, 1995, p. 39). Given the increased numbers of second-language learners in many countries and the current emphasis on high-stakes testing in mathematics, a greater understanding of the linguistic and cultural assumptions embedded in word problems and instruction in general is essential for the development of appropriate and effective mathematics instruction. In this paper, we have drawn on the analysis of problems from high-stakes tests, cognitive load theory, and a constructivist model of teaching to suggest a framework to help teachers reflect on materials and instruction in ways that incorporate issues of language and culture. It is important to note that each of these areas of research merits a much greater depth of treatment than space in this paper allows.

A fundamental question that influenced the development of the framework is how can we support teachers in their construction of more accurate mental models of the interplay between their student's perception of classroom experiences and the mathematics curriculum. To answer that question, we considered Simon's (1995) research on teachers' development of hypothetical learning trajectories (HLTs) of students' understanding and learning of mathematical concepts. We found that this research has not considered linguistic and cultural issues of mathematics teaching. Simon (1995) and others doing research on hypothetical learning trajectories (e.g., Clements & Sarama, 2004; Simon & Tzur 2004) are focused, for the most part, on a fine-grained analysis of learning trajectories related to specific content development with individual students. Clements, Wilson and Sarama (2004) recognized the need to bridge the gap between practice and this fine grained analysis by involving teachers in the research process to analyze children's individual learning trajectories and use their input in the development of mathematics curriculum. They, however, are not looking at how the teachers process the learning environment and how teachers develop their models to include the influence of student experience and knowledge on learning. These processes need to be examined in terms of the models created by teachers in many situations including classrooms with second-language learners. Understandings of two worlds (the world of the student learning mathematics and the world of the teacher learning to teach mathematics) need to be coordinated.

The next phase in the development of the framework discussed in this paper is to conduct research with the purpose of examining whether or not new habits of perception and instructional planning developed in teacher preparation courses will become habitual and be incorporated into a long-term classroom-based mathematics pedagogy. Effectiveness of instruction developed using the framework might be evaluated based on the ability to integrate variability in student interpretation of problem situations into the planning of mathematics instruction (Lamon, 2003). There is very little published data on the variability of ESL students' interpretations of mathematics word problems. Such data, from students and from teachers, especially from problems on high-stakes tests, would be most valuable to researchers because it would indicate what actually occurs in school test settings, as distinct from what test-writers might imagine about what should take place. Furthermore, research that investigates teachers' use of the framework and the effectiveness of their instruction on improving performance on high-stakes testing might enable us to more accurately assess students' mathematical learning and teachers' effect on that learning.

References

Adler, J. (1999). The dilemma of transparency: Seeing and seeing through talk in the mathematics classroom. *Journal for Research in Mathematics Education*, *30*(1), 47-64.

Berger. P., & Luckman, T. (1973). *The structures of the life-world*. Evanston, IL: Northwestern University Press.

Blumer, H. (1969). *Symbolic interactionism: Perspective and method.* Englewood Cliffs, NJ: Prentice-Hall.

Boero, P., Douek, N., & Ferrari, P. L. (2002). Developing mastery of natural language: Approaches to theoretical aspects of mathematics. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 241-268). Mahwah, NJ: Erlbaum.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.) (1999). *How People Learn: Brain, Mind, Experience, and School.* Washington DC: National Academy Press.

Buchanan, K., & Helman, M. (1993). Reforming mathematics instruction for ESL literacy students. *NCBE Program Information Guide Series, No. 15*. Rosslyn, VA: National Clearinghouse for Bilingual Education.

Cai, J., Jakabcsin, M., & Lane, S. (1996). Assessing students' mathematical communication. *School Sciences and Mathematics, 96*(5), 238-246.

Campbell, A. E. (1992). Once the door is closed: An ethnographic description of one content-based English language program as four teachers implemented it. (Doctoral dissertation, University of Florida, 1992). Dissertation Abstracts International, 54, 411.

Campbell, A. E. (1995). Academic culture and language: Implications for educating linguistically and culturally diverse students in the United States. *International Journal of the Humanities, 4*, 35-54.

Chamot, A. U., & O'Malley, M. J. (1986). *The Cognitive Academic Language Learning Approach.* Rosslyn, VA: National Clearinghouse for Bilingual Education.

Chamot, A. U., & O’Malley, M. J. (1994). *The CALLA handbook: Implementing the Cognitive Academic Language Learning Approach*. NY: Pearson Education/Longman.

Clements, D. H., & Sarama, J. (Eds.) (2004). Special issue: Hypothetical learning trajectories. *Mathematical Thinking and Learning, 6*(2), 81-260.

Clements, D. H., Wilson, D. C., & Sarama, J. (2004). Young children’s composition of geometric figures: A learning trajectory. *Mathematical Thinking and Learning, 6*(2), 163-184.

Cooper, B., & Dunne, M. (1998). Anyone for tennis? Social class differences in children’s responses to national curriculum mathematics testing. *The Sociological Review, 46*(1), 115-148.

Cuevas, G. (1984). Mathematics learning in English as a second language. *Journal for Research in Mathematics Education*, *15*, 35-144.

Cummins, J. (1979). Linguistic interdependence and the educational development of bilingual children. *Review of Educational Research*, *49*(2), 222-251.

Cummins, J. (1986). *Empowering minority students*. Teacher Training Monograph Number 5. Gainesville, FL: Teacher Training Project for Bilingual & English to Speakers of Other Languages Teachers.

Dale, T. C., & Cuevas, G. J. (1992). Integrating mathematics and language learning. In P. A. Richard-Amato & M. A. Snow (Eds.), *The Multicultural Classroom: Readings for Content Area Teachers* (pp. 358-270). White Plains, NY: Longman.

de Abreu, G. (2002). Mathematics learning in out-of-school contexts: A cultural psychology perspective. In L. D. English (Ed.), *Educational handbook of international research in mathematics education* (pp. 323-354). Mahwah, NJ: Erlbaum.

Echevarria, J., Vogt, M., & Short, D. (2004). *Making Content Comprehensible for English Learners: The SIOP Model*. Boston, MA: Pearson.

Education Commission on the States (2004). Recent State Policies/Activities. Retrieved March 25, 2005 from http://www.ecs.org/.

Freeman, D., Freeman, Y., & Mecuri, S. (2003). *Closing the achievement gap: How to reach limited-formal-schooling and long-term English learners*. Portsmouth, NH: Heinemann.

Gandara, P., Maxwell-Jolly, J., & Driscoll, A. (2005). *Listening to teachers of English language learners: A survey of California teachers' challenges, experiences, and professional development needs*. Santa Cruz, CA: The Center for the Future of Teaching and Learning.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*(3), 163-176.

Hirigoyen, H. (1997). Dialectal variations in the language of mathematics: a source for multicultural experiences. In J Trentacost & M. M. Kenny (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity 1997 yearbook* (pp. 164-168). Reston, VA: The National Council of Teachers of Mathematics.

Holmes, D., & Duron, S. (2000). LEP students and high-stakes assessment. *National Clearinghouse for Bilingual Education*. Retrieved May 21, 2004 from http://www.ncela.gwu.edu/ncbepubs/reports/highstakes/index.htm

Johnson-Laird, P. N. (1986). Mental models: Towards a cognitive science of language, inference and consciousness. *Cognitive Science, No 6*. Harvard, MA: Harvard University Press.

Kalyuga, S., Ayres, P., Chandler, P., & Sweller, J. (2003). The expert reversal effect. *Educational Psychology*, *38(*1), 23-31.

Kastberg, S., d'Ambrosio, B., McDermott, G., & Saada, N. (2005). Context matters in assessing students' mathematical knowledge. *For the Learning of Mathematics*, *25*(2), 10-15.

Khisty, L. L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 279-297). New York: Cambridge University Press.

Laborde, C. (1990). Language and mathematics. In P. Nesher & J. Kilpatrick (Eds.), Mathematics and cognition: A research synthesis by the International Group for the Psychology of Mathematics Education (pp. 53-69). New York: Cambridge University Press.

Lamon, S. (2003). Beyond constructivism: An improved fitness metaphor for the acquisition of mathematical knowledge. In R. Lesh & H. M. Doerr (Eds.) *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 435-447) Mahwah, NJ: Lawrence Earlbaum Associates.

Lesh, R., & Doerr, H. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.) *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33) Mahwah, NJ: Lawrence Earlbaum Associates.

Lesh, R., Lester, F., & Hjalmarson (2003). A models and modeling perspective on metacognitive functioning in everyday situations where problem solvers develop mathematical constructs. In R. Lesh & Doerr, H. (Eds.) *Beyond constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning and Teaching*. Mahwah: Erlbaum.

Lesh, R., & Yoon, C. (2004). Evolving communities of mind—in which development involves several interacting and simultaneously developing strands. *Mathematical Thinking and Learning, 6*(2), 205-226.

Lesh, R., & Zawojewski, J. (In Press). Problem solving and modeling. In F. Lester (Ed.), *Handbook of Research on Mathematics Education*. Greenwich, CT: Information Age Publishing.

MacDonald, V. (2004). *The status of English language learners in Florida: Trends and prospects policy brief*. Tempe, AZ: Education Policy Studies Laboratory. Retrieved May 28, 2004 from http://edpolicylab.org

Mace-Matluck, B. J., Alexander-Kasparik, R., & Queen, R, M. (1998). *Through the golden door: Educational approaches for immigrant adolescents with limited schooling*. McHenry, IL: Delta Systems Co.

Mohan, B. (1992). What are we really testing? In P. A. Richard-Amato & M. A. Snow (Eds.), *The Multicultural Classroom: Readings for Content Area Teachers* (pp. 358-270). White Plains, NY: Longman.

Moschkovich, J. (1999). Supporting the participation of English language learnings in mathematical discussions. *For the Learning of Mathematics, 19(*1*),* 11-19.

National Council for Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

National Coalition of Advocates for Students (1988). New voices: Immigrant students in U.S. public schools. Boston, MA: National Coalition of Advocates for students.

Nieto, S. (2000) *Affirming diversity: The sociopolitical context of multicultural education*. (3rd ed.). NY: Longman.

Nieto, S. (2004). *Affirming diversity: The sociopolitical context of multicultural education*. (4th ed.). Boston, MA: Pearson Education.

Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems? *Journal of Educational Psychology, 77*, 272-284.

Pass, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist*, *38*(1), 1-4.

Pass, F., Touvinen, J., Tabbers, H., & Van Gerven, P. (2003). Cognitive load measurement as a means to advance cognitive load theory. *Educational Psychologist*, *38*(1), 63-71.

Renkl, A., & Atkinson, R. (2003). Structuring the transition from example study to problem solving in cognitive skill acquisition: A cognitive load perspective. *Educational Psychologist*, *38*(1), 15-22.

Rieber, L. P., & Parmley, M.W. (1995). To teach or not to teach? Comparing the use of computer-based simulations in deductive versus inductive approaches to learning with adults in science. *Journal of Educational Computing Research*, *13*, 359-374.

Roy, F., & Rousseau, C. (2005). Student thinking as a context for high expectations. *For the Learning of Mathematics, 25*(2), 16-23.

Schoenfeld, A. H. (1985). Metacognitive and epistemological issues in mathematical understanding. In Edward A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 361-379). Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics and learning* (pp. 334-370). New York: Macmillan.

Schulte, B. (2002, June 9). Trapped between 2 languages: Poor and isolated, many immigrants' children lack English. *Washington Post*, p. A01.

Schutz, A. (1967). *The phenomenology of the social world*. (G. Walsh & F. Lehnert, Trans.). Evanston, IL: Northwestern University Press.

Silver, E., Shapiro, L., & Deutsch, A. (1993). Sense-making and the solution of division problems involving remainders: An examination of middle-school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education, 24*(2), 117-135.

Silver, E. A., Smith, M. S., & Nelson, B. S. (1995). The QUASAR Project: Equity concerns meet mathematics education reform in the middle school. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9-56). New York: Cambridge University Press.

Silver, E. A., & Stein, M. K. (1996). The QUASAR Project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education*, *30*(4), 476-521.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,* 114-145.

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6*(2), 91-104.

Steffe, L. P., & D' Ambrosio, B. S. (1995). Toward a working model of constructivist teaching: A reaction to Simon. *Journal for Research in Mathematics Education, 26,* 146-159.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12,* 257-285.

Sweller, J., & Low, R. (1992). Some cognitive factor relevant to mathematics instruction. *Mathematics Education Research Journal, 4*(1), 83-94.

Tikunoff, W. (1985). *Applying significant bilingual instructional features in the classroom.* Rosslyn, VA: National Clearinghouse for Bilingual Education.

van Merriënboer, J., Kirschner, P., & Kester, L. (2003). Taking the load off a learner's mind: Instructional design for complex learning. *Educational Psychologist*, *38*(1), 5-13.

van Merriënboer, J., & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational Psychology Review, 17*(2), 147-177.

Vygotsky, L. (1980). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wertsch, J. V. (1991). *Voices of the mind*. Cambridge, England: Cambridge University Press.

Wertsch, J. V., & Toma, C. (1995). Discourse and Learning in the classroom: A sociocultural approach. In L. P. Steffe & J. Gale (Eds.), *Constructivism in Education* (pp. 159-183). Hillsdale, NJ: Erlbaum.

Wilson, J., & Clark, D. (2004). Towards the modeling of mathematical metacognition. *Mathematics Education Research Journal, 16* (2), 25-48.

Wyndhamn, J., & Säljö, R. (1999). Quantifying time as a discursive practice: Arithmetics, calendars, fingers and group discussions as structuring resources. In J. Bliss, R. Säljö, & P. Light (Eds.), *Learning sites: Social and technological resources for learning (pp. 80-96, 267-268). Oxford, UK: Elsevier Science, Ltd.*