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Student Centered Approaches in High School Algebra

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Abstract

This quasi-experimental study evaluated the impact of four, student centered approaches on learning outcomes for high school math students, to determine if algebraic understanding improved in an active learning environment. Specifically, this study investigated high school algebra students' use of learning stations, hands on manipulatives, small group collaboration and mathematical games, to deepen their conceptual understanding of linear equations. Surveys revealed students prefered to work independently while being led by the teacher, yet this approach yielded the least amount of improvement. Pre- and post-tests showed students from the experimental groups out-performed peers in the control group on achievement of standards during each cycle of the experiment, with small group collaboration being the student centered approach yielding the highest mean. In conclusion, research findings revealed students need to become active participants in their learning in order to enrich their learning experience and enhance conceptual understanding in mathematics.

Keywords: Algebra, student centered approaches, high school, linear equations.

Introduction

What is algebra? Colin Maclaurin wrote, in his 1748 algebra text,

> Algebra is a general method of computation by certain signs and symbols which have been contrived for this purpose, and found convenient. It is called an universal arithmetic, and proceeds by operations and rules similar to those in common arithmetic, founded upon the same principles (Maclaurin, 1748).

Leonhard Euler, in his own algebra text of 1770 wrote, "Algebra has been defined as the science which teaches how to determine unknown quantities by means of those that are known" (Euler, 1984). That is, in the 18th century, algebra dealt with determining unknowns by using signs and symbols in multi-step equations to explore concepts like the Pythagorean Theorem. Can we say algebra has vastly evolved over the last 250 years? A dive into today's high school mathematics textbook found a wide variety of topics. These included the arithmetic of signed numbers, solutions of linear equations, quadratic equations, systems of linear and/or quadratic equations, and the manipulation of polynomials, including factoring and rules of exponents. Some textbooks also covered matrices, functions and graphs, conic sections, and other topics.

Traditionally, upper level, or secondary, mathematics has been taught from a textbook where students are responsible for memorizing a sequence of steps in order to achieve the correct answer. This method, memorizing algorithmic steps, often times allows students the ability to understand the concept procedurally (knowing how to go through a sequence of steps); however, most students will struggle conceptually (knowing why they are going through that sequence of steps). Researcher Rittle-Johnson (2017) reported , "learning techniques such as comparing, explaining, and exploring promote more than one type of knowledge about math, indicating that each is an important learning process" (p. 184). In the author's conclusion of her research she reported, "Mathematical competence rests on developing both conceptual and procedural knowledge" (p. 190). Conceptual knowledge is analytical in nature, basically the knowledge of why the concepts work. Procedural knowledge is knowing how to calculate, and knowing how to solve algebra problems algorithmically.

Since most algebra textbooks in America's high schools are procedurally driven, how does an algebra teacher make sure students are learning both procedurally and conceptually? The National Center for Educational Achievement (NCEA, 2009) examined higher performing schools in five states (California, Florida, Massachusetts, Michigan, and Texas) and determined that in terms of instructional strategies, higher performing middle and high schools use mathematical instructional strategies that include classroom activities which have a high level of student engagement, demand higher-order thinking and follow an inquiry-based model of instruction – "including a combination of cooperative learning, direct instruction, labs or hands-on investigations, and manipulatives." (p. 24) One could conclude the data obtained from the NCEA indicates today's algebra teachers need to reform their classrooms to incorporate these instructional strategies. This study seeks to explore if conceptual understanding of secondary algebra concepts is enhanced when the teacher uses math stations for collaborative learning, games to reinforce skills, and manipulatives for visualization of concepts. In addition the teacher will encourage dialogue amongst students to facilitate using peers as resources.

Literature Review

Mathematical Stations

One potential instructional strategy to support student learning is using math stations. Math stations, also known as math centers, were originally created as a means to accommodate a vast array of diverse learners. Carol Ann Tomlinson is known as the founder of the stations method in elementary and middle level classrooms. Wu (2013) reported, from an interview with Tomlinson, stations were found to be useful for differentiation. Defining stations as, “a place in the room where the kids go, to do specified work. Instructions at the station provide guidance on how to complete work appropriately, how to get help, where to put completed work and so on” (p. 125). Stations are designed to allow a few ability-grouped students to work together on concepts in an activity that interests them. When students are engaged in their station activities, the teacher is freed up to work with individual students or small groups of students to meet their individual needs. Unlike the traditional lecture method, the stations method allows teachers to know where his/her students are at in the development of conceptual understanding.

Chin, Daud and Zakaria (2010) reported stations encouraged collaborative learning that “provides children with the opportunity to share ideas, experiences and backgrounds with one another while they study various topics in the standard curriculum. When children work together to discover, create, solve problems, observe, and record data, they also learn how to communicate and cooperate with each other” (p.272). Multiple researchers have studied the benefits of students working together cooperatively to accomplish learning goals. Johnson and Johnson (1999) reported, “there are significant learning benefits for students who work together on learning activities” (p. 198) A comparison study by Johnson and Johnson (1999) presented students of various ages with four types of mathematical problems to be completed either individually or by working with peers. Researchers found that peer teams surpassed individuals on all types of problems and across all ages.

Games to Reinforce Skills

Another potential instructional strategy to support student learning is to incorporate games into the classroom structure. Games have been part of the American culture for centuries. Physical games originated when the Native Americans created stickball, now known today as lacrosse. Board games originated when the first European settlers brought checkers, chess, backgammon and card games across the Atlantic Ocean to their new home in America. To this day many families in America continue to enjoy both physical and mental games.

Not until recently have games found their way into the classroom. Milner (2014) wrote "mathematical/logical puzzles and games appeal to a wide range of people of all ages, backgrounds and perceived strengths" (p. 21). In the classroom, games can sometimes encourage students who are not normally interested in mathematics. In addition, games can allow students a way to show how they can logically solve a problem without using an algorithm or a finite set of rules. Furthermore, Milner reported, that "solving logical puzzles, or playing games, encourages all students to develop mathematical habits of mind such as tenacity, attention to detail, confirming that a solution is correct, tracking down errors, being willing to start over and so forth" (p.21).

Most elementary textbooks complete their conceptual units with a reinforcing game idea to be played with the students. One popular textbook series, Everyday Math, published by McGraw-Hill, uses games to reinforce concepts because, "Drill tends to become tedious and, therefore, gradually loses its effectiveness. Games relieve the tedium because children enjoy them" (University of Chicago School Mathematics Project, n.d.). Often times students will play games and don't even realize they are reinforcing or learning a new concept.

Despite the positive research citing the importance of using games for learning, high school classrooms remain lecture focused. There remains a lack of research supporting the use of games in a high school classroom to reinforce conceptual understanding of algebraic ideas.

Manipulatives to Visual Concepts

A third instructional strategy to support student learning is the use of manipulative materials. Manipulatives are concrete models that involve mathematical concepts, appealing to several senses that can be touched and moved around by the learners (Heddens, 2005). Tactile manipulatives are physical materials that student use in a variety of ways. They allow the learner to touch, rearrange, and see mathematics concepts. A virtual manipulative is defined as "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, 2002). Visual representations of concepts and relations help learners to gain insight in mathematics.

Instructional strategies that use manipulatives are often suggested as effective approaches to improve student mathematics achievement. Math manipulative-based instructional techniques are approaches that include opportunities for students to physically interact with objects to learn target information (Carbonneau & Marley, 2012). The National Council of Teachers of Mathematics (NCTM, 2000) has recommended that students be provided access to manipulatives in order to develop mathematical understanding. One could reason that the use of tactile or visual manipulatives may enhance the procedural and conceptual understanding for students, especially when the material being learned is abstract in nature. Since algebraic concepts are difficult for students to conceptualize, being able to see the math and move manipulatives can help to alleviate misunderstandings.

Despite the lack of tactile manipulatives in secondary classrooms, most are equipped with algebra tiles. Algebra tiles allow students to balance equations through the use of square and rectangular tiles. Heddens (2005) wrote, “concrete representations of abstract mathematical ideas, such as algebra tiles, offer opportunities for creation of situations to facilitate translations between manipulation of algebraic expressions and manipulation of concrete examples” (para.4). However, algebra tiles alone can not facilitate conceptual understanding, it is the teacher who orchestrates the appropriate environment to foster procedural and conceptual acquisition.

Small Group Collaboration

A fourth instructional strategy used to enhance learning is collaborating as small peer groups to enable the use of peers as resources. Students are valuable resources in the classroom whether it be in small group conversation, peer tutoring or critiquing the reasoning of others in regards to mathematical concepts, research continues to report students learn best from each other. The curriculum and evaluation standards for mathematical communication developed by the National Council of Teachers of Mathematics (2000) include the following statement:

> Small-group work, large group discussions, and the presentation of individual and group reports -- both written and oral -- create an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies. Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing (p. 78).

More recently, researcher Ebert (2017) reported, “talking and working together clearly has favorable effects on learning, especially conceptual learning” (p.171).

Encouraging students to look towards each other to test their own theories, or see if they are on the right track , is part of using your peers as resources. Often times students feel more comfortable communicating their thoughts to a person of their own age who is experiencing the same learning difficulties. Wiliam (2013) reported that peers can be very effective assessors of one another’s work, especially when the focus is on improvement rather than grading.

Summary and Conclusion

Researcher Golafshani (2013) reported “mathematics reformers and educators should use a variety of teaching strategies that will lead to effective teaching and learning” (p. 137). In addition others also believe that effective teaching includes teaching for understanding, teaching better mathematics (Friesen, 2005), and reversing mathematics misconceptions (Green, Flowers, & Piel, 2008). The teacher should not be a transmitter of knowledge but should instead act as a facilitator to the construction of knowledge for all learners. The fact that every classroom consists of students with different levels of ability to understand mathematical concepts means that teachers should focus on using multiple teaching strategies so that all students can benefit.

Much research has been done to support using math stations for collaborative learning, games to reinforce skills, and manipulatives for visualization of concepts, as well as using peers as resources for learning. However, the research to date focuses on the advantages of these instructional techniques in an elementary classroom. For this reason, this study will focus on the use of these strategies in a secondary mathematics algebra classroom. This quasi-experimental research study will be designed to examine how the implementation of various math methods will improve student performance in high school algebra. Specifically, this study investigated the research question:

How effective is the use of student centered approaches, during mathematics instruction, in improving students' conceptual understanding in a high school algebra class?

Research Design

Purpose of Research

The purpose of this research was to investigate how mathematics methods of instruction, like stations, using peers as resources, games, and working with manipulatives, enhanced the learning experience for high school algebra students. In an elementary setting, students generally work with their classmates to learn; whether it be working in stations, critiquing the reasoning of a peer, playing a game to learn a new concept, or working with beads to practice math facts. Dave Ebert (2017) reported, "stations allow students to work together in a small group to engage in challenging problems while also differentiating the tasks to meet the varied academic levels and interests of all students" (p.24). The phrase "all students" should mean students in all grades, not just students in Kindergarten through grade 5. Additional support comes from the National Council for Teachers of Mathematics where they report using peers as resources facilitates learning. It can "increase student learning, motivate students, support teachers in understanding and assessing student thinking and shift the mathematical authority from teacher (or textbook) to community" (p.1).

Working collaboratively, whether it be through stations, games or using manipulatives, has been the norm for how elementary teachers provide instruction to students. However, these strategies are rarely seen at a secondary level. The goal of this research was to see if the same positive results are yielded from students in a secondary mathematics classroom, specifically algebra 1.

This research benefitted teachers, students and school districts. Teachers gained knowledge of what potential new instructional strategies yielded the greatest gain of understanding from high school algebra students. Students benefited from being immersed in activities that allowed them to concretely understand the material taught to them, instead of skipping right to the abstract introduction of content. Lastly, school districts were able to use the research findings to support the implementation of elementary math strategies in their high school classrooms to support student understanding.

Research Question

This quasi-experimental research study was designed to examine how the implementation of various math methods improved student performance in high school algebra. Specifically, this study investigated the research question, how effective is the use of student centered approaches, during mathematics instruction, in improving students' conceptual understanding in a high school algebra class?

Students enrolled in algebra 1 traditionally receive instruction in an abstract way, by listening to teacher lecture and learning to follow a sequence of steps where they plug-and-chug numbers into formulas to get the desired outcome. As researcher Kaput (2010) found, United States classrooms today ask students to

> memorize procedures that they know only as operations on strings of symbols, solve artificial problems that bear no meaning to their lives, and are graded not on understanding of the mathematical concepts and reasoning involved, but on their ability to produce the right answers about which they have no reason to reflect and that they found (or as likely guessed) using strategies they have no need to articulate (p 2).

Through this research students were immersed into learning opportunities where they were able to visualize why formulas work.

Much of the data collected was qualitative in nature. Students were asked to express their opinion of the activities done in class. In addition, students were asked if they worked to correct the understanding of a peer, or if a peer helped them with their understanding of the topic. At the end of the teaching cycle for each concept, students were asked to complete a questionnaire to determine which student centered approach worked best for them. Students were encouraged to answer according to the method that helped their understanding, instead of the approach they had enjoyed the most. In addition, students were given a pre-test and a post-test to quantify their conceptual growth.

Central Concepts Related to the Investigation

Student-centered approaches, such as cooperative learning, improve mathematics achievement and attitudes towards mathematics among students (Chin, Daud and Zakaria, 2010). The United States Department of Education defines cooperative learning as a successful teaching strategy in which small teams, each with students of different levels of ability, use a variety of learning activities to improve their understanding of a subject. Dylan Wiliam (2013) noted peers can be very effective assessors of one another’s work, especially when the focus is on improvement. When students work collaboratively and use each other as resources they learn from each other and are able to see how others may approach a mathematical problem. In this approach the teacher no longer becomes the main facilitator of the knowledge, students can tap into using each other to learn. Often times students respond better to being taught by a fellow peer, and occasionally students will come up with their own way to solve a problem that was not even shown by the teacher.

High school algebra is abstract in nature and it comes at no surprise that students have a difficult time relating their learning to anything with meaning. Often times teachers are bombarded daily with questions from students like, “When will we ever use this?” If students can not see the reason for learning the content, and be able to attach their learning to prior knowledge or real world content, it makes it difficult for them to store the learning in their long term memory. Often times we find kids learn the material long enough to pass the test, but do not hold on to the learning. The results from high stakes exams support this claim.

Additionally students will sometimes hear rules in algebra that initiate misconceptions. One such rule, two positives makes a negative, is guilty of causing much angst with teachers and students. It is true that when multiplying or dividing integers, two negatives makes a positive, but when adding and subtracting two negatives the rules are different. When students lack the conceptual understanding of why rules are true, they are left confused and often times create a fixed mindset about these rules. Unfortunately, that mindset can follow students for a very long time. However, if students were taught to understand the rules by allowing them to work with number lines, positive and negative counters and algebra tiles they would be able to visualize the rule. This visualization would assist the student in going from the concrete stage (manipulatives) to the abstract stage (rules and formulas) with less angst.

The central concept related to this investigation is to enhance student understanding of abstract algebraic concepts through the use of manipulatives, games, using peers as resources and by allowing students to work in math stations. There is a lot of research supporting all of these instructional strategies, but most were administered with elementary aged students.

General Approach of the Investigation

In education, many conditions in the classroom require researchers to use intact groups. Creswell (2005) states, “this might happen because of the availability of the participants or because the setting prohibits forming artificial groups” (p.311). In this quasi-experimental study the researcher is unable to randomly assign groups, the groups of participants are created at the beginning of the school year due to class placement.

This quasi-experimental investigation took place over a period of six weeks, where the participants were taught by using hands-on mathematical strategies. The six weeks time frame was broken into two separate sections, 1st experimental group and 2nd experimental group. During the first three week timeframe, algebra period one and period five served as the experimental group, and the algebra period six served as the control group. Vice versa during the second three week period where the algebra period six served as the experimental group and the algebra period one and period five served as the control group.

During the six week period, the researcher was interested in students’ procedural knowledge and conceptual knowledge of algebraic concepts, as well as students’ perception of the effectiveness of hands on mathematical instructional strategies.

Methods of Inquiry

The researcher split three classes of algebra 1 into three groups: group A (period one=19 students), group B (period five=11 students) and group C (period six=7 students). During the first three weeks of the research, groups A and B served as the experimental group, meaning they received their algebra instruction using hands on approaches. These methods included, but were not limited to, cooperative learning, using peers as resources, stations, games, and tactile measures both virtually and kinesthetically. Meanwhile group C served as a control group during the same 3 weeks, where they received algebra instruction traditionally with lecture and worksheets. The next three weeks was a reverse of this process, where groups A and B served as the control group and group C was the experimental group. During the 6-week period, students' data was collected using an observation protocol, and student explanations of their learning was collected. Additionally students were asked to explain if the activity used in class helped them understand the algebraic concept. At the end of a 6-week period, all groups were statistically compared to determine if the hands on approaches provided deeper conceptual understanding of the algebra topics covered.

Creswell states, "in all experimental situations, you assess whether a treatment condition influences an outcome or dependent variable" (p. 302). The information gathered from the exit tickets and student surveys was used to assess whether or not the hands on strategies influenced conceptual understanding of the algebraic topic.

Research Methods

Setting

Research will be conducted at Central Aroostook Junior/Senior High School (CAHS) a part of MSAD 42, located in Mars Hill, Maine. CAHS consists of five small, rural towns, Mars Hill, Blaine, Robinson, E.Plantation and Bridgewater. The National Center for Education Statistics reports CAHS has 205 students in grades 7-12, with a student to teacher ratio of 11:1. The school is reported as having an enrollment of 97% Caucasian students. 53% of the students are male, and 47% are female. CAHS has a free and reduced lunch eligibility rate of 65%, which is higher than the state average of 47.1%. The United States Census Bureau reports citizens of Mars Hill as having a median household income of 38,231.

CAHS is an appropriate site since the researcher is employed as a high school mathematics teacher, and has been granted permission by both the building administrator and the district superintendent.

Participants

The participants consisted of ninth and tenth grade algebra students at Central Aroostook High School. Thirty Freshman and Sophomore algebra students were offered the opportunity to participate in the study, and were recruited as part of a convenience sample. Parent letters were sent home with students to seek consent for student involvement. A phone burst message was sent out through a program called Remind which notified parents to look for the consent for student participation. This text message was merely to notify parents of the form and was not meant to be coercive. Remind is a text-messaging service that parents of students in the Algebra classes were already familiar with. Since minors between the ages of 8-17 will be part of this study, parent consent was needed. In addition, students were asked to give written assent to be part of this study. All 30 students brought back the consent forms and were able to complete the study from start to finish.

Methodology

For this quasi-experimental study, the researcher collected primarily qualitative data. All research phases have been endured an IRB process through the University of Maine at Farmington to insure ethical procedures are upheld. During the experimental phase of the study, the researcher sent a Google form to each student daily. The form served as an exit ticket. The students were asked to identify the task that was used in class from the list below.

- Games
- Math stations
- Tactile manipulatives
- Peers as resources

The students were asked to explain if they felt the activity from class helped them understand the algebraic concept for the day. Then the students were asked to indicate whether or not they helped another student learn the concept or if a student helped them. These forms were looked at by the researcher daily and the student responses were categorized and stored in a Google spreadsheet on the researcher's school laptop. The laptop was password protected and the researcher was the only one with the password to ensure that the information remained confidential.

Quantitative data was collected in the form of a pre- and post test on algebraic concepts. All students, whether in the control or experimental group, were asked to complete a pre-test in order for the researcher to establish a baseline of conceptual understanding of the algebraic concepts. The same test was issued at the middle and end of the study to determine if conceptual gains had been made.

By collecting both qualitative and quantitative data the design of the research was strengthened. Creswell states, "the strength of this design is that it combines the advantages of each form of data; that is, quantitative data provides for generalizability, whereas qualitative data offer information about the context or setting" (p. 544).

Operational Measures

This research study began and ended with an assessment about the algebraic concept linear functions. The student learning goals were to (1) understand the functional relationships between two linear functions with the same representation and (2) understand the functional relationship between two quantities in a graph. Prior to the introduction of the learning goals, students were assumed to have little to no understanding of linear functions. However, these learning goals built upon students prior knowledge of plotting points from an input/output table into the coordinate system. This assessment allowed the researcher to collect quantitative data on procedural understanding of the algebraic concept of linear functions.

Qualitative data was collected by asking the students to articulate if the student centered approaches helped them learn about linear functions. Additionally, the students were asked to indicate if they used their peers as resources or if they assisted a peer in learning. This qualitative data was analyzed for commonalities among the student responses and informed the researcher of student perception of the activity.

Lastly, students were asked questions, called probes, electronically through Google docs. The probes typically included a prompt or question and a series of responses designed specifically to elicit prior understandings and commonly held misunderstandings, about linear functions, that may or may not have been uncovered during the instructional unit. Students were asked to choose from a selection of responses as well as write about how they determined their answer choice. These probes may come from Cheryl Tobey's book, *Uncovering Student Thinking About Mathematics in the Common Core*, or the probes were created by myself, the principal investigator.

Data Collection

Data was collected from the pre- and post test to determine procedural understanding of linear equations. In addition, diagnostic probes were collected from the experimental group to determine conceptual understanding of linear equations. Lastly, daily exit tickets were collected from the experimental group to gather students' perception of effectiveness from the hands-on math strategy.

The timeline of the study was as follows:

1. Obtained consent from the parents of the students in Algebra I.
2. Obtained written assent from students in Algebra I to participate in the research project.
3. Gave pretest to students in both groups: control and experimental.
4. Analyzed student responses from the pretest and recorded students' procedural knowledge on a Google spreadsheet.
5. Administered manipulatives and student centered strategies to the first experimental group and electronically collected daily exit tickets from students. Simultaneously

traditional teaching through the use of lecture and worksheets was provided to the control group. Recorded student responses on Google spreadsheet.

6. Gave students in the experimental group the diagnostic probe to test for conceptual understanding of linear functions. Recorded student responses on Google spreadsheet.
7. After three weeks, completed steps 5 and 6 with the second experimental group.
8. Once the six weeks were up, and the learning target timeline had been satisfied, the researcher proctored the post test to all students in the study.
9. Analyzed student responses from the post test and recorded students' procedural knowledge on a Google spreadsheet. Additionally, students' rate of growth in procedural knowledge was analyzed and documented.
10. Cross-referenced data from the experimental group and the control group. Documented significant differences.

Data Analysis

After collecting all of the data from both the control group and the experimental group, the information was analyzed by the primary researcher. The information from the pre- and post tests were recorded as an average or the mean growth, and the number of correct student responses was recorded. Both the experimental and control group took pre- and post tests at the end of each three week cycle. This information was analyzed to determine if students in the control group made greater gains from the pre-test, or if the students in the experimental group made greater gains from the pre-test. In addition, the information collected was cross-referenced with the qualitative survey to determine if there was a relationship between student centered approaches and concept attainment. For example, if student A4 scored a 2 on the beginning pre-test, an 8 on the three-week cycle test, and an 18 on the final post test, this student made 16 points of growth. This information was then cross-referenced, with the student centered approach the student felt worked best for their understanding, to determine if a correlation exists.

The Google survey information from the students were organized and coded. Creswell (2005) states, "preparing and organizing data for analysis in quantitative research consists of scoring the data and creating a codebook" (p. 173). To ensure meaningful labels, codes are assigned to chunks of data, usually phrases, sentences, or paragraphs that are connected to a specific context. Students were assigned a code; period one algebra students were A1-A19, period five students were B1-B11, and period seven students were C1-C7. These codes allowed the researcher to examine data while respecting student anonymity. The survey information pieces were also assigned a code; Stations (S), Games (G), Manipulatives (M), Using Peers as Resources (P), and Whole-Group (W). These codes were used daily when students were identifying which student centered approach they felt helped them learn the best. The teacher observation tool was coded as well; Student on Task (OT), Student off Task (OF), Student-to-student interactions academic (IA), Student-to-Student Interactions Non-Academic (INA), Students Reluctant to Join Activity (R), and Students Eager to Join Activity (E). These codes assisted the research when creating Google spreadsheets to store the many data points.

Expected Findings
(This section was written prior to the research phase)

Since I had taught mathematics to students in grades 2, 5 and 6 for the last ten years, I expected the findings to show higher achievement for the experimental group indicating a positive correlation between manipulatives in the classroom and student understanding of abstract mathematical concepts. In my experience, when students are able to work with manipulatives or work collaboratively through games or stations, they are able to visualize the mathematics and deepen their understanding of why the mathematics works.

Additionally, the pre- and post tests are expected to show procedural growth of all students whether they are in the control group or the experimental group. Despite whether or not the students have been exposed to student centered strategies or the traditional lecture and worksheet method, students will make growth. All students will be able to show understanding of the algebraic concepts, however I expect to find that the conceptual understanding of all students will increase.

Potential Issues and Weaknesses

Creswell states, “the quasi-experimental approach introduces considerably more threats to internal validity than the true experiment” (p. 311). When using this type of approach the researcher works with groups already created which can cause threats of maturation, selection and mortality. Maturation meaning the subjects can become bored with the research or act differently from day to day, selection meaning the researcher creates the groups instead of using random sampling, and mortality meaning subjects could drop out of the study leaving one of the groups, either control or experimental, impacted.

Another significant worry for the researcher was student participation. The sample size was significantly small, 37 students. One fear of the researcher prior to the study was that multiple students would opt out of the research phase, and the participant level would go so low it would have skewed the results of the study. However, all 37 students did participate in the study from start to finish.

Students in the high school setting were not used to working in collaborative groups, working with manipulatives or taking educational risks in front of their peers. Due to this, I expected to experience reluctance from the students. Most high school students were not willing to make mistakes in their learning in front of their peers for fear of ridicule or embarrassment. This culture change did inhibit the collection of accurate results. The combination of all of these factors could skew the results and prevent the generalization of the research to other populations.

Results

The purpose of this small-scale quasi-experimental study was to explore high school algebra students' use of learning stations, hands on manipulatives, small group collaboration and mathematical games, to determine if their conceptual understanding of linear equations increased in this setting versus the traditional teacher lecture with worksheets model. Specifically, this study sought to answer the following question:

- How effective is the use of student centered approaches, during mathematics instruction, in improving students' conceptual understanding in a high school algebra class?

Findings from the research surveys revealed students prefered to work independently while being led by the teacher, yet this approach yielded the least amount of conceptual improvement. Pre- and post tests showed students from the experimental groups out-performed peers in the control group on achievement of standards during each cycle of the experiment, with small group collaboration being the student centered approach yielding the highest mean.

Student Perceptions

Throughout each three-week cycle, the students were asked to indicate which of the four strategies they felt helped them best learn the algebraic concepts of the day. The survey was designed using a Likert scale, where a rating of one meant the student felt the approach was the least effective, and a rating of five meant the strategy was the most effective. Surveys were compiled at the end of each of the three-week experimental phases. The data was analyzed to determine what trends existed in regards to students' perceptions. Overall, both experimental groups of students voted the traditional method of sit-and-get (teacher led lecture with student worksheets) was the most effective approach that aided in their ability to learn about linear equations (Table 1).

Table 1:
Perceived Benefits of Student Centered Approaches on Student Learning of Algebra

	Sum	Mean	SD	Variance
Stations	66	2.36	1.09	1.20
Games	83	2.96	1.53	2.25
Manipulatives	72	2.57	1.03	1.07
Using Peers	90	3.33	1.44	2
Whole Group Facilitated by Teacher	102	3.64	1.49	2.24

Note. Participants (n=37) voted on a scale from 1-5 to indicate which strategy worked best for them. On the scale 1=least effective and 5=most effective. Five factors were given: working in mathematical stations around the room with very little guidance from the teacher, learning mathematical concepts through the use of games, using manipulatives to concretely visualize the concept, using peers as resources by talking through concepts, and sitting at desks working through algebra problems facilitated by the teacher.

Students were asked to explain their reasoning in regards to what student centered approach they chose. The survey question read, "I see you chose [insert student centered approach] as the strategy that works best for your learning. Can you please tell me why?" A sampling of the student responses, depending on the approach they enjoyed most are listed throughout the teachers' observations section below.

Overall, the students articulated their feelings in a manner that corresponded with the teacher's observations. Some students still prefer to sit and be fed the information from the teacher while working on a worksheet. However, throughout the research phase a sizeable shift was noted in regards to students' perceptions. Most students agreed that working with worksheets was not the most beneficial strategy to learn new content. Instead they began to see the effectiveness of the other approaches.

Teacher Observations

Observations of student engagement were recorded each day by the primary researcher. Three different students were observed each day during the six-week experimental phase and their behaviors were recorded (Appendix D). Forty-five students were observed during the first experimental phase and 45 more students were observed during the second experimental phase (n=90). Behaviors that were rated:

1. Were students on task or off task?
2. Were student to student interactions academic related or not?
3. Were students reluctant or eager to enter into the activity of the day?

Students were given zero points for being off task, having non-academic interactions with their classmates, and being reluctant to participate in the activity. Likewise, students were given one point for being on task, having academic interactions with fellow students, and being eager to participate in the activity.

The data collected from the observations was compiled to determine which student centered approach yielded the highest engagement rate from the students. The information is portrayed in Figure 1 below.

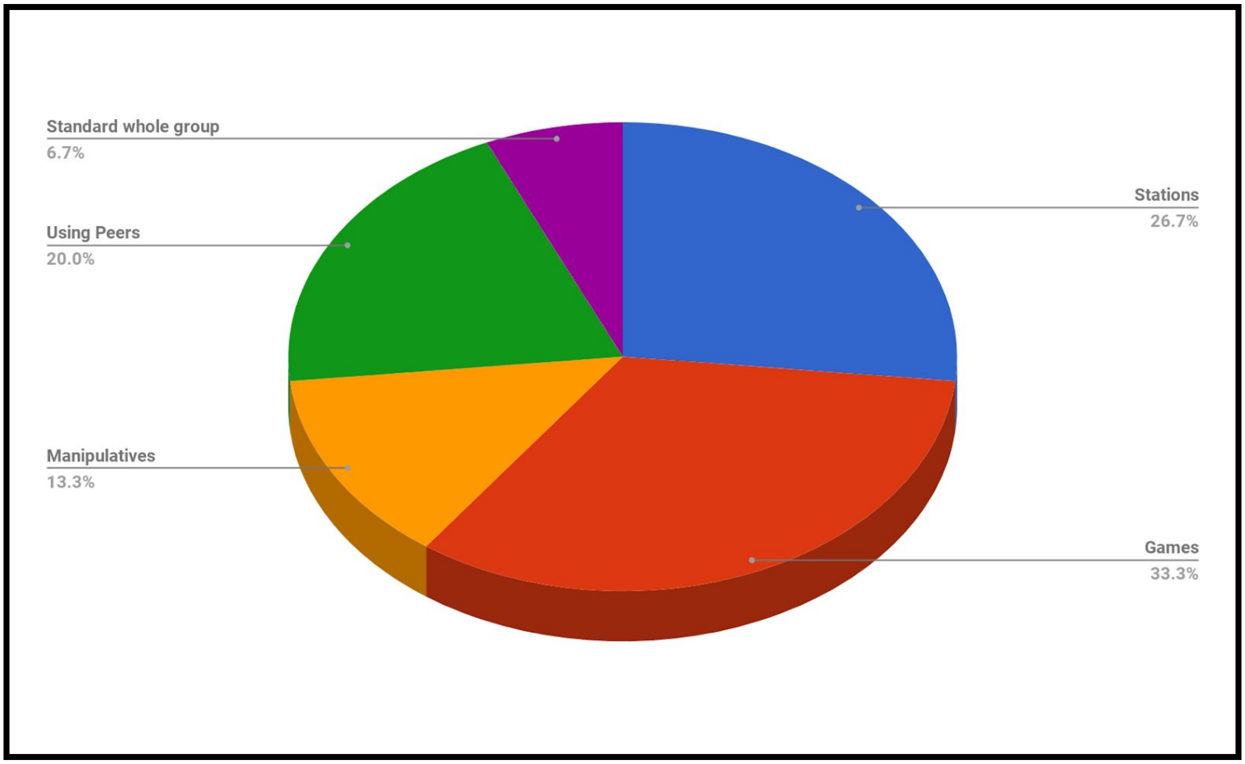


Figure 1. Observed Student Engagement During Experimental Phases

Algebraic Games

The student centered approach that yielded the highest mean for engagement was playing algebraic games at 33.3% (Figure 1). One of the games students enjoyed was a type of relay race, where students were able to move around the room quickly, answering questions. This relay race generated the highest rate of engagement from the students as they were rearranging linear equations from standard form into slope-intercept form in a timed group activity. Once a student solved their question, the next student in the group solved their question until all questions were solved and all students were cycled through. The first group to finish answering all of the questions, from the race checkpoints, was the group to win. This activity required all students in the group to participate, therefore the students tended to be more motivated to increase their groups' chances of winning.

When students were asked what student centered approach helped them learn the best, those who answered games said,

- "I think all strategies work, but I like games because it is fun."
- "The relay race was a blast!"
- "When I play games I get excited and it doesn't feel like learning."
- "Because it is fun and it makes most kids want to work harder."
- "I enjoy getting out of our seats and moving around in a fast paced game. Most of my classes make us sit and take notes."

These statements listed above are a sampling of the student responses, not all feedback is listed.

Stations

The second student centered approach generating the highest engagement was working in stations, where students' engaged behaviors were recorded 26.7% of the time (Figure 1). An example of station activities administered during the experimental phase was in regards to the concept slope. The students cycled through three designated areas named (1) slope name art, (2) find your slope match and (3) create a ramp. (These are simply samplings of what was offered to the students, this list is not comprehensive. For a full list of activities, please refer to Appendix F.) At the slope name art stations, students were instructed to write their name, then color the parts of the letters in their name according to what type of slope their letter had; positive, negative, no slope or undefined. At find your slope match, students were instructed to find a partner in the room who had the same slope as their linear equation. Each person was provided an index card where they needed to solve their linear equation to find their slope. Ideally, each student would find someone in the room who matched them. At create a ramp, the students used cardboard and masking tape in an attempt to create a ramp with a slope of their choosing. The students then sent a matchbox car down their ramp and recorded the time it took the car to go from the top of their ramp to the bottom. We then calculated the slope of their ramp and tried to make connections between the time it took the matchbox car to go down the ramp, to the slope. As the students cycled through the stations the researcher investigated the students behavior. Students were slightly scaffolded by the teacher when the students became frustrated with any of the activities, but the majority of the time the students were able to communicate within their groups to figure out what to do at the station. These station activities encouraged students to learn by following directions at the station, which were previously made by the teacher. Students weren't used to working in this manner which could be one of the reasons for the low engagement. Noted student behaviors were talking to a friend, saying, "I don't get it!", and refusal to work.

When students were asked what student centered approach helped them learn the best, those who answered stations said,

- "Stations helped me learn because I have a hard time sitting still. I learn best by moving."
- "I liked some of the station activities, they were fun."
- "Moving around the room was fun and fast, but sometimes it was difficult to settle down."

These statements listed above are a sampling of the student responses, not all feedback is listed.

Peers as resources

The third student centered approach yielding the highest engagement was students using their peers as resources. When students were allowed to communicate with their peers in a small group setting, their behaviors were engaged 20% of the time (Figure 1). Although students enjoyed working with their friends, they had a tendency to become off task if the teacher was not at their group. To remedy this problem, it might be suggested to set up classroom expectations in the beginning of the school year to promote appropriate behavior and alleviate student misconceptions about what the teacher may want from them. An example of an activity where students used their peers as resources was when students were taught a concept, they were then allowed to turn and talk to a friend, or buddy up to talk about their problem solving. In this type of learning environment, students had a tendency to ask questions to a peer that they felt would not invoke ridicule or make them feel silly for not knowing how to solve a problem. According the data analyzed by the researcher, students made the most conceptual gains when they worked with their peers (Figure 5).

When students were asked what student centered approach helped them learn the best, those who answered using peers as resources said,

- "It is easier for me to ask my classmates questions without being embarrassed by the whole group."
- "My peers are able to explain how to work through a problem in a way I can understand."
- "My friends explain it easier to me."
- "It works the best for me, because the teacher is only one person and for a large classroom there's more people available who might know the content but at the same time it's bad because it slows down others."
- "I like it the most because I like talking to classmates about the work and them telling me I work best from them."
- "I picked this [strategy] because if I forget what I'm doing I can ask the people around me for help or ask them if I'm doing it right."
- "This [strategy] works for me because I feel that I can learn from their mistakes and we get to fix them together."

The statements listed above are a sampling of the student responses, not all feedback is listed.

Working with manipulatives

The fourth student centered approach was working with manipulatives in the classroom, which yielded a 13.3% engagement rate among students (Figure 1). Students were introduced to algebra tiles to aid in their understanding of balancing linear equations. The tiles were made up of small, red and green squares, and rectangles. The number one was represented by the small square and the rectangle represented the variable x. Students were able to take an equation like 2x + 4 = 8, represent the 2x with two rectangles, the four with four small, green squares and the eight with eight small, green squares. The students were then instructed to eliminate the four on the left-hand side of the equation by subtracting four squares (red ones). The teacher reminded them that when they placed four red squares on the left they would need to put four red squares on the right to keep the equation balanced. Since the red squares represent negative numbers and the green squares represent positive numbers, the new equation would read 2x = 4. Lastly, the students were instructed to equally share the square tiles with the rectangle tiles. Since there were two rectangle tiles, the equal share would be two green square tiles for each rectangle, meaning x = 2. Students found this method to be confusing and they became easily frustrated with the process. It took several tries with this method before students began to see the benefits of this approach.

When students were asked what student centered approach helped them learn the best, those who answered working with manipulatives said,

- "Working with algebra tiles was very confusing at first, but now I love them!"
- "Using Desmos to graph equations was a lot of fun. It made it realistic"

The statements listed above are a sampling of the student responses, not all feedback is listed.

Teacher led instruction and worksheets

Lastly, students were engaged 6.7% of the time while working on worksheets during teacher led instruction (Figure 1). Although students preferred this method to obtain knowledge, the research findings do not support this type of instruction. Students were found to be inattentive, playing with gadgets in their desks, doodling on papers, and avoiding work altogether. It is the opinion of the researcher that students prefer this method of instruction because it enables them to appear to be busy, but allows them to be off task because the teacher is not up walking around the room to make sure the students are working. At a 6.7% engagement rate, three students were recorded being on task, and eager to participate at all times within the 42 minute algebra class.

When students were asked what student centered approach helped them learn the best, those who answered teacher-led worksheets said,

- "Working through teacher led instruction is much easier because I know what I'm doing is right, making it much simpler."
- "I don't like human contact or just people in general and this helps so I can see the steps as they go by."
- "I like working on worksheets because the classroom stays quiet."
- "Because the teacher goes through the steps and does them with us. If we have questions the teacher is there to demonstrate what we should be doing."
- "Because if you are doing it right you don't have to go at the same pace as other students. If you need help you can ask her."
- "I chose this [strategy] because I like following along with the teacher, you can ask for help, and you can see if you made a mistake."

The statements listed above are a sampling of the student responses, not all feedback is listed.

Overall, the teacher (the primary researcher) observed a total of 90 students within the six-week experimental period. However, the sampling of three students per day may have presented data that was not indicative of the whole group.

Student Achievement

Prior to the beginning of the experimental phase, students were administered a twenty question pre-test to determine if they had any prior knowledge of linear equations. Out of the 37 students who participated in the study, over half of the students did not demonstrate prior knowledge (Figure 2), scoring zero points out of 20. Only 2.7% of the students were able to answer four questions, 5.4% were able to answer three questions, 21.6% answered two questions and 18.9% answered one question. The researcher had expected the pre-test scores to be a little higher as students should have been able to demonstrate knowledge of how to calculate slope, when given a graph, considering most students had been marked proficient on this skill in 8th grade.

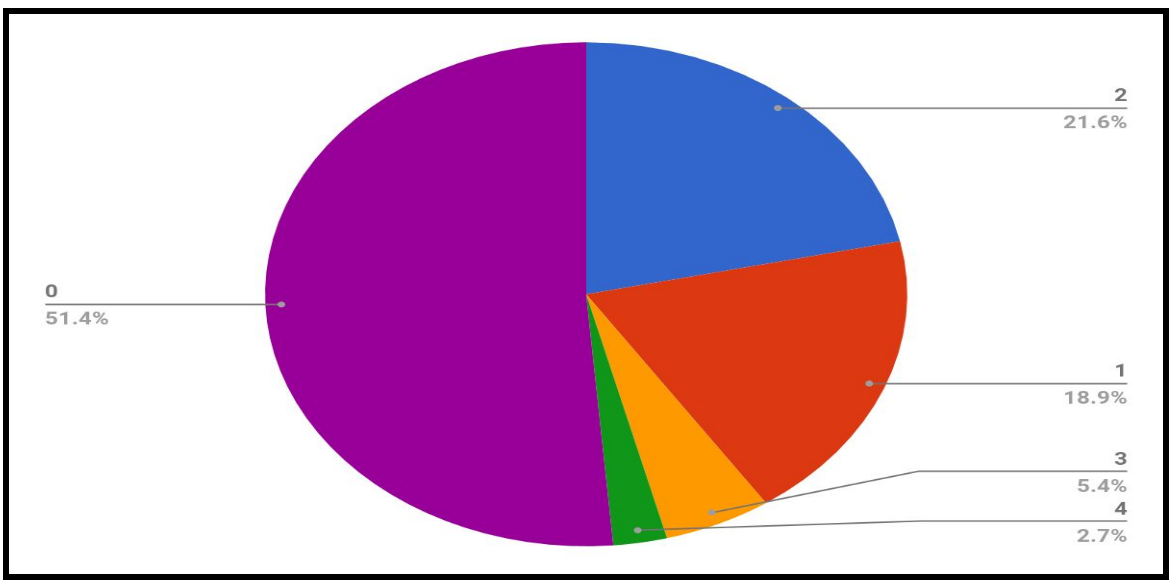
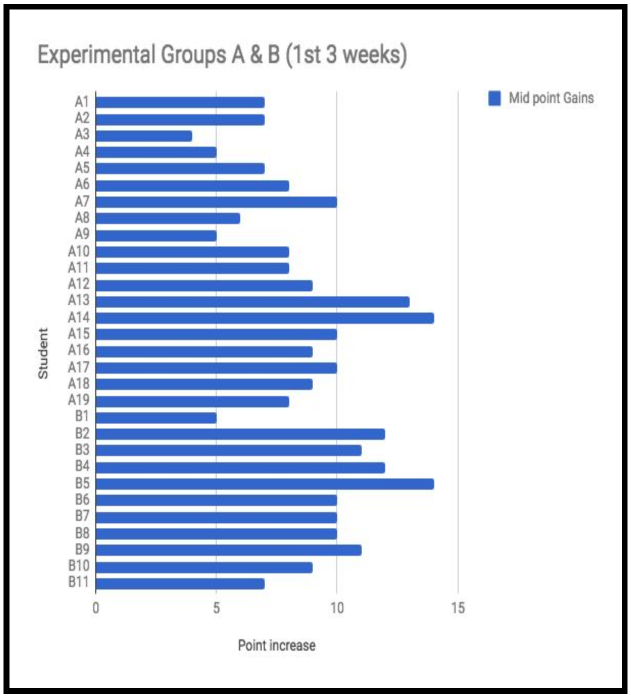


Figure 2. Student Pre-Test Knowledge of Linear Equations out of Twenty Questions

After the pre-test had been administered, the researcher incorporated student centered approaches in the classroom for the first experimental group (n=30). The next 15 learning days were dedicated to an equal amount of students collaborating in small groups with their peers (3 days), students working with tactile and virtual manipulatives to demonstrate balancing linear equations (3 days), students playing algebraic games (3 days), students working through stations in the classroom (3 days), and lastly students being instructed by the teacher while doing a worksheet (3 days). All students had a difficult time transitioning into this style of learning. It was not until half-way through the experimental phase that students began to see the impact the student centered approaches had on their learning. One students was quoted, “I prefer to use my peers as resources because, I feel my friends explain it easily to me and I don’t have to worry about feeling stupid in front of the whole class”. Another was quoted, “Games are fun and it makes me more motivated to learn”.

At the end of the first experimental phase, both the control group (n=7) and the experimental group (n=30) retook the twenty question pre-test to determine if any growth had been made by either group. (Figure 3) portrays the average growth for all students in both the experimental phase and the control phase. The experimental phase had a mean growth of 8.79 points, and the control group had a mean growth of 5.57 points. Meaning all students made growth from the first day of the experiment, despite which group they were in, but the experimental group made more growth than the control group. This information supports the researchers prediction that students would have greater conceptual gains when presented with algebraic information through student centered approaches rather than being led by the teacher through straight lecture and student worksheets. Although the lecture model does produce growth, the growth of the student centered approaches was approximately three points higher in all instances.

Mean Growth 8.79	Mean Growth 5.57



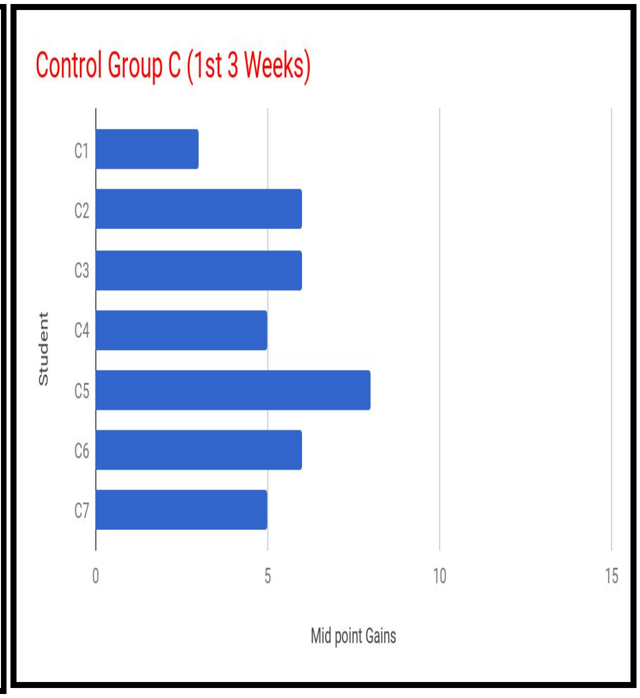


Figure 3. First Experimental Round Data

Upon completion of the first experimental round, the second experimental group (n=7) proceeded with the same student centered approaches in the classroom while the first experimental group became the control group (n=30). Now three weeks deep into the learning goal of linear equations, the experimental group transitioned into the student centered approaches with learning about how to graph linear equations. Some of the activities for this round included graphing with yarn around the room, where the room became the coordinate plane and the students became the coordinate points; linear equation scavenger hunts; rearranging equations from point-slope form and standard form into slope-intercept form; and linear equation BINGO.

(These are simply samplings of what was offered to the students, this list is not comprehensive. For a full list of activities, please refer to Appendix F.) The information collection process for experimental group two was the same as the process for group one. (Figure 4) portrays the average growth for all students in both the experimental phase and the control phase. The experimental phase had a mean growth of 7.43 points, and the control group had a mean growth of 5.37 points. Experimental group two gained on average one point less than the experimental group one, but both times the experimental groups outperformed the control groups by an average of 2.64 points. This outperformance confirmed the researcher's projection that students would make higher conceptual gains when presented new material through student centered approaches.

Mean Growth = 5.37	**Mean Growth = 7.43**

Figure 4. Second Experimental Round Data

Lastly, the researcher analyzed the pre-test with the post-test data to look for trends between student growth and the student centered approach that the students felt helped them learn the best. Students were coded A1-A20, B1-B10 and C1-C7. The students' rate of growth from the pre-test to the post-test was cross-referenced with the student centered approach they felt helped them learn the best. This analysis was done to determine how their rate of growth correlated with the student centered approach they chose, and to determine which student centered approach yielded the highest gains from the pre-test to the post-test. (Figure 5) depicts the information found by the researcher. Students who chose using their peers as resources in a small group setting increased their pre-test score by an average of 15.4 points. Likewise, students who chose working with manipulatives increased their score by 14.5 points, playing algebraic games increased by 14.3 points, working at stations increased by 13.2 points, and teacher lecture increased by 11.7 points. All students, despite the student centered approach they chose, made gains from the pre-test to the post-test.

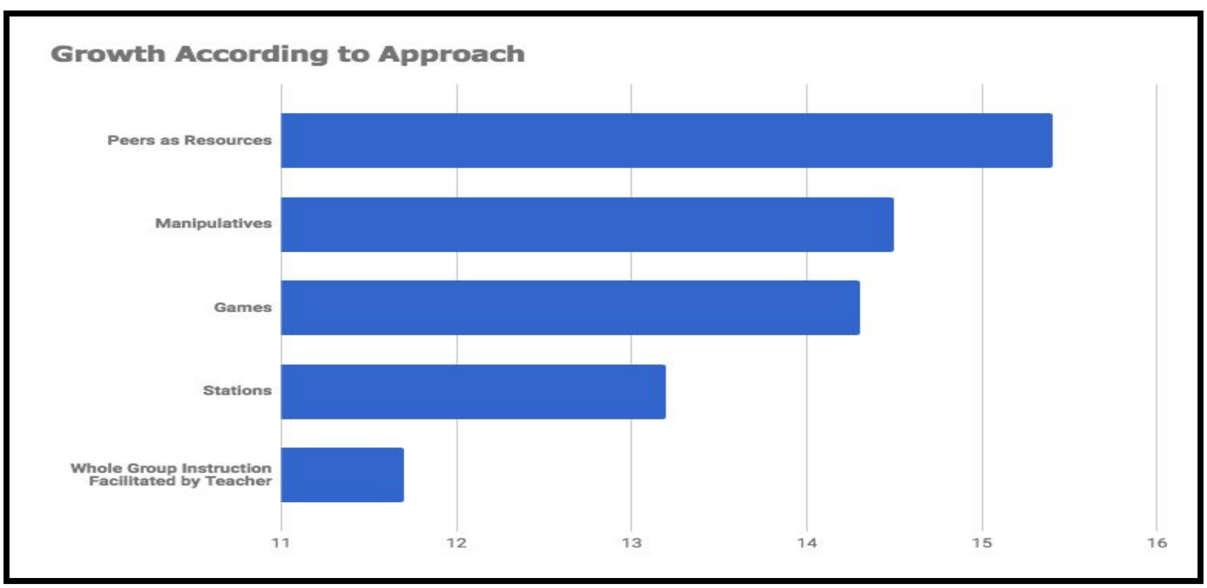


Figure 5. Pre- and Post-Test Growth During Experimental Phases

In conclusion, all student centered approaches yielded conceptual gains from students in regards to the learning goal, linear equations. As you can tell from the research findings, some approaches yielded higher conceptual gains than others, but one must take into consideration the engagement factor as well. When you cross-reference conceptual gains with student engagement, a conclusion could be drawn that games might be the most important approach in order to increase students' understanding. This could be because games had the highest engagement rate of 33.3% and games on average increased students' scores by 14.3 points from the pre-test to the post-test. Considering it would be near impossible to play algebraic games in the classroom each day it is recommended by the researcher to incorporate a healthy ratio of all student centered approaches, learning stations; hands on manipulatives; small group collaboration; and mathematical games, to enhance the students' learning experience and increase conceptual gains.

Discussion

This quasi-experimental study confirms the finding from Chin, Daud and Zakaria (2010) where they noted student-centered approaches, such as cooperative learning, improve mathematics achievement and attitudes towards mathematics among students. Engagement and aptitude did increase throughout the entire study for all students, despite the student centered approach they chose as the one that best suited their academic needs. However, it is important to note the challenges and successes of the research.

Challenges

Students had a difficult time transitioning into the student centered approaches I had prepared for them. They were so used to robotically following the algorithmic steps I put in front of them, that they did not enjoy having to think, collaborate and solve problems on their own. We often hear of the expression "students are being spoon fed information" and that rang very true throughout my research process. Students were extremely content with sitting in their seats and following along with me to complete problems on their worksheet. This process felt very safe for them. They were able to hide their misunderstandings with the material, pretend that they knew what they were doing, and protect their image among their peers. However, this process has been repeatedly shown to be the least ineffective way to learn mathematics (Chin, Daud & Zakaria, 2010).

Aside from student motivation, another challenge that was encountered was in regards to working with stations. By design, station activities should be something students would be able to do with little to no guidance from the teacher. However, this did not prove to be true with this study as students struggled with being independent. I speculate this may be due to lack of classroom expectations being established prior to the activity. Students really needed to know what was expected of them during this activity, and they needed to be taught how to persevere through difficult situations. Instead, students were found talking, and off task, during the station activities. Multiple questions arose and the teacher was caught having to give explicit instructions to each group before they could proceed with the activity. This took up too much time and was very frustrating to everyone involved. I recommend incorporating classroom behavior expectations prior to implementing station activities in a high school algebra classroom.

However, as researchers Chin, Daud and Zakaria (2010) reported stations encourage collaborative learning which “provides children with the opportunity to share ideas, experiences and backgrounds with one another while they study various topics in the standard curriculum. When children work together to discover, create, solve problems, observe, and record data, they also learn how to communicate and cooperate with each other” (p.272). Therefore, it is in my opinion the advantages far outweigh the disadvantages.

Successes

A prior research study completed by Ebert (2017) reported, “talking and working together clearly has favorable effects on learning, especially conceptual learning” (p.171). This resonated throughout my whole research study as well. Students became owners of their own thinking. They began to converse with their peers on a different level, one which focused on the why of mathematics. Students began to see the importance of talking through their learning instead of begin spoon fed the material. Near the completion of the study, one student was quoted, “Do we need to go back to just doing worksheets? They are so boring!” It was then I knew the research project was a success, both for me and for them.

The data collected from the study indicates students were most engaged while working with games. Milner (2014) reported, “mathematical/logical puzzles and games appeal to a wide range of people of all ages, backgrounds and perceived strengths” (p. 21). This was true for my study as well, where I saw students at all academic levels work to solve problems. Many came up with their own method to solve, instead of following the scripted algorithmic rules. This was powerful for me as a teacher to watch students who do not normally experience success feel like an equitable partner in the math classroom.

Implications for Practice

The implications from this study suggest student centered approaches be a part of every high school mathematics classroom. Researcher Golafshani (2013) reported "mathematics reformers and educators should use a variety of teaching strategies that will lead to effective teaching and learning" (p. 137). In addition, others also believe that effective teaching includes teaching for understanding, teaching better mathematics (Friesen, 2005), and reversing mathematics misconceptions (Green, Flowers, & Piel, 2008). This philosophy suggests that the teacher should not be a transmitter of knowledge, but should instead act as a facilitator to the construction of knowledge for all learners. Using the approaches outlined in this study the teacher can achieve just that.

School leaders should ensure their teachers have adequate planning time to implement these strategies. Materials will need to be made for the games and stations, forethought will need to be put into how to effectively use small groups to encourage good dialogue for the students, and manipulatives will either need to purchased or virtual sites will need to be researched. These methods are not something a teacher can just decide to implement on a whim, it will take time to create the resources, but the benefits far outweigh the hindrances.

Future Research

From this study, future research would need to be done to determine which student centered approach, in isolation, produces the most student growth. It may be beneficial to spend several weeks on each strategy, rather than incorporating all of the strategies within a three week time frame, much like this research project did. Since this study was only six weeks in length, it may be valuable to compare a similar, long-term study's findings to see if students continue to prevail when immersed in student centered approaches over a school year. In addition, research may need to be done to find ways to offset the cost of these approaches, considering many of the approaches need materials to be created and/or purchased.

Conclusion

Of the 37 students who participated in this study, 100% of the students made conceptual gains in regards to linear equations. Students who chose using their peers as resources in a small group setting increased their pre-test score by 77%, students who chose working with manipulatives increased by 73%, playing algebraic games increased by 72%, working at stations increased by 66%, and teacher lecture increased by 59%. These results clearly depict the advantages of incorporating these strategies into high school mathematics classrooms.

Change is not something to be obtained overnight; it takes time to cultivate and nurture. High school algebra classrooms will not be able to fix the many deficits our students have, but with the incorporation of a few of these student centered approaches, I believe we will change the way students think and feel about mathematics. It is time to show our students that mathematics is not a simple set of procedures and rules to be followed; it is an art meant to be interpreted.

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Appendix A: ADMINISTRATOR CONSENT

Dear Dr. Kay York/Elaine Boulier,

I am writing to request your consent to conduct research that will take place between January and April of 2018. Findings from the research will be presented to my peers at an open symposium in April of 2018. In my research I will be exploring the relationship between elementary math strategies and conceptual understanding of high school algebra.

Using a quasi-experimental design, students will be placed in three groups: Group A (18 students), Group B (11 students) and Group C (7 students). During the first three weeks of my research groups A and B will serve as the experimental group, meaning they will be receiving their algebra instruction using elementary math methods. These methods may include, but are not limited to, cooperative learning, using peers as resources, stations, games, and tactile measures both virtually and kinesthetically. Meanwhile group C will serve as a control group during the same 3 weeks, where they will receive algebra instruction traditionally with lecture and worksheets. The next three weeks will be a reverse of this process, where groups A and B will serve as the control group and group C will be the experimental group. At the end of a 6-week period, all groups will be statistically compared to determine which elementary math method provided the deepest conceptual understanding of the algebra topics covered.

Parents will be asked to sign a consent form for their child to participate in the research study. Written assent from students will also be collected. Student participation is voluntary and participants can opt out of the study at any time.

I will not share identifiable data about specific students or parents involved in the study. If you have any questions about the research, I am the principal investigator to contact at jennifer.mahan@maine.edu or at 207-551-8042. You may also reach the faculty advisor on this study, Dr. Johanna Prince, at (207) 778-7066 or johanna.prince@maine.edu.

Thank you for considering my request to conduct research,
Jennifer Mahan

By signing below I acknowledge I have reviewed Jennifer Mahan's research plan for, "Elementary Math Strategies and Conceptual Understanding in a High School Algebra Class". I give my consent to conduct this research at the beginning of the 2018 year. I am aware that I may also be asked to view the report at the end of the study.

________ ____________________________ ____________________________

Date Name Position in District/Site

Appendix B: PARENTAL INFORMED CONSENT FORM

Dear Parents,
Your child is invited to participate in a research project being conducted by Jennifer Mahan. I am your child's Algebra I teacher and I am also a student at the University of Maine at Farmington. I am currently studying educational leadership and as part of my studies I will be conducting a research study as my final project. I will be researching how elementary math strategies, hands on approaches, impact conceptual understanding of high school algebra. Through that research I will be incorporating cooperative learning, math stations, tactile and visual manipulatives, math games and training students to use each other as peer resources.

What Will Your Child Be Asked to Do?

If you consent for your child to participate, your child will

- Participate in math instruction using hands on materials.
- Be observed by the teacher who will chart student behavior in the activities.
- Be asked to explain their thinking about concepts in addition to answering algebraically.
- Be asked to fill out google surveys about their learning experience with the hands on approaches.

Risks

There are no known risks.

Benefits: Your child may learn more about algebraic concepts in a deeper way. Additionally this study may help future students at school and in other classrooms, as I hope to learn more about the implementation of elementary math strategies in high school mathematics.

Confidentiality: The information collected from this study will be utilized by me for my research purposes. Your child's name or other identifying information will not be reported in any publications.

Voluntary: Participation is voluntary. If you choose to have your child take part in this study, s/he may stop at any time. Whether or not your child participates will not impact your child's relationship with the school, his classroom teacher or any other teachers, or the instruction he/she will receive in the algebra classroom. Your child may skip any questions he does not wish to answer or withdraw from the study. If your child wishes to withdraw from the study, he/she will continue to receive instruction just like everyone in the study, except his/her information won't be included in the research data.

Contact Information: If you have any questions about this study, please contact me, Jennifer Mahan, at jmahan@sad42.us or 207-425-2811. You may also reach the faculty advisor, Johanna Prince on this study at johanna.prince@maine.edu or at 207-778-7066. You may also contact the Chair of the IRB, Karol Maybury at karol.maybury@maine.edu or at 207-778-7067.

Your signature below indicates that you have read and understand the above information. You will receive a copy of this form.

__________________________________ _______________

Signature Date

____ Yes, I agree to allow my child to participate in the research.

____ No, I do not give permission for my child to participate in the research.

Appendix C: WRITTEN ASSENT CONSENT SCRIPT
Children 8-17

Hello,

My name is Jennifer Mahan, and I am a student at the University of Maine at Farmington. As part of my master's program I am asked to conduct a research project to collect information about a topic that is meaningful to me. I have decided to conduct a research study on the use of elementary math strategies in a high school algebra classroom.

If you agree to be in our study, we are going to work together in groups in the algebra classroom. We will be working with tactile manipulatives and virtual manipulatives to increase our understanding of algebra. We will also be working collaboratively in groups, play games to enrich our understanding, and learn to talk about our learning. For example, I might ask you to work at a math station where you are using algebra tiles to show how to solve a two-step equation.

You can ask questions about this study at any time. If you decide at any time not to finish, you can ask to stop being part of the study. If you do take part in the study any information reported won't contain your name or any identifiable information.

If you sign this paper, it means that you have read this and that you want to be in the study. If you don't want to be in the study, don't sign this paper. Being in the study is up to you, and no one will be upset if you don't sign this paper or if you change your mind later.

Contact Information: If you have any questions about this study, please contact me, Jennifer Mahan at jmahan@sad42.us or 207-425-2811. You may also reach the faculty advisor, Johanna Prince on this study at johanna.prince@maine.edu or 207-778-7066. You may also contact the Chair of the IRB, Karol Maybury at karol.maybury@maine.edu or 207-778-7067.

Your signature: ____ Date ______________
Your printed name: ____ Date ______________
Signature of person obtaining consent: _____________________________ Date ______________
Printed name of person obtaining consent: __________________________ Date ______________

Appendix D: OBSERVATION PROTOCOL

Teacher: Mrs. Mahan Date: ______________ Algebra Group: A B C

Learning Goal:

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Intent of Student Activity:

- Using peers as resources (P)
- Games to enhance understanding (G)
- Manipulatives to provide concrete knowledge (M)
- Station Activities (S)
- Whole Group Learning (W)
- Other: ______________________

Materials used by teacher:

____

Assessment strategies used: (Formative)

____

____

Number of students present: ______________________________ Male: _____ Female: _____

Student behaviors: (Circle applicable behavior)

Students are on task ---Students are off task

Student interactions ---Student interactions
are not academic are focused on content

Students are reluctant ---Students are eager
to enter into activity to enter into activity

Learning emphasis: (Circle applicable emphasis)

Foundational Concrete --Foundational Abstract

Conceptual Concrete --Conceptual Abstract

This form will be used to chart student behaviors in the classroom. Since there are three large classes participating in the study, student behavior will be charted by dividing the students into groups. I will be monitoring three different students daily per class for their behavior.

Appendix E: STUDENT ACTIVITY EXIT TICKET

Name: ____ Date: _____________
Target # __________

Activity: (Circle one) Students will be told which method was used in class.

- Games
- Math stations
- Tactile manipulatives
- Virtual manipulatives
- Math tools __________________
- Peers as resources

Do you feel the activity that was used in class today helped you understand the algebra concept? Please explain:

____
____
____

If you worked with a peer today, did that peer help your algebraic understanding or did you help someone else in the class understand today's concept?

____
____
____

****This form will be distributed daily to students that are in the experimental group. The data will be analyzed using google survey. The intent is to qualitatively analyze students thinking.

Appendix F: STUDENT CENTERED APPROACHES

Name: ______________________________ Date: ___________________
Target #: ____________

Over the last six weeks we have had the opportunity to participate in a research project. During that time you were involved in several activities that took you away from the usual worksheet and allowed you to participate in games or to work with manipulatives to learn mathematical concepts. Please circle all of the activities you enjoyed.

Slope Name Tags	Creating slope out of cardboard and timing the rate of the car.	Simon Says with positive, negative, zero and undefined slope
Desmos computer activity calculating slope	Slope index cards with yarn (Slope necklaces)	Slope partner card sort activity
Whole group collaboration with worksheets guided by the teacher.	Small group collaboration with your table set.	Relay race at the board
Mathematical Recipes with Popcorn	Graphing Lines and Killing Zombies	Whose Line Is It? With crazy animals
Task Cards Scavenger Hunt around the room	Slope Sorting Activity	Algebra Tiles
Equation Change	Buddy Up / Turn and Talk	

Of all the activities listed above, which one was your favorite?
Why?____