

LIQUIDITY RISK IMPLICATIONS FOR MARKET RISK ASSESSMENT IN EMERGING MARKETS

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Abstract

Classical financial market theories built upon the assumption of a perfect market have been coping with frictions on both developed and emerging markets. There are numerous factors affecting the operation of financial markets and their participants' behavior, but illiquidity is a continuous problem that has important consequences on the financial asset prices and the degree of competition between market participants. Moreover, investments that yield high profits are often the ones related to less liquid financial assets from emerging markets. Since investment decisions are based on risk preferences and investors are commonly risk averse, they tend to limit their risk exposure while defining their investment strategy. Various risk measures can be used to estimate the level of risk. Value at Risk (VaR) is a widely accepted summary measure of market risk that is also recommended by the financial industry regulatory authorities as a risk management tool. The usage of VaR models is rapidly expanding; thus, it is used by both financial and non-financial institutions in order to estimate exposure to financial risks, complement allocation of capital, set trading position limits and evaluate performance of trading strategies. However, the last global financial crisis that occurred in 2007-2008 highlighted some of the weaknesses of this measure as a measure of market risk. The lack of a liquidity parameter in methodologies used to compute VaR significantly decreased the effectiveness of this measure. Therefore, the objective of this research is to examine the implications of asset liquidity risk on market risk assessment, which is obtained by using VaR.

The most frequently used technique for VaR estimation is the parametric (analytic) method, but the constant search for precise prediction models results in a large number of variations of basic parametric and non-parametric methods. Thus, in this research, the parametric VaR and volatility models are implemented on a sample representing the stock indices of the European emerging markets in the period from 2009 to 2017.

The results of this study indicate that the application of a liquidity constraint in the VaR

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model provides more accurate assessment of potential loss, especially in emerging markets, and enables investors to detect the liquidity risk and its effect in comparison with a conventional VaR.

Keywords: *liquidity risk, Value at Risk, emerging markets.*

JEL classification: *C22, G11, G15*

Introduction

The liquidity of financial markets has many dimensions and can be analyzed from different perspectives. Therefore, this problem remains elusive, despite the fact that liquidity is often considered to represent an important feature of the investment environment. Considering the fact that liquidity shortage had an important role in the development of many financial crises, illiquidity of financial markets could be observed as a key determinant of macro economy as a whole. Although the causes of crises cannot be generalized, the analyses of impact of various factors indicate that financial systems, particularly financial systems of the developing economies, are vulnerable to an abrupt change of the dynamics of capital flows (Tirol, 2002). Recent researches generally perceived the liquidity problem through the forms of central bank liquidity, market liquidity and funding liquidity (Nikolaou, 2009). Due to the financial services convergence, these forms of liquidity could be closely interrelated. During the global financial crisis of 2007-2008, many central banks assisted in maintaining liquidity through the prevention of bankruptcy of systemically important institutions, maintaining the liquidity of the interbank market and increasing the liquidity in financial markets (BIS, 2017). Sudden shifts in market liquidity and funding liquidity are mutually reinforcing and could lead to a liquidity spiral. Therefore, the central bank policy could have important implications on market liquidity during a liquidity crisis (Brunnermeier and Pedersen, 2009).

Market liquidity is an important factor for portfolio managers and large institutional investors and it refers to the ability to execute a trade promptly, at low cost or no cost, risk or inconvenience (Dowd, 2002; Roy, 2004). This notion implies that the degree of market liquidity is determined by the following determinants: (1) tightness, which refers to low transaction costs, (2) depth, which indicates the market ability to absorb the orders without making price change, and (3) resilience of the market, which refers to the speed with which underlying prices are restored after a disturbance (Kyle, 1985) and (4) immediacy, which refers to the time needed to complete a trade (Black, 1971). In order to estimate the liquidity risk, various measures can be applied – either dimensional or multi-dimensional. However, the Amihud's measure (2002) is considered the most generalized one which follows the presented Kyle's (1985) price impact definition of liquidity (Minović, 2011) more closely.

Nevertheless, investments that yield high profits are often the ones in less liquid financial assets from emerging markets. Since investment decisions are based on risk preferences and investors are commonly risk averse, they tend to limit their risk

exposure while defining their investment strategy. Various risk measures can be used to estimate the level of risk and Value at Risk (VaR) as a widely accepted summary measure of market risk that is also recommended by the financial industry regulatory authorities as a risk management tool. In terms of illiquidity, the conventional VaR fails to capture the costs of investing in illiquid financial assets. Therefore, the aim of this paper is to complement the existing research by testing the performance of the liquidity-adjusted VaR model in emerging and frontier markets. The group of selected markets consists of Serbian, Croatian, Greek and Romanian stock markets.

The paper is structured as follows. In the second part of the paper, the existing methodologies for incorporating liquidity risk in VaR models are discussed. The third part presents a liquidity-adjusted Value at Risk model (L-VaR), while the results of the analyses are discussed in the fourth part. The fifth part of the paper presents the concluding remarks.

Literature review

Following classical financial market theories built upon the assumption of a perfect market, the conventional VaR provides assessment of market risk by assuming that assets will be liquidated at mid-price. In reality, most markets, especially the emerging ones, are less than perfectly liquid. In those terms, neglecting the liquidity risk leads to an underestimation of the overall market risk, which could, due to the regulatory recognition of VaR measure, result in inaccurate assessment of capital for the safety of financial institutions (Le Saout, 2002). From the perspective of an investor and risk manager, liquidity risk is the potential loss due to the time-varying cost of trading, which is often ignored. Research studies reveal that liquidity can have significant impact in market risk estimation, but liquidity risk modeling and predictability is difficult in spite of the numerous models proposed in literature.

The selection of a liquidity risk model is determined by the purpose and type of asset, as well as the data available. Prior research proposed the theoretical model of an optimal trading strategy for liquidating portfolios in order to find an optimal balance between price impact costs and delay cost caused by timing a transaction. The study of Lawrence and Robinson (1995) appeared among the first ones to investigate the impact of liquidity risk on VaR calculation and proposed a simple rule that adds the time estimated to liquidate the investor's position to the time horizon of VaR calculation. If the time horizon increases, due to the illiquidity of the portfolio, VaR will also increase in order to reflect higher risk. This model, however, assumes that investor's position can be liquidated in a single transaction and ignores bid-ask spread volatility over time. Similarly, Haberle and Persson (2000) propose a method of VaR calculation based on the assumption of orderly liquidation that assumes the investor's ability to liquidate a fraction of the daily trading volume without a significant impact on the market price. A solution of an optimal trading strategy within a given liquidation horizon is proposed by Almgren and Chriss (2000), who construct a liquidity-adjusted VaR. This approach was extended by including non-linearity in the price impact (Almgren, 2003) and considering the special case of "coordinated variation" in which liquidity

and volatility vary together (Almgren, 2012). However, the implementation of these models does not provide assertive and consistent results on market illiquidity.

Depending on the data used in measuring the liquidity risk, there are models based on bid-ask spread data, models based on volume or transaction data and models based on limit order book data (Stange and Kaserer, 2009).

Considering the fact that data on bid and ask prices are available for most assets, Bangia et al. (1999) defined a parametric liquidity-adjusted VaR by adding the time-varying empirical bid-ask spread to the price risk modeled using the mean-variance approach. The empirical application of this model showed that ignoring the liquidity risk could underestimate the total VaR by 25-30% in emerging market currencies (Bangia et al., 1999), but also the total VaR of illiquid stocks on developed markets by more than 50% (Le Saout, 2002). In the Indian debt market, the liquidity risk component can be as high as 20% of the total VaR (Roy, 2004). If liquidity risk is considered on the level of the intraday time horizon, liquidity can constitute on average 30% of the VaR for small-price stocks in Hong Kong stock market (Lei and Lai, 2007). Estimating the spread distribution and providing the more precise results is achieved by the application of a Cornish-Fisher approximation (Ernst et al. 2009).

Nevertheless, the availability of high-frequency and detailed financial data on emerging economies is often limited. The insufficiencies of data, as well as the characteristics of emerging markets, could lead to an inadequate liquidity estimation if the liquidity measures from developed markets are used. Specifically, the bid-ask spread is the most used measure and the most demonstrable indicator of overall liquidity, but data on bid and ask prices are not always available for all assets or for all time periods. Therefore, the usual approach for liquidity risk modeling in emerging and frontier markets uses proxies. Some of the most used liquidity measures calculated on the basis of low-frequency data are: Roll's spread (Roll, 1984), LOT measure and zero return days (Lesmond et. al, 1999), the Amihud's measure (Amihud, 2002), the Amivest measure (Cooper et al., 1985) and the Pastor-Stambaugh measure (Pastor and Stambaugh, 2003). Although alternative measures of liquidity reflect different aspects of liquidity, in the emerging markets there is a significant within-country correlation between all liquidity proxies, especially during a period of crisis (Lesmond, 2005; Yeyati et al., 2008).

In case of the Serbian capital market, results of illiquidity estimation using two measures - zero rates and price pressure of non-trading, showed that this market was low liquid and that this lack of liquidity was persistent during the period 2005-2009 (Minović, 2011). During the crisis period, the level of market illiquidity measured by illiquidity of stocks constituting Belexline index, as well as the level of illiquidity of the most liquid stocks constituting Belex15 index, increased and it caused the sudden increase in systematic risk by 58.7% in the post-crisis period in Serbia (Minović and Živković, 2010). The Croatian capital market has been facing significant changes in recent years. Despite its numerous improvements, the market remains insufficiently liquid. Although the applied measures of liquidity showed certain variations and inconsistency of achieved results, they undoubtedly implied higher levels of illiquidity

in Croatian capital market compared to the developed markets (Benić and Franić, 2009). However, the Croatian capital market is less illiquid than the Serbian capital market according to the values of the zero rates measure (Minović, 2012).

The Bucharest Stock Exchange is a large and important part of Romanian capital market. However, this exchange is characterized by a relatively small number of days when trades of both liquid and illiquid stocks were recorded, and it is also characterized by an inconsistent pattern of relationship between illiquidity and stock returns (Vidović et al., 2014). According to the number and value of transactions in the period of the financial crisis, it could be concluded that market liquidity did not reduce, and that the Bucharest Stock Exchange remained an attractive investment opportunity (Geambasu and Stancu, 2010). On the other hand, the Athens Stock Exchange is considered an emerging market, which implies that it improves its efficiency as time goes by and especially with respect to their degree of global integration (Bekaert and Harvey, 1997). Although all emerging markets may not experience the same degree of liquidity improvements, the Greece stock market in the pre-crisis period improved its liquidity significantly (Jun et al., 2003).

Realizing the importance of liquidity risk in investment decision-making and considering the applicability of existing liquidity-adjusted risk models in the observed developing markets, in this study we will use the widely accepted parametric (analytic) method of VaR calculation.

Data and Methodology

Considering the lack of transparency and readily accessible information about financial assets traded in the selected frontier and emerging markets and the relatively short time-series samples, this analysis is adjusted to the obstacles observed. Therefore, the liquidity proxy is determined by the available data and implemented in the recognized market risk assessment model – liquidity-adjusted Value at Risk proposed by Bangia et al. (1999).

In order to estimate the model parameters, data sets were divided into two sub-samples. To allow enough data for fitting the volatility models, the first sub-sample period starts at the beginning of October 2009 and lasts until the end of 2016. It is considered an in-sample period. The second sub-sample period starts at the beginning of 2017 and lasts until the beginning of October 2017. It is considered an out-of-sample period. On the in-sample data, we estimate parameters of log-returns and cost of liquidity distributions, while the second period data are used for the validation of all proposed types of VaR models.

Data description

The Value at Risk model in this study is applied to the stock indices of capital markets that are categorized as emerging markets and frontier markets according

to the Standard and Poor's (S&P) and MSCI (Morgan Stanley Capital International) categorization. Regarding indicators that include measures of size (market capitalization and the number of listed domestic companies) and liquidity (the value of traded shares), selected stock markets of the Republic of Serbia, the Republic of Croatia and the Republic of Romania are classified as frontier markets, while the Greece stock market is classified as emerging¹. As data presented in Table 1 indicate, the market capitalization of selected frontier markets is rather low. If it is compared to the market capitalization of regarded emerging market expressed in US dollars, the market capitalization of the frontier stock exchanges is smaller by a factor ranging from 2.08 to 6.02. The gap is even bigger if we consider market liquidity (smaller by a factor ranging from 4.69 to 7.62) and the turnover ratio (smaller by a factor ranging from 2.92 to 14.61) of the Serbian, Croatian and Romanian stock market compared to the aforementioned emerging market. However, the relevant indices of these stock markets are rated as blue-chip indices and included in the MSCI Frontier Emerging Market Index and SandP frontier indices. Thus, it can be concluded that these markets can be investment-grade, mostly because they provide significant diversification benefits for international investors due to the low correlation between the frontier and developed markets (Speidell and Krohne, 2007; Jayasuriya and Shambora 2009; Berger et al. 2011).

Table 1. Stock market development indicators for 2012

Country	Stock exchange	Market capitalization		Market liquidity [*]	Turnover ratio ^{**}	Number of listed domestic companies	S&P/Global Equity Indices ^{***}
		\$ millions	% of GDP				
Greece	Athens Stock Exchange	44,876	18.3	6.1	33.6	262	24.7
Serbia	Belgrade Stock Exchange	7,451	19.9	0.8	3.7	751	-
Romania	Bucharest Stock Exchange	15,925	9.4	1.3	11.5	77	9.8
Croatia	Zagreb Stock Exchange	21,560	36.4	0.8	2.3	211	-5.2

^{*}Value of shares traded presented as a percentage of GDP

^{**}Value of shares traded presented as a percentage of market capitalization

^{***}S&P Global Equity Indices measure the U.S. dollar price change in the stock markets covered by the S&P/IFCI and S&P/Frontier BMI country indices

Source: The World Bank (available at <http://wdi.worldbank.org/table/5.4>, retrieved on September 20, 2017)

In order to evaluate the performance of financial markets as a whole, we use the daily closing values of the selected stock exchange indices that represent the most liquid stocks on the market: for the Athens Stock Exchange – Athex, Belgrade Stock Exchange – Belex15, the Zagreb Stock Exchange – Crobex10 and the Bucharest Stock Exchange – BET. The data period is 8 years long – from October 2009 until October 2017. The data sets are obtained from the official stock exchanges' websites and are expressed in national currencies. In this study, we use continuously compounded returns r_t on stock market indices calculated as a change in logarithms

of the daily stock index level.

The statistical analysis of the observed stock market indices (Table 2) shows that the mean of the daily return series of all indices is very low - approximately zero, while the volatility of these returns measured by the standard deviation is relatively high. The kurtosis of all index returns is higher than three, which means that extreme values are observed more frequently than for the normal distribution. The right-skewed distribution of daily lognormal returns on the Crobex10 index indicates that there is a probability greater than normal to achieve extreme gains, while the left-skewed distributions of returns of the Athex, Belex15 and BET indices indicate that there are substantial probabilities of extreme negative returns. The results of the Jarque-Bera test of normality prove that the null hypothesis of normal distribution in the case of every index can be rejected.

Table 2. The descriptive statistics of selected indices log-return series in the period October 2009 – October 2017

	Athex	BET	Belex15	Crobex10
<i>N</i>	1970	2014	2018	2000
Mean	-0.000645	0.000286	-0.000068	-0.000037
Median	-0.000101	0.000453	-0.000074	-0.000150
Max	0.134311	0.076737	0.082290	0.115709
Min	-0.177129	-0.110125	-0.074080	-0.040754
Standard deviation	0.022577	0.010872	0.008621	0.007586
Skewness	-0.304937	-0.738114	-0.037003	1.638302
Excess Kurtosis	5.389181	12.478579	11.569365	28.273119
Jarque-Bera Test <i>p</i> -value	2399.75 (0.000000)	13178.72 (0.000000)	11193.68 (0.000000)	67158.21 (0.000000)
ADF Test <i>p</i> -value	-11.52 (0.01)	-11.99 (0.01)	-11.63 (0.01)	-10.91 (0.01)
Ljung-Box Test <i>p</i> -value	49.70 (0.000000)	17.56 (0.06279)	87.54 (0.000000)	22.58 (0.01234)
ARCH Test <i>p</i> -value	80.15 (0.000000)	287.32 (0.000000)	615.41 (0.000000)	5.59 (0.8485)

Source: Authors' calculation

Further statistical tests provide more insight into the characteristics of the financial time series observed. The results of the Augmented Dicky-Fuller (ADF) unit root tests indicate that all series are first difference stationary. The Ljung and Box Q-statistics on the 10th lag of the return series sample autocorrelation functions indicate significant serial correlation for all markets. However, the ARCH effects are evident in all return series, except the returns on Crobex10 index, according to the results of Engle's ARCH tests (1982). Therefore, the returns on this index will be modeled under the assumption that there is no volatility clustering using the Exponentially Weighted Moving Average (EWMA) model. Considering the results of the analysis conducted, the returns on the Athex, BET and Belex15 indexes will be modeled using the non-linear ARMA(m,n)-GARCH(p,q) model that can include a wide range of characteristics

of volatility. The implementation of the GARCH(p,q) model of order $p \leq 2$ and $q \leq 2$ in most cases confirms that modeling volatility using this model provides satisfactory results (Xiao and Aydemir, 2007). Therefore, the most commonly applied model GARCH (1,1) will be used in this study. Since the Gaussian GARCH model could not explain the leptokurtosis exhibited by returns on the stock indices analyzed, we will use two types of this model replacing the assumption of conditional normality of innovations with that of conditional Student's t distribution.

Therefore, the following conditional variance specifications were adopted:

$$\text{EWMA: } \sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2 \tag{1}$$

$$\text{GARCH(1,1): } \sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 \tag{2}$$

where $0 \leq \lambda \leq 1$ is the smoothing parameter and its value is set to $\lambda=0.94$; $\omega>0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ are parameters of the used GARCH model that should meet the following stationary condition $\alpha_1 + \beta_1 < 1$; error term ε_t is a function of z_t , which is a random component with the properties of white noise.

The conditional mean will be modeled as a linear process due to a significant return autocorrelation, whereby the number of lags is limited to 2.

Table 3. The parameters of ARMA(m,n)-GARCH(p,q) models applied on the selected indices log-return series in the period Oct. 2009 – Dec. 2016

Index	Model	Parameters	
Athex	ARMA(2,2)- GARCH(1,1) (Normal distribution)	AR ₁ = 1.170734 AR ₂ = -0.846087 MA ₁ = -1.127093 MA ₂ = 0.797536	$\omega = 0.000018$ $\alpha_1 = 0.099878$ $\beta_1 = 0.871892$
	ARMA(2,2)- GARCH(1,1) (Student's t distribution)	AR ₁ = 1.116292 AR ₂ = -0.839086 MA ₁ = -1.067576 MA ₂ = 0.804878	$\omega = 0.000027$ $\alpha_1 = 0.099568$ $\beta_1 = 0.851874$
BET	ARMA(2,2)- GARCH(1,1) (Normal distribution)	AR ₁ = 0.106013 AR ₂ = -0.880503 MA ₁ = -0.065377 MA ₂ = 0.895344	$\omega = 0.000005$ $\alpha_1 = 0.188192$ $\beta_1 = 0.780888$
	ARMA(2,2)- GARCH(1,1) (Student's t distribution)	AR ₁ = 0.456507 AR ₂ = -0.997423 MA ₁ = -0.452884 MA ₂ = 0.996286	$\omega = 0.000005$ $\alpha_1 = 0.150047$ $\beta_1 = 0.813198$

Belex15	ARMA(2,2)-GARCH(1,1) (Normal distribution)	AR ₁ = 1.328373 AR ₂ = -0.362489 MA ₁ = -1.195407 MA ₂ = 0.250217	ω = 0.000004 α ₁ = 0.133084 β ₁ = 0.815944
	ARMA(2,2)-GARCH(1,1) (Student's <i>t</i> distribution)	AR ₁ = 1.136307 AR ₂ = -0.185718 MA ₁ = -1.009894 MA ₂ = 0.091118	ω = 0.000003 α ₁ = 0.127721 β ₁ = 0.827727

Source: Authors' calculation

According to the determined parameters of the models (Table 3), it can be concluded that the stationary condition for GARCH(p,q) model is met. Regarding the ARCH parameter (α_1) in the cases of BET and Belex15 index, its value is greater than 0.1, which implies the fact that the volatility of these series is very sensitive to changes on observed stock markets (Alexander, 2008). The value of the GARCH parameter (β_1) is ranging from 0.7809 in the case of BET index to 0.8719 in the case of Athex index, which implies that there are different levels of volatility convergence to the long-term mean value. However, the long-term volatility effect ($\beta_1 > 0.9$) is not observed in any series analyzed.

Cost of Liquidity

Since liquidity is difficult to observe directly, literature about market liquidity focuses on one or several kinds of liquidity proxy. Each of the proxies provides information on different aspects of liquidity. On the other hand, market liquidity indices can combine different aspects of liquidity and they are usually based on measures of tightness and depth dimensions of market liquidity. The widely used liquidity measure in the liquidity-adjusted VaR is the bid-ask spread. However, in emerging markets, detailed transaction data on bid-ask spreads are not widely available, especially for long time series. Hence, we employ Amihud (2002)'s illiquidity measure that can be calculated using only daily data. Regarded as a ratio of the daily absolute return to the trading volume in monetary units, this measure of liquidity reflects a generalized approach to liquidity that captures both the exogenous illiquidity and the endogenous illiquidity. Applying this liquidity proxy, the illiquidity of stock i in day t is calculated in the following manner:

$$ILLIQ_t^i = \frac{|r_t^i|}{V_t^i} \quad (3)$$

where r_t and V_t are the return and volume (in ten millions of monetary units) for stock i on day t , respectively. In this paper, we use stock market indices and therefore, we will calculate the average liquidity measure for the stock markets selected.

Due to market capitalization changes during the observed period of time, we construct the scaled series $(m_h / m_1)ILLIQ_t^i$, where m_h is the total value of market capitalization

at the end of period h corresponding to day t , and m_1 is the total value of market capitalization at the beginning of October 2009. Finally, the applied illiquidity measure is calculated in the following manner:

$$CoL_t^i = \min \left\{ \frac{m_h}{m_1} ILLIQ_t^i, 10.00 \right\} \quad (4)$$

The statistical analysis of the observed market liquidity proxies presented in Table 4 shows that the mean of the daily liquidity cost varies depending on the market. The average daily cost of liquidity is relatively low on Athens Stock Exchange and Bucharest Stock Exchange, while on Belgrade Stock Exchange and Zagreb Stock Exchange, the recorded values are higher, as well as the volatility of these proxies measured by the standard deviation. The kurtosis of all liquidity cost indices is higher than three, which means that extreme values, i.e. the periods of illiquidity, are observed more frequently than for the normal distribution.

Table 4. The descriptive statistics of the average liquidity cost for selected markets in the period Oct. 2009 – Oct.2017

	Athex	BET	Belex15	Crobex10
<i>N</i>	1970	2014	2018	2000
Mean	0.003347	0.003902	0.021546	0.028404
Median	0.002467	0.002697	0.014870	0.021042
Max	0.036824	0.059383	0.302384	0.618868
Min	0.000005	0.000001	0.000000	0.000052
Standard deviation	0.003293	0.004378	0.023221	0.028969
Skewness	2.655806	3.858059	3.773677	5.524178
Excess Kurtosis	13.712940	27.364393	27.530531	88.831179
Jarque-Bera Test <i>p</i> -value	17634.22 (0.000000)	67501.47 (0.000000)	68182.89 (0.000000)	664411.75 (0.000000)
ADF Test <i>p</i> -value	-8.24 (0.01)	-8.22 (0.01)	-9.62 (0.01)	-9.323 (0.01)
Ljung-Box Test <i>p</i> -value	351.72 (0.000000)	890.98 (0.000000)	652.72 (0.000000)	224.39 (0.000000)
ARCH Test <i>p</i> -value	80.19 (0.000000)	287.33 (0.000000)	615.41 (0.000000)	5.59 (0.8485)

Source: The authors' calculation

Selected statistical tests prove that the characteristics of the observed liquidity cost series are very similar to the characteristics of the respective series of log-returns. According to the results of these statistical tests, it can be concluded that the liquidity cost series are first difference stationary, but significant serial correlation can be observed. On the other hand, the ARCH effects are evident in the series of liquidity cost on the Greek, Serbian and Romanian stock exchanges and the conditional standard

deviation in these cases will be modeled using the aforementioned model GARCH (1,1). In the case of Croatian stock exchange, the conditional standard deviation of the liquidity cost will be modeled using the EWMA model. The conditional mean will be modeled as a linear process due to the significant return autocorrelation, whereby the number of lags is limited to 2.

Table 5. The parameters of ARMA(m,n)-GARCH(p,q) models applied on the average cost of liquidity for selected markets in the period Oct. 2009 – Dec. 2016

Index	Model	Parameters	
Athex	ARMA(1,2)-GARCH(1,1) (Normal distribution)	AR ₁ = 0.963565 MA ₁ = -0.952457 MA ₂ = 0.069665	$\omega = 0.000000$ $\alpha_1 = 0.118552$ $\beta_1 = 0.855067$
	ARMA(1,2)-GARCH(1,1) (Student's <i>t</i> distribution)	AR ₁ = 0.928010 MA ₁ = -0.911016 MA ₂ = 0.049594	$\omega = 0.000001$ $\alpha_1 = 0.092486$ $\beta_1 = 0.827463$
BET	ARMA(2,2)-GARCH(1,1) (Normal distribution)	AR ₁ = 1.537242 AR ₂ = -0.545388 MA ₁ = -1.391565 MA ₂ = 0.427536	$\omega = 0.000001$ $\alpha_1 = 0.178504$ $\beta_1 = 0.811220$
	ARMA(2,2)-GARCH(1,1) (Student's <i>t</i> distribution)	AR ₁ = 1.306964 AR ₂ = -0.340071 MA ₁ = -1.208105 MA ₂ = 0.283971	$\omega = 0.000001$ $\alpha_1 = 0.102582$ $\beta_1 = 0.852534$
Belex15	ARMA(2,2)-GARCH(1,1) (Normal distribution)	AR ₁ = 1.823969 AR ₂ = -0.826516 MA ₁ = -1.706360 MA ₂ = 0.717409	$\omega = 0.000036$ $\alpha_1 = 0.132643$ $\beta_1 = 0.784878$
	ARMA(2,2)-GARCH(1,1) (Student's <i>t</i> distribution)	AR ₁ = 1.354941 AR ₂ = -0.378114 MA ₁ = -1.246051 MA ₂ = 0.298331	$\omega = 0.000021$ $\alpha_1 = 0.094400$ $\beta_1 = 0.870893$

Source: Authors' calculation

The covariance stationary condition for GARCH(p,q) model is met, since the sum of determined parameters of the models is less than 1 in all cases (Table 5). Regarding the ARCH parameter (α_1), in the case of BET index, its value is greater than 0.1, which implies the fact that the volatility of this series is very sensitive to changes on the observed stock market. The value of the GARCH parameter (β_1) implies that there are different levels of volatility convergence to the long-term mean value, but the long-term volatility effect ($\beta_1 > 0.9$) is not observed in any series analyzed.

Liquidity-Adjusted Value at Risk

In order to incorporate the liquidity risk into the VaR model, we use the liquidity-adjusted VaR (L-VaR) methodology developed by Bangia et al. (1999), which calculates L-VaR

in the following manner:

$$L - VaR_t^i = VaR_t^i(r_t^i) + \frac{1}{2} VaR_t^i(CoL_t^i) \tag{5}$$

where the symbols used represent VaR_t – conventional VaR for each stock index i in time t , CoL_t – cost of liquidity of each index i in time t that is constructed under the assumption that the liquidity cost is calculated using Amihud’s measure of illiquidity (formula 4).

Using the mean-variance framework for measuring the market risk, the relative liquidity-adjusted VaR measure can be calculated using the following formula:

$$L - VaR_t^i = \mu_t^r + z_{1-\alpha}^r \sigma_t^r + \frac{1}{2} (\mu_t^P + \hat{z}_{1-\alpha}^P \sigma_t^P) \tag{6}$$

The conventional parametric VaR model used in this study is given by the first two terms on the right-hand-side of equation (6), where μ_r is the mean of daily log-returns, σ_r is the volatility of daily log-returns and $z_{1-\alpha}(r)$ is the standard normal variation for the chosen confidence level for VaR calculation. The VaR estimated in this manner is then augmented with a time-varying liquidity proxy by deducting the cost of half the worst values of the proxy, which implies that only one-way transaction costs are considered, as determined by μ_p - the liquidity proxy mean, σ_p - the liquidity proxy volatility and $z_{1-\alpha}(P)$ - the standard normal variation for the chosen confidence level for VaR of liquidity cost calculation.

Table 6. The conventional VaR prediction performance – results of the unconditional and conditional tests

Index	1- α %	%Violation		LR _{UC}		LR _{CC}		VaR _{AVG}	
		VaR _{Gn}	VaR _{Gt}	VaR _{Gn}	VaR _{Gt}	VaR _{Gn}	VaR _{Gt}	VaR _{Gn}	VaR _{Gt}
Athex	5.0	2.12	1.06	4.18602	8.99346	7.75404	9.03647	-2.22	-2.37
	2.5	0.53	0.53	4.41906	4.41906	4.42976*	4.42976*	-2.65	-2.83
	1.0	0.00	0.00	3.79903*	3.79903*	3.79903*	3.79903*	-3.14	-3.36
BET	5.0	1.60	1.06	6.17406	6.17406	6.27189	6.27189	-1.20	-1.22
	2.5	0.53	0.53	4.37906	4.37906	4.38982*	4.38982*	-1.44	-1.46
	1.0	0.00	0.00	3.77893*	3.77893*	3.77893*	3.77893*	-1.71	-1.74
Belex15	5.0	1.06	1.06	8.91221	8.91221	8.95545	8.95545	-1.05	-1.05
	2.5	1.06	1.06	2.02192*	2.02192*	2.06516*	2.06516*	-1.25	-1.26
	1.0	0.00	0.00	3.77893*	3.77893*	3.77893*	3.77893*	-1.48	-1.49
Index	1- α %	EWMA VaR							
		%Violation		LR _{UC}		LR _{CC}		VaR _{AVG}	
Crobex10	5.0	4.79		0.022904*		0.647826*		-1.13	
	2.5	4.25		1.933736*		2.889228*		-1.35	
	1.0	2.66		3.560592*		3.833850*		-1.60	

LR_{UC} – the unconditional coverage test Likelihood Ratio statistic, LR_{CC} – the conditional coverage test Likelihood Ratio statistic, VaR_{AVG} – average value of VaR

Source: Authors’ calculation

In order to highlight the importance of liquidity risk for market risk assessment, we decompose the total risk into components and define the relative liquidity impact as:

In this study, we predicted a one-day-ahead VaR for confidence levels of 95%, 97.5% and 99%, using a sliding window of 7 years (the number of trading days is different for each market). The results of EWMA and ARMA-GARCH-type models applied in modeling the conventional VaR were tested using the unconditional (Kupiec, 1995) and conditional (Christoffersen, 1998) test. In all cases, ARMA-GARCH-type models provide acceptable results of the calculated VaR, except for a VaR with a 99% confidence level (Table 6). The GARCH-type volatility models with Student's t distribution of innovations provide more adequate results for the calculation of a VaR with a 97.5% confidence level, especially in the case of log-returns on Belex15 index.

The back testing conducted on the predicted conventional VaR of log-returns on Crobex10 index proved that the applied EWMA model is adequate for modeling VaR with all levels of confidence.

$$l_t^i = \frac{L - VaR_t^i - VaR_t^i}{VaR_t^i} \quad (7)$$

This ratio can be observed as the measure of liquidity's relative significance in market risk assessment.

Results and discussion

In this study, we analyzed the significance of liquidity risk for the adequate estimation of market risk using the widely accepted parametric Value at Risk model. The worst liquidity cost was modeled using Amihud's measure and added to the conventional VaR model in order to calculate the liquidity-adjusted VaR as it was shown in the previous section. For the estimation of price risk and liquidity risk, we used the analytical VaR model and estimated the parameters of the model using the EWMA and ARMA-GARCH-type of models. In this study, we used two GARCH(1,1) models for volatility modeling: the GARCH(1,1) model that assumes that the innovation term follows a standard normal distribution and the GARCH(1,1) model with Student's t distributed innovations. By combining the estimations of these models, we obtained two models for L-VaR in the case of Athex, Belex15 and BET index and one EWMA model for calculation of L-VaR in the case of Crobex10 index.

Following similar researches on liquidity risk in developed and developing markets (Lesmond, 2005; Stange and Caserer, 2009), we can conclude that more developed markets have lower liquidity risk. In this study, the lowest liquidity risk is observed in the Athens Stock Exchange, since the relative liquidity impact on total market risk is in the range of 3.04 to 5.12% (Table 7). All tested L-VaR models are acceptable regarding the results of conditional and unconditional back tests. However, the standard deviation of the results achieved is smaller in the case of ARMA(1,2)-GARCH(1,1) model with Student's t distribution of innovations and it ranges from 0.38 to 0.53%.

Table 7. The mean predicted value of L-VaR and the liquidity component / in the case of Athex index

Index	1- α %	Liquidity-adjusted Value at Risk model			
		ARMA(1,2)-GARCH(1,1)		ARMA(1,2)-GARCH(1,1)	
		(Normal distribution)		(Student's <i>t</i> distribution)	
		L-VaR	/	L-VaR	/
Athex	5.0	-2.28	3.04	-2.47	4.01
	2.5	-2.75	3.77	-2.96	4.62
	1.0	-3.28	4.37	-3,53	5.12

Source: Authors' calculation

Depending on the size and level of development, there is also a significant difference in the liquidity risk on the Belgrade Stock Exchange and the Zagreb Stock Exchange on one side, and the Bucharest Stock Exchange on the other side. The illiquidity of the Bucharest Stock Exchange can increase the value of total liquidity-adjusted VaR by 8.07 to 13.48% (Table 8). According to the results of used back tests, both types of models are acceptable for modeling the market risk including liquidity implications, but results obtained by implementing the ARMA(2,2)-GARCH(1,1) model, assuming that innovations are Student's *t* distributed, are more stable, since the standard deviation is ranging from 0.38 to 0.54% compared to the standard deviation of L-VaR ranging from 0.44 to 0.64% in the case of Gaussian distributed innovation process assumption.

Table 8. The mean predicted value of L-VaR and the liquidity component / in the case of Belex15 and BET index

Index	1- α %	Liquidity-adjusted Value at Risk model			
		ARMA(2,2)-GARCH(1,1)		ARMA(2,2)-GARCH(1,1)	
		(Normal distribution)		(Student's <i>t</i> distribution)	
		L-VaR	/	L-VaR	/
Belex15	5.0	-1.62	55.64	-1.83	73.88
	2.5	-2.09	68.14	-2.31	83.95
	1.0	-2.64	78.37	-2.87	92.20
BET	5.0	-1.20	8.07	-1.35	10.82
	2.5	-1.58	9.88	-1.64	12.29
	1.0	-1.91	11.35	-1.97	13.48

Source: Authors' calculation

Analyzing the liquidity of the least developed frontier markets – Serbian and Croatian, we can conclude that those markets have remained very low liquid even in the post-crisis period. The relative liquidity impact in the case of the Belgrade Stock Exchange is ranging from 55.64 to 92.20%, while on the Zagreb Stock Exchange, the liquidity

cost can increase the price risk by 53.27 to 94.92%. The L-VaR models applied on the log-returns of indices of these markets are acceptable regarding the results of the unconditional and conditional back tests. In the case of Belgrade Stock Exchange, more stable results are achieved using the ARMA(2,2)-GARCH(1,1) model with Gaussian distributed innovation process (standard deviation ranges from 0.33 to 0.50%). However, the L-VaR in the case of Crobex10 index (Table 9) exhibit more volatility compared to the other markets, since the standard deviation of results obtained is ranging from 1.23 to 1.75%.

Table 9. The mean predicted value of L-VaR and the liquidity component l in the case of Crobex10 index

Index	$1-\alpha_{\%}$	Liquidity-adjusted Value at Risk model	
		EWMA volatility model	
		L-VaR	l
Crobex10	5.0	-1.89	53.27
	2.5	-2.52	76.14
	1.0	-3.25	94.92

Source: Authors' calculation

Although the results imply that liquidity risk still affects the evaluation of market risk in frontier markets significantly, recognizing the liquidity risk is important for a financial stability analysis. Considering the fact that the econometric estimation of VaR can be determined by the volatility model chosen (Bucevska, 2013; Miletić and Miletić, 2013), the proposed model for predicting the value of L-VaR can be considered adequate in the scope of this research. Regarding the importance of volatility modeling in emerging markets, future research will include testing of asymmetric GARCH-type models. However, according to the backtest results, the findings reported in this study can be used to detect the emerging vulnerabilities and define mitigating actions on the stock markets analyzed.

Conclusion

Liquidity risk is an aspect of market risk that has been largely neglected by conventional Value at Risk models. This negligence is partly due to the fact that there is a large number of various liquidity measures, but also a theoretical discussion on the effectiveness of a single measure to capture the various aspects of liquidity in financial markets. Having in mind that the Amihud's measure is considered the most generalized one, which more closely follows the presented Kyle's (1985) price impact definition of liquidity, in this study, we used this measure to estimate liquidity risk and apply it in the liquidity-adjusted Value at Risk model provided by Bangia et al. (1999) to measure market risk on the emerging and frontier European stock markets.

The results achieved show that frontier markets remain low liquid despite their constant improvements and development. Liquidity risk can increase the estimation

of market risk in these cases up to more than 90%. On the other hand, as the market develops and trades become more frequent, liquidity risk is lower. Therefore, more developed markets show a significantly lower level of liquidity risk that can increase the estimation of market risk by 14%. However, it can be observed that the preciseness of liquidity risk measurement is determined by the sample and liquidity proxy used. A longer sample with more varieties of events in the market will deliver a more accurate result. This conclusion opens some future research questions.

Given the data limitations in the stock markets analyzed, we can conclude that the historical data used may not contain the extreme shocks, since data samples started after the financial crisis. The severity of the liquidity problems observed and showed by the used L-VaR model is relative to the severity of the problems included in the considered sample. In order to obtain a more accurate prediction of L-VaR, extreme shocks could be simulated using the experiences from other developed and developing markets and in that manner, the robustness of the proposed model can be tested.

Also, it is important to recognize that there is no universal proxy that best captures liquidity across different emerging markets. Considering the availability of data, the proxies used in emerging markets are low-frequency proxies, but certain proxies are more suitable to a specific region or country than others. Although the Amihud's measure is the most effective price-impact proxy, further research may analyze other proxies in order to obtain more accurate results that can be implemented in financial decision-making.

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ⁱ Available at http://www.msci.com/products/indexes/market_classification.html, retrieved on September 20, 2017.