

OPTIMAL BAND SELECTION FOR THE CALCULATION OF PLANCK MEAN ABSORPTION COEFFICIENTS

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Abstract. Radiative heat transfer is a major heat loss mechanism in thermal plasmas generated during arc flashes/faults in switchgear applications or during high current interruption in low voltage circuit breakers. A common way to calculate the radiation balance is by means of approximate non-gray radiation models like P1 or discrete ordinates (DOM), where the frequency dependent absorption and emission are described in a number of frequency intervals (bands) using a constant absorption coefficient in each band. Current work is focused on finding the optimal number of bands as well as band interval boundaries that provide a reasonable level of accuracy in comparison to a full spectral solution. An optimization procedure has been applied to different SF₆ and copper vapor gas mixtures for an assumed temperature profile. Radiation model results using optimized band averaged absorption coefficients as well as spectral values are provided and discussed for the exemplary temperature profile.

Keywords: radiation, thermal plasma, circuit breaker, SF₆.

1. Introduction

In various applications in the area of low-, medium-, and high-voltage power distribution, thermal plasmas are either used as switching elements for current interruption (e.g. circuit breakers) or arcs can occur as fault conditions (arc flash in switchgear). In order to enhance the design and performance of these devices, modeling approaches that describe the behavior of thermal plasmas have been developed [1] and enhanced in order to include more physical phenomena and cover a broader range of applications [2]. Hereby a set of coupled equations for flow and electrodynamic processes is usually solved (magneto-hydrodynamic approach), in combination with one of various simplified models to consider energy transfer due to radiation.

The simplest approach to model the effect of radiation is the net emission coefficient (NEC) [1], which describes the net radiation from an isothermal sphere or cylinder. But in various applications it is necessary to model not only the energy sink in the arc core due to the net emission, but also the reabsorption in the colder arc fringe and the radiative flux impinging on surfaces needs to be described to achieve higher model fidelity. Radiative heat flux contributes to the ablation of plastic materials, which can substantially influence the arc behavior [3].

Commonly used approximate radiation transfer models are the P1 and the discrete-ordinates model (DOM) [4], where the radiation spectrum is divided into several gray bands. A constant Planck or Rosseland averaged absorption coefficient is derived from the spectral absorption coefficient. Instead of resolving the whole spectrum, the radiation transfer is solved for a small number of bands only, which does reduce

the computational effort dramatically while providing reasonable accuracy.

The selection of the number and location of the band limits has an influence on the achievable accuracy and it is not obvious how many bands are sufficient and where to place the bands across the spectrum. To address these questions, numerical optimization procedures have been developed to find an optimal number and distribution of the bands [5, 6]. In this paper, we apply the optimization procedure as introduced in [5, 7] to a specific mixture of SF₆ and copper vapor. Since the amount of metal vapor in the arc chamber or switchgear can vary from 0 to 100% in the real application, it is necessary to quantify the error that is introduced when using the same band limits for various mixture ratios. By using the same band intervals for different mixtures and solving radiation transfer for the full spectrum as well as for the identified bands, the error introduced by the band selection can be identified.

2. Optimization and verification

Planck averaging is used to calculate the mean absorption coefficient $k_{Planck,i}$ for each band i according to:

$$k_{Planck,i} = \frac{\int_{\nu_i}^{\nu_{i+1}} k_{\nu} B_{\nu} d\nu}{\int_{\nu_i}^{\nu_{i+1}} B_{\nu} d\nu}, \quad (1)$$

where k_{ν} is the spectral absorption coefficient and B_{ν} represents the black body radiative intensity. The lower and upper interval boundaries (band limits) are denoted by ν_i and ν_{i+1} respectively. We renormalize the line contribution of the spectral absorption coefficients according to [8] with a characteristic plasma absorption length $H = 3 \cdot R_p$, as suggested in [7]. The

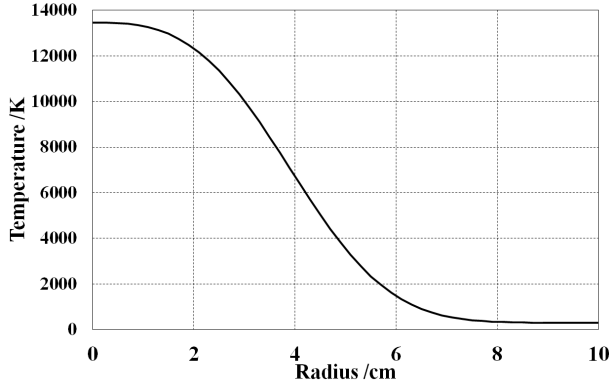


Figure 1. Assumed temperature profile.

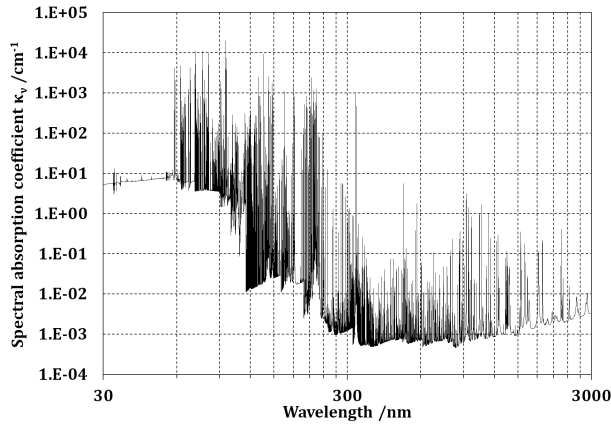


Figure 2. Spectral absorption coefficient for a 90% SF₆ and 10% Cu vapor mixture at 10 000 K and 1 atm.

plasma radius R_p is defined as radius value where the temperature decreased to 50% of the core temperature.

In our optimization procedure, we assume an infinitely long cylindrical calculation domain with 10 cm radius. The domain is evenly filled with a 90% SF₆ and 10% Cu vapor mixture at 1 atm pressure (all gas mixture proportions are noted as mass fractions in this paper). This mixture could be exemplary for the initial phase during an arc flash event in gas insulated medium voltage switchgear. The plasma inside this domain has a temperature distribution along the radius as shown in Fig. 1 with 13460 K maximum in the arc core and 300 K at the outside boundary. This temperature profile represents a plasma radius of 4 cm according above mentioned definition. The equilibrium species composition for this gas mixture and other mixtures of SF₆ and copper vapor in the temperature range of interest is calculated by minimization of Gibbs free energy. The species composition is the input for the calculation of the spectral absorption coefficients k_ν in a frequency range that covers 10^{12} Hz to 10^{16} Hz with frequency step size of $2 \cdot 10^{10}$ Hz. The calculated spectral absorption coefficient for a 90% SF₆ and 10% Cu vapor mixture at 10 000 K and 1 atm is shown in Fig. 2.

A baseline calculation of the radiation transfer re-

solving the whole spectrum is the first step in the optimization procedure (spectral solution [5]). This is a computationally expensive step because the radiation transfer needs to be solved for each of the approximately 500 000 points in the spectrum. A spatial discretization of 3.45 mm is used hereby (30 points). In the second step, the radiation transfer is solved for a fixed number of spectral bands using Planck mean absorption coefficients. The optimization algorithm varies the band limits in order to minimize the value of objective function ΔF_ν according to Eq. (2):

$$\Delta F_\nu = \frac{1}{30} \sqrt{\sum_{i=1}^{30} (\nabla \cdot F_{\text{exact},i} - \nabla \cdot F_{\text{mean},i})^2}, \quad (2)$$

where $\nabla \cdot F_{\text{exact},i}$ represents the divergence of the radiative flux calculated in the spectral solution for each point i and $\nabla \cdot F_{\text{mean},i}$ represents the divergence of radiative flux calculated using the line limited Planck mean absorption coefficients. This objective function describes the difference between the spectral and the band averaged solution, in particular the energy source/sink term due to radiation transfer. Alternatively, the radiative intensity could be used to define the objective function as well [6].

Based on the variation of ΔF_ν due to varying band limits, the procedure is able to determine an optimal set of band limits. Although the algorithm would be able to find a global minimum, we limit the computational time and abort the calculations if a defined maximum number of iterations is reached. This procedure is repeated for a different number of bands (2 to 6) to analyze the achievable accuracy when using smaller or larger number of bands. Since a larger number of bands increases the computational effort when solving radiation transfer in complex three dimensional CFD models, it is beneficial to use the smallest number of bands that still provides sufficient accuracy.

The optimization procedure delivers a set of band limits for the specified uniform gas mixture, but in realistic applications any mixture of gas and metal vapor could be present. Thus, we need to quantify the accuracy of the radiation transfer model using the band averaged absorption coefficients for various other mixtures. For this evaluation, the difference between spectral solution and band averaged solution is calculated using the 2D radiation transfer code RAT [4], which is enhanced to calculate radiation transfer resolving the whole spectrum or utilizing band averaged coefficients. In this verification step, the same temperature profile as shown in Fig. 1 is used with a spatial resolution of 100×100 points. A relative error norm $\Delta F'_\nu$ is used to quantify the difference between the two approaches [5], where the error is evaluated along the radius only:

$$\Delta F'_\nu = \sqrt{\frac{\sum_i (\nabla \cdot F_{\text{exact},i} - \nabla \cdot F_{\text{mean},i})^2}{\sum_i (\nabla \cdot F_{\text{exact},i})^2}}. \quad (3)$$

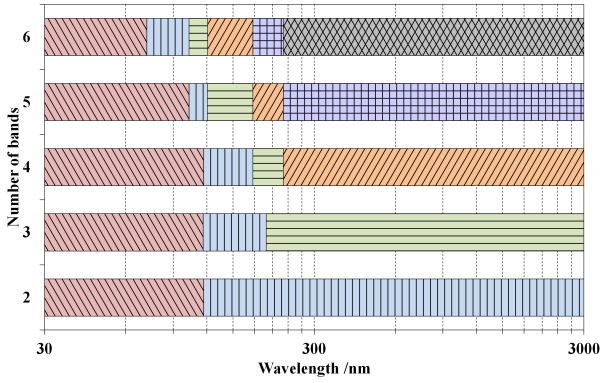


Figure 3. Optimized mean absorption coefficient band boundaries for 90% SF₆ – 10% Cu vapor mixture.

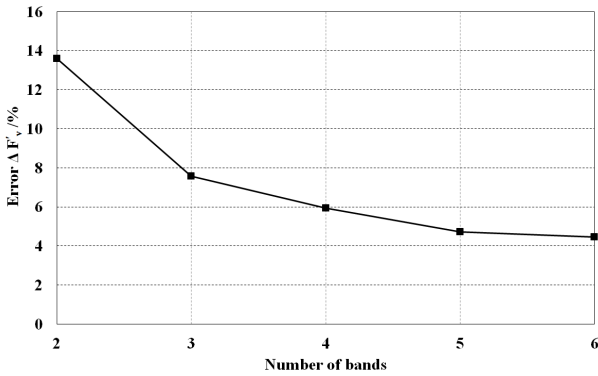


Figure 4. Error norm $\Delta F'_\nu$ for 90% SF₆ – 10% Cu vapor mixture.

3. Results

The optimization procedure results for 90% SF₆ – 10% Cu vapor mixture at 1 atm are shown in Fig. 3. The derived band boundaries for the different total number of bands are all located in UV region of the spectrum. In case 2 to 4 band numbers, the first interval boundary is located at around 116.5 nm. Also, in case of 5 or 6 bands, the same interval boundary is selected, but more intervals are introduced below and above that wavelength. Comparing the band boundaries with spectral absorption coefficient as shown in Fig. 2, the 116.5 nm interval boundary can be identified as larger change in the continuum contribution. The other interval boundaries cannot be associated with the spectral absorption coefficient that distinctively.

Corresponding error norm values according to Eq. 2 are provided in Fig. 4. The 2–band approximation has an error of 13.6 % which is decreased with increasing number of bands down to 4.4 % for the 6–band approximation. A larger number of bands does improve the accuracy of the radiation model as expected, but the difference between 5–band and 6–band approximation is only about 0.3 %. For this specific gas mixture and temperature profile, solving a 6–band model would not provide a benefit over a 5–band model.

The influence of copper vapor on the radiation trans-

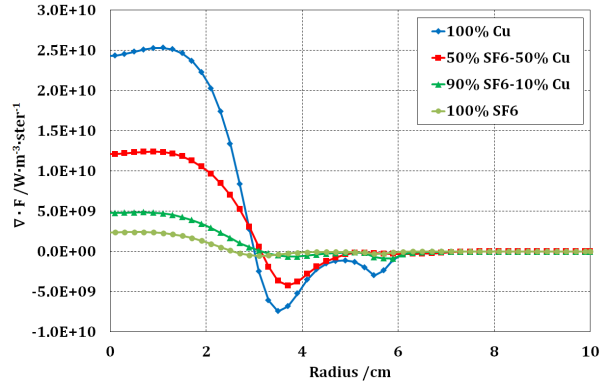


Figure 5. Divergence of radiative flux for various gas mixtures, spectral resolved model.

| Band | 90% SF ₆ 10% Cu | 50% SF ₆ 50% Cu | 100% SF ₆ | 100% Cu |
|--------|-------------------------------|-------------------------------|----------------------|---------|
| 6 band | 4.4% | 13.6% | 12.0% | 14.5% |
| 3 band | 8.1% | 14.9% | 11.9% | 16.4% |

Table 1. Error norm values for different mixtures and band approximations at 1 atm.

fer can be derived from Fig. 5. The divergence of the radiative flux along the domain radius as result of the spectrally resolved model is shown for these gases/gas mixtures: 100% SF₆, 90% SF₆ – 10% Cu, 50% SF₆ – 50% Cu, and 100% Cu. The divergence of radiative flux in the arc core is increased with larger copper vapor fraction. Compared to pure SF₆, the cooling term is increased by an order of magnitude for pure copper vapor, due to the presence of strong metallic lines in the spectrum.

The influence of different mixtures on the accuracy of the band averaged models is depicted in Fig. 6 for the same four gases/gas mixtures at 1 atm. Spectrally resolved model results as well as the 3–band and 6–band model results are provided, where band averaging is done using band limits according to Fig. 3 (optimized for 90% SF₆ – 10% Cu vapor mixture). In case of pure SF₆, the 3–band model overpredicts the emission by 13.7% ($\nabla \cdot F$ in the arc core), although the reabsorption in the fringe is represented well. This can lead to an overestimation of the radiation losses in an arc model later on. But in case of 50% copper vapor admixture, an underprediction of the reabsorption in the fringe of the temperature profile is noticeable. This can influence the temperature profile as result of an arc model. The error values as provided in Tab. 1 can be used to quantify the accuracy for the whole profile.

The 90% SF₆ – 10% Cu vapor mixture and 6–band approximation has the lowest error value of 4.4%, because this mixture was used to find optimal band boundaries. The error value has a maximum of 16.4% in case of pure copper vapor and 3–band approximation. Reducing the number of bands from 6 to 3

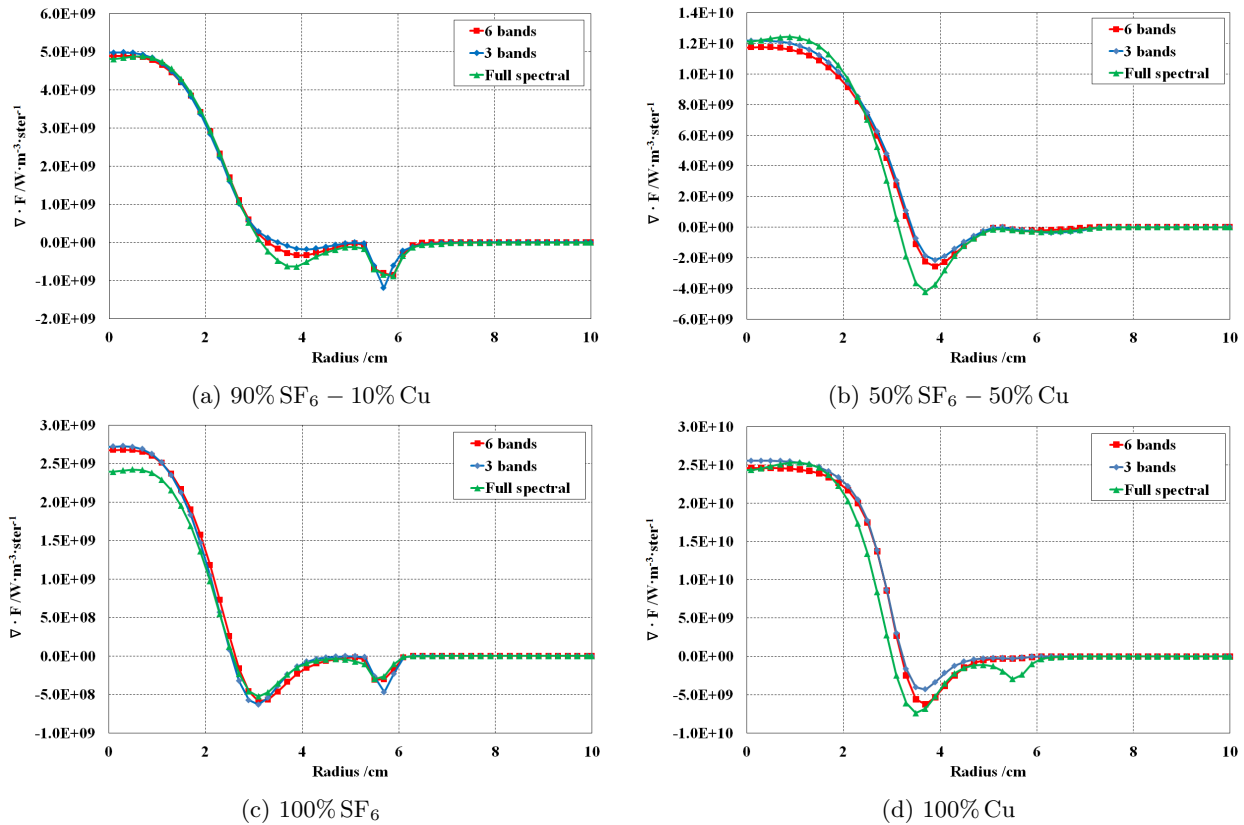


Figure 6. Divergence of radiative flux for different mixtures at 1 atm.

in case of 90% SF₆ – 10% Cu vapor mixture almost doubles the error from 4.4% to 8.1%, but the error resulting from different gas composition is larger. In this case, a larger number of bands does help to limit the error if gas mixture is varied as well.

4. Conclusions

Approximate radiation transfer models are used in arc modeling and band averaged absorption coefficients are necessary model input. We presented a numerical approach towards an optimal selection of band boundaries for the calculation of Planck mean absorption coefficients. The optimization was performed for a 90% SF₆ – 10% Cu vapor mixture and a defined temperature profile. The comparison between band averaged radiation model and spectral model results allows the quantification of error introduced by band averaging. Based on the results, it is feasible to perform the optimization for a 90% SF₆ – 10% Cu vapor mixture and apply the band limits to various mixtures of SF₆ and copper, at least for the investigated temperature profile. The influence of different temperature profiles is a topic for further investigations.

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