BENCHMARK EXAMPLE PROBLEMS FOR BEAMS
At Elevated Temperatures

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Abstract

The paper presents development of a series of solutions for beams at elevated temperatures which are supposed to serve as benchmark problems for applications of computational models in fire structural engineering. Three cases of loading i.e. pure bending, central force, and uniformly distributed loading, are considered for a simply supported, and fixed on both ends beams at uniformly distributed elevated temperature varying in time. The results are provided in terms of the midspan deflection for specified loading levels and temperatures. The results mainly obtained using finite element (FE) models and two commercial codes, are verified through comparison with analytical solutions for simplified cases and through parametric study aimed to examine the effect of modelling parameters. The numerical results are subjected to mesh density study using the grid convergence index (GCI) concept.

Keywords: beam, benchmark, fire, finite element, verification

INTRODUCTION

Nowadays verification and validation (V&V) is recognized as the primary method for evaluating the confidence of computer simulations (Oberkampf et al, 2004). V&V is especially important in the research areas where complex, highly nonlinear structural behaviour is considered. One of those is the structural fire engineering where interaction of additional effects due to elevated temperatures has to be considered such as reduction of material properties and generation of additional forces due to constrained thermal elongation.

Validation in the structural fire engineering through comparison between numerical results and experimental data obtained using furnace tests is difficult and has many limitations due to inevitable uncertainties characterising the specimen behaviour (Gillie, 2009). This fact enhances the importance of verification which is sought as a comparison of computational solutions with highly accurate (analytical or numerical) benchmark solutions and among themselves, for example using mesh density study (Santiago et al, 2009).

The benchmark solutions can serve for both code developers, to check the corrections of new features introduced in the code, and for code users who can check if their models are developed correctly. They should represent a good balance between simplicity and applicability. Simpler the problem considered, more reliable solution can be obtained, but too simplified problem can miss some important features which need to be taken into account in the study. On the other hand, more complex problem, less reliable solution can be provided. This fact is reflected often when different codes are used for the same problem (Santiago et al, 2009) or different modelling parameters such as finite element formulation, solving procedures, material models are used within the same code. To solve this dilemma a hierarchical approach is proposed in which a series of solutions is developed starting from the simplest cases towards more complex.

The paper presents application of such strategy to the problem of beams at elevated temperatures. The objective of the presented study is to provide not only solutions but also accumulate the evidence that they are correct.

1 PROBLEM DESCRIPTION

To define a family of cases characterizing the behaviour of beams at elevated temperatures, several parameters need to be identified as listed below. Some of them are taken from the existing studies (Gillie, 2009) and present a good balance between simplicity and correspondence with the real world. The stress is put on description of the numerical models to allow other users to follow calculations and check their simulations. For better comparison of results and for understanding the factors affecting differences between them and analytical solution, two commercial FE codes are used: ABAQUS, very commonly used in such cases (Santiago et al, 2009) and LS-DYNA, less often used in similar problems.

1.1 Geometry

Geometry means here a set of data defining all dimensions and shape of the beam. For all the solutions presented in this study a 1000x50x30 mm beam was considered. The ratio of length to depth is 20, so it is assumed that the shear force effect on deflection is negligible.

1.2 Material

Similar as in (Gillie, 2009), elastic-perfectly plastic material is considered, with the difference that here both elastic modulus and yield stress are temperature dependent, comparable to (Lin et al, 2010). Stress-strain relationship and temperature dependence are shown schematically in Fig. 1. Elastic modulus at 0°C is E_0=200 GPa and the corresponding yield stress σ_{y0}=200 MPa. Material model is simple enough to allow easy FE modelling but also reflects material properties of structural steel at elevated temperature. In all cases υ=0.3 and α_t=1.2·10^{-5} K^{-1}, which corresponds well with steel properties.

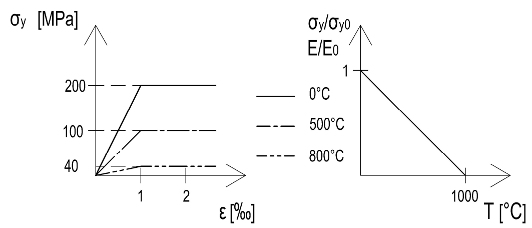

Fig. 1 Material model

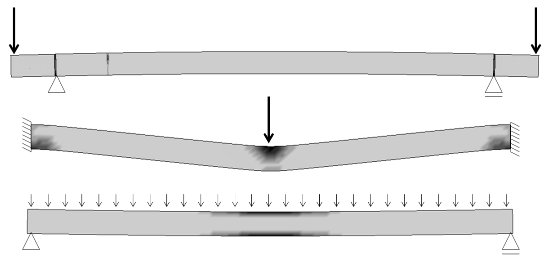

Fig. 2 Plastic regions, loading and boundary conditions – simplified drawings

1.3 Mechanical Loading

Three loading cases are considered: pure bending, central force, and uniformly distributed load, see Fig. 2. Pure bending is applied by four point bending test where two equal forces are applied at the top of cantilevers with bigger stiffness than the main beam. The central force is applied at the neutral axis of the beam, and uniformly distributed loading is applied as increased gravity. The first case is considered mainly to verify FE results with analytical solutions for elastic-plastic bending. This solution is also used for the mesh density study. The last case is more realistic and more related to experiments. The magnitude of the loads was chosen to produce bending moment of magnitude 700 Nm in the most stressed cross-section of simply supported beam. This lies between moments in which outer fibres yields (500 Nm) and whole section reaches plasticity (750 Nm) at 800°C, giving good elastic-plastic response of the beam. This gives force of 7000 N at the ends of 100 mm cantilevers in pure bending, 2800 N in point force load and equivalent of 5.6 N/mm distributed loading.

1.4 Boundary Conditions

Only two idealized boundary conditions are taken into account: a simply supported beam, where constraints are applied at the neutral axis, and a beam fully fixed on both ends. In the presented study only planar bending is considered with symmetry constraints (transverse y-displacement constrained) applied to all nodes in the vertical symmetry plane.

1.5 Temperature variation

Uniformly distributed temperature within the beam (the main span) is assumed. For FE mesh studies with constant temperature, the temperature effect is modelled directly through variation of the material properties and thermal expansion is not taken into account. At the beginning of the simulations the loading is applied gradually and then it is kept constant. For temperature varying in time, the simulations are divided into three steps. In the first step temperature 0°C is constant while the mechanical loading grows from zero to the full magnitude. In next two phases the loading is kept constant and the temperature grows linearly to 800°C and back to 0°C,(Gillie, 2009). All calculations are static, without inertia effects, and with time serving as nonphysical loading parameter.

1.6 FE Meshes

Only 3D meshes, as most general, are considered. It is supposed that FE models built of solid elements are able to capture more comprehensively local effects, and eventual deviation of plane cross sections (deplanation) due to shear deformation. Dimensions of FEs for three subsequent meshes used for the mesh density study are 16.67x12.5x7.5, 8.33x6.25x3.75 and 4.17x3.125x1.185 mm. Each of three meshes is built of the same elements with denser meshes generated through dividing each edge in two (one solid is divided into eight).

2 ANALYTICAL SOLUTIONS

For validation of FE models, analytical solutions for simply supported beams under pure bending, point force and uniformly distributed loading were obtained below.

2.1 Pure bending

Analytical solutions are obtained based on the assumptions that cross sections stay planar and the effect of shear is neglected. It is also assumed that the approximate formula (second derivative) for the curvature can be applied to find beam deflection. For elastic-ideal plastic material with yield stress σ_y the bending moment can be given by

$$M = \sigma_y b\left(\frac{h^2}{4} - \frac{\sigma_y^{\ 2}}{3E^2}\rho^2\right) \text{if } \sigma_y \frac{bh^2}{6} \le M \le \sigma_y \frac{bh^2}{4} \tag{1}$$

where ρ is the radius of curvature which can be approximated by

$$\frac{1}{\rho} = w''(x), \quad w(0) = 0, \quad w(l) = 0 \tag{2}$$

To obtain displacement function $w(x)$ it is necessary to solve differential *Eq. (2)* for $M = const$. The maximum deflection f in the midspan is

$$f = \frac{1}{8\sqrt{3}} \frac{\sigma_y l^2}{Eh}\left(\frac{1}{4} - \frac{1}{6}\mu\right)^{-1/2} \text{ and } \mu = \frac{M}{\sigma_y \frac{bh^2}{6}} \tag{3}$$

2.2 Concentrated force

For point load *Eqs. (1)*and *(2)* are still valid. Maximum bending moment $M(x)$ and the maximum elastic moment M_s (at the cross-section $x = x_0$) are given as

$$M(x) = \frac{P}{2}x\,,\ M_s = \frac{\sigma_y bh^2}{6} \quad \text{and} \quad M_s = \frac{P}{2}x_0 \tag{4}$$

Beam deflection shape $w(x)$ is divided into $w_1(x)$ for $x \le x_0$ (elastic behaviour of the cross-section) and $w_2(x)$ for $x_0 \le x \le l/2$ (elastic-perfectly plastic behaviour of the cross-section). Parameter x_0 results from *Eqs.(4.2)* and *(4.3)*.Displacement function $w(x)$ is obtained after solving *Eq. (5)*.

$$\begin{aligned} &-EIw_1''(x) = \frac{P}{2}x, \quad w_2''(x) = \frac{1}{\rho} = \frac{\sigma_y}{\sqrt{3}Eh}\left(\frac{1}{4} - \frac{P}{2\sigma_y bh^2}x\right)^{-1/2}, \\ &w_1(0) = 0, \quad w_1(x_0) = w_2(x_0), \quad w_1'(x_0) = w_2'(x_0) \quad w_2'\left(\frac{l}{2}\right) = 0 \end{aligned} \tag{5}$$

Maximum deflection of the beam can be written as *Eq. (6)*.

$$f = \frac{1}{6}\frac{\sigma_y l^2}{Eh}\frac{3\sqrt{3-2\psi} + \psi\sqrt{3-2\psi} - 5}{\psi^2} \quad \text{and} \quad \psi = \frac{P}{\frac{2}{3}\frac{\sigma_y bh^2}{l}} \tag{6}$$

3 MESH DENSITY STUDY

For the mesh density study, the maximum deflection for cases with pure bending at 0°C and 800°C was used. The procedure called Grid Convergence Index GCI (Slater, 2008), (Kwasniewski, 2013) was applied. Using concept of Richardson extrapolation, the order of convergence and asymptotic solution is found based on results obtained from three subsequent meshes. The meshes are constructed with a constant grid refinement ratio $r = 2$

$$r = \frac{h_3}{h_2} = \frac{h_2}{h_1} = const \tag{7}$$

where h_1, h_2, h_3 are measures of mesh size (e.g. the largest element egde) and $h_1 < h_2 < h_3$. In this paper the convergence rate, given as (Slater, 2008)

$$p = \frac{\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right)}{\ln(r)} \tag{8}$$

where f_1, f_2, f_3 are the results from three subsequent meshes. Next, the asymptotic solution is obtained as

$$f_{h=0} \cong f_1 + \frac{f_1 - f_2}{r^p - 1} \tag{9}$$

The GCI is defined as (Slater, 2008):

$$GCI = \frac{F_s|\varepsilon|}{r^p - 1}100\% \tag{10}$$

where $F_s = 1$ is a safety factor, and ε defines relative difference between subsequent solutions

$$\varepsilon = \frac{f_1 - f_2}{f_1} \tag{11}$$

As can be seen (Tab. 1) a good convergence and correlation with analytical solutions are obtained for constant 0°C, 800°C (no thermal expansion) and for temperature varying from 0 to 800°C and deflection recorded at 800°C. For all next cases, the finest mesh is applied.

Tab. 1 GCI results for simply supported beam with pure bending

Solver	Temperature [°C]	Result [mm]			p	$f_{h=0}$ [mm]	GCI_{12}	GCI_{23}	$\frac{GCI_{23}}{r^p GCI_{12}}$
		f_3	f_2	f_1					
ABAQUS	0constant	1.367	1.391	1.398	1.778	1.401	0.206	0.710	1.005
	800constant	9.256	10.490	10.970	1.362	11.276	2.786	7.489	1.046
	800 varying	8.478	9.636	9.963	1.824	10.092	1.292	4.729	1.034
LS-DYNA	0constant	1.369	1.393	1.400	1.778	1.403	0.206	0.709	1.005
	800constant	9.640	11.023	11.818	0.799	12.893	9.095	16.963	1.072
	800 varying	8.384	9.228	9.814	0.526	11.145	13.562	20.774	1.064

4 NUMERICAL RESULTS FOR BEAMS UNDER FIRE

Tab. 2 Deflection of simply supported beam with pure bending under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.400	1.750	2.800	3.500	4.667	11.180	-	-	-	-	-
ABAQUS	1.398	1.747	2.796	3.495	4.569	9.963	7.635	6.469	5.770	4.727	4.372
LS-DYNA	1.400	1.743	2.773	3.461	4.609	9.814	7.512	6.358	5.667	4.626	4.275

Tab. 3 Deflection of simply supported beam with concentrated force under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	0.933	1.167	1.866	2.333	3.111	4.667	-	-	-	-	-
ABAQUS	0.940	1.174	1.882	2.352	3.136	5.074	3.507	2.723	2.253	1.548	1.312
LS-DYNA	0.940	1.172	1.869	2.333	3.108	5.008	3.462	2.688	2.223	1.526	1.293

Tab. 4 Deflection of fixed beam with concentrated force under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
ABAQUS	0.239	9.693	29.964	36.008	41.846	47.716	44.456	40.732	36.669	24.472	16.235
LS-DYNA	0.239	9.667	29.877	35.915	41.752	47.627	44.372	40.656	36.608	24.491	16.359

Tab. 5 Deflection of simply supported beam with distributed loading under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
Analytical	1.167	1.458	2.333	2.917	3.889	5.833	-	-	-	-	-
ABAQUS	1.172	1.466	2.347	2.934	3.912	7.019	5.065	4.088	3.501	2.621	2.328
LS-DYNA	1.173	1.462	2.331	2.911	3.877	6.920	4.991	4.024	3.444	2.571	2.279

Fig. 3-7 present numerical and analytical results for five selected cases: simply supported beam subjected to four point bending, simply supported and fixed beams subjected to concentrated force and uniformly distributed loading. For all cases the time variation of temperature defined in Section 1.5 is considered. Deflection f is shown in the relation to the displacement f_0 at 0°C, which makes variation of material properties under temperature more visible.

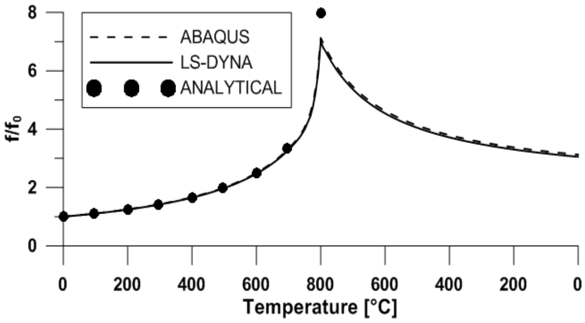

Fig. 3 Deflection of simply supported beam under pure bending and elevated temperature

Tab. 6 Deflection of fixed beam with distributed loading under fire [mm]

	0°C	200°C	500°C	600°C	700°C	800°C	700°C	600°C	500°C	200°C	0°C
ABAQUS	0.239	9.649	29.946	35.996	41.839	47.714	44.449	40.734	36.669	24.386	16.060
LS-DYNA	0.239	9.607	29.810	35.840	41.664	47.514	44.261	40.555	36.503	24.311	16.110

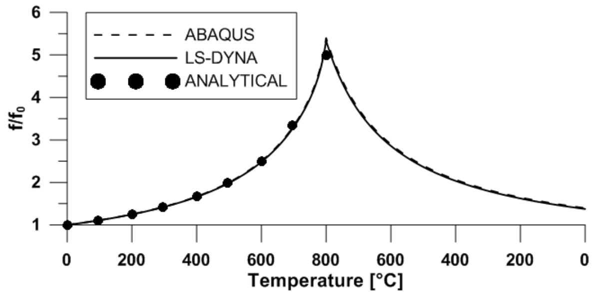

Fig. 4 Deflection of simply supported beam under concentrated force and elevated

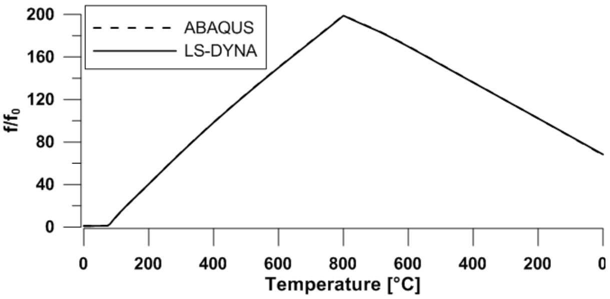

Fig. 5 Deflection of fixed beam under concentrated force and elevated temperature

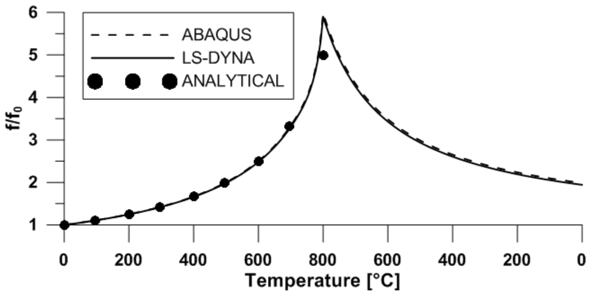

Fig. 6 Deflection of simply supported beam with distributed load and elevated temperature

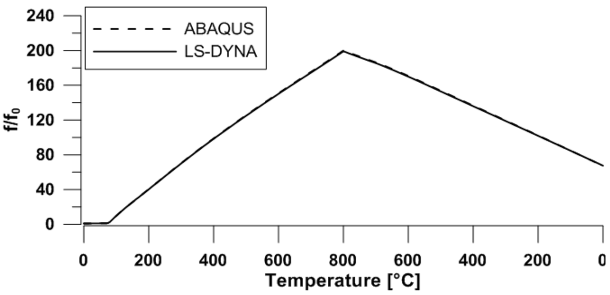

Fig. 7 Deflection of fixed beam under distributed load and elevated temperature

5 SUMMARY

A series of solutions for beams at elevated temperatures are presented as benchmark problems for applications of computational models in fire structural engineering. Presented numerical solutions have been verified through comparison with analytical solutions for limited number of simple cases and through comparison of the numerical results obtained using two FE codes. The mesh density study based on the grid convergence index (GCI) concepts is also presented for beams subjected to pure bending. Comparison of the results shows very good correlation for elastic range however, when plastic deformation is present there is a clear difference between FE and analytical solutions.

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