INTERACTIVE SHEAR RESISTANCE OF CORRUGATED WEB IN STEEL BEAM EXPOSED TO FIRE

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Abstract

The design approach to shear buckling resistance evaluation for corrugated web being a part of a steel beam exposed to fire is presented and discussed in detail. It is based on the experimentally confirmed interaction between the local and global elastic instability failure modes as well as on the possible yielding of the whole web cross – section during fire. Conclusively, the new formulae, adequate for specification of the suitable shear buckling coefficients depend not only on the web slenderness but also on the temperature of structural steel. The methodology proposed by the authors can be added to the current European standard recommendations given in EN 1993-1-2 as a well-justified design algorithm helpful in reliable evaluation of safety level for steel beams with slender corrugated webs subject to fire exposure. It seems to be highly desirable, because at present there are no detailed instructions in this field.

Keywords: steel beam, corrugated web, shear buckling coefficient, fire, interaction formulae.

INTRODUCTION

It is a common knowledge that in the case of a steel beam with corrugated web such web fails due to shear buckling or yielding while its flanges resist the whole bending moment. Therefore, it is usually assumed that the considered web can carry only shear forces applied to the beam and estimation the value of its shear resistance V_{Rd} becomes the main goal of many analytical models. According to the approach given in EN 1993-1-5 the design value of this resistance is assessed by the specification of the effective shear buckling coefficient χ_c:

$$V_{Rd} = \chi_c h_w t_w \frac{f_{yw}}{\sqrt{3}\gamma_{M1}} = \chi_c V_{R,pl,d} \tag{1}$$

In this formula h_w and t_w are the web height and its thickness respectively, f_{yw} is the yield point of the steel the web is made of and γ_{M1} is the suitable partial safety factor. It is important that:

$$\chi_c = min(\chi_{cL}, \chi_{cG}) \tag{2}$$

where the first coefficient χ_{cL} is connected with the local shear buckling failure mode (with buckled areas limited to the single web panels or half-waves), whereas the second one χ_{cG} deals with the global shear buckling failure mode. To evaluate the values of such both instability coefficients the web relative slendernesses, $\bar{\lambda}_{cL}$ and $\bar{\lambda}_{cG}$, are defined as follows:

$$\bar{\lambda}_{cL} = \sqrt{\frac{\tau_{pl}}{\tau_{cr,L}}} = \sqrt{\frac{f_{yw}}{\sqrt{3}\tau_{cr,L}}} \quad \text{and} \quad \bar{\lambda}_{cG} = \sqrt{\frac{\tau_{pl}}{\tau_{cr,G}}} = \sqrt{\frac{f_{yw}}{\sqrt{3}\tau_{cr,G}}} \tag{3}$$

Finally:

$$\chi_{cL} = \frac{1{,}15}{0{,}9 + \bar{\lambda}_{cL}} \le 1 \qquad \text{and} \qquad \chi_{cG} = \frac{1{,}5}{0{,}5 + \bar{\lambda}_{cG}^{2}} \le 1 \tag{4}$$

1 INTERACTIVE SHEAR BUCKLING COEFFICIENT

The standard approach presented above seems to be very simple and easy to use; however, it is necessary to say that it is not fully correct in mathematical sense. Let in random realisation the resistance $V_R = min(V_{RL}, V_{RG})$ be the random variable, usually being compared with the associated shear force treated as the concurrent random action effect. Such assumption does not lead to the similar conclusion, dealing with the suitable design values, that $V_{Rd} = min(V_{RL,d}, V_{RG,d})$, suggested by the Eurocode. In this paper the authors propose to replace the classical instability factor χ_c (taken from Eq. (2)) by another factor, named the equivalent interactive shear buckling coefficient $\chi_{c,int}$. It should give the web shear buckling resistance assessment being significantly more precise because its defining formula is based on the experimentally confirmed interactive relations between the potential elastic – plastic shear stresses. There are a lot of various interactive relations between $\tau_{cr,L} - \tau_{cr,G} - \tau_y$ (or only between $\tau_{cr,L} - \tau_{cr,G}$), proposed by many authors (Eldib, 2009). In further analysis two most popular of them are considered in detail. The first one is written as follows:

$$\frac{1}{(\tau_{int})^n} = \frac{1}{(\tau_{cr,L})^n} + \frac{1}{(\tau_{cr,G})^n} + \frac{1}{(\tau_y)^n} \tag{5}$$

The exponent n is here most frequently adopted as $n = 2$ (El-Metwally, 1998) or even as $n = 3$ (Sayed-Ahmed, 2001). Let us notice that in case when only interaction between global and local elastic instability failure modes is considered, whereas the influence of web yielding is neglected, this formula is shortened to the following form:

$$\frac{1}{(\tau_{cr,int})^n} = \frac{1}{(\tau_{cr,L})^n} + \frac{1}{(\tau_{cr,G})^n} \tag{6}$$

in which also various values of exponent n can be used, particularly $n = 1$ (Bergfelt) (Driver et al., 2006), $n = 2$ (Abbas) (Abbas et al., 2002) and $n = 4$ (Hiroshi) (Hiroshi et al., 2003). Multiplication of both sides of Eq. (5) by $(\tau_y)^n$ gives:

$$\frac{(\tau_y)^n}{(\tau_{int})^n} = \frac{(\tau_y)^n}{(\tau_{cr,L})^n} + \frac{(\tau_y)^n}{(\tau_{cr,G})^n} + 1 \tag{7}$$

Let $\chi_{c,int} = \tau_{int} / \tau_y$, then:

$$\left(\frac{1}{\chi_{c,int}}\right)^n = \bar{\lambda}_{cL}^{2n} + \bar{\lambda}_{cG}^{2n} + 1 \tag{8}$$

Consequently:

$$\chi_{c,int} = \left(\bar{\lambda}_{cL}^{2n} + \bar{\lambda}_{cG}^{2n} + 1\right)^{-(1/n)} \tag{9}$$

Similarly, starting from Eq. (6) one have obtained:

$$\chi_{c,int} = \left(\overline{\lambda}_{cL}^{2n} + \overline{\lambda}_{cG}^{2n} \right)^{-(1/n)} \tag{10}$$

2 GENERALIZATION OF THE STANDARD APPROACH TO THE FIRE CASE

To evaluate considered web shear resistance under fully developed fire conditions not only the yield point reduction specified for steel the beam is made of must be regarded through the substitution in Eq. (1) the value f_{yw} by the product $f_{yw,\Theta} = k_{y,\Theta} f_{yw}$ (where reduction factors $k_{y,\Theta} = f_{y,\Theta} / f_y$ are given in EN 1993-1-2 for particular values of material temperature Θ_a) but also the reliable specification of the relation $\chi_{c,\Theta} = \chi_c(\Theta_a)$ should be made and effectively applied to the analysis. Moreover, the suitable partial safety factor have to be changed, from γ_{M1} into $\gamma_{M,fi}$, however in practice such conversion does not give any quantitative difference because in the standard EN 1991-1-2 it is suggested to be used $\gamma_{M,fi} = 1{,}0$. Conclusively, Eq. (1) is rearranged to the form:

$$V_{Rd,\Theta} = \chi_{c,\Theta} V_{R,pl,d,\Theta} \frac{\gamma_{M1}}{\gamma_{M,fi}} = \chi_{c,\Theta} h_w t_w \frac{k_{y,\Theta} f_{yw}}{\sqrt{3}\gamma_{M,fi}} \tag{11}$$

To study the influence of fire temperature on instability coefficients it is convenient to rewrite Eqs. ($2 \div 4$) into the alternative formulae being easier for interpretation:

$$\chi_c = \chi_{cL} \qquad \text{if} \qquad \overline{\lambda}_{cG} < \overline{\lambda}_c^* = \sqrt{0{,}674 + 1{,}304\overline{\lambda}_{cL}} \tag{12}$$

$$\chi_c = \chi_{cG} \qquad \text{if} \qquad \overline{\lambda}_{cG} \geq \overline{\lambda}_c^* = \sqrt{0{,}674 + 1{,}304\overline{\lambda}_{cL}} \tag{13}$$

The resultant dependence obtained for persistent design situation (without any fire influence), between the coefficient χ_c and the suitable relative web slendernesses, $\overline{\lambda}_{cL}$ and $\overline{\lambda}_{cG}$, is shown in detail in Fig. 1a.

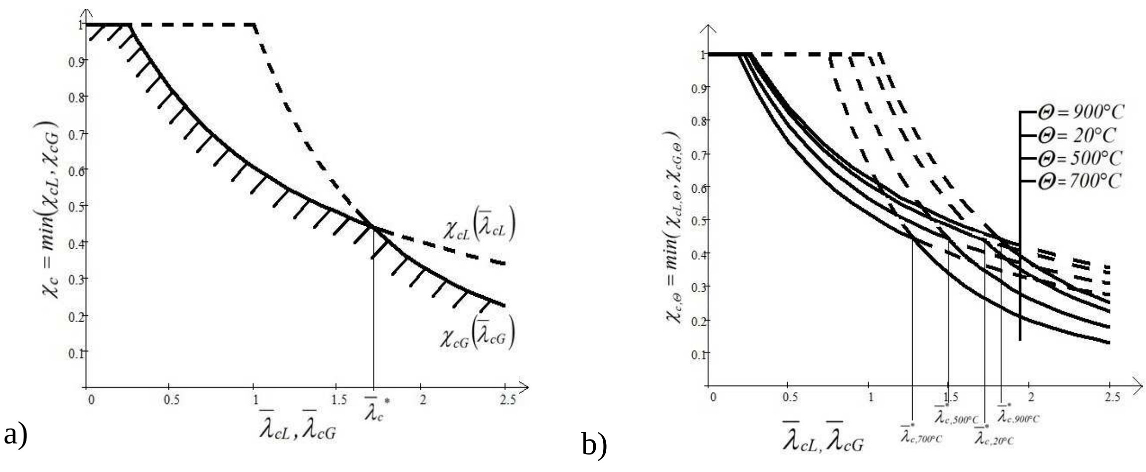

Fig. 1 Relations between shear buckling coefficient χ_c and relative slendernesses $\overline{\lambda}_{cL}$ and $\overline{\lambda}_{cG}$ a) in persistent design situation - according to Eq. (4), b) under fire conditions - according to Eq. (22).

In further analysis relations between the steel temperature Θ_a and the ultimate critical shear stresses are examined, both for local $\tau_{cr,L,\Theta}$ and for global $\tau_{cr,G,\Theta}$ instability failure modes. Denotations applied in considered formulae, describing the corrugated web geometry, are illustrated in detail in Fig. 2. Taking from the standard EN 1993-1-5 one can obtain:

$$\tau_{cr,L} = 4{,}83 E_a \left(\frac{t_w}{a_{max}} \right)^2 \quad \text{where} \quad a_{max} = max(a,c) \tag{14}$$

It is easy to show that, for fire conditions:

$$\tau_{cr,L,\Theta} = k_{E,\Theta} \tau_{cr,L} \quad \text{where} \quad k_{E,\Theta} = E_{a,\Theta} / E_a \tag{15}$$

Values of the reduction factor $k_{E,\Theta}$ are also collected in the standard EN 1993-1-2.

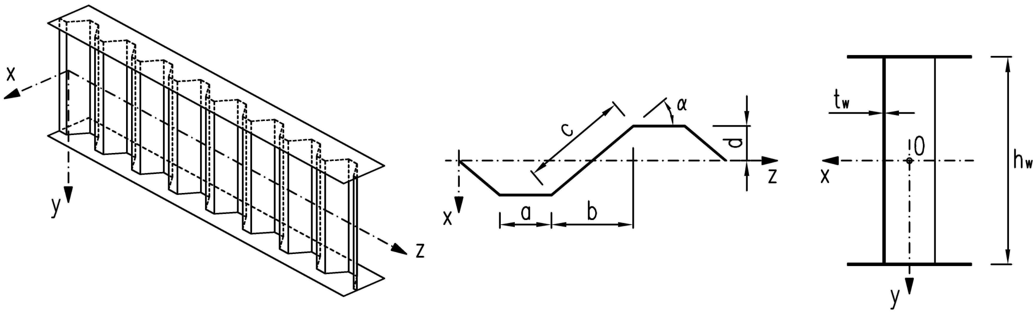

Fig.2. Interpretation of the web dimensions applied to Eqs. (14, 16 and17).

Similarly, on the base of EN 1993-1-5 occurs:

$$\tau_{cr,G} = \frac{32{,}4}{t_w h_w^2} \sqrt[4]{D_x D_z^3} \tag{16}$$

where:

$$D_x = \frac{a+b}{a+b \cdot sec(\alpha)} \frac{E_a t_w^3}{12(1-\nu^2)} \quad \text{and} \quad D_z = \frac{E_a}{a+b} \left(\frac{a t_w (b \cdot tan(\alpha))^2}{4} + \frac{t_w (b \cdot tan(\alpha))^3}{12 \cdot sin(\alpha)} \right) \tag{17}$$

which means that:

$$\tau_{cr,G,\Theta} = k_{E,\Theta} \tau_{cr,G} \tag{18}$$

Let us notice that both Eq. (15) and Eq. (18) are fully adequate not only in relation to trapezoidally but also to sinusoidally corrugated web.

Identification of the relations presented above allows to generalize the definition of the suitable relative web slendernesses, previously specified by Eq. (3). They are now expressed by the following formulae:

$$\bar{\lambda}_{cL,\Theta} = \sqrt{\frac{k_{y,\Theta} f_{yw}}{\sqrt{3} k_{E,\Theta} \tau_{cr,L}}} = \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}} \bar{\lambda}_{cL} \quad \text{and} \quad \bar{\lambda}_{cG,\Theta} = \sqrt{\frac{k_{y,\Theta} f_{yw}}{\sqrt{3} k_{E,\Theta} \tau_{cr,G}}} = \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}} \bar{\lambda}_{cG} \tag{19}$$

In consequence, Eqs. 12 and 13 are changed into the form:

$$\chi_{c,\Theta} = \chi_{cL,\Theta} \quad \text{when} \quad \bar{\lambda}_{cG} < \bar{\lambda}_c^* = \frac{k_{E,\Theta}}{k_{y,\Theta}} \sqrt{0{,}674 + 1{,}304 \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}} \bar{\lambda}_{cL}} \tag{20}$$

$$\chi_{c,\Theta} = \chi_{cG,\Theta} \quad \text{when} \quad \bar{\lambda}_{cG} \geq \bar{\lambda}_c^* = \frac{k_{E,\Theta}}{k_{y,\Theta}} \sqrt{0{,}674 + 1{,}304 \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}} \bar{\lambda}_{cL}} \tag{21}$$

Finally, having substituted both above equations to Eq. (4), the relations between the instability coefficients and the relative web slendernesses, adequate for fire conditions, can be obtained. They are given as follows:

$$\chi_{cL,\Theta} = \frac{1{,}15}{0{,}9 + \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}}\,\overline{\lambda}_{cL}} \leq 1 \qquad \text{and} \qquad \chi_{cG,\Theta} = \frac{1{,}5}{0{,}5 + \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}\,\overline{\lambda}_{cG}^{2}}} \leq 1 \tag{22}$$

and both are illustrated in Fig. 1b for selected values of steel temperature Θ_a.

3 ALTERNATIVE SHEAR BUCKLING COEFFICIENTS SPECIFIED FOR FIRE CONDITIONS

Regarding an alternative design approach, proposed by the authors (Maślak, Łukacz, 2012) and dealing with the concept of interactive shear buckling coefficients, suitable generalization of the factor defined by Eq. (9), or in the simpler version by Eq. (10), should be made, with respect to the solution obtained by Eq. (19). Consequently, Eq. (9) can be rearranged to the following form:

$$\chi_{c,int,\Theta} = \left(\sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}}^{\,2n} \overline{\lambda}_{cL}^{2n} + \sqrt{\frac{k_{y,\Theta}}{k_{E,\Theta}}}^{\,2n} \overline{\lambda}_{cG}^{2n} + 1 \right)^{-(1/n)} \tag{23}$$

This formula is possible to be written in a simpler way:

$$\chi_{c,int,\Theta} = \frac{k_{E,\Theta}}{k_{y,\Theta}} \left[\overline{\lambda}_{cL}^{2n} + \overline{\lambda}_{cG}^{2n} + \left(\frac{k_{E,\Theta}}{k_{y,\Theta}} \right)^{n} \right]^{-(1/n)} \tag{24}$$

Similarly, considering of Eq. (10) leads to the conclusion that:

$$\chi_{c,int,\Theta} = \frac{k_{E,\Theta}}{k_{y,\Theta}} \left[\overline{\lambda}_{cL}^{2n} + \overline{\lambda}_{cG}^{2n} \right]^{-(1/n)} \tag{25}$$

4 CONCLUDING REMARKS

Both Eq. (22) and Eq. (24) (or alternatively Eq. (25)) give the opportunity to study the beam corrugated web behaviour under fire conditions. In general, the value of the shear buckling coefficient, both $\chi_{c,\Theta}$ and $\chi_{c,int,\Theta}$, decreases when the steel temperature Θ_a grows; however, this comment is not accurate when the web temperature is very high ($\Theta_a \cong 900\ ^{\mathrm{o}}\mathrm{C}$), because the inequality $k_{y,\Theta} < k_{E,\Theta}$ occurs in such circumstances. Nevertheless, the considered web shear resistance is monotonically diminishing in the whole time of fire duration. This effect is not very intense if $\Theta_a \leq 400\ ^{\mathrm{o}}\mathrm{C}$, because then $k_{y,\Theta} = 1{,}0$, but it is significantly strengthened when the web temperature becomes higher.

The alternative approach proposed by the authors to the assessment of the value of shear buckling coefficient specified for fire conditions, in which the experimentally confirmed failure modes interaction formula is taken into account, seems to be more satisfactory and better justified in relation to the commonly used classical standard design technique, when the global and local buckling modes are examined separately.

Detailed relations being the result of application of the formulae recommended in the presented paper are shown in the diagrams completed below (Figs. 3 and 4).

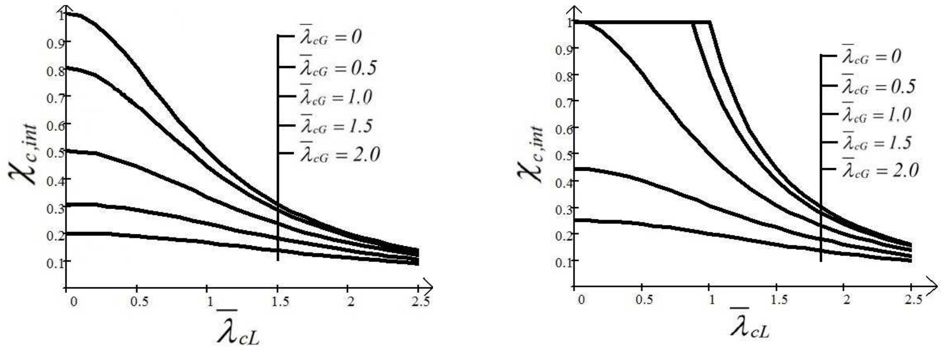

Fig. 3. Relations between the coefficient $\chi_{c,int}$ and the relative slenderness $\bar{\lambda}_{cL}$ resulted from the application of Eq. (9) - in the left side, and Eq. (10) - in the right side.

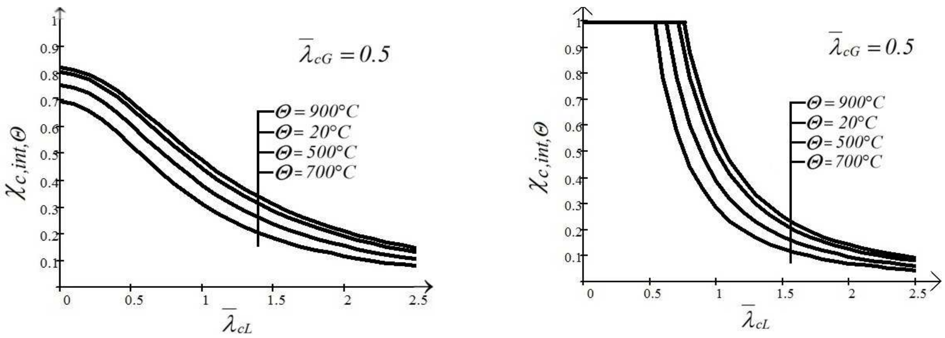

Fig. 4**.** Relations between the coefficient $\chi_{c,int,\Theta}$ and the relative slenderness $\bar{\lambda}_{cL}$, obtained for selected values of web temperature Θ_a, resulted from the application of Eq. (24) – in the left side, and Eq. (25) – in the right side (it is assumed that $\bar{\lambda}_{cG} = 0.5$).

Suitable relations prepared for fixed value of $\bar{\lambda}_{cL}$ and being the function of $\bar{\lambda}_{cG}$ are similar to those, illustrated in the diagrams presented above.

REFERENCES

Eldib M.E.A.-H., Shear buckling strength and design of curved corrugated steel webs for bridges, Journal of Constructional Steel Research, 65, 2009,

El-Metwally A.S., Pre-stressed composite girders with corrugated steel webs, Thesis for degree of master, University of Calgary, Calgary, Alberta, Canada, 1998,

Sayed-Ahmed E.Y., Behaviour of steel and composite girders with corrugated steel webs, Canadian Journal of Civil Engineering, 28, 2001,

Driver R.G., Abbas H.H., Sause R., Shear behaviour of corrugated web bridge girder, Journal of Structural Engineering, 132(2), 2006,

Abbas H.H., Sause R., Driver R.G., Shear strength and stability of high performance steel corrugated web girders, Proceedings of Structural Stability Research Council Conference, Seattle, Washington, USA, 2002,

Hiroshi Shiratoni, Hiroyuki Ikeda, Yohiaki Imai, Koichi Kano, Flexural shear behaviour of composite bridge girder with corrugated steel webs around middle supports, Japan Society of Civil Engineers J, 724(I-62), 2003,

Maślak M., Łukacz M., Shear buckling resistance of steel beam with trapezoidally corrugated web exposed to fire, Proceedings of International Jubilee Conference UACEG 2012: Science & Practice, University of Architecture, Civil Engineering and Geodesy, Sofia, Bulgaria, November 15-17, 2012.