STOCHASTIC FINITE ELEMENT METHODS FOR THE RELIABILITY-BASED FIRE-RESISTANT DESIGN OF STRUCTURES

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Abstract

A reliability-based design methodology is needed to reconcile the uncertainty and ensure a consistent level of safety is provided in engineered structural fire designs. This paper presents the application of two stochastic finite element methods, namely the First- and Second-Order Reliability Methods and Monte Carlo Simulation, to the design of structures subjected to fire. An example of a protected steel column subjected to natural fire is presented. A numerical investigation of the evolution of the failure probability with time demonstrates that analytical reliability methods improve the efficiency of the simulation, although significant errors arise when treating the fuel load as a random parameter. Further analysis reveals a "kink" in the response surface due to the lack of sensitivity to the fuel load during the heating phase of fire development. Utilization of an alternative fire model overcomes this limitation.

Keywords: stochastic finite element method, first-order reliability method, Monte Carlo simulation, structural reliability

INTRODUCTION

The primary objective of engineering design is to produce a system that has strength that exceeds the load demand. However, both the strength and demand of a system naturally exhibit a large amount of randomness. In the fire-resistant design of building structures, the fuel load density, thermal and mechanical properties of materials, and mechanical loads are random in time, leading to a considerable amount of uncertainty in the structural response. Safety factors are often used in engineering design to limit the failure probability in light of uncertainty. Although such design philosophies generally lead to an acceptable level of safety and are easy to implement due to the straightforward manner of the design, they are only the first level of reliability-based design, i.e., the randomness has been taken into account but the reliability of the system is not explicit quantified.

In recent decades, with major developments in the field of structural fire engineering, there is a tendency to replace current prescriptive codes with performance-based design codes. The performance-based design focuses on meeting target levels performance for given design events, thus encouraging engineers to apply new materials and technologies to achieve solutions that are beyond the prescriptive codes. In performance-based design, reliability evaluation plays an important role in ensuring that the limit state requirements are achieved with an acceptable level of safety. A number of researchers have conducted reliability analyses and safety assessments for structures exposed to fire in recent years (Magnusson and Petterson 1980/81; Mehaffey and Harmathy 1984; Beck, 1985; Shetty et al., 1998; Fellinger and Both, 2000; Iqbal and Harichandran, 2010, 2011; Khorasani et al., 2012; Guo et al., 2012; Guo and Jeffers, 2012)

This paper presents the application of two stochastic finite element methods, namely the First- and Second-Order Reliability Methods (FORM/SORM) and Monte Carlo Simulation (MCS), to evaluate the reliability of a column that was designed according to the prescriptive code in the U.S. A comparison was conducted between the analytical reliability methods (FORM/SORM) and Monte Carlo Simulation. The SORM shows a potential to evaluate the reliability problem of a nonlinear limit state curve with improved accuracy and computational efficiency. However, the following study demonstrates that challenges arise when treating the fuel load density as a random parameter for a parametric fire model (e.g., the Eurocode parametric fire) that is not sensitive to the fuel load during the heating phase.

1 METHODS OF ANALYSIS

Three sequentially coupled processes are needed to simulate the structural response under fire: (1) a fire simulation (i.e., parametric fire curve, zone model, or computational fluid dynamics model) to determine the thermal boundary conditions at the structural surface, (2) a heat transfer analysis to determine the temperatures within the structure under the specified boundary conditions, and (3) a structural analysis to determine the mechanical response of the structure. Uncertain parameters appear in each domain, and all of them will affect the final structural response due to the coupling of the various models.

Monte Carlo Simulation (MCS) is the most popular sampling method to evaluate the reliability of structures in fire. It is a versatile tool that can account for uncertainty in any number of parameters as well as the coupling between various domains. However, MCS requires a large sample of parameters, particularly for quantifying the reliability in systems with low probabilities of failure. MCS therefore involves excessive computational costs, although some advanced MCS methods have been introduced in recent years to improve the efficiency of the method.

The First-Order and Second-Order Reliability Methods (FORM/SORM) are a class of analytical reliability methods. These two methods simplify the limit state function by a first-order and second-order Taylor expansion about the “design point,” as shown in Fig. 1. The design point is defined as the point on the limit state curve which has the shortest distance to the origin in standard normal space. The probability of failure evaluated by FORM and SORM is equal to the integration of the joint probability density function on one side of the approached limit state function. As all parameters have been transformed to standard normal space as independent, normally distribution parameters, the integral can be simplified as

$$P_{f\_FORM} = 1 - \Phi(\beta) \tag{1}$$

and

$$P_{f\_SORM} = \Phi(-\beta)\prod_{i=1}^{n-1}\frac{1}{\sqrt{1+\psi(\beta)\kappa_i}} \tag{2}$$

where β is the distance from the design point to the origin, Φ is cumulative density function for a standard normal distribution, κ_i is the principle curvature, and $\psi(\beta)$ is given as

$$\psi(\beta) = \frac{\varphi(\beta)}{\Phi(-\beta)}, \tag{3}$$

where φ is the probability density function for a standard normal distribution.

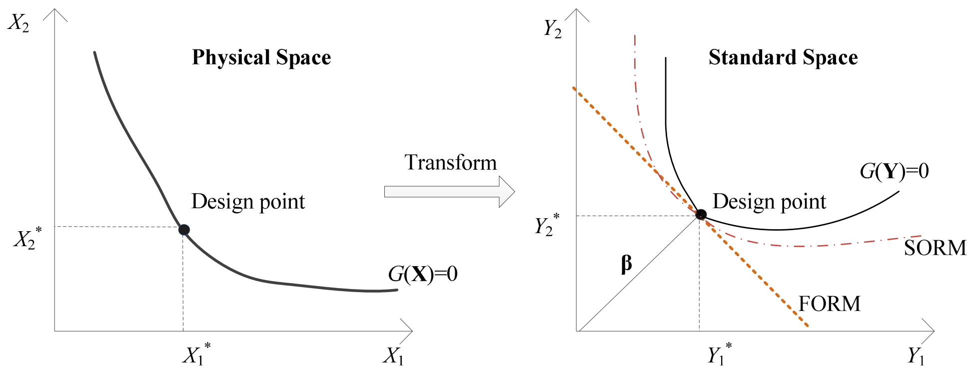


Fig. 1 Calculation of failure probability using FORM/SORM (Haldar and Mahedevan, 2000)

2 CASE STUDY

A protected and ideally pinned steel column subjected to natural fire is analysed here. As shown in Fig. 2, the column is the interior column D2 in the second floor of a four-story building introduced in AISC (2011). According to the design requirement, a W12x65 section was chosen for strength, and its geometric properties are shown in Fig. 3. The fire resistance design used a cementitious spray-applied fire resistant material (SFRM). In order to achieve a 2-hour fire resistance rating, the thickness of 28.6mm (9/8 in.) was selected from UL fire resistance directory, although in the reliability analysis the thickness was taken to be 1.6mm greater than the design thickness based on the fact that the actual thickness tends to be larger than the design value in construction (Iqbal and Harichandran, 2010).

Natural fire exposure was modelled by the Eurocode parametric fire (EC1, 2007). The column shown in Fig. 3 was first modelled deterministically to evaluate response under natural fire exposure. The column was given a rather large initial imperfection of $L/100$ to increase the failure probability of the structure; this was done to ensure that discrepancies in the computed failure probabilities were due to the FORM/SORM calculation rather than inadequate sampling in the MCS. The opening factor O was assumed to be 0.04 $m^{1/2}$ to ensure that the fire was ventilation controlled. The values of thermal inertia b and fuel load density e_t shown in Table 1 were based on the mean values reported in (Culver, 1976; Iqbal and Harichandran, 2010). In the heat transfer analysis, the exposed surface was heated by convection and radiation assuming that the convection heat transfer coefficient h was 35 W/m^2K and the effective emissivity ε of the structural surface was 0.8. The SFRM was assumed to have a density of 300 kg/m^3, a conductivity of 0.12 W/mK, and a specific heat capacity of 1200 J/kgK (Buchanan, 2002). In the structural model, the design dead and live loads were 1226 kN and 605 kN respectively. In the reliability analysis, the design dead load was multiplied by a factor of 1.05 and the design live load was multiplied by a factor of 0.24 to get the values at an arbitrary point in time (Ellingwood, 2005). The total column load w was calculated by

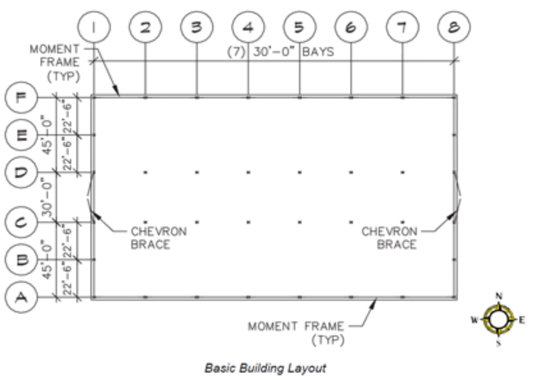


Fig. 2 Floor plan

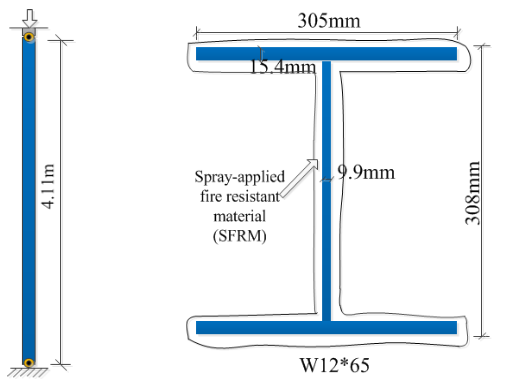


Fig. 3 Geometric properties of the column

Table 1. Statistical properties for the uncertain parameters

Parameter		Mean Value	Distribution	COV	Sensitivity
Room Properties	Fuel Load	564 MJ/m^2	Gumbel	0.62	1.161
	Thermal Initial	423.5 $Ws^{0.5}/m^2K$	Normal	0.09	0.0058
Properties of SFRM	Thickness	30.2 mm	Lognormal	0.2	-1.561
	Conductivity	0.120 W/mK	Lognormal	0.12	1.342
Parameters in structural model	Yield (at 20C)	359 MPa	Normal	0.08	-1.740
	Dead Load	1287.6 kN	Normal	0.1	2.729
	Live Load	145.3 kN	Gamma	0.24	0.308
	A Factor	1	Normal	0.04	2.729
	B Factor	1	Normal	0.2	0.308
	E Factor	1	Normal	0.05	3.037

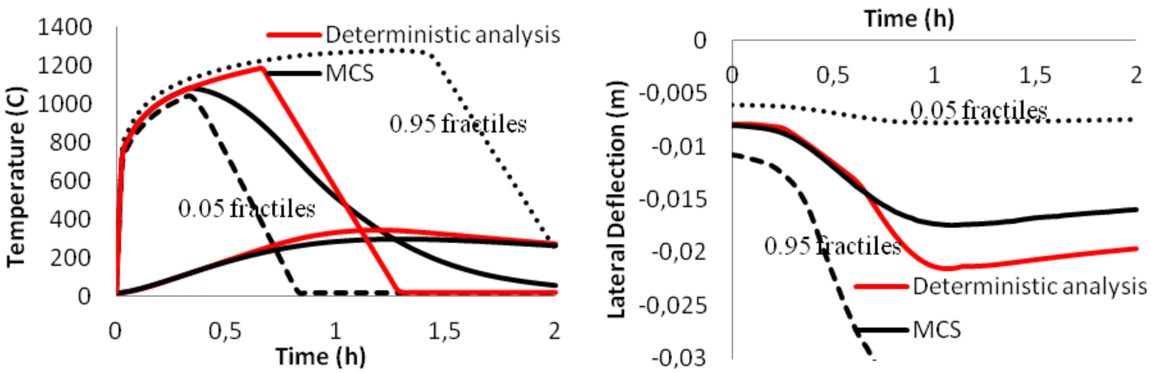


Fig. 4 Simulation results: (a) Fire and column temperatures, and (b) Structural response

$$w = E \times (A \times P_{DL} + B \times P_{LL}) . \quad (4)$$

where *A*, *B*, and *E* are load factors given by (Iqbal and Harichandran, 2010).
The fire and steel temperature from the deterministic analysis (using mean values for the input parameters) are shown in Fig. 4a as the red solid line. The maximum fire temperature arrived around 45 min and was approximately $1100\,^{\circ}C$. Under this heating, the column reached a maximum temperature of approximately $400\,^{\circ}C$ around 75 min. As shown in Fig. 4b, the column maintains structural stability for the duration of the fire exposure.
A Monte Carlo Simulation with a sampling space of 10,000 was conducted here. In order to reduce the number of random parameters, a sensitivity analysis had been conducted and ten "key" parameters (given in Table 1) were chosen as random based on their sensitivity coefficient and range of variability. A group of natural fire curves were obtained using the statistical properties for the fire parameters. The mean fire temperature with the 0.05 and 0.95 fractiles is shown in Fig. 4a. The maximum fire temperature exceeded $1{,}200\,^{\circ}C$ in the most severe case. The mid-height lateral deflection is plotted in Fig. 4b for the mean value as well as the 0.05 and 0.95 fractiles.
To evaluate the reliability of the system, failure was defined as the maximum mid-height lateral deflection of L/150$\approx$0.028m. In the MCS, the probability of failure was calculated by evaluating the total ratio of failed simulations, i.e.

$$P_f = \frac{n_f}{n} , \quad (5)$$

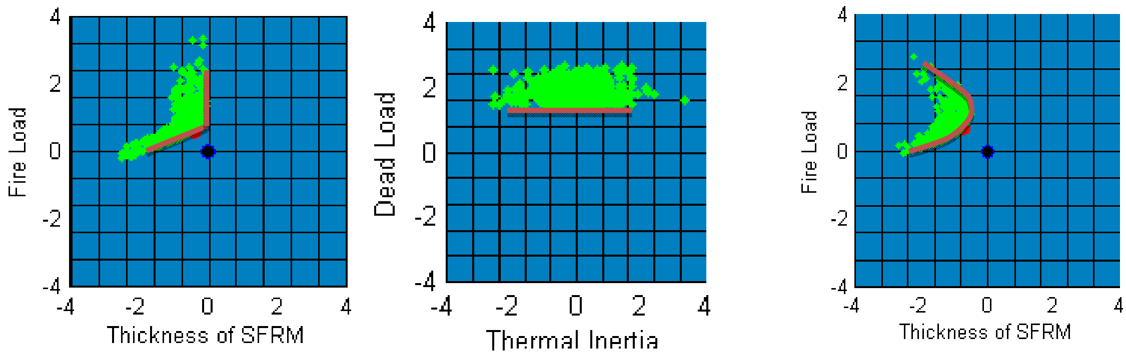
where n_f is the number of failed simulations and *n* is the total number of simulations. The failure probability as a function of time is shown in Fig. 5a for both the MCS and FORM analysis. As the FORM uses a first-order Taylor expansion of the limit surface, there is significant error in the calculation, although it is noted that the result is conservative. It should also been noticed that the column which was considered to be safe by the deterministic analysis had a 30% probability of failure under natural fire exposure. However, the large probability of failure was likely due to the large initial imperfection that was assumed.

3 COMPARISON OF METHODS

To better understand the source of the discrepancies between the FORM and MCS results, an in-depth study of the response and response surface was conducted for various combinations of the parameters given in Table 1. It was found that the FORM gave excellent agreement with the MCS for all combinations of parameters *except* when the fuel load e_t was treated as a random parameter. It was hypothesized that the error could result from the fact that the response surface was possibly nonlinear. However, a second-order reliability analysis using the SORM gave equally poor results for this case, as illustrated in Fig. 5b.
The problem was reduced to two random parameters and the response surface plotted in Fig. 6. In Case 1, the fuel load e_t and the thermal inertia of the compartment *b* were treated as random parameters. To simplify the analysis, the distribution of the fire load was assumed to

follow a lognormal distribution rather than Gumbel distribution. The response surface for Case 1 illustrates a "kink" in the response surface, which resulted in a limit state function that was practically bilinear. This resulted in significant error in the failure probability for both FORM and SORM, as illustrated in Fig. 5b. For comparison, Case 2 considered the dead load P_D and thermal inertia b as random. It can be seen that the response surface for Case 2 was close to linear, giving reason as to why the FORM was able to yield a high level of accuracy for this case.

The source of the bilinear nature of the response surface was suspected to stem from the fact that the fire temperature from the parametric fire curve was not sensitive to the fuel load during the heating phase (i.e., the fuel load only affects the *duration* of heating in the Eurocode parametric fire). Therefore, the problem was reanalysed using the fire model proposed by Ma and Makelainen (2000), which uses an exponential function that depends on the fuel load during both heating and cooling phases of development. The response surface for this case (Case 3) is shown in Fig. 6c. It can be seen that the fire temperature computed by the Ma and Makelainen model leads to a smooth (albeit nonlinear) response surface. The failure probability for this case (shown in Fig. 6c) illustrates that the FORM is not able to capture the nonlinear behaviour of the response. However, the SORM yields excellent agreement with the MCS, as shown in Fig. 6c. The FORM and SORM are more efficient than the MCS. In case 3, FORM and SORM spent around 2.6 hours and 3.3 hours respectively to obtain the reliability curve, but the MCS need 7 hours running in 20 parallel high-performance computing nodes.



(a) (b) (c)

Fig. 6 Response surface (a) Case 1, (b) Case 2, (c) and Case 3

4 CONCLUSION

Two kinds of reliability methods were used to analyse the structural response in fire where uncertain parameters appeared in multiple physical domains. The comparison between the analytical method (FORM) and the Monte Carlo simulation demonstrates that the FORM exhibits acceptable accuracy and offers significant saving in computational cost. Furthermore, the accuracy of the analytical reliability methods can be improved by the SORM with a second-order approximation of the limit state function. However, the accuracy of both FORM and SORM is dependent on the shape of the response surface. For the Eurocode parametric fire curve, the response surface has a kink, which leads to significant error that cannot be resolved with a higher order analysis. Nevertheless, this novel application of the analytical reliability method provides an efficient computation of the time-variant probability of failure, which allows the realization of high-level reliability-based designs in fire engineering.

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