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Dynamic Effect of Discharge Flow of a Rushton Turbine Impeller on a Radial Baffle

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This paper presents an analysis of the mutual dynamic relation between the impeller discharge flow of a standard Rushton turbine impeller and a standard radial baffle at the wall of a cylindrical mixing vessel under turbulent regime of flow of an agitated liquid. A portion of the torsional moment of the baffle corresponding to the region of the force interaction of the impeller discharge stream and the baffle is calculated under the assumption of constant angular momentum in the flow region between the impeller and the baffles. This theoretically obtained quantity is compared with the torsional moment of the baffles calculated from the experimentally determined distribution of the peripheral (tangential) component of dynamic pressure along the height of the radial baffle in pilot plant mixing equipment. It follows from the results of our calculations that for both investigated impeller off-bottom clearances the theoretically determined transferred torsional moment of the baffles in the area of interference between the impeller discharge flow and the baffles agrees fairly well with experimentally determined data and, moreover, that more than 2/3 of the transferred torsional moment of the baffles as a whole is located in the above mentioned interference area.

Keywords: Rushton turbine impeller, impeller discharge stream, dynamic pressure.

1 Introduction

An axially located standard Rushton turbine impeller in a cylindrical vessel with radial baffles exhibits the main force effects – radial and peripheral [1]. The distribution of the peripheral (tangential) components of dynamic pressure affecting a radial baffle at the wall of a cylindrical pilot plant vessel with an axially located axial or radial flow rotary impeller under turbulent regime of flow of agitated liquid was determined experimentally [2, 3, 4]. The experiments [3, 4] were carried out in a flat-bottomed cylindrical pilot plant mixing vessel with four baffles at its wall (see Fig. 1) of diameter $T = 0.3$ m filled with water ($\mu = 1$ mPa·s) or a water-glycerol solution of dynamic viscosity $\mu = 3$ mPa·s and $\mu = 6$ mPa·s, respectively. The impeller was a standard Rushton turbine disk impeller (see Fig. 2) with six flat plane blades [5]. The range of impeller frequency of revolution was chosen in the interval $N = 3.11\text{–}5.83\ \mathrm{s}^{-1}$.

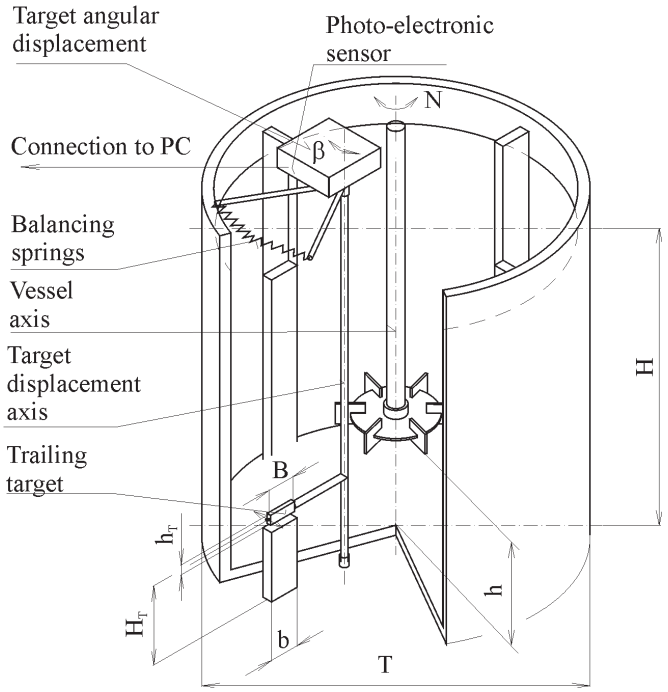

Fig. 1: Sketch of a flat-bottomed agitated pilot plant mixing vessel with four radial baffles at the wall and axially located standard Rushton turbine impeller and sketch of measurement of local peripheral force affecting the trailing target ($H/T = 1$, $h/T = 0.33$, 0.48, $b/T = 0.1$, $h_T = 10$ mm, $B = 28$ mm)

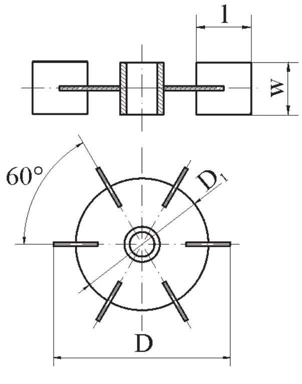

Fig. 2: Standard Rushton turbine disk impeller ($D/T = 1/3$, $w/D = 0.2$, $D_1/D = 0.75$, $l/D = 0.25$)

The originally developed technique for measuring the peripheral component of dynamic pressure affecting the baffle [4] is illustrated in Fig. 1. One of the baffles was equipped with a trailing target of height h_T and width B enabling it to be rotated along the axis parallel to the vessel axis with a small eccentricity and balanced by a couple of springs. Eleven positions of the target along the height of the baffle were examined, above all in the region of the interference of the baffle and the impeller discharge flow. The angular displacement of the target β is directly proportional to the peripheral force F affecting the balancing springs (see Fig. 3). The flexibility of the springs was selected in such a way that the maximum target displacement was reasonably small compared with the vessel dimensions (no more than 5 % of the vessel perimeter). By means of a small photo-electronic device, composed of two photo diodes, the angular displacement was scanned and the output signal was treated, stored and analysed by the computer.

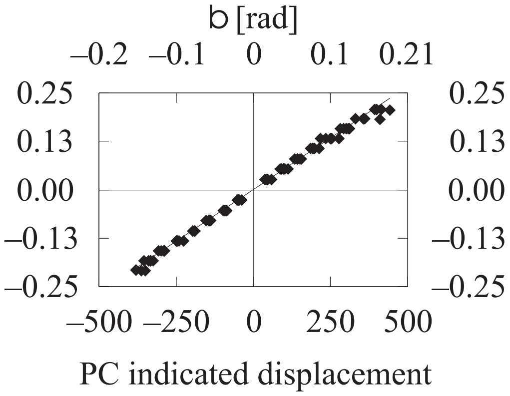

Fig. 3: Results of mechanical calibration of balancing springs

The vertical (axial) distribution of the peripheral component of dynamic pressure along the height of the radial baffle coincides with the flow pattern in an agitated system with a standard Rushton turbine impeller [5]. The aim of this study was to analyse the force interaction of the impeller discharge stream and the corresponding part of the radial baffle and to compare the results of such an analysis with available experimental data.

2 Theoretical

Let us consider a flat-bottomed cylindrical vessel filled with a Newtonian liquid and provided with four radial baffles at its wall. At the axis of vessel symmetry a standard Rushton turbine rotates in such a way that the flow regime of the agitated liquid can be considered turbulent. In such a system the radially-tangential impeller discharge stream leaves the rotating impeller and reaches the baffles (see Fig. 4). For the balance region considered between the impeller and baffles we can assume that the angular momentum flow is constant [6], i.e.,

$$\rho \cdot Q \cdot \overline{w_t} = const., \quad R \in \langle D/2, T/2 \rangle . \tag{1}$$

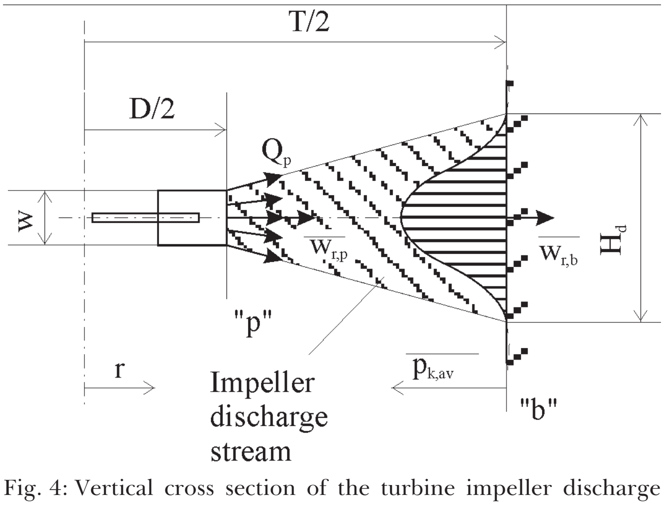

Fig. 4: Vertical cross section of the turbine impeller discharge stream region

Eq. (1) can also be used for the relation between the values of the angular momentum in the impeller discharge stream and around the baffles

$$\frac{\overline{w_{t,\,p}}}{\overline{w_{t,\,b}}} = \frac{T}{D} \cdot \frac{Q_b}{Q_p} , \tag{2}$$

where index "b" characterises the baffle area and index "p" the flow leaving the rotating impeller (impeller discharge flow). Eq. (2) can be rearranged into the form

$$\rho \cdot Q_b \cdot \left(T/2\right) \cdot \overline{w_{t,\,b}} = \overline{M_{b,\,d,\,th}} , \tag{3}$$

where quantity $\overline{M_{b,\,d,\,th}}$ represents a theoretically considered portion of the time averaged (mean) torsional moment of the baffles corresponding to the area of the force interaction of the force action of the impeller discharge stream and the baffle (see shaded area below the curves in Figs. 6 and 7).

By combining Eqs. (2) and (3) we can eliminate the unknown value of the tangential component of the mean velocity $\overline{w_{t,\,b}}$ in the baffle area, and so we have finally

$$\overline{M_{b,\,d,\,th}} = \left(D/2\right) \cdot \rho \cdot \overline{w_{t,\,p}} \cdot Q_p . \tag{4}$$

Eq. (4) can be used for estimating the torsional moment of the baffles transferred via the impeller discharge flow. The tangential component of the mean velocity in the impeller discharge stream $\overline{w_{t,\,p}}$ is related with the radial component $\overline{w_{r,\,p}}$ by the equation

$$\overline{w_{t,\,p}} = \overline{w_{r,\,p}} \cdot \tan(\alpha) , \tag{5}$$

where α is the angle between the horizontal velocity component in the impeller discharge stream and its radial component [7]. It can be calculated from the equation

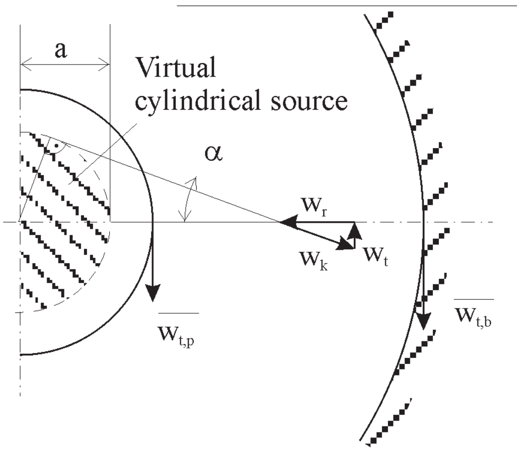

Fig. 5: Turbine impeller as a cylindrical tangential jet

$$\alpha = \arcsin\left[a/(D/2)\right], \qquad (6)$$

where parameter a is the radius of the cylindrical jet, i.e., the virtual cylindrical source of the impeller discharge stream (see Fig. 5). Considering the relation between the impeller pumping capacity Q_p and the average value of the radial component of the mean velocity over the cross section of the impeller discharge flow

$$Q_p = \overline{w_{r,p}} \cdot \pi \cdot D \cdot w\,, \qquad (7)$$

where D and w are the diameter of the impeller and the width of the impeller blade, respectively, we can rearrange Eq. (4) into the form

$$\overline{M_{b,d,th}} = \rho \cdot Q_p^2 \, \frac{\tan\left\{\arcsin\left[a/(D/2)\right]\right\}}{2\pi \cdot w}\,. \qquad (8)$$

Let us consider the flow rate number [8, 9] expressing the quantity Q_p in dimensionless form

$$N_{Q_p} = \frac{Q_p}{N \cdot D^3}\,. \qquad (9)$$

After substitution into Eq. (8) we have after a simple rearrangement

$$\overline{M^*_{b,d,th}} = \frac{\overline{M_{b,d,th}}}{\rho \cdot N^2 \cdot D^5} = N_{Q_p}^2 \cdot \frac{\tan\left[\arcsin(2 \cdot a/D)\right]}{2\pi \cdot (w/D)}\,, \qquad (10)$$

where N is the impeller frequency of revolution.

3 Results and discussion

It follows from the experimental data that the radius of the virtual cylindrical jet [8] is

$$a = 0.34 \cdot D, \left[D/T = 1/3,\ h/T \in \langle 1/3, 1/2\rangle\right]\,, \qquad (11)$$

and the Rushton turbine impeller flow rate number [8, 9]

$$N_{Q_p} = 0.80, \left[D/T = 1/3,\ h/T \in \langle 1/3, 1/2\rangle\right]\,. \qquad (12)$$

The ratio of the impeller blade width according to the impeller geometry (see Fig. 2) corresponds to the relation [5]

$$w/D = 1/5\,. \qquad (13)$$

Now, we can substitute all the numerical values into Eq. (10), and have finally in dimensionless form the theoretically considered portion of the impeller torque transferred by the baffles via the impeller discharge stream

$$\overline{M^*_{b,d,th}} = 0.472, \left[D/T = 1/3,\ h/T \in \langle 1/3, 1/2\rangle\right]\,. \qquad (14)$$

The mean dimensionless pressure affecting the part of the baffle of dimensionless height H^*_{d} can be calculated from the known experimental dependence (see Figs. 6 and 7)

$$\overline{p^*_{k,av}} = \overline{p^*_{k,av}}(H_T/T)\,. \qquad (15)$$

Eq. 15 consists of the dimensionless axial position of the target centre of gravity H_T related to the vessel diameter T as an independent variable and the dimensionless peripheral component of the local mean dynamic pressure affecting the baffle

$$\overline{p^*_{k,av}} = \frac{\overline{p_{k,av}}}{\rho \cdot N^2 \cdot D^2}\,, \qquad (16)$$

considered as a dependent variable. Then the average mean dynamic pressure corresponding to the hatched surface in Figs. 6 and 7, i.e., the region along the baffle between the lower and upper intersections of the curve $\overline{p^*_{k,av}} = \overline{p^*_{k,av}}(H_T/T)$ and zero quantity $\overline{p^*_{k,av}}$ below and above its peak, is

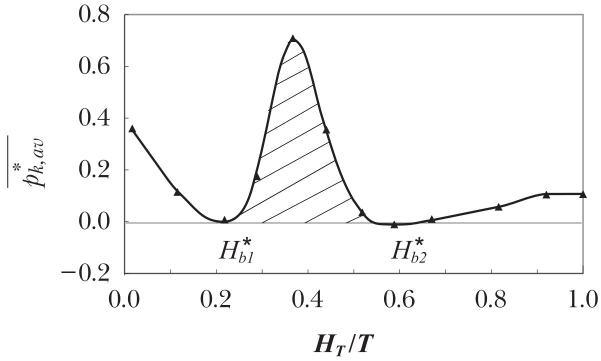

Fig. 6: Axial profile of the dimensionless peripheral component of dynamic pressure affecting a radial baffle along its height (h/T = 0.33)

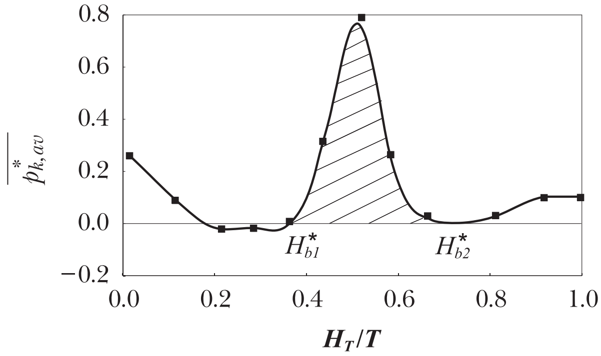

Fig. 7: Axial profile of the dimensionless peripheral component of dynamic pressure affecting a radial baffle along its height (h/T= 0.48)

$$\overline{p^*_{d,av}} = \frac{1}{H^*_d}\int_{H^*_{b1}}^{H^*_{b2}} \overline{p^*_{k,av}}\,\mathrm{d}(H_T/T)\,, \qquad (17)$$

where

$$H^*_d = H^*_{b2} - H^*_{b1}\,. \qquad (18)$$

Dimensionless coordinates H^*_{b1} and H^*_{b2} depict the above mentioned intersections of the curve $\overline{p^*_{k,av}} = \overline{p^*_{k,av}}(H_T/T)$ with the zero values of the quantity $\overline{p^*_{k,av}}$ (see Figs. 6 and 7). The total dimensionless mean peripheral force affecting the baffle along its interference region with the impeller discharge stream can be calculated from the relation

$$\overline{F^*_{d,av}} = \frac{\overline{F_{d,av}}}{\rho \cdot N^2 \cdot D^4} = (T/D)^2 \cdot b^* \cdot H^*_d \cdot \overline{p^*_{d,av}}\,, \qquad (19)$$

where the dimensionless width of the radial baffle

$$b^* = b/T\,. \qquad (20)$$

Similarly the total mean dimensionless peripheral force affecting the whole baffle can be calculated from the relation

$$\overline{F^*_{av}} = \frac{\overline{F_{av}}}{\rho \cdot N^2 \cdot D^4} = (T/D)^2 \cdot b^* \cdot H^*_d \cdot \overline{p^*_{av}}\,, \qquad (21)$$

where the dimensionless total liquid depth in the mixing vessel

$$H^* = H/T \qquad (22)$$

The average mean dynamic pressure over the whole baffle can be calculated by integration

$$\overline{p_{av}^{*}} = \frac{1}{H^{*}} \int_{0}^{\overline{H^{*}}} \overline{p_{k,\,av}^{*}}\,\mathrm{d}\left(H_T/T\right). \qquad (23)$$

From the momentum balance of the mechanically agitated system [1] it follows that the mean impeller torque

$$\overline{M} = \frac{P}{2\pi \cdot N}\,, \qquad (24)$$

where P is the impeller power input, should correspond to the sum of the mean reaction moments of the baffles, bottom and walls. The mean impeller torque can be expressed in the dimensionless form

$$\overline{M^{*}} = \frac{\overline{M}}{\rho \cdot N^{2} \cdot D^{5}} = \frac{P}{2\pi \cdot \rho \cdot N^{3} \cdot D^{5}} = \frac{Po}{2\pi}\,, \qquad (25)$$

where the standard Rushton turbine impeller power number under conditions $Re > 10^{4}$ and $D/T \in \langle 0.25,\ 0.70\rangle$ can be expressed by means of the equation [8, 10]

$$Po = \frac{P}{\rho \cdot N^{3} \cdot D^{5}} = 2.5\left(x_1/D\right)^{-0.2}\left(T/T_0\right)^{0.065}, \qquad (26)$$

where x_1 is the thickness of the turbine disk and $T_0 = 1$ m.

From knowledge of the quantity $\overline{F_{av}^{*}}$, the mean dimensionless torsional moment of the baffle can be calculated. We only need to know the arm corresponding to the centre of gravity of the linear dynamic pressure along the width of the baffle (see Fig. 8) from the axis of symmetry of the cylindrical mixing vessel:

$$R_b = \left(T/2\right) - \left(2/3\right)b\,. \qquad (27)$$

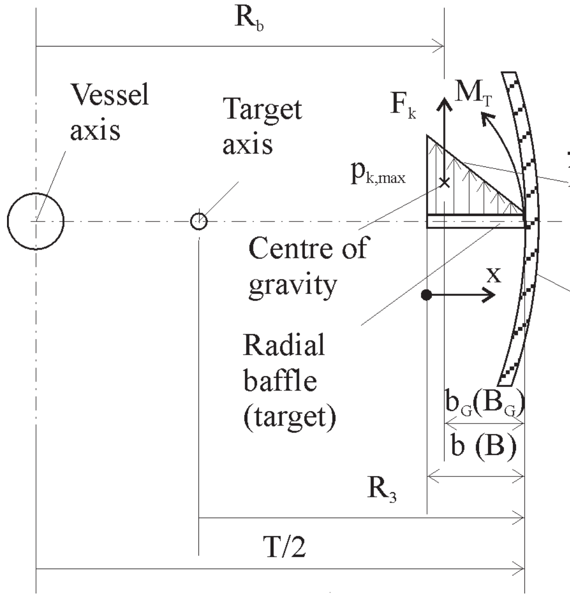

Fig. 8: Radial profile of baffle loading

If we consider the total number of baffles n_b, the moment transferred by the baffles is

$$\overline{M_b} = \overline{F_{av}} \cdot R_b \cdot n_b\,, \qquad (28)$$

and finally in dimensionless form

$$\overline{M_b^{*}} = \frac{\overline{F_{av}^{*}} \cdot R_b \cdot n_b}{\rho \cdot N^{3} \cdot D^{5}} = n_b \cdot \overline{F_{av}^{*}} \cdot \frac{R_b}{D}\,. \qquad (29)$$

Similarly the portion of the reaction moment of the baffles corresponding to the mutual interference of n_b baffles and the impeller discharge stream can be expressed in dimensionless form as

$$\overline{M_{b,d}^{*}} = n_b \cdot \overline{F_{d,av}^{*}} \cdot \frac{R_b}{D}\,, \qquad (30)$$

where the quantity $\overline{F_{d,av}^{*}}$ was calculated according to Eq. (19). Table 1 contains a comparison of the impeller torque and the calculated reaction moment of the baffles, and a comparison of the experimentally determined quantity $\overline{M_{b,\,d}^{*}}$ and the theoretically found quantity $\overline{M_{b,\,d,\,th}^{*}}$ [see Eq. (14)]. The power number Po was calculated for the impeller tested from Eq. (26).

It follows from Table 1 that theoretical considerations about the character of the discharge flow leaving a standard turbine impeller were fairly well confirmed experimentally. Theoretically and experimentally determined values of the reaction moments of the baffles corresponding to the mutual interference of the baffles and the impeller discharge stream practically coincide. From the results summarised in Table 1 it also follows that most of the turbine impeller torque is transferred via the agitated liquid by the radial baffles: more than 3/4 of the impeller torque is indicated as a reaction moment of the baffles. Moreover, this reaction moment is concentrated mainly in the impinging region of the impeller discharge stream and the vessel wall: more than 2/3 of the total baffles reaction moment affects this narrow part of the baffles, i.e., more than 2/3 of the reaction moment takes place along one third of the height of the baffle. This knowledge plays a significantly role in the design of industrial mixing units with a standard Rushton turbine impeller and baffles, where the maximum of fatigue stress can be considered in the above mentioned region with consequences for the baffle fixing using the corresponding welding technique.

Fig. 9 illustrates the axial profiles of the dimensionless peripheral component of dynamic pressure affecting a radial baffle for a pitched blade impeller and for a standard Rushton turbine impeller at the same off-bottom clearances when the pitched blade impeller is pumping liquid down towards the bottoms. We can clearly distinguish between the distribution of the force effect of the agitated liquid on the radial baffles: the pitched blade impeller exhibits the main

Table 1: Transfer of the impeller torque by radial baffles in an agitated system with a standard Rushton turbine disk impeller (thickness of the impeller separating disk $x_1 = 0.55$ mm)

h/T	Po	$\overline{M^{*}}$	$\overline{M_b^{*}}$	$\overline{M_{b,\,d}^{*}}$	$\overline{M_{b,\,d,\,th}^{*}}$	H_d^{*}	$\overline{M_b^{*}}/\overline{M^{*}}$	$\overline{M_{b,d}^{*}}/\overline{M^{*}}$
0.33	5.289	0.842	0.713	0.464	0.472	0.33	0.847	0.651
0.48	5.289	0.842	0.641	0.475	0.472	0.30	0.762	0.741

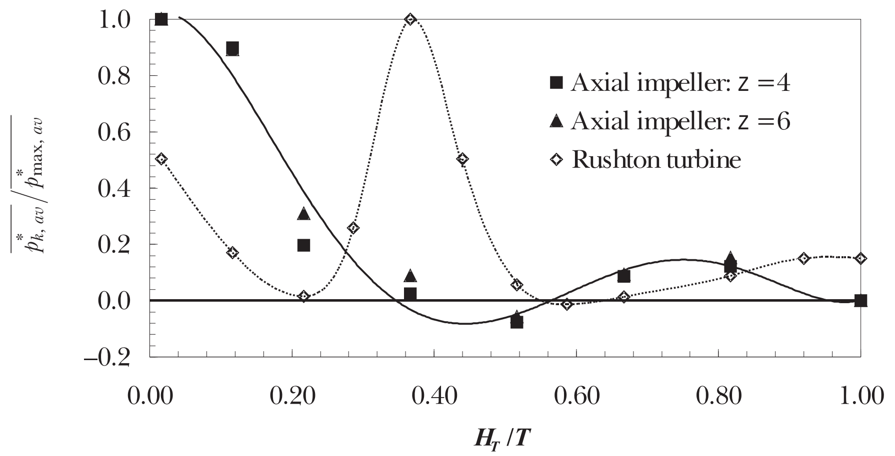

Fig. 9: Comparison of axial profiles of the dimensionless peripheral component of dynamic pressure affecting radial baffle along its height for down pumping pitched blade impeller and standard Rushton turbine impeller ($D/T = 1/3$, $h/T = 1/2$, pitched blade impeller $z = 4$: $\overline{p^*_{\max,av}} = 0.195$, $z = 6$: $\overline{p^*_{\max,av}} = 0.230$, Rushton turbine impeller: $\overline{p^*_{\max,av}} = 0.716$)

effect at the bottom while the turbine impeller effects mainly the region around the horizontal plane of its separating disk.

4 Conclusions

- More than 3/4 of the turbine impeller torque is transferred via agitated liquid by the baffles.
- More than 2/3 of the baffle reaction moment is located in the impinging region of the turbine impeller discharge stream and the vessel wall.
- The mean flow characteristics of the turbine impeller discharge stream allow us to estimate the mutual force interference of the baffles and the impeller discharge stream.

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List of symbols

a	radius of the virtual cylindrical jet, m
B	target width, m
b	baffle width, m
D	impeller diameter, m
D_1	diameter of the impeller separating disk, m
F	peripheral component of the force affecting the radial baffle, N
H	total liquid depth, m
H_d	height of the area of the impeller discharge stream interference region with the baffles, m
H_T	height of the target above the bottom, m
H	impeller off-bottom clearance, m
$\overline{M}$	mean impeller torque, N·m
$\overline{M_b}$	mean reaction moment of the baffles, N·m
$\overline{M_{b,d}}$	mean torsional moment of the baffles transferred via impeller discharge flow, N·m
N	impeller speed, s^{-1}
N_{Qp}	flow rate number
n_b	number of baffles
P	impeller power input, W
Po	power number
p	peripheral component of the dynamic pressure affecting the radial baffle, Pa
Q_p	impeller pumping capacity, $m^3 \cdot s^{-1}$
R	radial coordinate, m
Re	impeller Reynolds number
R_b	radial coordinate of the dynamic pressure linear profile centre of gravity, m
T	vessel diameter, m
T_0	diameter of the standard cylindrical vessel, 1m
x_1	thickness of the impeller separating disk, m
z	number of impeller blades
w	width of the impeller blade, m
α	angle between the horizontal velocity component in the impeller discharge stream and its radial component, °
β	target angular displacement, °
μ	dynamic viscosity of agitated liquid, Pa·s
ρ	density of agitated liquid, $kg \cdot m^{-3}$

Subscripts and superscripts

av	average value
b	related to the baffle
k	related to the position of the target
th	theoretically calculated value
$*$	dimensionless value
$\overline{}$	mean (time average) value

References

[1] Standart, G.: *General Relations Concerning Forces and Torques in Mixing Vessel*. Collect. Czech. Chem. Commun. 23, 1958, pp. 1163–1168

[2] Kipke, H. D.: *Mechanische Belastung von strömungslenkenden Einbauten in gerührten Behältern*. Chemie-Technik, 6 (11), 1977, pp. 467–472

[3] Kratěna, J., Fořt, I., Brůha, O., Pavel, J.: *Local Dynamic Effect of Mechanically Agitated Liquid on a Radial Baffle.* In: "Proceedings of the 10th European Conference on Mixing, Delft" (Editors: H. E. A. van den Akker and J. J. Derksen), Amsterdam, Elsevier Science, 2000, pp. 369–376

[4] Kratěna, J., Fořt, I., Brůha, O., Pavel, J.: *Distribution of Dynamic Pressure along a Radial Baffle in an Agitated System with Standard Rushton Turbine Impeller*. In: "Proceedings of the 4th International Symposium on Mixing in Industrial Processes – ISMIP 4, Toulouse", in press

[5] *Turbine Disk Impeller with Flat Plane Blade*. Czech Standard CVS 691021, Mixing Equipment, Brno, 1997

[6] Biting, H., Zehner, P.: *Power and Discharge Numbers of Radial Flow Impellers and Baffles*. Chem. Eng. and Proc., 33, 1994, pp. 295–301

[7] Drbohlav, J., Fořt, I., Krátky, J.: *Turbine Impeller as a Tangential Cylindrical Jet*. Collect. Czech. Chem. Commun., 43, 1978, pp. 696–712

[8] Rutherford, K., Mahmoudi, S. M. S., Lee, K. C., Yianneskis, M.: *The Influence of Rushton Impeller Blade and Disk Thickness on the Mixing Characteristics of Stirred Vessel*. Trans. Inst. Chem. Engrs., 74 (part A), 1996, pp. 369–378

[9] Stoots, C. M., Calabrese, R. V.: *Mean Velocity Field Relative to a Rushton Turbine Blade*. AIChE Jour., 41 (1), 1995, pp. 1–11.

[10] Bujalski, W., Nienow, A. W., Chatwin, S., Cooke, M.: *The Dependency on Scale of Power Numbers of Rushton Disk Turbine*. Chem. Eng. Sci., 42 (2), 1987, pp. 317–326

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