ON RENORMALIZATION OF POISSON–LIE T-PLURAL SIGMA MODELS

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Abstract. Covariance of the one-loop renormalization group equations with respect to Poisson–Lie T-plurality of sigma models is discussed. The role of ambiguities in renormalization group equations of Poisson–Lie sigma models with truncated matrices of parameters is investigated.

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1. Introduction

One-loop renormalizability of Poisson–Lie dualizable σ-models and their renormalization group equations were derived in [1]. Covariance of the renormalization group equations with respect to Poisson–Lie T-duality was proven in [2]. This suggests that also properties of quantum σ-models can be given in terms of Drinfel'd doubles and not their decompositions into Manin triples. This was indeed claimed in [3] where a renormalization on the level of sigma models defined on Drinfel'd double was proposed. A natural way to independently verify this claim would be to extend the proof of covariance of [2] to Poisson–Lie T-plurality.

Unfortunately, transformation properties of the structure constants and the matrix M (parameters of the models) under the Poisson–Lie T-plurality are much more complicated than in the case of T-duality. That's why we decided to check it first on examples using our lists of 4- and 6-dimensional Drinfel'd doubles and their decompositions into Manin triples [4, 5].

It turned out that the renormalization group equations of [1, 2] are indeed invariant under Poisson–Lie T-plurality. The equivalence of the renormalization flows of the models on the Poisson–Lie group of [2] and on the Drinfel'd double [3] also holds in all cases studied so far provided one is careful in interpreting of the formulas in different parts of [3], see Section 3.

An assumption in the renormalizability proof [1] is that there is no a priori restriction on elements of matrix M that together with the structure of the Manin triple determine the models. It was noted in [2, 6] that the renormalization group equations need not be consistent with truncation of the parameter space. On the other hand there is some freedom in the renormalization group equations and we are going to show how they can be used in the choice of one-loop β functions for a given truncation.

2. Review of Poisson–Lie T-plurality

For simplicity we will consider σ-models without spectator fields, i.e. with target manifold isomorphic to a group. Let G be a Lie group and $\mathcal{G}$ its Lie algebra. The σ-model on group G is given by the classical action

$$S_E[g] = \int d^2x\, R_-(g)^a E_{ab}(g) R_+(g)^b, \qquad (1)$$

where $g : \mathbf{R}^2 \to G, (\sigma_+, \sigma_-) \mapsto g(\sigma_+, \sigma_-)$, $R_\pm(g)^a$ are components of the right-invariant fields $\partial_\pm g g^{-1}$ in the basis T_a of the Lie algebra $\mathcal{G}$,

$$\partial_\pm g g^{-1} = (R_\pm(g))^a T_a \in \mathcal{G}$$

and $E(g)$ is a certain bilinear form on the Lie algebra $\mathcal{G}$, to be specified below.

The σ-models that can be transformed by the Poisson–Lie T-duality are formulated (see [7, 8]) by virtue of the Drinfel'd double $D \equiv (G|\tilde{G})$ — a Lie group whose Lie algebra $\mathcal{D}$ admits a decomposition $\mathcal{D} = \mathcal{G} \dotplus \widetilde{\mathcal{G}}$ into a pair of subalgebras maximally isotropic with respect to a symmetric ad-invariant nondegenerate bilinear form $\langle\,.\,,.\,\rangle$. These decompositions are called Manin triples.

The matrices $E(g)$ for such σ-models are of the form

$$\begin{aligned} E(g) &= (M + \Pi(g))^{-1}, \\ \Pi(g) &= b(g) \cdot a^{-1}(g) = -\Pi(g)^t, \qquad (2) \end{aligned}$$

where M is a constant matrix, the superscript t means matrix transposition and $a(g), b(g)$ are submatrices of the adjoint representation of the subgroup G on the Lie algebra $\mathcal{D}$ defined as

$$\begin{aligned} gTg^{-1} &\equiv Ad(g) \triangleright T = a^{-1}(g) \cdot T, \\ g\tilde{T}g^{-1} &\equiv Ad(g) \triangleright \tilde{T} = b^t(g) \cdot T + a^t(g) \cdot \tilde{T}, \qquad (3) \end{aligned}$$

where T_a and $\tilde{T}^a$ are elements of dual bases of $\mathcal{G}$ and $\widetilde{\mathcal{G}}$, i.e.

$$\langle T_a, T_b \rangle = 0, \quad \langle \tilde{T}^a, \tilde{T}^b \rangle = 0, \quad \langle T_a, \tilde{T}^b \rangle = \delta_a^b.$$

The origin of the Poisson–Lie T-plurality [7, 9] lies in the fact that in general several decompositions (Manin triples) of the Drinfel'd double may exist. Let $\mathcal{D} = \hat{\mathcal{G}} \dotplus \bar{\mathcal{G}}$ be another decomposition of the Lie algebra $\mathcal{D}$ into maximal isotropic subalgebras. The dual bases of $\mathcal{G}, \widetilde{\mathcal{G}}$ and $\hat{\mathcal{G}}, \bar{\mathcal{G}}$ are related by the linear transformation

$$\begin{pmatrix} T \\ \tilde{T} \end{pmatrix} = \begin{pmatrix} K & Q \\ W & S \end{pmatrix} \begin{pmatrix} \hat{T} \\ \bar{T} \end{pmatrix}, \tag{4}$$

where the matrices K, Q, W, S are chosen in such a way that the structure of the Lie algebra $\mathcal{D}$ in the basis $(T_a, \tilde{T}^b)$

$$[T_a, T_b] = f_{ab}{}^c T_c,$$
$$[\tilde{T}^a, \tilde{T}^b] = \tilde{f}^{ab}{}_c \tilde{T}^c,$$
$$[\tilde{T}^a, T_b] = f_{bc}{}^a \tilde{T}^c - \tilde{f}^{ac}{}_b T_c \tag{5}$$

transforms to a similar one where $T \to \hat{T}$, $\tilde{T} \to \bar{T}$ and the structure constants $f, \tilde{f}$ of $\mathcal{G}$ and $\widetilde{\mathcal{G}}$ are replaced by the structure constants $\hat{f}, \bar{f}$ of $\hat{\mathcal{G}}$ and $\bar{\mathcal{G}}$. The duality of both bases requires

$$\begin{pmatrix} K & Q \\ W & S \end{pmatrix}^{-1} = \begin{pmatrix} S^t & Q^t \\ W^t & K^t \end{pmatrix}. \tag{6}$$

The σ-model obtained by the Poisson–Lie T-plurality is defined analogously to (1)-(2) where

$$\hat{E}(\hat{g}) = (\hat{M} + \hat{\Pi}(\hat{g}))^{-1},$$
$$\hat{\Pi}(\hat{g}) = \hat{b}(\hat{g}) \cdot \hat{a}^{-1}(\hat{g}) = -\hat{\Pi}(\hat{g})^t,$$
$$\hat{M} = (M \cdot Q + S)^{-1} \cdot (M \cdot K + W)$$
$$= (K^t \cdot M - W^t) \cdot (S^t - Q^t \cdot M)^{-1}. \tag{7}$$

The transformation (7) $M \mapsto \hat{M}$ is obtained when the subspaces $\mathcal{E}^{\pm} = span\{E_a^{\pm}\}_{a=1}^n$ spanned by

$$E_a^+ := T_a + M_{ab}^{-1}\tilde{T}^b, \quad E_a^- := T_a - M_{ba}^{-1}\tilde{T}^b \tag{8}$$

are expressed as

$$\mathcal{E}^+ = \{\hat{T}_a + \hat{M}_{ab}^{-1}\bar{T}^b\}_{a=1}^n,$$
$$\mathcal{E}^- = \{\hat{T}_a - \hat{M}_{ba}^{-1}\bar{T}^b\}_{a=1}^n.$$

Classical solutions of the two σ-models are related by two possible decompositions of $l \in D$,

$$l = g\tilde{h} = \hat{g}\bar{h}. \tag{9}$$

Examples of explicit solutions of the σ-models related by the Poisson–Lie T-plurality were given in [10]. The Poisson–Lie T-duality is a special case of Poisson–Lie T-plurality with $K = S = 0$, $Q = W = \mathbf{1}$.

It is useful to recall that several other conventions are used in the literature. E.g., the action in [2, 3] is defined as

$$S[g] = \int d^2x L_+(g) \cdot (M + \bar{\Pi}(g))^{-1} \cdot L_-(g), \tag{10}$$

where $\bar{\Pi}(g) = b^t(g) \cdot a(g) = \Pi(g^{-1})$. The transition between actions (1) and (10) is given by $g \leftrightarrow g^{-1}$, $M \leftrightarrow M^t$.

The one-loop renormalization group equations for Poisson–Lie dualizable σ-models were found in [1]. In our notation it reads

$$\frac{dM^{ba}}{dt} = r^{ab}(M^t). \tag{11}$$

Note that equation (11) appears in [1, 2] without transposition of M on both sides of the equation due to different formulations of the σ-model action (1) vs. (10).

The matrix valued function r^{ab} is defined as

$$r^{ab}(M) = R^{ac}{}_d(M)L^{db}{}_c(M), \tag{12}$$
$$R^{ab}{}_c(M) = \frac{1}{2}(M_S^{-1})_{cd}\big(A^{ab}{}_e M^{de}$$
$$+ B^{ad}{}_e M^{eb} - B^{db}{}_e M^{ae}\big), \tag{13}$$
$$L^{ab}{}_c(M) = \frac{1}{2}(M_S^{-1})_{cd}\big(B^{ab}{}_e M^{ed}$$
$$+ A^{db}{}_e M^{ae} - A^{ad}{}_e M^{eb}\big), \tag{14}$$
$$A^{ab}{}_c = \tilde{f}^{ab}{}_c - f_{cd}{}^a M^{db},$$
$$B^{ab}{}_c = \tilde{f}^{ab}{}_c + M^{ad} f_{dc}{}^b, \tag{15}$$
$$M_S = \frac{1}{2}(M + M^t). \tag{16}$$

It was shown in [2] that equation (11) is covariant with respect to the Poisson–Lie T-duality, i.e., it is equivalent to

$$\frac{d\tilde{M}^{ba}}{dt} = \tilde{r}^{ab}(\tilde{M}^t) \tag{17}$$

obtained by

$$f \to \tilde{f}, \quad \tilde{f} \to f, \quad M \to \tilde{M} = M^{-1}. \tag{18}$$

One expects that equations (11) are covariant also with respect to the Poisson–Lie T-plurality when

$$f \to \hat{f}, \quad \tilde{f} \to \bar{f}, \quad M \to \hat{M}, \tag{19}$$

where the transformation of $\hat{M}$ under plurality is given by (7). We have checked the invariance on numerous examples of Poisson–Lie T-plurality using 4- and 6-dimensional Drinfel'd doubles and their decompositions into Manin triples of [4, 5], and, have found no counterexamples.

3. RELATION TO THE RENORMALIZATION GROUP EQUATIONS ON THE DRINFEL'D DOUBLE

The renormalization equation (11) presented above will be compared to the renormalization group equations derived in [3] on the whole Drinfel'd double

$$\frac{dR_{AB}}{dt} = S_{AB}(R, h) = \frac{1}{4}(R_{AC}R_{BF} - \eta_{AC}\eta_{BF})$$
$$\cdot (R^{KD}R^{HE} - \eta^{KD}\eta^{HE})h_{KH}{}^C h_{DE}{}^F \quad (20)$$

for the symmetric matrix R, indexes $A, B, \ldots$ refer to Drinfel'd double Lie algebra $\mathcal{D}$ spanned by the basis $T_A = \{T_i, \tilde{T}^j\}$. For a given decomposition of the Drinfel'd double into a Manin triple $(G|\tilde{G})$, the structure constants h of the Drinfel'd double are given by the structure constants $f, \tilde{f}$ of the subalgebras of the Manin triple $h = h(f, \tilde{f})$ as in equation (5). Matrix R is related to the matrix M, which defines the σ-model on the group G, by

$$R_{AB} = \rho_{AB}(M) = \begin{pmatrix} \tilde{M}_s - B\tilde{M}_s^{-1}B & -B\tilde{M}_s^{-1} \\ \tilde{M}_s^{-1}B & \tilde{M}_s^{-1} \end{pmatrix},$$
$$(21)$$

where

$$B = \frac{1}{2}\left[M^{-1} - (M^{-1})^t\right], \quad \tilde{M}_s = \frac{1}{2}\left[M^{-1} + (M^{-1})^t\right],$$
$$R^{AB} = (R^{-1})^{AB}, \quad R^{-1} = \eta \cdot R \cdot \eta,$$

and

$$\eta_{AB} = \langle T_A | T_B \rangle = \begin{pmatrix} \mathbf{0} & \mathbb{I}_{d_G \times d_G} \\ \mathbb{I}_{d_G \times d_G} & \mathbf{0} \end{pmatrix}. \quad (22)$$

It is easy to show that due to (21) the equivalence of (20) and (11) where $r^{ab} = r^{ab}(M, f, \tilde{f})$ requires

$$S_{AB}\left(\rho(M), h(f, \tilde{f})\right) = \frac{\partial \rho_{AB}}{\partial M^{ab}}(M)\, r^{ba}(M^t, f, \tilde{f}).$$
$$(23)$$

Note the presence of transpositions on the right-hand side.

By construction — cf. equation (4.15) of [3] — matrix M which is put into equation (21) (and thus appears in equation (20)) transforms under T-plurality as in (7), i.e. agrees with the convention used here for the sigma model of the form (1). However, the sigma models on the Poisson–Lie groups in [3] are expressed in a different convention, as in equation (10) here. Thus, a tacit transposition of matrix M is necessary when comparing the renormalization group flows on the double and on the individual Poisson–Lie subgroup in [3]. Taking this fact into consideration we were able to recover the examples presented in [3] and also confirm the conjectured equivalence of the renormalization group equations (20) and (11) in all the investigated 4- and 6-dimensional Drinfel'd doubles.

4. NON-UNIQUENESS OF THE RENORMALIZATION GROUP EQUATIONS

It was noted in the paper [1] that there is a certain ambiguity in the one-loop renormalization group equations. Namely, the flow given by equation (11) is physically equivalent to the flow given by the equation

$$\frac{dM^{ba}}{dt} = r^{ab}(M^t) + R^{ab}{}_c(M^t)\,\xi^c, \quad (24)$$

where ξ^c are arbitrary functions of the renormalization scale t and $R^{ab}{}_c(M)$ were defined in (13).

The origin of this arbitrariness in ξ^c lies in the fact that the metric and B-field are determined up to the choice of coordinates, i.e. up to a diffeomorphism, of the group G viewed as a manifold. In our case we may in addition require that the transformed action again takes the form (1)–(2) for some matrix M'. On the other hand, we do not have to require the diffeomorphism to be a group homomorphism because the group structure plays only an auxiliary role in the physical interpretation.

For example, in the particular case of the semi-Abelian double, i.e. $\tilde{f} = 0$, $\Pi = 0$, with a symmetric matrix M, the left translation by an arbitrary group element $h = \exp(X) \in G$, i.e. replacement of g by hg in the action (1), leads to the new matrix $M' = Ad(h) \cdot M \cdot Ad(h)$, specifying a metric physically equivalent to the original one. Such a diffeomorphism is generated by the flow of the left-invariant vector field X. For general Manin triples and matrices M similar transformations are generated by more complicated vector fields parameterized by ξ^c, as was found in [1]. Thus the renormalization group flows (24) differing by the choice of ξ^c are physically equivalent. Consistency under the Poisson–Lie T-plurality requires that the functions $\hat{\xi}^c$ for the plural model satisfy

$$\hat{R}(\hat{M}^t) \cdot \hat{\xi} = (S - M^t \cdot Q)^{-1}$$
$$\cdot \left(R(M^t) \cdot \xi\right) \cdot (K + Q \cdot \hat{M}^t). \quad (25)$$

For the Poisson–Lie T-duality this formula simplifies to

$$\tilde{R}(\tilde{M}^t) \cdot (\tilde{\xi} + \tilde{M}^t \cdot \xi) = 0.$$

Freedom in the choice of functions ξ^a can be employed when compatibility of the renormalization group equation flow with a chosen ansatz (truncation) for the matrix M is sought.

4.1. RENORMALIZABLE σ-MODELS FOR M PROPORTIONAL TO THE UNIT OR DIAGONAL MATRIX

The simplest ansatz for the constant matrix is $M = m\mathbf{1}$ where $\mathbf{1}$ is the identity matrix and $m \neq 0$. As mentioned in the Introduction, truncation or symmetry of the constant matrix M that determines the background of the σ-model often contradicts the form of the r.h.s. of the renormalization group equations

Manin triple	Conditions on ξ^1 and/or m and their duals		
$(1\vert 1)$	$\frac{dm}{dt} = 0,$	$\xi^1 = 0,$	
$(3\vert 3.i\vert b)$	$\frac{dm}{dt} = 0,$	$\xi^1 = 0,$	$m = \pm b,$
$(5\vert 1)$	$\frac{dm}{dt} = 2m^2,$	$\xi^1 = 2m,$	
$(6_0\vert 5.iii\vert b)$	$\frac{dm}{dt} = 0,$	$\xi_1 = 0,$	$m = \pm b,$
$(6_a\vert 6_{1/a}.i\vert b)$	$\frac{dm}{dt} = 0,$	$\xi_1 = 0,$	$m = \pm b/a,$
$(6_a\vert 6_{1/a}.i\vert b)$	$\frac{dm}{dt} = 2b^2(a^2 - \frac{1}{a^2}),$	$\xi_1 = -2b(a + \frac{1}{a}),$	$m = -b,$
$(7_a\vert 1)$	$\frac{dm}{dt} = 2a^2 m^2,$	$\xi^1 = 2am,$	$a \geq 0,$
$(7_a\vert 7_{1/a}\vert b)$	$\frac{dm}{dt} = 2(m^2 - b^2),$	$\xi^1 = 2(m - b),$	$a = 1,$
$(9\vert 1)$	$\frac{dm}{dt} = -m^2/2,$	$\xi^1 = 0,$	
$(9\vert 5\vert b)$	$\frac{dm}{dt} = -\frac{1}{2}m^2 - 2b^2,$	$\xi^1 = -2b$	

TABLE 1. Conditions for consistency of the one-loop renormalization group equations for three-dimensional σ-models with M proportional to the unit matrix (for notation of $(X\vert Y)$ or $(X\vert Y\vert b)$ see [5]).

(11). On the other hand, the freedom in the choice of ξ^c in (24) may help to restore the renormalizability. It is therefore of interest to find consistency conditions for the renormalization group equations for the σ-models given by this simple M.

Two-dimensional Poisson–Lie σ-models are given by Manin triples generated by Abelian or solvable Lie algebras with Lie products

$$[T_1, T_2] = a\, T_2, \quad [\tilde{T}^1, \tilde{T}^2] = \tilde{a}\, \tilde{T}^2,$$
$$a \in \{0, 1\},\ \tilde{a} \in \mathbb{R} \quad (26)$$

or

$$[T_1, T_2] = T_2, \ [\tilde{T}^1, \tilde{T}^2] = \tilde{T}^1. \quad (27)$$

In the former case, equation (24) for $M = m\mathbf{1}$ reads

$$\begin{pmatrix} \frac{dm}{dt} & 0 \\ 0 & \frac{dm}{dt} \end{pmatrix} = \begin{pmatrix} a^2 m^2 - \tilde{a}^2 & (a\, m + \tilde{a})\xi^2 \\ 0 & -(a\, m + \tilde{a})\xi^1 \end{pmatrix} \quad (28)$$

so that we generically get $\xi^1 = \tilde{a} - a\, m$, $\xi^2 = 0$ and the renormalization group equation is $dm/dt = a^2 m^2 - \tilde{a}^2$. In the special case $a = 1$, $m = -\tilde{a}$ the r.h.s. of the equation (28) vanishes for all choices of ξ^k, i.e. there is no renormalization. Notice that had we allowed a diagonal ansatz

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad (29)$$

instead of the multiple of the unit matrix, the restriction on the value of ξ^1 would disappear and the renormalization group equation would take the form

$$\frac{dm_1}{dt} = -\tilde{a}^2 + m_1^2 a^2, \quad \frac{dm_2}{dt} = -\frac{m_2}{m_1}\xi^1(\tilde{a} + m_1 a). \quad (30)$$

For the Manin triple (27), the equation (24) reads

$$\begin{pmatrix} \frac{dm}{dt} & 0 \\ 0 & \frac{dm}{dt} \end{pmatrix} = \begin{pmatrix} m^2 + \xi^2 & m\,(\xi^2 - 1) \\ m - \xi^1 & -1 - m\,\xi^1 \end{pmatrix} \quad (31)$$

and no choice of ξ^1, ξ^2 satisfies the equation (31). Therefore the Poisson–Lie σ-model given by Manin triple (27) is not renormalizable with M kept proportional to the unit matrix. The situation changes when we allow general diagonal form (29) of matrix M. Then the renormalization group equation becomes

$$\begin{pmatrix} \frac{dm_1}{dt} & 0 \\ 0 & \frac{dm_2}{dt} \end{pmatrix} = \begin{pmatrix} m_1^2 + \frac{m_1}{m_2}\xi^2 & m_1\,(\xi^2 - 1) \\ m_1 - \xi^1 & -1 - m_2\,\xi^1 \end{pmatrix} \quad (32)$$

which allows the flow

$$\frac{dm_1}{dt} = m_1^2 + \frac{m_1}{m_2}, \qquad \frac{dm_2}{dt} = -1 - m_1 m_2$$

respecting the diagonal ansatz (29) for the unique choice $\xi^1 = m_1$, $\xi^2 = 1$.

Consistency of the one-loop renormalization group equations for three-dimensional Poisson–Lie σ-models with M proportional to the unit matrix fixes $\xi^3 = 0$ and is consistent with the choice $\xi^2 = 0$ (unique in some cases). It exists for Manin triples and choices of ξ^1 and/or m and their duals summarized in Table 1. Renormalization of the Poisson–Lie σ-models given by other six-dimensional Manin triples is not consistent with the assumption M proportional to identity, i.e. renormalization spoils the ansatz.

We have also investigated three-dimensional σ-models with general diagonal matrices M but the list of renormalizable models is rather long, so that we do not display it here.

We note that the list of renormalizable three-dimensional Poisson–Lie σ-models with M proportional to the unit matrix is in agreement with the results obtained in [11]. There the conformally invariant Poisson–Lie σ-models, i.e. those with vanishing β-function, were studied and the sigma models with diagonal M and constant dilaton field were obtained. They appear in the above constructed list with vanishing r.h.s. of the renormalization group equation.

5. CONCLUSIONS

We have discussed the transformation properties of the renormalization group flow under Poisson–Lie T-plurality.

Originally, on the basis of our previous experience with the Poisson–Lie T-duality and T-plurality, we expected that it should possible to generalize the proof of the equivalence of the renormalization group flows (11) of Poisson–Lie T-dual sigma models [2] to the case of Poisson–Lie T-plurality. Unfortunately, this task proved to be beyond our present means due the relative complexity of the transformation formula (7) compared to the duality case (18). Thus, we resorted to investigation of the invariance properties of the renormalization group flows on low-dimensional examples. We have found no contradiction with the hypothesis that the renormalization group flows as formulated in [2] are equivalent under the Poisson–Lie T-plurality and with the claim that the renormalization renormalization flows of the models on the Poisson–Lie group and on the Drinfel'd double are compatible.

Next, we studied whether the freedom in the choice of functions ξ^c in the renormalization group equations (24) can be employed to preserve chosen ansatz of the matrix M during the renormalization group flows. It turned out that indeed this ambiguity often enables one to stay within the diagonal ansatz for matrix M.

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