THE SENSITIVITY OF THE GREENHOUSE EFFECT TO CHANGES IN THE CONCENTRATION OF GASES IN PLANETARY ATMOSPHERES

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Abstract. We present a radiative transfer model for Earth-Like-Planets (ELP). The model allows the assessment of the effect of a change in the concentration of an atmospheric component, especially a greenhouse gas (GHG), on the surface temperature of a planet. The model is based on the separation between the contribution of the short wavelength molecular absorption and the long wavelength one. A unique feature of the model is the condition of energy conservation at every point in the atmosphere. The radiative transfer equation is solved in the two stream approximation without assuming the existence of an LTE in any wavelength range.

The model allows us to solve the Simpson paradox, whereby the greenhouse effect (GHE) has no temperature limit. On the contrary, we show that the temperature saturates, and its value depends primarily on the distance of the planet from the central star.

We also show how the relative humidity affects the surface temperature of a planet and explain why the effect is smaller than the one derived when the above assumptions are neglected.

Keywords: greenhouse effect, radiative transfer, earth like planets, surface temperature.

1. Introduction

The influence of concentration changes of atmospheric gases on the surface temperature of planetary atmospheres is a leading thread in current planetary research [7]. Due to the importance of the problem, it is desirable to have a model which can predict correctly the greenhouse effect and its dependence on various changes, while at the same time being sufficiently simple to provide a tool to understand the details of the **radiative transfer** physical processes affecting this problem. We devised such a model. The model consists of two parts – radiative transfer and the molecular absorption dependent optical depth.

We find that the minimum number of bands needed for a model to be faithful to the underlying physics, is a two-band semi-grey model. Consequently, we first solve the radiative transfer problem in terms of two optical depths in the two chosen bands, chosen in such a way as to provide a faithful representation of the underlying radiative transfer. We denote the two bands by *"vis"* and *"fir"* and we will specify them shortly. The radiative transfer equation is then solved in terms of the optical depths τ_{vis} and τ_{fir} to yield a universal function $T_{\mathrm{surf}}(\tau_{\mathrm{vis}}, \tau_{\mathrm{fir}})$. As the solution is found in terms of dimensionless quantities, it is universal and does not depend on many of the planetary parameters such as its mass or specific composition of the atmosphere.

Once we obtain the universal solution to the radiative transfer problem, we calculate the optical depths τ_{vis} and τ_{fir} from the basic molecular absorption data. As the molecular absorption is predominantly line absorption with wide windows, special care must be exercised in devising the algorithm which converts the molecular absorption coefficients $\kappa(\lambda)$ into the above optical depths.

With the universal radiative transfer solution and the optical depths, a change ΔX in the concentration of a certain gas yields a change ΔT according to:

$$\frac{\partial T_{\mathrm{surf}}}{\partial X} = \frac{\partial T_{\mathrm{surf}}}{\partial \tau_{\mathrm{vis}}}\frac{\partial \tau_{\mathrm{vis}}}{\partial X} + \frac{\partial T_{\mathrm{surf}}}{\partial \tau_{\mathrm{fir}}}\frac{\partial \tau_{\mathrm{fir}}}{\partial X}, \qquad (1)$$

where the optical depths τ_{vis} and τ_{fir} will be defined shortly.

Our study is unique is several ways. In particular, we define the two wavelength bands according to physical properties. Since the original treatment by Simpson [10, 11], the semi-grey model has only been applied to RT problems by Thomas and Stamnes [12] and practically no other similar distinction was made. It is also crucial to note the fundamental difference between stellar and planetary atmospheres. The molecular absorption, with its large variation in wavelength, and the total optical depth of planetary atmospheres, are such that the atmosphere contains spectral windows through which the planetary radiation can leak to space almost freely. Such a phenomenon does not exist in stellar atmospheres which are hotter and in which the absorption is due to ions. Consequently, the assumption of LTE, frequently implemented in stellar atmospheres, is not really justified in planetary

atmospheres. Note that the assumption of LTE is usually referred to the distribution of the electrons in the various levels and to the distribution of the radiation field with wavelength. Many treatments treat the visible as a heat source and the IR in the diffusion approximation. We assume that the electrons satisfy the Boltzmann distribution but we do not assume that the radiation field is Planckian. In this sense, the radiation field in illuminated planetary atmospheres is not in LTE and it is improper to assume that $F = \nabla B(T)$ where F is the radiation flux.

Separate treatment of the radiative transfer in the visible and infrared was already carried out in the past (cf. [12, section 12.3]). Our treatment differs in several points: We impose the energy conservation at each layer, we redefine the absorption coefficients, we apply the radiative transfer and not the diffusion approximation, and we find the temperature feedback directly from the radiative equation and not from the diffusion approximation for the far infrared part of the spectrum.

2. BASIC ASSUMPTIONS

We show in Fig. 1 the specific intensities of the insolating star and the thermal emission of the planet. We define $\lambda_{\rm rad}$ as the wavelength at which the two intensities are equal, namely

$$I_{\rm p}(\lambda_{\rm rad}) = \frac{(1-a)}{4}\left(\frac{R_*}{d}\right)^2 I_*(\lambda_{\rm rad}), \qquad (2)$$

where $I_{\rm p}$ and I_* are the specific intensities of the planet and the insolating star at the top of the planetary atmosphere. a is the mean albedo. Energy conservation implies that

$$\frac{1}{C_V}\frac{{\rm d}Q(z)}{{\rm d}t} = \int_0^\infty \kappa(\lambda, z)\left[J(\lambda, z) - B(\lambda, z)\right]{\rm d}\lambda, \quad (3)$$

where J is the mean specific intensity. A further requirement of steady state implies that ${\rm d}Q(z)/{\rm d}t = 0$. As apparent in the figure, $J(\lambda) \gg B(\lambda(T_{\rm atm}))$ for $\lambda < \lambda_{\rm rad}$. Consequently, we can split the energy integral, and write

$$\int_0^{\lambda_{\rm rad}} \kappa(\lambda, z)J(\lambda, z){\rm d}\lambda$$
$$= \int_{\lambda_{\rm rad}}^\infty \kappa(\lambda, z)\left[J(\lambda, z) - B(\lambda, z)\right]{\rm d}\lambda \qquad (4)$$

The first term is positive definite and hence always represents heating. The second term must therefore be negative and thus represents cooling. Clearly, the two wavelength ranges describe different phenomena. At this point, we can also state that $J = B$, which is the condition for LTE, only if the first integral vanishes, namely there is no heating of the atmosphere by absorbed radiation.

Our semi-grey model therefore has two bands, the *vis* band in the range up to $\lambda_{\rm rad}$, and the *fir* band for
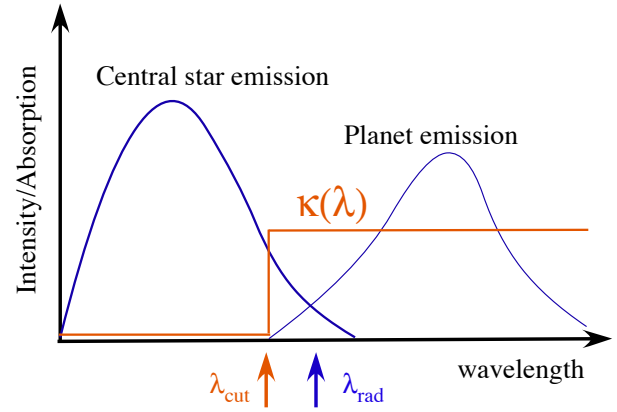

FIGURE 1. The definition of $\lambda_{\rm rad}$, where the specific intensity of insolation equals the specific intensity of the planetary thermal emission. Also shown is the definition of $\lambda_{\rm cut}$, the wavelength above which the molecular absorption (denoted by κ) becomes significant.

$\lambda > \lambda_{\rm rad}$. The radiative transfer equation is solved in the two stream approximation. However, while the optical depths $\tau_{\rm vis}$ and $\tau_{\rm fir}$ are constant over the respective wavelengths, we allow $I(\lambda)$ to change with λ. We assumed in the calculations reported here: $N_{\rm vis} = 200$ wavelengths in the range $(10^3 {\rm \AA} \div \lambda_{\rm rad})$ and $N_{\rm fir} = 400$ wavelengths in the range $(\lambda_{\rm rad} \div 8 \times 10^5 {\rm \AA})$. The atmosphere was divided into 50 slabs. Each slab has an optical depth of $\tau_{\rm vis}/50$ for $\lambda < \lambda_{\rm rad}$ and an optical depth $\tau_{\rm fir}/50$ for $\lambda > \lambda_{\rm rad}$. The $N_{\rm vis}$ and $N_{\rm fir}$ wavelengths were distributed logarithmically. The energy condition was used to calculate the temperature of slab i.

The energy condition in our calculation is given by:

$$\frac{\tau_{\rm vis}}{N_{\rm atm}}\sum_{j=1}^{j=N_{\rm vis}}\left[J_{i,j} - B(T_i, j)\right] \qquad (5)$$

$$+ \frac{\tau_{\rm fir}}{N_{\rm atm}}\sum_{j=1}^{j=N_{\rm fir}}\left[J_{i,j} - B(T_i, j)\right] = 0$$

where $N_{\rm atm}$ is the number of layers in the atmosphere (50 in our case) and the index i runs over all layers in the atmosphere. The above condition must be satisfied for every i. $N_{\rm vis}$ and $N_{\rm fir}$ are the number of wavelengths we use in the respective wavelength range. $J_{i,j}$ is J at atmospheric layer i and wavelength j and $B(T_i, j)$ is the Planck function for temperature T_i at layer i and wavelength λ_j. Although it can be eliminated, $N_{\rm atm}$ is kept for clarity. There are $N_{\rm atm}$ such equations to solve for the temperature T_i in each layer i. Note that we kept the Planck function in the $\lambda < \lambda_{\rm rad}$ range despite the fact that it is very small.

We assume that the total optical depth is divided equally in all the layers of the atmosphere. This implies that the physical width of the atmosphere varies with height. Moreover, we work with the two τ's as the independent variables, not κ.
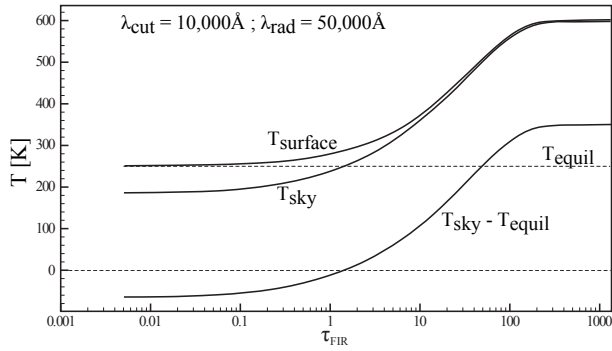
Figure 2. The surface temperature as a function of $\tau_{\rm fir}$ for a fixed $\tau_{\rm vis}$. Also shown is the sky temperature $T_{\rm sky}$.
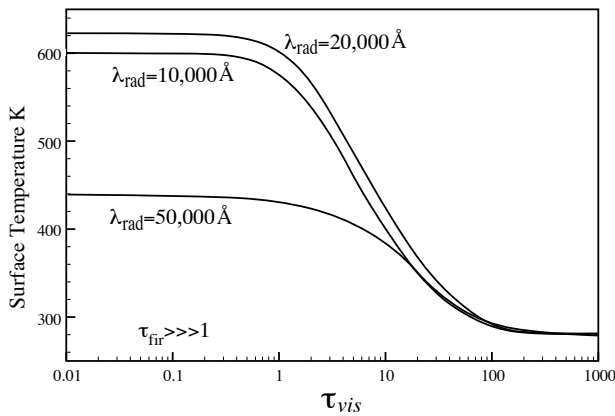

Figure 3. The effect of $\tau_{\rm vis}$ on $T_{\rm surf}$ for three cases.

3. Greenhouse and Anti-Greenhouse models

Figure 2 illustrates a typical case, where $\tau_{\rm fir}$ increases indefinitely while keeping $\tau_{\rm vis}$ fixed. The surface temperature increases slowly for small $\tau_{\rm fir}$'s (i.e., per given logarithmic increase of $\tau_{\rm fir}$), but when $\tau_{\rm fir} \sim 1$, the rate of increase becomes larger, up to about $\tau_{\rm fir} \sim 100$, where the surface temperature saturates and levels off. The reason is that as the temperature rises, the peak of the Planck spectrum progressively moves towards shorter wavelengths and thermal radiation leaks through the *vis* range (cf. [9]). Thus, the greenhouse effect does not experience a runaway when the concentration of any gas, and its corresponding optical depth, increase indefinitely. Clearly, Simpson's paradox does not exist when the problem is solved properly. Previous attempts to solve the paradox assumed the existence of windows in the absorption coefficient. The saturation is obtained, however, even if no windows are assumed. Moreover, the saturation appears even without assuming various feedbacks like increased albedo etc.

The figure also depicts the sky temperature $T_{\rm sky}$, defined such that $\sigma T_{\rm sky}^4$ is equal to the radiation flux from the atmosphere to the surface of the planet. As $T_{\rm sky} < T_{\rm surf}$ in this equilibrium model, the surface cools by exchanging energy with the atmosphere.

Figure 3 demonstrates the effect of a changing $\tau_{\rm vis}$ on the surface temperature, for extremely large $\tau_{\rm fir}$ for which the saturation temperature is reached. We show that $\tau_{\rm vis}$ is sufficiently powerful to create an anti-GHE even under the most adverse condition. As long as $\tau_{\rm vis} \leq 1$ the effect of $\tau_{\rm vis}$ is negligible, but for larger values, the effect is very noticeable. In the limit of $\tau_{\rm vis} \gg 100$, the decrease in temperature approaches an asymptotic value. It is interesting to note that irrespective of $\lambda_{\rm rad}$, the minimal temperature reached is the same. Finally, the saturation temperature is not a monotonic function of $\lambda_{\rm rad}$.

4. The resolution of the Simpson paradox

In 1927 Simpson treated the radiative transfer problem in planetary atmospheres, and tacitly assumed: (a) that $\lambda_{\rm cut} \equiv \lambda_{\rm rad}$. (b) the existence of LTE for the long wavelength range. Under these assumptions, he found that the temperature of the atmosphere is given by:

$$T_{\rm p}^4(\tau) = \left(1 + \frac{3\tau}{4}\right) \frac{(1 - \langle a \rangle)}{4} \left(\frac{R_*}{d}\right)^2 T_*^4, \quad (6)$$

where τ is the mean optical depth for $\lambda \geq \lambda_{\rm rad}$, which is measured from the top of the atmosphere downward. $\langle a \rangle$ is the mean albedo. Simpson did not specify how τ is evaluated, presumably it was the Planck mean. In particular, the temperature near the surface is obtained by substituting $\tau = \tau_{\rm tot}$. Obviously, as $\tau_{\rm tot} \to \infty$, so does $T_{\rm p}$. For sufficiently large $\tau_{\rm tot}$ ($\tau_{\rm tot} \approx 3.8 \times 10^5$), the temperature of the planet reaches the temperature of the central star and can even surpass it. We note that there are particular wavelength ranges in the far IR, for which the total optical depth is as high as 10^4. The possibility of a temperature runaway, as predicted by the simple Simpson solution, was coined the Simpson paradox. There were attempts to resolve the paradox by assuming the existence of windows in the *fir*. However, it is obvious that as long as the *vis* band is ignored and the radiative transfer is solved with a single band, there is no way to eliminate the paradox. Our treatment (see [9]) solves the problem, as we demonstrate that for moderate optical depths of $\tau_{\rm fir}$, the temperature saturates.

5. The $(\tau_{\rm vis}, \tau_{\rm fir})$ plane

The main result is shown in Fig. 4 where the contour lines of constant $T_{\rm surf}$ in the $(\tau_{\rm vis}, \tau_{\rm fir})$ plane are plotted. The classical picture of the greenhouse effect, where $\tau_{\rm vis}$ does not exist or is merged with $\tau_{\rm fir}$, is along the $\tau_{\rm fir}$ axis, where the surface temperature rises monotonically and saturates. As long as $\tau_{\rm vis} < 1$, its effect is small, but as $\tau_{\rm vis}$ increases, its effect becomes more prominent. For $\tau_{\rm vis} \sim 10$, it reduces the saturation temperature.
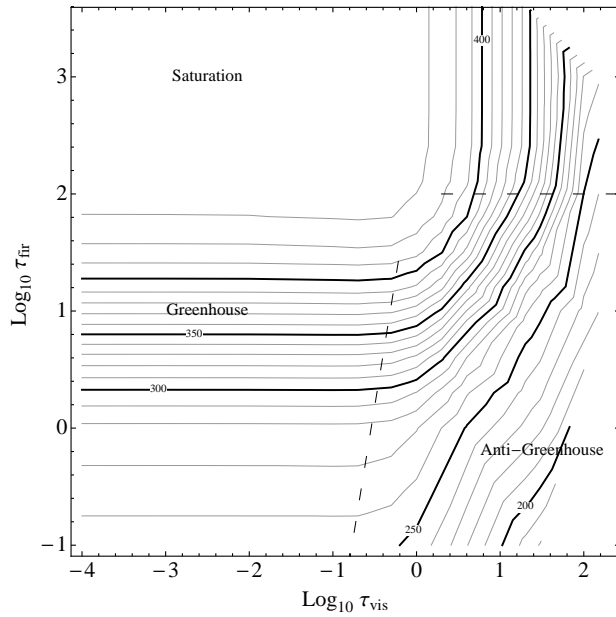
FIGURE 4. Curves of constant T_{surf} in the $(\tau_{\mathrm{vis}}, \tau_{\mathrm{fir}})$ plane. The regions of saturation, greenhouse and anti-greenhouse are marked.

The $(\tau_{\mathrm{vis}}, \tau_{\mathrm{fir}})$ plane allows us to determine the effect that the change in concentration of an atmospheric constituent has, once we evaluate the two optical depths τ_{vis} and τ_{fir} and their changes with concentration. In the next section we show how to evaluate the mean optical depth of the relevant bands.

6. THE ALGORITHM FOR THE OPTICAL DEPTH

Both the Planck and the Rosseland means are poor averages of the absorption when it comes to planetary atmospheres, where molecular absorption dominates. Two factors play here, the existence of spectral windows and the large variation as a function of wavelength. In any averaging of this sort, the optimal weight function is the one which is the closest to the actual solution, which is of course not known. Nevertheless, it is imperative to have a good guess for the weight function, or else the results may be completely skewed. The failure of the Rosseland mean is a good example for a poor weight function which yields an unacceptable result.

Consider first the *vis* range. Since the temperature of the radiation is that of the central star and hence very high relative to that of the planetary atmosphere, the zeroth solution for the transmission of specific intensity $I(z, \nu)$ is given by

$$I(z, \nu) = I_{*,\nu}^{\mathrm{TOA}} e^{-\kappa(\nu)z}. \qquad (7)$$

Here $I_{*,\nu}^{\mathrm{TOA}}$ is the stellar specific intensity at the top of the atmosphere. To secure the energy flux transfer through the atmosphere, we therefore write that:

$$\int_{\nu_1}^{\nu_2} I_{*,\nu}^{\mathrm{TOA}} e^{-\kappa(\nu)z} \mathrm{d}\nu = e^{-\langle \tau_{\mathrm{vis}}\rangle} \int_{\nu_1}^{\nu_2} I_{*,\nu}^{\mathrm{TOA}} \mathrm{d}\nu. \qquad (8)$$

Next we note that the radiation interacts with the atmosphere, which is at temperature T_{atm}, and the stellar radiation is to a good approximation that of a black body at temperature T_*, so we have:

$$\langle \tau_{\mathrm{vis}}\rangle = -\log\left(\frac{\int_{\nu_1}^{\nu_2} B(T_*, \nu) e^{-\tau(\nu, T_{\mathrm{atm}})} \mathrm{d}\nu}{\int_{\nu_1}^{\nu_2} B(T_*, \nu) \mathrm{d}\nu} \right), \qquad (9)$$

where ν_1 and ν_2 correspond to the boundaries of the *vis* range, T_{atm} is the temperature of the atmosphere and τ_{vis} is the total average optical depth for this range. It is important to note that the temperature in the weighting function is not that of the atmosphere through which the radiation passes, but that of the insolating star – the sun in the particular case of the Earth or the star in the general case. The optical depth, $\int_0^Z \kappa(\lambda, T, P)\mathrm{d}\lambda$, however, is calculated with the temperature of the atmosphere. The expression so obtained reduces to the trivial results in various limits. If there is no absorption in the *vis* range, then $\tau_{\mathrm{vis}} = 0$. When the optical depth is constant, then τ_{vis} is equal to this constant as well.

6.1. TRANSITION IN THE FAR INFRARED DOMAIN

Consider now the radiative transfer in the *fir* range. Let τ be measured from the top of the atmosphere downwards. If we write $I_+(\tau)$ as the thermal flux towards larger optical depths (downwards) and $I_-(\tau)$ as the flux towards smaller optical depths, then the solutions for $I_{\pm}(\tau)$ under the above approximations are:

$$F(\tau) \equiv I_-(\tau) - I_+(\tau) = \mathrm{const.} \qquad (10)$$

$$E(\tau) \equiv I_-(\tau) + I_+(\tau) \qquad (11)$$

$$= [I_-(\tau) - I_+(\tau)]\,\tau + \mathrm{const.}$$

By comparing the conditions at the top ($\tau = 0$) to the bottom ($\tau = \tau_{\mathrm{tot}}$), we obtain

$$I_-(\tau_{\mathrm{tot}}) - I_+(\tau_{\mathrm{tot}}) = I_-(0) - I_+(0), \qquad (12)$$

$$I_-(\tau_{\mathrm{tot}}) + I_+(\tau_{\mathrm{tot}}) = [I_-(0) - I_+(0)]\,\tau_{\mathrm{tot}} + [I_-(0) + I_+(0)].$$

The boundary conditions we have are:

$$I_+(0) = I_{*,\mathrm{fir}} \quad \text{and} \quad I_-(\tau_{\mathrm{tot}}) = I_{\mathrm{p,fir}}, \qquad (13)$$

where $I_{*,\mathrm{fir}}$ is the insolation for $\lambda > \lambda_{\mathrm{rad}}$ and $I_{\mathrm{p,fir}}$ is the planet's emission at $\lambda > \lambda_{\mathrm{rad}}$. It is generally assumed that $I_{*,\mathrm{fir}} = 0$. However, if λ_{rad} decreases significantly, this assumption may no longer be justified.

Next we consider the thermal equilibrium of the surface, i.e., total absorption equals the total emission:

$$(1-a(\lambda))I_{*,\mathrm{vis}} + I_{\mathrm{atm},\downarrow} = (1-a(\lambda))I_{\mathrm{p,vis}} + I_{\mathrm{p,fir}}, \quad (14)$$

where $I_{*,\mathrm{vis}}$ is the insolation for $\lambda < \lambda_{\mathrm{rad}}$, $I_{\mathrm{atm},\downarrow}$ is the emission of the atmosphere towards the surface

(at $\lambda > \lambda_{\rm rad}$), $I_{\rm p,vis}$ the planet's emission at $\lambda < \lambda_{\rm rad}$ and a is the albedo at the short wavelengths. Our main point is that $I_{\rm p,vis}$ must be included at relatively high surface temperatures.

Using the two sets of Eqs. 12 and 13, the thermal equilibrium becomes:

$$(1 - a(\lambda))\,(I_{*,\rm vis} - I_{\rm p,vis}) = \frac{2(I_{\rm p,fir} - I_{*,\rm fir})}{2 + \tau_{\rm fir,tot}}. \quad (15)$$

From the above set of equations, we can derive an expression for the average net *fir* flux (per unit frequency) over a finite band $\Delta\nu$, and define an effective opacity through the following:

$$
\begin{aligned}
\overline{\Delta I}_{\rm fir} &= \frac{1}{\Delta\nu}\int_{\nu_1}^{\nu_2}\frac{[I_{\rm p,fir}(T_{\rm p}) - I_{*,\rm fir}]}{1 + 3\tau(\nu)/4}{\rm d}\nu \quad (16)\\
&\approx \frac{[\overline{I}_{\rm p,fir}(T_{\rm p}) - \overline{I}_{*,\rm fir}]}{\Delta\nu}\int_{\nu_1}^{\nu_2}\frac{{\rm d}\nu}{1 + 3\tau(\nu)/4}\\
&\equiv \frac{[\overline{I}_{\rm p,fir}(T_{\rm p}) - \overline{I}_{*,\rm fir}]}{\Delta\nu}\frac{1}{1 + 3\tau_{\rm fir,tot}/4},
\end{aligned}
$$

that is,

$$\tau_{\rm fir,tot} = \frac{4}{3}\left[\frac{\int_{\nu_1}^{\nu_2} B(T_{\rm atm},\nu){\rm d}\nu}{\int_{\nu_1}^{\nu_2}\frac{B(T_{\rm atm},\nu){\rm d}\nu}{1 + 3\tau_{\rm tot}(\nu)/4}} - 1\right]. \quad (17)$$

This expression for the grey absorption is useful because it encapsulates the different behaviors in the *fir*. In particular, wavelength regions for which the optical depth is small allow for a larger flux and therefore receive a larger weight in the averaging. This is present in the Rosseland mean as well. However, unlike the Rosseland mean, the flux does not diverge if the wavelength region becomes optically thin. In such a case, the emission saturates at its surface emission. Thus, the mean can adequately describe the effect of spectral windows.

7. ACTUAL CALCULATION

The data of molecular absorption was taken from the HITRAN compilation [6]. The procedure for calculating the line absorption is described in the manual of this compilation.

8. THE EFFECT OF WATER VAPOR

We calculated the optical depths for increasing degrees of column densities of water molecules. The results are shown in Fig. 5. The effect of increasing the column density of water vapor is shown by the green line. The interesting phenomenon is that the curve starts in the region for greenhouse effect and continues for sufficiently high amounts of water, towards the anti-greenhouse domain. The water curve is not vertical but has a slope. The pink arrow denotes the result that a model which does not distinguish between the two domains, the *vis* and *fir*, would yield. As we can see, the inclusion of the effect of $\tau_{\rm vis}$ lowers the predicted
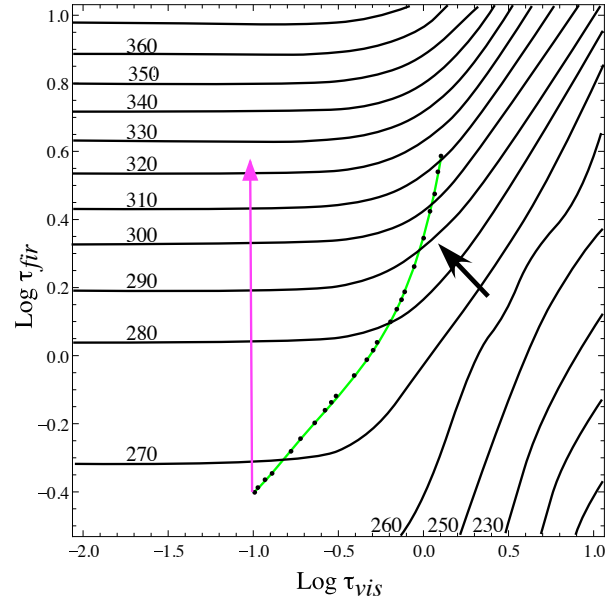
FIGURE 5. The effect of gaseous water molecules in the $(\tau_{\rm fir}, \tau_{\rm vis})$ plane. The points are increasing column density starting from zero to the maximum amount of water vapor (relative humidity of 100%). The green line depicts the increase in $T_{\rm surf}$ while the pink arrow depicts the result that would have been obtained from the classical approach, which neglects $\tau_{\rm vis}$. The arrow marks the location of the Earth according to the Earth mean column density of water vapor as given by Crisp [2].

surface temperature. The arrow marks the location of the Earth for a column density of $8.12 \times 10^{22}\#/{\rm cm}^2$, as given by Crisp [2] for the mean Earth.

8.1. LINE-BY-LINE MODELS

Of all the methods applied to describe the greenhouse effect of any gas on the atmospheric temperature [3] the Line-by-line (LBL) models are the most elaborate. These are not full radiative transfer models but radiation transmittance models. These models calculate the absorbed energy by each absorption line. The LBL model treats only one downward stream of radiation. In the *vis*, it is pure insolation, whereas beyond $\lambda_{\rm rad}$, it is the downward self emission from the top of the atmosphere (TOA) and subsequent lower layers, in response to the long wavelength radiation emitted from the planetary surface.

The absorbed energy so calculated (in terms of W/m^2) is transferred to a global circulation model (GCM), where the effect on the temperature is calculated. Since in this way the increase in concentration of any absorber always leads to increased energy absorbed, the results are always positive, namely heating. Less detailed methods, like the correlated-k distribution are even less accurate.

9. A MAXIMUM TEMPERATURE FOR A PLANET

The fact that the greenhouse saturates implies that a planet at a given distance from the central star has a maximal temperature, the temperature of saturation. This is a strict limit which does not depend on the parameters of the planet like atmospheric composition or the mass or structure of the atmosphere (like pressure and density). The saturation temperature depends only on the distance of the planet from the central star and the mean albedo. Hence, this limit can serve to distinguish between brown dwarfs revolving around a central star and a planet. This model should be further developed for jovian planets, to distinguish them from brown dwarfs. This should be done by changing the lower boundary conditions.

10. SUMMARY

The radiative transfer model for planetary atmospheres presented here enjoys simplicity yet it does not compromise the fundamental physics of the greenhouse effect. The prediction of the greenhouse effect is correct and reliable, as we saw with the prediction of the location of the Earth in the (τ_{vis}, τ_{fir}) plane. In general, having the universal solution $T_{surf}(\tau_{vis}, \tau_{fir})$ allows an easy determination of the surface temperature of the atmosphere, irrespective of the particular composition. We demonstrated the solution in the case of water vapor, and have shown that water vapor do not lead to a runway greenhouse effect, and even does not drive an Earth-like atmosphere into the saturation region.

The model generalizes the greenhouse and the anti-greenhouse effects and generates a comprehensive picture of the phenomenon.

Radiative models which are essentially transmittance models like the LBL, cannot predict the resulting temperature changes, as they do not impose an energy balance equation. Such models can predict how much energy is absorbed by a given atmospheric structure but they do not have the feedback to evaluate the resulting temperature changes. For this purpose, one has to resort to a General Circulation Model, which is a dynamic model and not static.

The saturation of the greenhouse effect leads to the existence of a strict limit to the temperature of a planet. This limit can help to distinguish between planets and brown dwarfs.

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DISCUSSION

Jim Beall — There is a correlation between temperature and CO_2 in the industrial period between 1880–2004. The CO_2 goes from 280 ppm to 380 ppm. But the temperature only increased by 3/4 °C during the interval from 1880–2004.

Smadar — Different analyses give a wide range for the anthropogenic contribution to the temperature rise, and even the measured temperature rise itself has a large uncertainty, such that our results all fall within these limits. It is only with better and more detailed models that we will be able to rule out (through this methodology) possible misconceptions about the extent that anthropogenic CO_2 is the sole cause for heating of the atmosphere.

Maurice van Putten — Is it known how much CH_4 is released when there is a 1 °C increase? The result, I expect, can be readily included in your model.

Smadar — You are correct that CH_4 is the next runner up in our model after CO_2, being the third important greenhouse gas in Earth's atmosphere. The answer to your question can be obtained using our model in a similar manner to that of CO_2 and water. On the one hand, we expect a smaller effect due to the still relatively low column density of CH_4 (about 200 times less than CO_2). Its strong absorption, on the other hand, might compensate for this, and it is hard to predict what will be its final effect without calculating it. We have developed a greenhouse indicator aimed exactly at comparing different greenhouse gases by their slope over the τ_{fir} and τ_{vis} plane, due to changes in concentration. For example, from the presented data, it seems that CO_2 has a stronger greenhouse effect than

water, at least for some humidity regions. So we should wait and see.

Bozena Czerny — 1. What about cloud coverage? This varies across the surface and it is also coupled vertically with high cloud reflectivity. 2. What about mechanic effects like convection or winds? This likely predominantly cools the surface.

Smadar — 1. Cloud coverage has not yet been considered, as it is a complicated matter: clouds at different heights have different albedos, and also the complication of combining the surface albedo with partial cloud coverage, changing the effective albedo. Initial experiments with changing surface albedo, though, show very low sensitivity of the model to even 30 % changes, but this is yet to be determined carefully in further calculations. 2. Convection was still not considered in this preliminary model, only pure radiative transfer. As convection will onset only at

the steepest temperature gradient, which is the adiabatic limit, our calculation yields the upper limit of the temperature. We can parameterize convection from our model by translating the $dT/d\tau_{\mathrm{fir}}$ into dT/dz through a linear scale factor

$$l = \left(\frac{\mathrm{d}\ln\tau}{\mathrm{d}z} \right)^{-1}. \tag{18}$$

This will be carried out in further studies. As to winds, our model is not suitable for calculating the effect of global winds and wind jets (in gravitationally locked planets especially), and it requires coupling with climate global circulation models (GCM), which are used today to calculate the greenhouse temperatures from LBL-models, which yield radiation fluxes in $\mathrm{W/m^2}$. We hope to match our radiative transfer software in the future with a good GCM, in order to feed our results to climatic models. As you see, there is still much work to be done! We are only at the beginning . . .