Multi-Particle Universal Processes

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We generalize bipartite universal processes to the subclass of multi-particle universal processes from one to N particles. We show how the general statement for a multi-particle universal process can be constructed. The one-parameter family of processes generating totally anti-symmetric states has been generalized to a multi-particle regime and its entanglement properties have been studied. A view is given on the complete positivity and the possible physical realization of universal processes.

Keywords: universal processes, quantum entanglement, complete positivity.

1 Introduction

In recent years much effort has been invested in an investigation of how to employ quantum systems as parts of a computer. It has been demonstrated that quantum information is different from classical information and that the essence of this difference lies in the entanglement of quantum systems. Entanglement is a simple consequence of the linearity of quantum mechanics, hence it does not have a classical counterpart. This effect equips a quantum computer with massive parallelism and hence could be used to speed up computation. Therefore quantum information processing could be more efficient than classical information processing. On the other hand, it has become clear that the linear character of quantum theory imposes severe restrictions on the character of elementary tasks of quantum information processing. For example it is impossible to clone an arbitrary quantum state perfectly [1].

Nevertheless, imperfect copies can be made [2, 3]. This particular process of cloning belongs to the class of so called universal processes. These processes act on all input states of a quantum system in a 'similar' way. For universal processes working with one quantum system in a pure state and finishing in an N-particle state this property is mathematically described by the so called covariance condition. Two-particle processes fulfilling this condition were analyzed in [4]. In this paper a theoretical framework was developed within which all possible two-particle universal processes can be described and those compatible with the linear character of quantum theory were determined. Of special interest were universal processes generating entangled two-particle output states which do not contain any separable components. It has been shown that this particular subclass forms a one-parameter family of totally anti-symmetric states with respect to permutations of the two particles.

The aim of this paper is to generalize the results obtained for two-particle universal processes to the subclass of multi-particle universal processes from one to N particles. For this purpose we use theoretical framework developed in [4]. We show how the general statement for a multi-particle universal process can be constructed. The one-parameter family of processes generating totally anti-symmetric states was generalized to a multi-particle regime and its properties were studied, mainly bipartite entanglement. For this purpose we use the concept of negativity [5]. Finally we briefly review the question of the complete positivity of the obtained universal processes.

2 General structure of a universal process

We will now derive the general structure of the N-particle covariant mapping using the statement for the two-particle case [4]. In the following we assume that the D-dimensional one-particle Hilbert space $\mathcal{H}$ is the same for all N particles. An arbitrary one-particle pure state can be described by D-dimensional generalized Bloch vector $\mathbf{p}$. Such a state will be simply denoted by $|\mathbf{p}\rangle$.

Consider a linear map $\mathcal{P}$ from 1 to N particles, i. e.

$$\mathcal{P}:\rho_{in}(\mathbf{p})\mapsto\rho_{out}(\mathbf{p}),\ \rho_{in}(\mathbf{p})\in\mathcal{B}(\mathcal{H}),\ \rho_{out}(\mathbf{p})\in\mathcal{B}(\mathcal{H}^{\otimes N}),\quad(1)$$

where the density matrix $\rho_{in}(\mathbf{p})$ has the form

$$\rho_{in}(\mathbf{p})=|\mathbf{p}\rangle\langle\mathbf{p}|\otimes\frac{1}{D^{(N-1)}}I^{\otimes(N-1)},\quad(2)$$

i.e. the first particle is in the pure state $|\mathbf{p}\rangle$, and all others are in the state of a complete mixture.

We will call this map universal (or covariant), if it possesses the following covariance property

$$\rho_{out}(\mathbf{p})=(U(\mathbf{p}))^{\otimes N}\rho_{out}(\mathbf{p}_0)(U(\mathbf{p})^{\dagger})^{\otimes N}\quad(3)$$

where the one-particle unitary transformation $U(\mathbf{p})$ maps the state $|\mathbf{p}_0\rangle$ to the state $|\mathbf{p}\rangle$, i. e.

$$|\mathbf{p}\rangle=U(\mathbf{p})|\mathbf{p}_0\rangle.\quad(4)$$

To construct the general statement for covariant mapping with N particles we will use the results for the two-particle [4]. We will now summarize the main results.

An arbitrary density operator of a D dimensional quantum system can be represented in terms of some basis of the $su(D)$ algebra. In order to implement the covariance condition (3) we used the basis $\{\mathbf{A}_{ij}\},(i,j=1,\ldots,D)$, which fulfills the following commutation relations

$$[\mathbf{A}_{ij},\mathbf{A}_{mn}]=\mathbf{A}_{ab}(\delta_{jm}\delta_{ai}\delta_{bn}-\delta_{in}\delta_{am}\delta_{bj}),\quad(5)$$

(we have used the Einstein summation convention in which we have to sum over all indices which appear in an expression twice from one to D). A representation of these generators is given by the $D\times D$ matrices

$$(\mathbf{A}_{ij})^{(kl)}=\delta_{ik}\delta_{jl}-\frac{1}{D}\delta_{ij}\delta_{kl}.\quad(6)$$

The density matrix $\rho_{in}(\mathbf{p})$ can be written in the form

$$\rho_{in}(\mathbf{p})=\frac{1}{D}(\mathbf{I}+p_{ij}A_{ij}).\quad(7)$$

In terms of matrices (6) the most general output state is represented by the density matrix

$$\rho_{out}(\mathbf{p}) = \frac{1}{N^2}\mathbf{I}\otimes\mathbf{I} + \alpha_{ij}^{(1)}(\mathbf{p})\mathbf{A}_{ij}\otimes\mathbf{I} + \alpha_{ij}^{(2)}(\mathbf{p})\mathbf{I}\otimes\mathbf{A}_{ij} + \beta_{ijkl}(\mathbf{p})\mathbf{A}_{ij}\otimes\mathbf{A}_{kl}, \tag{8}$$

where the linearity requirement of quantum mechanics implies that $\alpha_{ij}^{(1)}(\mathbf{p})$, $\alpha_{ij}^{(2)}(\mathbf{p})$ and $\beta_{ijkl}(\mathbf{p})$ have to be linear with respect to the parameters of the Bloch vector $\mathbf{p}$. To fulfill the covariance condition (3) the matrix (8) has to involve only terms invariant under the transformation of the form $U\otimes U$, i.e. the scalar part, or terms which transform like the generators of the $su(D)$ group $\mathbf{A}_{ij}$, i.e. the vector part. It was shown in [4] that the nontrivial scalar is $\mathbf{A}_{ij}\otimes\mathbf{A}_{ji}$ and the nontrivial vectors have the form $\mathbf{A}_{ki}\otimes\mathbf{A}_{kj}$, $\mathbf{A}_{ik}\otimes\mathbf{A}_{kj}$. From these facts, the most general output matrix that fulfills the covariance condition (3) and depends linearly on the input must have the form

$$\rho_{out}(\mathbf{p}) = \frac{1}{N^2}\mathbf{I}\otimes\mathbf{I} + \alpha^{(1)}p_{ij}\mathbf{A}_{ij}\otimes\mathbf{I} + \alpha^{(2)}p_{ij}\mathbf{I}\otimes\mathbf{A}_{ij} + C\mathbf{A}_{ij}\otimes\mathbf{A}_{ji} + \beta p_{ij}\mathbf{A}_{ik}\otimes\mathbf{A}_{kj} + \beta^{*}p_{ji}\mathbf{A}_{ki}\otimes\mathbf{A}_{jk}. \tag{9}$$

where $\alpha^{(1,2)}$ and C are real parameters and β is complex. The ranges of these parameters are restricted by the fact that $\rho_{out}(\mathbf{p})$ must be a density matrix, i. e. a positive operator with a unit trace.

This statement can be easily generalized to the N-particle case. The output density matrix has to involve only the scalar terms (multiplied by an arbitrary constant) and the vector terms (multiplied by a constant with the parameters of the initial state p_{ij}). These terms can be constructed by the tensor product of the one-particle scalar I and vector $\mathbf{A}_{ij}$, and the two particle scalar and vector terms. To obtain a scalar term we have to sum over all free indices, and for a vector term two indices must remain free (these are later summed with the parameters p_{ij}). The summation is done in such a way that one summation index is in the first position while the second is in the second position of the generators (6).

For example we will show the explicit form of the scalar and vector terms for the case $N = 3$ (for more information on three-particle universal processes see [6]):

- Scalar terms
  - $\mathbf{A}_{ij}\otimes\mathbf{A}_{ji}\otimes\mathbf{I}$, $\mathbf{A}_{ij}\otimes\mathbf{I}\otimes\mathbf{A}_{ji}$, $\mathbf{I}\otimes\mathbf{A}_{ij}\otimes\mathbf{A}_{ji}$,
  - $\mathbf{A}_{ij}\otimes\mathbf{A}_{jk}\otimes\mathbf{A}_{ki}$ + hermitian conjugate
- Vector terms
  - $\mathbf{A}_{ij}\otimes\mathbf{I}\otimes\mathbf{I}$, $\mathbf{I}\otimes\mathbf{A}_{ij}\otimes\mathbf{I}$, $\mathbf{I}\otimes\mathbf{I}\otimes\mathbf{A}_{ij}$,
  - $\mathbf{A}_{ik}\otimes\mathbf{A}_{kj}\otimes\mathbf{I}$, $\mathbf{A}_{ik}\otimes\mathbf{I}\otimes\mathbf{A}_{kj}$, $\mathbf{I}\otimes\mathbf{A}_{ik}\otimes\mathbf{A}_{kj}$ +hermitian conjugate
  - $\mathbf{A}_{ij}\otimes\mathbf{A}_{kl}\otimes\mathbf{A}_{lk}$, $\mathbf{A}_{kl}\otimes\mathbf{A}_{ij}\otimes\mathbf{A}_{lk}$, $\mathbf{A}_{kl}\otimes\mathbf{A}_{lk}\otimes\mathbf{A}_{ij}$,
  - $\mathbf{A}_{ik}\otimes\mathbf{A}_{kl}\otimes\mathbf{A}_{lj}$, $\mathbf{A}_{kl}\otimes\mathbf{A}_{ik}\otimes\mathbf{A}_{lj}$, $\mathbf{A}_{ik}\otimes\mathbf{A}_{lj}\otimes\mathbf{A}_{kl}$ +hermitian conjugate

In this case the output density matrix $\rho_{out}(\mathbf{p})$ depends on 23 real parameters, which are restricted by the positivity of this operator. Compared with just five real parameters in the two-particle case, we see that the complexity of the N-particle covariant mapping grows rapidly with the number of particles N.

If we want to study the properties the covariant mappings in more detail, we can simply study the resulting density matrix for an arbitrary input state. Due to the covariance (3), functions like the entropy or the entanglement measures of the particular universal process will have the same value for all possible input states, because these functions are invariant under local transformations.

The universal processes generating entangled two-particle output states which do not contain any separable components were studied in [4]. It has been shown that this particular subclass forms a one-parameter family of totally anti-symmetric states with respect to permutations of the two particles. We will now give a generalization of this class for the case of N-particle processes.

3 Generalization of the universal process generating anti-symmetric states to N particles

In this section we propose a generalization of the two-particle universal process generating totally anti-symmetric states to an arbitrary number of particles. The initial state of the first qudit is assumed to be $|1\rangle$ without loss of generality, due to the covariant condition (3). We will use the following notation

$$A_D = \{\vec{j}\in Z^N \,|\, \vec{j} = (1, j_2, \ldots, j_N), 2\le j_2 < j_3 < \ldots < j_N \le D\}$$
$$B_D = \{\vec{j}\in Z^N \,|\, \vec{j} = (j_1, \ldots, j_N), 2\le j_1 < j_2 < \ldots < j_N \le D\} \tag{10}$$
$$\left|\{\vec{j}\}\right| = \frac{1}{\sqrt{N!}}\sum_{\pi\in P_N}\mathrm{sgn}(\pi)\left|\pi(j_1)\ldots\pi(j_N)\right\rangle,$$

where P_N is a group of permutations of N elements. The output states of the N-particle universal process generating totally anti-symmetric states form a one-parameter family and can be written in the form

$$\rho_N^{(anti)} = (1-p_{(N)})\,\sigma_N^{(anti)} \oplus p_{(N)}\,\tau_N^{(anti)}, \quad p_{(N)}\in\langle 0,1\rangle, \tag{11}$$

where the matrices $\sigma_N^{(anti)}$ and $\tau_N^{(anti)}$ are given by

$$\sigma_N^{(anti)} = \frac{(N-1)!}{(D-1)\ldots(D-N+1)}\sum_{\vec{j}\in A_D}\left|\{\vec{j}\}\right\rangle\left\langle\{\vec{j}\}\right|,$$
$$\tau_N^{(anti)} = \frac{(N-1)!}{(D-1)\ldots(D-N)}\sum_{\vec{j}\in B_D}\left|\{\vec{j}\}\right\rangle\left\langle\{\vec{j}\}\right|. \tag{12}$$

In the case $N = D$, i.e. when the number of particles is equal to the dimension of the one-particle Hilbert space, we have to put $p_{(N)} = 0$ and the one-parameter family collapses to a single process generating an N-particle singlet state. This is the only case when the one-parameter family produces a pure state, while in all other possible cases the output state $\rho_N^{(anti)}$ is always mixed. The maximally mixed state is described by the parameter $p_{(N)} = \frac{D-N}{D}$ and the output density matrix involves all $\binom{D}{N}$ N-particles anti-symmetric states with the same probability. In this case the output density matrix has the form

$$\rho_N^{(anti)}\left(p_{(N)} = \frac{D-N}{D}\right) = \binom{D}{N}^{-1}\sum_{\vec{j}\in C_D}\left|\{\vec{j}\}\right\rangle\left\langle\{\vec{j}\}\right|, \tag{13}$$

where the set C_D is defined as

$$C_D = \{\vec{j}\in Z^N \,|\, 1\le j_1 < j_2 < \ldots < j_N \le D\}. \tag{14}$$

This state is invariant under the transformation of the type $U^{\otimes N}$ and therefore this particular process involves only the scalar part.

Reduced states of K particles are the same for all K-particle subsystems and have the same form as the output state of the K-particle universal process. The only difference is that the parameter $p_{(K)}$ has to be replaced by $p_{(N)}^{(K)}$, which is given by

$$p_{(N)}^{(K)} = \frac{K p_{(N)} + (N-K)}{N}. \tag{15}$$

The properties of the two-particle subsystems are described by the parameter $p_{(N)}^{(2)}$. The entanglement of two-particle subsystems is quantified by the negativity [5]. For this one-parameter family this function is given by (c.f. [4])

$$N(p_{(2)}) = \frac{p_{(2)}}{2(D-1)} + \frac{1}{2}\sqrt{\frac{p_{(2)}^2}{(D-1)^2} + \frac{\left(1-p_{(2)}\right)^2}{D-1}}. \tag{16}$$

The negativity of the two-particle subsystems of the N-particle universal process generating totally anti-symmetric states is then given by

$$N(p_{(N)}) = \frac{1}{2N(D-1)}\Big(N-2+2p_{(N)} + \sqrt{4Dp_{(N)}^2 + 4(N-2D)p_{(N)} + N^2 - 4N + 4D}\Big). \tag{17}$$

This function has a minimum for

$$p_{(N)} = \frac{D-N}{N}, \tag{18}$$

i.e. the scalar process that produces the most disordered state also produces the state with the least entangled two-particle subsystems.

The maximal value of negativity is obtained for $p_{(N)} = 1$ or $p_{(N)} = 0$, depending on the relation between N and D. For $D > 2N+1$ the maximum is at the point $p_{(N)} = 0$ and has the value

$$N_{\max}(D > 2N+1) = \frac{N-2+\sqrt{4D+N(N-4)}}{2N(D-1)}. \tag{19}$$

For $D < 2N+1$ the maximum is at the point $p_{(N)} = 1$ and is given by

$$N_{\max}(D < 2N+1) = \frac{1}{D-1}. \tag{20}$$

In the case when $D = 2N+1$ the values of negativity at the points $p_{(N)} = 0$ and $p_{(N)} = 1$ are equal, so the function $N(p_{(N)})$ has two equal peaks.

The output density matrix (11) has the form of the convex sum of two density matrices, $\sigma_N^{(anti)}$ is the sum of the projectors onto the anti-symmetric subspace involving the initial state $|1\rangle$, $\tau_N^{(anti)}$ involves the complementary projectors to those in $\sigma_N^{(anti)}$ on the N-particle anti-symmetric subspace. For the special case of $p_{(N)} = 0$ only the matrix $\sigma_N^{(anti)}$ is involved and the output density matrix can be written as

$$\sigma_N^{(anti)} = \frac{(N-1)!}{(D-1)\ldots(D-N+1)} A_N \rho_{in} A_N, \tag{21}$$

where A_N is the projection operator onto the anti-symmetric subspace of the N-particle Hilbert space. Hence this particular process can be realized by a certain projector acting on the initial state. Universal processes with this property will be studied in the following section.

4 Realization of universal processes

As was shown above, some important universal processes can be realized by the action of certain projectors on the input state. Besides the example in the previous section, where the projector on the anti-symmetric subspace of the N-particle Hilbert space was used, we can mention the optimal universal copying process from one to N particles, which can be implemented by using a projector on the symmetric subspace of the N-particle Hilbert space [2]. These projectors play the role of Kraus operators for a given processes and can be used to construct a unitary evolution operator in the canonical way [7], as will be shown below.

The natural question arises what class of projectors have this property of realizing covariant processes. Let T be a projector on some subspace of the N-particle Hilbert space and ρ_{in}. The mapping on the density operators $\rho_{out} = \mathcal{N} T \rho_{in} T$, where $\mathcal{N}$ is normalizing factor ρ_{out} to be a density matrix, fulfills the covariant condition if the following equality is satisfied for all one-particle unitary transformations U and for all one-particle pure input states $|\mathbf{p}\rangle\langle\mathbf{p}|$:

$$\begin{aligned} &T(U|\mathbf{p}\rangle\langle\mathbf{p}|U^{\dagger} \otimes I^{\otimes(N-1)})T \\ &= U^{\otimes(N)} T|\mathbf{p}\rangle\langle\mathbf{p}| \otimes I^{\otimes(N-1)} T U^{\dagger\otimes N} \end{aligned} \tag{22}$$

A sufficient condition for T to satisfy (22) is

$$U^{\otimes(N)} T = T U^{\otimes(N)}, \tag{23}$$

i.e. T must commute with all unitary transformations of the form $U^{\otimes(N)}$.

All N-particle transformations $U^{\otimes(N)}$ form an N-particle representation of a one-particle unitary group. This representation is generally reducible for $N \geq 2$. From Schur's lemma, which states that a representation is irreducible if and only if the only matrix commuting with all members of the representation is proportional to the identity, we can deduce that if T is proportional to the projector on invariant subspace of the reducible representation then condition (23) is satisfied.

More generally, let $\{P_\alpha\}$ be a set of all projectors on the minimal invariant subspaces, then operator T, which is defined as

$$T = \sum_\alpha \sigma_\alpha P_\alpha, \tag{24}$$

commutes with all unitary operators of the form $U^{\otimes(N)}$, for arbitrary $\sigma_\alpha \in \mathbb{C}$. Now it is obvious that the mapping $\widetilde{T}$

$$\rho_{out} = \widetilde{T}(|\mathbf{p}\rangle\langle\mathbf{p}|) = \frac{T(|\mathbf{p}\rangle\langle\mathbf{p}| \otimes I^{\otimes(N-1)})T^{\dagger}}{\mathrm{Tr}\,[T(|\mathbf{p}\rangle\langle\mathbf{p}| \otimes I^{\otimes(N-1)})T^{\dagger}]}, \tag{25}$$

where T is of the form (24), from the pure one-particle input states to the N-particle output density operators satisfies condition (22). The next simple consequence of the presented construction is that any convex combination of transformations of the form (25) also satisfies (22).

Even though transformation (25) fulfils covariant condition (22) it need not be completely positive, which is a necessary condition for the physical acceptability of this mapping.

From the Kraus theorem we have the following criterion [8]: linear mapping P from the density matrices on the input Hilbert space to the density matrices on the output Hilbert space is completely positive if and only if there exists a so called Kraus decomposition of a given transformation, i.e. if there exists a set of linear mappings, Kraus operators, $\{E_k\}$ satisfying the two following conditions for any input density operator σ,

$$P(\sigma) = \frac{\sum_k E_k \sigma E_k^\dagger}{\mathrm{Tr}\left[\sum_k E_k \sigma E_k^\dagger\right]} \tag{26}$$

$$\sum_k E_k^\dagger E_k \le I. \tag{27}$$

The definition of transformation $\widetilde{T}$ guarantees fulfillment of the first condition (26). To satisfy (27) we must restrict the range of parameters σ_α in (24) to inequality $|\sigma_\alpha| \le 1$ for all α.

The construction of the completely positive universal processes which was described above can still be generalized. The transformation of the form (25) describes the universal process from one to N particles. By applying the partial trace operation over arbitrary M particles, $M < N$, to the N-particle output state of transformation (25) we get the transformation from one particle pure states to an $(N-M)$-particle. It is easy to see that in this case the covariant condition remains satisfied and we get the completely positive universal process on $N-M$ particles. The last form of the universal process on N particles built up of projectors is therefore the following

$$\rho_{out} = \frac{\mathrm{Tr}_{i_1,\ldots,i_M}\left[T^{(L)}(\rho_{in} \otimes I^{\otimes(L-1)})T^{(L)\dagger}\right]}{\mathrm{Tr}\left[T^{(L)}(\rho_{in} \otimes I^{\otimes(L-1)})T^{(L)\dagger}\right]}, \tag{28}$$

where $M \in \mathbb{N}_0$, $L = N + M$, $\{i_1,\ldots,i_M\} \subset \{1,\ldots,L\}$ and $T^{(L)}$ is the transformation of the form (24) on the L-particle Hilbert space, where the condition $|\sigma_\alpha| \le 1$ is required for all α. Note that by tracing out different particles we can generally get a different classes of processes [9]. The simple consequence of the linearity of quantum theory is that the convex combination of the processes of the form (28) is also a completely positive universal process.

In the case of formula (28) the mapping $T^{(L)}$ does not play the role of the Kraus operator of the universal process (except the case $M = 0$), but (28) can be further rewritten in the following way:

$$\rho_{out} = \frac{\sum_{\substack{j_1,\ldots j_M=1 \\ k_1,\ldots k_M=1}}^{D} \mathcal{K}^{i_1,\ldots i_M}_{j_1,\ldots j_M,k_1,\ldots k_M}\, \rho_{in} \otimes I^{\otimes(N-1)} \mathcal{K}^{i_1,\ldots i_M\,\dagger}_{j_1,\ldots j_M,k_1,\ldots k_M}}{\mathrm{Tr}\left[T^{(L)}(\rho_{in} \otimes I^{\otimes(L-1)})T^{(L)\dagger}\right]} \tag{29}$$

where D is a dimension of the one-particle Hilbert space and $\mathcal{K}^{i_1,\ldots i_M}_{j_1,\ldots j_M,k_1,\ldots k_M}$ are the linear operators on the N-particle Hilbert space, which are defined

$$\mathcal{K}^{i_1,\ldots i_M}_{j_1,\ldots j_M,k_1,\ldots k_M}|\varphi\rangle = \langle j_1|_{i_1} \otimes \ldots \otimes \langle j_M|_{i_M} P^{(L)}|\varphi\rangle \otimes |k_1\rangle \otimes \ldots \otimes |k_M\rangle. \tag{30}$$

Subscription by the bra-vector $\langle i|_j$ means that $\langle i|$ acts on the j-th component of the tensor product, i.e. on the j-th particle. The operators defined by equation (30) form the set of Kraus operators of the universal process (28).

We generalize the bipartite universal processes to the subclass of the multi-particle universal processes from one to N particles. We show how the general statement for a multi-particle universal process can be constructed. The one-parameter family of processes generating totally anti-symmetric states was generalized to a multi-particle regime and its entanglement properties were studied. A view on the complete positivity and the possible physical realization of the universal processes is given.

5 Conclusions

Universal processes may play an important role in various branches of quantum information processing, e. g. in preparation of entangled states or copies of the input state. In addition to operations with two particles, multi-particle operations are also of interest. For this purpose we have generalized the two-particle universal processes to a multi-particle regime and we have shown how the general statement for the multi-particle universal process can be constructed using the results for two-particle universal processes. For the preparation of multi-particle entangled states we generalized the one-parameter family of processes generating totally anti-symmetric states to a multi-particle regime and studied its bipartite entanglement properties. This one-parameter class generates entangled states with equal reduced states, thus the entanglement is shared uniformly between all pairs of particles. A particular process of this one-parameter family can be realized by a simple action of a projection operator. For processes of this kind we can in a canonical way construct a unitary evolution operator, thus they are completely positive. An open question is whether other universal processes can be performed by an action of a certain projector, possibly on a larger Hilbert space (i. e. with more particles) and then tracing over some of the particles. Such processes will also be completely positive, and therefore physically feasible, which makes them very interesting for the possible future realization of a quantum computer.

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