Dynamical Symmetry Breaking In R^N Quantum Gravity

A. T. Kotvytskiy, D. V. Kruchkov

Abstract

We show that in the R^N gravitation model, there is no dynamical symmetry breaking effect in the formalism of the Schwinger-Dyson equation (in flat background space-time). A general formula for the second variation of the gravitational action is obtained from the quantum corrections $h_{\mu\nu}$ (in arbitrary background metrics).

Keywords: quantum gravity, Schwinger-Dyson equations formalism, dynamical symmetry breaking.

The study of dynamical mass generation and dynamical symmetry breaking in different external fields [1, 2], including the gravitational field [3–5] is an important step in studying fundamental interactions, which can either be considered in field theory and an attempt at quantization can be made or considered as an external interaction. This paper is devoted to the possibility of dynamical symmetry breaking in such gravitation models using the Schwinger-Dyson equation formalism [6–9]. The primary goal is a study of some gravity properties of $f(R)$-gravity [10], where R is a scalar curvature. One of the most discussed variants of such models is R^N gravity [11, 12]. In particular, we are interested in a possible effect of dynamical chiral symmetry breaking in this model under graviton and fermion interaction, which leads to mass formation of fermions. This problem is important for several reasons. First, quantum corrections should be taken into consideration from different scenarios describing the evolution of the early universe. Second, an understanding of the dynamical symmetry breaking mechanism is important for black hole physics. Third, R^N — gravity, as a modified theory of General Relatity, is also interesting from the phenomenological point of view, the peculiarity of which appears in various cosmological models. In particular, it is necessary to introduce either dark energy or a quintessence with a much more exotic state equation to explain the accelerated expansion of the observed universe (being within the limits of relativity theory). To sum up, it is significant for a description of the future of the universe. However, if one modifies gravity in a proper way, it is also possible to receive interesting cosmological dynamics without introducing new concepts. Here, we provide a comparative analysis of the above formalism both in the case of flat background metrics (Minkowski space) and in the case of an arbitrary background.

Let us expand the four-dimensional space-time metric as follows

$$\tilde{g}_{\mu\nu} \simeq g_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $\tilde{g}_{\mu\nu}$ is perturbed metric, $g_{\mu\nu}$ is background metric, $h_{\mu\nu}$ are quantum corrections. This definition leads to the following results. In the case of flat background space-time in the R^N gravity model there is an existence of corrections like $O(h^N)$ order and higher that gives no possibility to get the necessary equations. We can speak about the absence of a symmetry breaking effect (at least in the Schwinger-Dyson equation formalism). However, if the background metric is curved, then corrections of order appear and, particularly, specifically quadric ones $O(h^2)$. They allow us to obtain the Schwinger-Dyson equations and to test them for possible dynamical symmetry breaking. An important conclusion is the following: if our real Universe is described by a curved metric, then while constructing quantum gravity theory, we should take into consideration the summands of all powers in R, as they provide the same degree in small quantum corrections h.

1 Schwinger-Dyson equations

One possible method for a dynamical symmetry breaking study is the Schwinger-Dyson equation formalism. Since it is impossible to write closed system equations for all elements of the Feynman diagram, we have to use some approximations, which allow us to solve the Schwinger-Dyson equation and to find the type of exact propagator. Exact propagators and the vertex part are connected by the integral relation,

$$S^{-1} - S_0^{-1} = i\frac{\delta\Gamma_2}{\delta S} \tag{2}$$

where S, S_0 – exact and free fermion propagators, $\dfrac{\delta\Gamma_2}{\delta S}$ – describes two particle and irreducible interaction diagrams, and Γ_2 is a part of the effective action

$$\Gamma[S] = -i\mathrm{Sp}\left(\mathrm{Log}\, S^{-1} + S_0^{-1}S\right) + \Gamma_2[S]. \quad (3)$$

Here we confine ourselves to the exact fermion propagator, which is written down in original type

$$S(p) = \frac{1}{A(p^2)p^\mu\gamma_\mu - B(p^2)}, \quad (4)$$

where $A(p^2)$, $B(p^2)$ are some unknown functions of the fourth momentum p, γ_μ are Dirac matrices. Then the Schwinger-Dyson equation for this propagator can be put down like [13–15]

$$\begin{aligned}(A-1)p^\mu\gamma_\mu - B \;=\; & \int \frac{\mathrm{d}^4 q}{(2\pi)^4 i}\Gamma_{\alpha\beta}(p, q-p)S'(q)\cdot \\ & \Gamma'_{\mu\nu}(q, p-q)G'^{\alpha\beta\mu\nu}(p-q), \quad (5)\end{aligned}$$

where Γ', G' are an exact vertex function and an exact graviton propagator.

The infinite set of SDEs determining the exact fermion and boson Green functions, as well as the full interaction vertex, can be solved within some truncation version only. This means that the only subset of Feynman graphs is taken into account, which naturally leads to the disappearance of the magical cancellation of gauge-dependent terms in the S-matrix expansion. For this reason, the most widespread ladder approximation gives gauge-dependent results, when the fermion Green function is treated to be exact only in the case when the boson propagator and the interaction vertex are taken to be free [14]. In this way, we can define the functions $A(p^2)$, $B(p^2)$ for the quantum R^N-gravity.

The following important remark should also be made. We have to decide what kind of gravity action and gauge-fixing term should be used. It is convenient to bring the following condition

$$\begin{aligned}S_{gf} \;=\; & \frac{-\beta_1}{2M^2}\int \mathrm{d}^4 x\sqrt{-g}\left(\nabla_\lambda h^\lambda_\mu - \beta_2\nabla_\mu h\right)\cdot \\ & \left(g^{\mu\nu}\nabla_\rho\nabla^\rho + \beta_3\nabla^\mu\nabla^\nu\right)\times \quad (6) \\ & \left(\nabla_\sigma h^\sigma_\nu - \beta_2\nabla_\nu h\right),\end{aligned}$$

where $\beta_1, \beta_2, \beta_3$ are arbitrary parameters. Then, putting down the second variation of the full action which we can describe as the sum of the gravitational field action and the gauge-fixing action in the form

$$\delta^2\left(S_g + S_{gf}\right) = \frac{1}{2M^2}\int \mathrm{d}^4 x h^{\mu\nu}H_{\mu\nu\rho\sigma}h^{\rho\sigma}. \quad (7)$$

Then the gravitational field propagator is defined as an operator inverse to $H_{\mu\nu\rho\sigma}$, that is

$$G^{\mu\nu\rho\sigma} = M^2\left(H^{-1}\right)^{\mu\nu\rho\sigma}. \quad (8)$$

Let us include the gravitational field in our consideration.

2 Flat background metric

Here we single out the part of the quadratic action according to corrections h, in order to put down the Schwinger-Dyson equations. Instead of (1) the perturbed metric will be written down as

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}, \quad (9)$$

where $\eta_{\mu\nu}$ is the Minkowski metric (we choose the signature $(+---)$).

We consider the following action

$$S_g = \frac{1}{2M^2}\int \mathrm{d}^4 x\sqrt{-g}R^N. \quad (10)$$

Note that action (10) does not have to be a full action for the gravitational field, while the Einstein linear gravitation on the curvature and Λ — term, and other possible variants, can be also included. However, a discussion of the effects caused by the form (10) is the main goal of this paper.

In the case of a flat background metric, we have the following expansion (with accuracy up to the second order approximation)

$$\sqrt{-g} \simeq 1 + \frac{1}{2}h - \frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \frac{1}{8}h^2, \quad (11)$$

where raising and lowering the index is by $\eta_{\mu\nu}$ background metric and $h = \eta^{\mu\nu}h_{\mu\nu}$.

The Riemann tensor is now

$$R^\mu_{\nu\rho\sigma} = \partial_\rho\Gamma^\mu_{\nu\sigma} - \partial_\sigma\Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\tau\rho}\Gamma^\tau_{\nu\sigma} - \Gamma^\mu_{\tau\sigma}\Gamma^\tau_{\nu\rho}, \quad (12)$$

while the Ricci tensor is determined by the Riemann tensor convolution according to the first and the third indices.

Then, for the scalar curvature we get

$$\begin{aligned}R \;=\; & g^{\mu\nu}R_{\mu\nu} \simeq \frac{1}{2}\left(\eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\rho}h^\nu_\rho\right)\times \quad (13) \\ & \left(\partial_\alpha\partial_\mu h^\alpha_\nu - \partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu h + \partial_\alpha\partial_\nu h^\alpha_\mu + \right. \\ & \left. O(h^2)\right) = \partial_\alpha\partial^\nu h^\alpha_\nu - \partial_\alpha\partial^\alpha h + O(h^2).\end{aligned}$$

This implies, in the case of R^N gravitation, the development according to h has the form

$$R^N \simeq \left(\partial_\alpha\partial^\nu h^\alpha_\nu - \partial_\alpha\partial^\alpha h + O(h^2)\right)^N \sim O(h^N) \quad (14)$$

That is, the smallest order has a power N in quantum corrections. Physically this means that the graviton propagator is missed in such a gravitation model, and only N-particle vertex functions exist. Therefore, it is not essential for the study of dynamical symmetry breaking (DSB) in this formalism. In other words, we can say that there is no dynamical symmetry breaking effect in this approximation.

Now we proceed to the quite different situation of non-flat background space-time.

3 Curved background metric

Let us choose the action for the gravitational field in the form of

$$S_g = \frac{1}{2M^2} \int \mathrm{d}^4x \sqrt{-\tilde{g}} \tilde{R}^N, \tag{15}$$

where tilde denotes a perturbed metric, and is determined by (1).

Since the background metric is not the Minkowski one, the summands of all h orders appear in all redefined constructions. So the expressions for Christoffel symbols can be reduced to the form (with accuracy within the second order on quantum corrections)

$$\begin{aligned} \tilde{\Gamma}^{\mu}_{\nu\rho} \simeq {} & \Gamma^{\mu}_{\nu\rho} + \frac{1}{2}\left(g^{\mu\gamma} - h^{\mu\gamma}\right) \cdot \\ & \left(\nabla_\rho h_{\gamma\nu} + \nabla_\nu h_{\gamma\rho} - \nabla_\gamma h_{\nu\rho}\right), \end{aligned} \tag{16}$$

where $\Gamma^{\mu}_{\nu\rho}$ are the Christoffel symbols calculated according to the unperturbed metric $g_{\mu\nu}$, and ∇_ρ is the covariant derivative relative to the same metric.

The complete expression for the Riemann tensor (in this approximation) is

$$\begin{aligned} \tilde{R}^{\mu}_{\nu\rho\sigma} \simeq {} & R^{\mu}_{\nu\rho\sigma} + \frac{1}{2}\left(g^{\mu\tau} - h^{\mu\tau}\right) \times \\ & \big(h_{\tau\varepsilon} R^{\varepsilon}_{\nu\sigma\rho} + h_{\varepsilon\nu} R^{\varepsilon}_{\tau\sigma\rho} + \nabla_\rho \nabla_\nu h_{\tau\sigma} - \\ & \nabla_\rho \nabla_\tau h_{\nu\sigma} - \nabla_\sigma \nabla_\nu h_{\tau\rho} + \nabla_\sigma \nabla_\tau h_{\nu\rho}\big) + \\ & \frac{1}{4} g^{\mu\gamma} g^{\tau\varepsilon} [(\nabla_\sigma h_{\varepsilon\nu} + \nabla_\nu h_{\sigma\varepsilon} - \nabla_\varepsilon h_{\nu\sigma}) \cdot \\ & (\nabla_\tau h_{\gamma\rho} - \nabla_\rho h_{\gamma\tau} - \nabla_\gamma h_{\tau\rho}) + \\ & (\nabla_\sigma h_{\tau\gamma} - \nabla_\tau h_{\sigma\gamma} + \nabla_\gamma h_{\tau\sigma}) \cdot \\ & (\nabla_\rho h_{\nu\varepsilon} + \nabla_\nu h_{\rho\varepsilon} - \nabla_\varepsilon h_{\nu\rho})]. \end{aligned}$$

Here, the procedure for raising and lowering the indices is provided by the background metric.

Hence, we find the scalar curvature

$$\tilde{R} = \tilde{g}^{\nu\sigma} \tilde{R}^{\rho}_{\nu\rho\sigma} \simeq \left(g^{\nu\sigma} - h^{\nu\sigma} + h^{\nu\alpha} h^{\sigma}_{\alpha}\right) \tilde{R}^{\rho}_{\nu\rho\sigma}. \tag{17}$$

Introduce the following symbols

$$\mathsf{h}_1 \equiv -R_{\nu\sigma} h^{\nu\sigma} + \nabla^\nu \nabla^\sigma h_{\nu\sigma} - \nabla^\nu \nabla_\nu h, \tag{18}$$

$$\begin{aligned} \mathsf{h}_2 \equiv {} & R_{\nu\sigma} h^{\nu\alpha} h^{\sigma}_{\alpha} - h^{\rho\tau}\left(\nabla_\rho \nabla^\sigma + \nabla^\sigma \nabla_\rho\right) h_{\tau\sigma} - \\ & h^{\rho\tau} \nabla_\rho \nabla_\tau h + h^{\rho\tau} \nabla_\sigma \nabla^\sigma h_{\rho\tau} + \\ & \frac{1}{2} \nabla_\sigma h^{\tau\sigma} \nabla_\tau h - \nabla_\sigma h^{\tau\sigma} \nabla^\gamma h_{\gamma\tau} - \\ & \frac{1}{4} \nabla^\tau h \nabla_\tau h + \frac{1}{2} \nabla^\tau h \nabla^\gamma h_{\gamma\tau} + \\ & \frac{3}{4} \nabla_\sigma h^{\varepsilon\rho} \nabla^\sigma h_{\rho\varepsilon} - \frac{1}{2} \nabla^\varepsilon h^{\rho}_{\sigma} \nabla^\sigma h_{\rho\varepsilon}. \end{aligned} \tag{19}$$

The expression for scalar curvature is

$$\tilde{R} \simeq R + \mathsf{h}_1 + \mathsf{h}_2, \tag{20}$$

where h_1 contains the first power of the quantum corrections only, and h_2 contains the second powers.

Then, the N-th power of the scalar curvature becomes

$$\begin{aligned} \tilde{R}^N \simeq {} & R^N + N R^{N-1}\left(\mathsf{h}_1 + \mathsf{h}_2\right) + \\ & \frac{1}{2} N(N-1) R^{N-2} \mathsf{h}_1^2. \end{aligned} \tag{21}$$

In the case of arbitrary background space-time we should take into account the expansion

$$\begin{aligned} \det(\tilde{g}) \simeq {} & \det(g) + h_{\mu\nu} K^{\mu\nu}(g) + \\ & h_{\mu\nu} h_{\alpha\beta} F^{\mu\nu\alpha\beta}(g), \end{aligned} \tag{22}$$

where

$$\begin{aligned} K^{\mu\nu}(g) = {} & \varepsilon^{\alpha\beta\rho\sigma}\big(\delta^{\mu}_{0}\delta^{\nu}_{\alpha} g_{1\beta} g_{2\rho} g_{3\sigma} + \delta^{\mu}_{1}\delta^{\nu}_{\beta} g_{0\alpha} g_{2\rho} g_{3\sigma} + \\ & \delta^{\mu}_{2}\delta^{\nu}_{\rho} g_{0\alpha} g_{1\beta} g_{3\sigma} + \delta^{\mu}_{3}\delta^{\nu}_{\sigma} g_{0\alpha} g_{1\beta} g_{2\rho}\big). \end{aligned}$$

We underline the fact that if the background metric is the Minkowski one $\eta_{\mu\nu}$, then function $K^{\mu\nu} = -\eta^{\mu\nu}$ and with accuracy within the first order we get the well-known formula $-\tilde{g} \simeq 1 + h$. The expression for $F^{\mu\nu\alpha\beta}(g)$ is cumbersome, and we do not present it here.

To consider the possibility of dynamical symmetry breaking, it is necessary to have an expression for the second variation of the gravitational action. Let us take into consideration the following expansion

$$\sqrt{a + x_1 + x_2} \simeq \sqrt{a} + \frac{x_1 + x_2}{2\sqrt{a}} - \frac{x_1^2}{8\sqrt{a^3}}, \tag{23}$$

where $x_1 + x_2 \ll a$, and also $(x_1)^2 \sim x_2$.

Then,

$$\begin{aligned} \sqrt{-\tilde{g}} \simeq {} & \sqrt{-g - h_{\mu\nu} K^{\mu\nu} - h_{\mu\nu} h_{\alpha\beta} F^{\mu\nu\alpha\beta}} \simeq \\ & \sqrt{-g} - \frac{h_{\mu\nu} K^{\mu\nu} + h_{\mu\nu} h_{\alpha\beta} F^{\mu\nu\alpha\beta}}{2\sqrt{-g}} - \\ & \frac{\left(h_{\mu\nu} K^{\mu\nu}\right)^2}{8\sqrt{-g^3}}. \end{aligned} \tag{24}$$

Thus, we obtain the final form of the second variation

$$\begin{aligned} \delta^{(2)} S_g = {} & \frac{1}{2M^2} \int \mathrm{d}^4x \Bigg(\sqrt{-g} \cdot \\ & \left(N R^{N-1} \mathsf{h}_2 + \frac{1}{2} N(N-1) R^{N-2} \mathsf{h}_1^2\right) - \\ & \frac{N}{2\sqrt{-g}} h_{\mu\nu} K^{\mu\nu} R^{N-1} \mathsf{h}_1 - \\ & \frac{h_{\mu\nu} h_{\alpha\beta} F^{\mu\nu\alpha\beta}}{2\sqrt{-g}} R^N - \frac{\left(h_{\mu\nu} K^{\mu\nu}\right)^2}{8\sqrt{-g^3}} R^N\Bigg). \end{aligned} \tag{25}$$

Note, that in the case of the quadric gravitation ($N = 2$) and a flat background ($R = 0$), instead of (25) we obtain

$$\delta^{(2)} S_g = \frac{1}{2M^2} \int \mathrm{d}^4 x \sqrt{-g} \mathsf{h}_1^2, \qquad (26)$$

which coincides with [13].

So, it has been shown that in the case of any arbitrary background metric for any N there is an equation (25). Hence, we obtain the propagator (8) and the Schwinger-Dyson equations (5). This indicates that the effect of dynamical symmetry breaking is possible in R^N-gravity.

4 Conclusions

Some quantum properties of model R^N-gravity have been considered in the paper. A comparative analysis for two cases: a) a flat background space-time, and b) an arbitrary curved background has been carried out.

Expanding this model of gravity with quantum corrections $h_{\mu\nu}$, we found that in the first case the smallest order of quantum corrections is N. This means that in the quantum theory of the R^N-gravity graviton propagator (for $N > 2$) does not exist, and there is a vertex function of graviton-graviton interaction that is not used in this formalism. Thus, there is no Schwinger-Dyson equation (5) and, therefore, there is no effect to discuss.

In case b, a general formula (25) for the second variation of gravitational action on the quantum corrections $h_{\mu\nu}$ is obtained, which in the limit $R \to 0$ coincides with the previously known results. It is determined that in this formulation of the problem under the effects of dynamical symmetry breaking research the terms of all powers from the scalar curvature should be considered in action for the gravitational field, because they give exactly the same order in quantum fluctuations as the Einstein action ($N = 1$).

Therefore, if we represent a full gravitational action in the form of $L = \alpha_1 R^1 + \alpha_2 R^2 + \alpha_3 R^3 + \ldots$, where α_i are some factors of a necessary dimension, then in the case of quantum gravity, each term will contribute to the propagator of a graviton ($\sim h^2$). And we cannot neglect any term. In fact, this means that there exist Schwinger-Dyson equations for any N, and, hence, the effect of dynamical symmetry breaking is possible.

We would like to note the analogy with the Fierz-Pauli model, in which all undesirable degrees of freedom in the flat metric background are cancelled, whereas in a curved background one undesirable degree of freedom (the Boulware-Deser mode) appears again in the spectrum [16].

References

[1] Sitenko, Yu. A.: Polarization of the Massless Fermionic Vacuum in the Background of a Singular Magnetic Vortex in 2 + 1 Dimensional Space-Time, *Ukr. J. Phys.* **43** (1998), 1 513–1 525.

[2] Sitenko, Yu. A.: Induced vacuum condensates in the background of a singular magnetic vortex in 2 + 1 dimensional space-time, *Phys.Rev.* **D60** (1999), 125 017.

[3] Miransky, V. A.: *Dynamical symmetry breaking in quantum field theories*, Singapore : World Scientific, 1993.

[4] Inagaki, T., Odintsov, S. D., Shil'nov, Yu. I.: Dynamical symmetry breaking in the external gravitational and constant magnetic fields, *Modern Phys. A.* **12** (1999), 481–503.

[5] Gitman, D. M., Odintsov, S. D., Shil'nov, Yu. I.: Chiral symmetry breaking in $d = 3$ NJL model in external gravitational and magnetic fields, *Phys. Rev.* **D54** (1996), 2 968–2 970.

[6] Odintsov, S. D., Shil'nov, Yu. I.: Schwinger-Dyson equations in QED at non-zero temperature, *Modern Phys. Lett. A.* **6** (1991), 707–710.

[7] Bashir, A., Diaz-Cruz, J. L.: A study of Schwinger-Dyson Equations for Yukawa and Wess-Zumino Models, *J. Phys.* **G25** (1999), 1 797–1 805.

[8] Elizalde, E., Odintsov, S. D., Romeo, A., Shil'nov, Yu. I.: Schwinger-Dyson Equations and Chiral Symmetry Breaking in 2D Induced Gravity, *Mod. Phys. Lett.* **A10** (1995), 451–456.

[9] Šauli, V., Adam, J., Jr., Bicudo, P.: Dynamical chiral symmetry breaking with Minkowski space integral representations, *Phys. Rev.* **D75** (2007), 087 701.

[10] De Felice, A., Tsujikawa, S.: $f(R)$ Theories, *Living Rev. Relativity* **13** (2010), 1–161, 3. http://www.livingreviews.org/lrr-2010-3

[11] Kotvytskiy, A. T., Kryuchkov, D. V.: The generalized Equation of Gravitation in Type Models, *The Journal of Kharkiv National University, ser. Nuclei, Particles and Fields* **4(40)** (2009), 29–33.

[12] Bertolami, O., Boehmer, C. G., Harko, T., Lobo, F.: Extra force in $f(R)$ modified theories of gravity, *Phys. Rev.* **D75** (2007), 104 016.

[13] Shil'nov, Yu. I., Chitov, V. V., Kotwicki, A. T.: Equations Schwinger-Dyson and dynamical symmetry breaking in two-dimension quantum gravity, *Russian Phys. J. (Izv. Vuzov. Fizika)* **3** (1997), 40–44.

[14] Shil'nov, Yu. I., Chitov, V. V., Kotwicki, A. T.: Chiral Symmetry breaking in quantum — gravity, *Modern Phys. Lett. A.* **12** (1997), No 34, 2 599–2 612.

[15] Shil'nov, Yu. I., Chitov, V. V., Kotwicki, A. T.: Dynamical symmetry breaking in quadratic quantum gravity, *Nuclei Physics* **60** (1997), No 8, 1 510–1 517.

[16] Rubakov, V. A., Tinyakov, P. G.: Infrared-modified gravities and massive gravitons, *Usp. Fiz. Nauk* **51** (2008), 759–792.

A. T. Kotvytskiy
D. V. Kruchkov
E-mail: kotw@mail.ru
Department of Theoretical Physics
V. N. Karzin Kharkov National University
Svobody Sq. 4, 61077 Kharkov, Ukraine