

# On Deterministic Chaos in Microdischarge Phenomena

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*Time series of pulsating microdischarges were analysed. The results showed that deterministic chaos is present in these series. The estimated values of the correlation coefficients indicated a strong chaotic discharge behaviour.*

*Keywords: microdischarges, deterministic chaos, fractals.*

## 1 Introduction

The external partial microdischarges which appear at the insulating barriers of high voltage components are in fact of the same origin as the so-called “barrier” microdischarges studied in the field of discharge physics. The only difference is that the external partial microdischarges in high voltage technology are considered to be an undesirable phenomenon that is suppressed as much as possible, while the barrier microdischarges in discharge physics are treated as a useful phenomenon with many industrial applications (ozone generators, modern flat plasma displays, excimer lamps, etc.).

Regardless of the diverse approaches to the microdischarges running in the close vicinity of insulation barriers, it is worth studying them since both the mentioned approaches (elimination versus utilisation) require a thorough knowledge of the basic physics underlying their mechanism.

The fractal properties of electrical pre-breakdown phenomena like electrical trees in high voltage insulation are sometimes explained [1] as a consequence of competitive acting of positive and negative feedbacks. The positive feedback accelerates degradation of the insulation and is mainly governed by an electric field. Under its influence the degradation proceeds straightforwardly through local inhomogeneities without producing “fractally” branched filamentary structures. Conversely, the negative feedback tends to decelerate the degradation and to facilitate fractal branching of the filamentary structures. The negative feedback tends to equilibrate the system at low electrical fields and may even cause the system to attain the regime of deterministic chaos [2]. Since a similar situation, i.e. competition between accelerating and decelerating factors, may also be encountered with the microdischarge phenomenon, the question arises whether deterministic chaos also appears with the microdischarges that are in fact precursors to electrical trees. The goal of this paper is to find an answer to this question.

## 2 Experiment

The sandwiched plane-to-plane electrode system was employed. All experiments were carried out at normal atmospheric conditions. Polyethylene terephthalate sheets 0.2 mm in thickness were inserted between two brass electrodes to serve as insulating barriers. The metal-insulator-metal interfaces created in this way were loaded with a uniform high voltage of 4 kV to study the microdischarge signal. The whole experimental arrangement is depicted in Fig. 1.

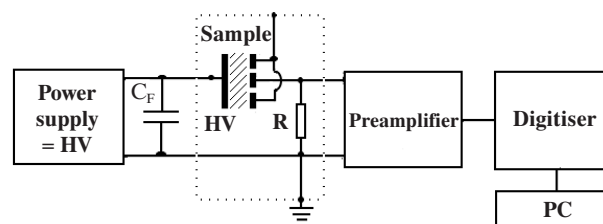


Fig. 1: Experimental arrangement

Microdischarges were detected as voltage pulses across the resistor  $R = 10\text{ k}\Omega$ . The resistor is connected in series with the electrode system and experiences electric pulses initiated by microdischarges. The pulsating signal was amplified and digitised in a digitiser of unique construction [3, 4] and then processed by a special software implemented on a PC. The pulsating signal has the character of a discrete time series  $\{U_i\}_{i=1}^N$  with a finite number  $N$  of voltage pulses  $U_i$  registered at moments  $t_i$  (see Fig.2). The number  $N$  of registered pulses amounted to about 60 000.

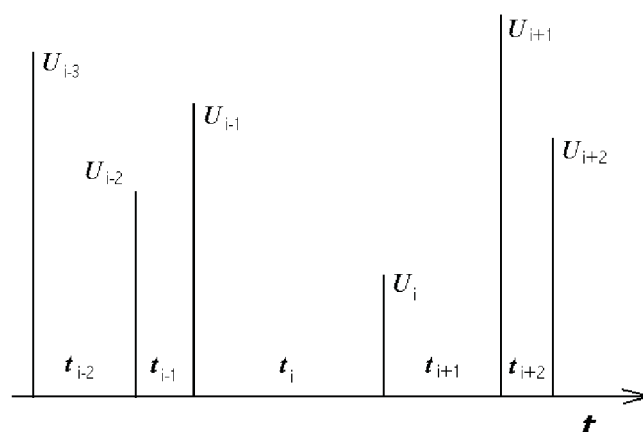


Fig. 2: Microdischarge time series

## 3 Computational model

Investigations of deterministic chaos within a time series usually consist of three separate steps: determining the correlation coefficient (correlation dimension), specifying the Lyapunov exponent, and reconstructing the attractor. The present analysis of the discrete microdischarge series  $\{U_i\}_{i=1}^N$  focuses on determining the correlation coefficient. For this purpose the series of heights (amplitudes)  $\{U_i\}_{i=1}^N$  was rearranged into a sequence of  $d$ -dimensional pseudo-vectors  $\omega(d)$

with the interval  $\Delta N > 1$  between their components, i.e., the components are not successive measurements. For example, the three-dimensional ( $d=3$ ) pseudo-vectors  $\omega(3)$  with the component interval  $\Delta N = 2$  can be written as follows

$$\begin{aligned} \omega_1(3) &= U_1 \mathbf{i} + U_3 \mathbf{j} + U_5 \mathbf{k} \\ \omega_2(3) &= U_2 \mathbf{i} + U_4 \mathbf{j} + U_6 \mathbf{k} \\ &\dots \\ \omega_n(3) &= U_n \mathbf{i} + U_{n+2} \mathbf{j} + U_{n+4} \mathbf{k}, \\ n &= N - (d - 1)\Delta N. \end{aligned} \tag{1}$$

Between the  $i$ -th and  $j$ -th pseudo-vectors  $\omega_i(d)$  and  $\omega_j(d)$  there is a scalar separation

$$\begin{aligned} |\omega_i(d) - \omega_j(d)| &= \left[ (U_i - U_j)^2 + (U_{i+\Delta N} - U_{j+\Delta N})^2 + \dots \right. \\ &\quad \left. + (U_{i+(d-1)\Delta N} - U_{j+(d-1)\Delta N})^2 \right]^{\frac{1}{2}}. \end{aligned} \tag{2}$$

A double sum running over all the  $n$  measured values determines the correlation integral  $C(r)$

$$C(r) = \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j=1, j \neq i}^n H(r - |U_i - U_j|), \tag{3}$$

where  $H(x)$  is the Heavyside step function

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \tag{4}$$

Repeating the procedure for a range of  $r$ , the correlation integral in the form of the power law can be obtained

$$C(r) \approx r^v. \tag{5}$$

By carrying the whole procedure for increasing values of  $d$ , deterministic chaos – if present – will manifest itself in a constant value of  $v$ . This constant value is just the fractal correlation dimension  $v_0$ . If the sequence is the result of random noise,  $v$  increases in proportion to  $d$ .

The strange attractor with the largest value of  $v_0$  shows the greatest excursions away from its centre of attraction. Therefore, larger values of  $v_0$  correspond to a more chaotic behaviour while smaller values are related to a reduction of chaocity and to a movement towards determinicity.

### 4 Results

By carrying the computational procedure described above for a set of  $d$  dimensions, the graphs of the functions  $C(r)$  were plotted in a bilogarithmic co-ordinate system (see Fig. 3) to obtain the correlation coefficients  $v$ . These coefficients were

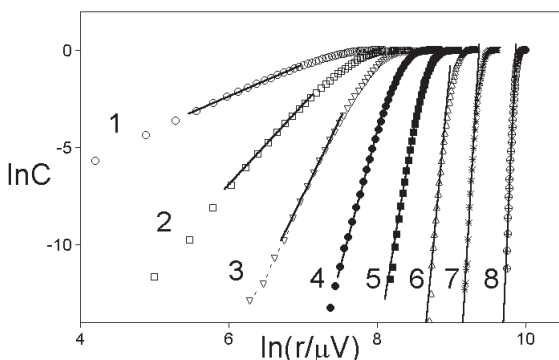


Fig. 3: Graphs of correlation integrals  $C(r)$

calculated as the slopes from the linear sections of the graphs  $C(r)$ , using the least square method. The resulting dependences  $v(d)$  showed a monotonically increasing behaviour with an asymptote, and showed a quite satisfactory resemblance to an exponential pattern with two parameters  $v_0$  and  $d_0$

$$v = v_0 \left[ 1 - \exp\left(\frac{-d}{d_0}\right) \right]. \tag{6}$$

The asymptotes  $v_0$  are accepted as the best approximation for the correlation dimensions. The existence of the asymptotes indicates the occurrence of deterministic chaos rather than a random noise. In Fig. 4 the corresponding value of  $v_0$  is close to 89. Such a value is a consequence of strong deterministic chaos, i.e., pronounced chaotic behaviour of the studied discharge sequence. Similar results have been reported for electrical trees [1], in which microdischarges run in the very tips of the discharge branches.

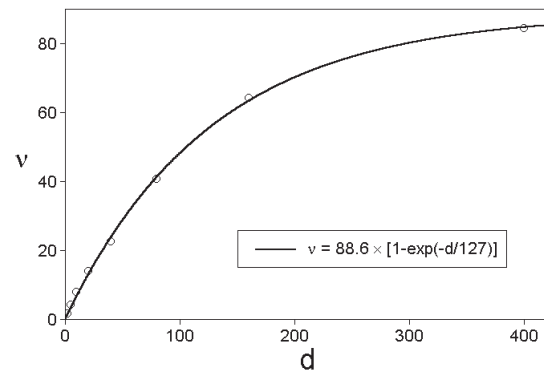


Fig. 4: Correlation coefficients  $v$  in dependence on  $d$ -dimensions

### 5 Conclusion

Deterministic chaos seems to be one of the characteristic features of microdischarge phenomena. It manifests itself not only in electrical treeing, which is a pre-breakdown phenomenon appearing in high voltage insulations, but it is also characteristic for the microdischarges that precede treeing. The study of deterministic chaos in microdischarge time series would deserve more attention since it could provide missing information on possible connections between fractal morphology [5–7] and chaotic properties of microdischarge phenomena.

### 6 Acknowledgment

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