Credibility of Design Procedures

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Theory of structural reliability enables comprehensive analysis of structural elements with respect to various limit states, and provides valuable insights into the methodology of applied standards. In addition to reliability analysis of the structural element, a new concept of the credibility of theoretical models used to calculate the design value of basic variables is introduced. The presented example of structural verification for limit states of cracking shows that the credibility of commonly applied formulas and reliability of a reinforced concrete slab have a great scatter and are in some cases inadequate.

Keywords: credibility of design procedures, reliability of elements, basic variables, model uncertainties, probability of failure, reliability index.

1 Introduction

Construction works are commonly designed using various operational (deterministic) methods specified in national or international standards or other prescriptive documents. Consequently, the actual reliability of a designed structure depends on the applied standards, their principles and application rules including quality requirements specified for design and verification of the structure with respect to the ultimate and serviceability limit states.

Previous experience and performed analysis show that the ultimate resistance, serviceability or durability of a given structure designed in accordance with various standards is to be expected within a broad range. The actual structural resistance may depend not only on the used theoretical models, but also on appropriate detailing and other rules recommended in the applied standards. Moreover, theoretical models specified in various standards for determining structural resistance provide considerably different probabilities of exceeding the calculated design value.

Theory of structural reliability and mathematical statistics enable a comprehensive analysis of structural elements with respect to both the ultimate and serviceability limit states. It is shown that the credibility of the design formulas and the reliability of structural elements may have a great scatter and are in some cases inadequate. In particular the credibility of the design formulae with respect to serviceability limit states appears to be very low.

2 Design procedure

Two limit states should generally be considered in the design of construction works – the ultimate and serviceability limit states. Reaching the ultimate limit states leads to structural failure, e.g., due to loss of overall equilibrium, by reaching critical strain conditions at a certain part of the cross-section or by fatigue of the construction materials. The serviceability limit states characterise a structural condition in which, where it is reached, the specified requirements are not satisfied. This can be caused by cracking, deformation or sensitivity to vibration. In the case of reinforced or prestressed structures, the stresses in concrete, reinforcing and prestressing steel should be limited. Uncontrolled stresses in concrete under serviceability conditions can lead to excessive cracking, high levels of creep, and plastic deformations negatively influencing the properties of the whole stucture. Tensile stresses in reinforcements should be checked to avoid inelastic strain, unacceptable cracking and deformations.

To evaluate the reliability or probability of failure of a structure it is necessary to specify basic variables describing the load and resistance parameters and their relationship corresponding to the relevant performance criterion of a structure. This relationship, called performance function (limit state function) G, is given as

$$G = \mathrm{g}(\boldsymbol{X}) \tag{1}$$

where the vector of basic variables $\boldsymbol{X}$ represents random variables, which may be time dependent. The limit state of a structure for random realisation $\boldsymbol{x}$ of the vector of basic variables can be defined as $\mathrm{g}(\boldsymbol{x}) = 0$. It represents a state beyond which a structure can no longer fulfil the function for which it was designed. The method of partial factors (Level I design method), used in most current European countries for structural design, and which is also a basis of the new European standards (Eurocodes), deals with influences of uncertainties and randomness of basic variables by means of design values assigned to the variables. The design limit state function is expressed in terms of the design values of basic variables as

$$G_{\mathrm{d}} = \mathrm{g}(\boldsymbol{x}_{\mathbf{d}}) \tag{2}$$

where $\boldsymbol{x}_{\mathbf{d}}$ is the vector of design values of basic variables represented, e.g., by design values of actions F_{d}, design material properties f_{d}, design models of uncertainties θ_{d}, design values of geometrical quantities a_{d}, serviceability constraints C_{d} and importance coefficients γ_{n}, in accordance with ISO 2394 [1]. The design condition is expressed as

$$\mathrm{g}(\boldsymbol{x}_{\mathbf{d}}) \geq 0 \tag{3}$$

and the design vector of basic variables $\boldsymbol{x}_{\mathbf{d}}$ can be obtained on the basis of characteristic values of variables and of a set of partial factors γ for actions and material properties. The values of the partial factors depend on the design situation and the considered limit state. The design procedure, recommended values of partial factors and other reliability elements are described in various standards for structural design used throughout the world. The partial factors are based on previous experience and calibration using methods of structural reliability. Knowledge about the reliability level of a structure designed according to current standards and also the credibility of calculation models recommended by the standards can be used for optimisation of design procedures.

3 Reliability

In accordance with traditional reliability theory, a structure can be considered as reliable if the following condition is satisfied

$$P_{\mathrm{f}} < P_{\mathrm{t}}, \text{ or } \beta > \beta_{\mathrm{t}} \tag{4}$$

where the probability of failure P_{f} is given as

$$P_{\mathrm{f}} = P\{\mathrm{g}(\boldsymbol{X}) < 0\} = \int_{\mathrm{g}(\boldsymbol{X})<0} \varphi_X(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x} \tag{5}$$

where $\varphi_X(\boldsymbol{x})$ is the joint probability density function of the vector $\boldsymbol{X}$ of basic variables. The probability of failure P_{f} can be expressed by the reliability index $\beta = -\boldsymbol{\Phi}^{-1}(P_{\mathrm{f}})$, where Φ is the distribution function of the standardised normal variable. The probability of failure P_{t} and reliability index β_{t} are the specified (target) values that should not be exceeded during the intended period of time, as shown by Haldar, Mahadevan [2].

If for example the reliability of a structural element stems from a comparison between action effects E and resistance R, then the probability of structural failure can be expressed as

$$P_{\mathrm{f}} = P\{E < R\} = \int_{R-E<0} \varphi_E(\xi)F_R(\xi)\,\mathrm{d}\xi \tag{6}$$

where $\varphi_E()$ is the density function of variable E, $F_R()$ is the distribution function of variable R, and ξ denotes a generic point of E and R.

4 Credibility

The accuracy of the calculation models given in standards can be examined using the credibility analysis proposed in [10]. Credibility is defined as the probability P_{c} of design value $\mathrm{g}(\boldsymbol{x}_{\mathrm{d}})$ being exceeded by random variable $\mathrm{g}(\boldsymbol{x})$. Thus, the probability P_{c} is given as

$$P_{\mathrm{c}} = \mathrm{P}\{\mathrm{g}(\boldsymbol{X}) > \mathrm{g}(\boldsymbol{x}_{\mathrm{d}})\} = \int_{\mathrm{g}(\boldsymbol{X})>\mathrm{g}(\boldsymbol{x}_{\mathrm{d}})} \varphi_X(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}\,. \tag{7}$$

Thus, similar general principles can be used to determine the credibility of prescriptive formulae and the reliability of the structure. It should be mentioned, that unfavourable changes in the properties of a significant basic variable can dramatically influence the credibility as well as the reliability of the element in both the ultimate and serviceability limit states.

An example of reliability analysis of a reinforced concrete element with respect to crack width and credibility analysis of selected theoretical models recommended for verification of the limit state of cracking are presented in the following, for the sake of illustration.

5 Verification of crack width regarding selected standards

Cracking in reinforced concrete elements due to load effects can be controlled by applying the calculation models recommended in various standards or by fulfilling appropriate practical rules (e.g., for the position of reinforcement, size of bars, area of reinforcement). Many theoretical models exist for predicting crack width. Almost any standard for the design of concrete structures contains some calculation formulae, as is also shown in Structural Concrete [3]. The following condition should be satisfied in the process of crack width verification

$$w(\boldsymbol{x}_{\mathrm{k}}) \le w_{\mathrm{lim}} \tag{8}$$

where $w(\boldsymbol{x}_{\mathrm{k}})$ is the calculated crack width and w_{lim} is the crack width limit. Most current standards recommend various theoretical models for crack width. The structural element is designed and verified using the specific methodology provided by the relevant set of national or international standards. It is known that the vector of characteristic values $\boldsymbol{x}_{\mathrm{k}}$ and design values $\boldsymbol{x}_{\mathrm{d}}$ of basic variables may differ from country to country (e.g., due to the different geometrical requirements, material properties defined through non consistent classes of concrete, different properties of reinforcement). The design and verification of structures is influenced not only by prescribed values of basic variables, but also by specified design procedures (e.g., different load combinations used for checking ultimate and serviceability criteria), and by detailing rules.

Results for the credibility of theoretical models given in the prestandard ENV 1992-1-1 [4], in CEB FIP Model Code 1990 [5] and also in its previous proposal (marked prMC 90 in all Figures), in the working draft of new operational document prEN 1992-1-1 [6], in BS 8110 [7], in ACI 318-89 [8] and in ČSN 73 1201 [9] are presented here, based on previous works by Marková and Holický [10, 11].

A reinforced concrete slab subjected to bending moment is selected to analyse theoretical models for crack width. The slab is from 0.19 to 0.29 m in depth, with a span of 5 m. It is loaded by one permanent load and one imposed load. Note that in all cases the same material and concrete reinforcement cover are assumed. The following alternatives are considered:

1. The reinforced concrete slab is designed for the ultimate limit states according to the Eurocodes. The crack width is verified taking into account the theoretical models for crack width recommended in the above mentioned standards, considering the quasi-static combination of actions of Eurocodes. Calculated crack width is compared with crack width limit $w_{\mathrm{lim}} = 0.3$ mm (0.2 mm for long-term load effects in the case of ČSN 73 1201 [9]). The requirement of standards for crack width limit is almost identical for similar types of environment. Thus, these study cases mutually differ only by the theoretical models for crack width. The resulting quantities are shown in Fig. 1 to 3 and Fig. 5 (the symbols of the used models are indicated without brackets).
2. Considering three standards (BS 8110 [7], ACI 318-89 [8] and ČSN 73 1201 [9]) the slab is designed for the ultimate limit state using appropriate loading requirements (including partial factors, combination of actions) and

verified for serviceability limit states using recommended loading and a theoretical crack width model. The results are also shown in Fig. 1 to 3 and Fig. 5 (the symbols of the standards are indicated in brackets), in all cases the limit crack width $w_{\text{lim}} = 0.3$ mm.

The resulting crack width $w(\boldsymbol{x}_k)$ shown for a slab 0.25 m in depth in Fig. 1 is in a broad range. It is obvious that crack width $w(\boldsymbol{x}_k)$ represents only a theoretical value calculated on the basis of different assumptions using the influence coefficients considered in the standards. However, these calculated values of crack width $w(\boldsymbol{x}_k)$ determined on the basis of a broad range of normative recommendations are compared with the same limiting value w_{lim}. Classic deterministic methods do not enable deeper analysis of particular influences and consequently detailed determination of structural reliability with respect to crack width.

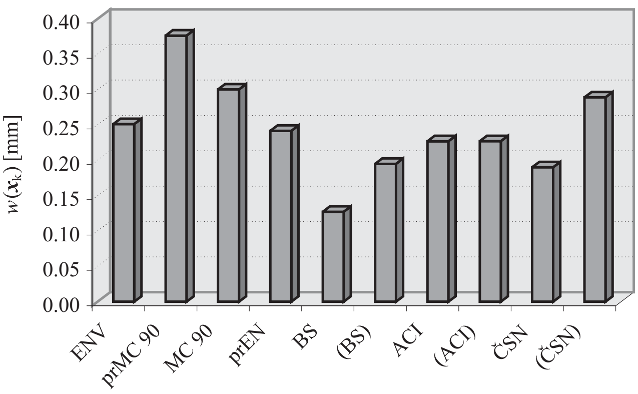

Fig. 1: Crack width $w(\boldsymbol{x}_k)$ for a reinforced concrete slab 0.25 m in depth calculated according to the selected theoretical models considering two alternatives: 1. slab is designed and verified following loading requirements of ENV [4] – the symbols of standards are given without brackets, 2. slab is designed and verified for loading recommendations of BS [7], ACI [8] and ČSN [9] – the symbols of the standards are given in brackets.

6 Credibility analysis of crack width model

The credibility of the design value of crack width wd is verified using methods of structural reliability. The probability of the random variable $w(\boldsymbol{X})$ exceeding the calculated value of crack width $w(\boldsymbol{x}_k)$ determined in accordance with a particular standard is expressed as

$$P_c = \mathrm{P}\{w(\boldsymbol{x}_k) - \xi_w\, w(\boldsymbol{X}) < 0\} \tag{9}$$

where $\boldsymbol{x}_k$ is the vector of characteristic values of basic variables and the coefficient ξ_w represents the uncertainties of action effects and the inaccuracy of the theoretical model for crack width.

The probabilistic models of basic variables entering equation (9) are recommended on the basis of the working materials of JCSS (Joint Committee for Structural Safety) and previous reliability analyses. Some of the models applied in the reliability analyses are assumed to be deterministic values, while the others are considered as random variables having a normal distribution, lognormal, beta or gamma distribution. Statistical properties of basic variables are described using the moment characteristics (by mean, standard deviation), lower and upper bounds, and they are listed in Marková and Holický [10, 11]. The significant basic variable influencing the resulting crack width is the concrete reinforcement cover. Its probabilistic model is based on measurements carried out at the Klokner Institute of CTU in Prague, in the United Kingdom, and on working materials of JCSS [12] and Vrouwenvelder et al [13].

The theoretical models for crack width presented in current standards are based on various presumptions. They are often based on physical models and modified by influencing coefficients taking into account experimental data. In some cases they are assessed on the basis of experimental research or in combination with an empirical relationship based on previous experience. The selected theoretical models assume different probabilities of exceeding the design value of crack width, or maximum crack spacing. The probability of exceeding the calculated crack width $w(\boldsymbol{x}_k)$ is 5 % according to ENV 1992-1-1 [4] and ČSN 73 1201 [9], 10 % in CEP FIP Model Code 1990 [5] and prEN 1992-1-1 [6], 20 % in BS 8110 [7]. The following relationship between the average crack width w_m and the characteristic crack width w_k can be considered

$$w_k = w_m\left(1 + u_p v\right), \tag{10}$$

where v is a coefficient of variation expressing up to 40 % variability of crack width and u_p is an upper fractile of the standardised normal variable for probability p based on the assumption of normal distribution of crack widths. The relationship for calculating the average crack width introduced in ACI 318-89 [8] was derived on the basis of experimental measurements, and is given in Marková [10].

The probability P_f of exceeding the design crack width wd according to relationship (9) is determined by the FORM method using Comrel software and expressed here by reliability index β_c, as illustrated in Fig. 2. Further, the SORM method and the Importance Sampling method were also used to check the results of reliability analysis.

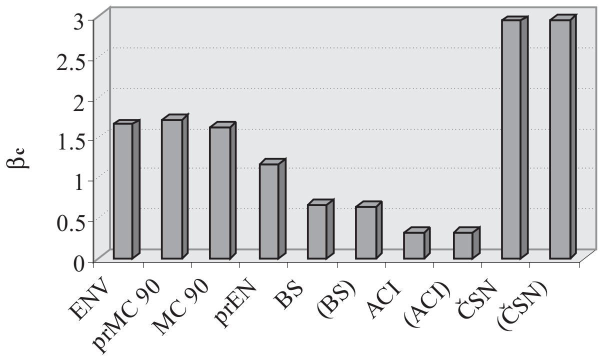

Fig. 2: The credibility of the design value of crack width w_d for the selected theoretical models, or the slab depth of 0.25 m and two considered alternatives

Analysis of the credibility of the design values of crack width shows that the reliability index β_c determined for a slab depth from 0.19 m to 0.29 m is low for the theoretical model introduced in the American standard (reliability index β_c is about 0.3), in the British standard ($0.5 \le \beta_c \le 0.9$), in the working draft of Eurocode 2 ($0.4 \le \beta_c \le 1.4$), and the credibility is high for the Czech standard (about 2.9). The credibility of theoretical models seems to be sufficient for ENV 1992-1-1 [4] and in most cases also for CEB FIP Model

Code 1990 [5], greater than the reliability index $\beta_d = 1.5$ recommended for serviceability limit states in prEN 1990 [14].

Analysis of the sensitivity factors α of the basic variables indicates that the significant basic variables influencing the credibility of the design crack width include permanent and variable loads, thickness of the reinforcement cover, tensile strength of concrete and influence coefficients (e.g., expressing bond strength, duration of loading, shape of the strain across the cross-section). However, theoretical models for crack width give different significance to some basic variables, as shown in Fig. 3.

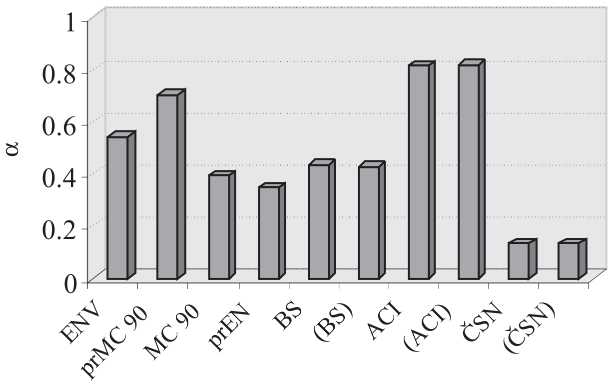

Fig. 3: Sensitivity factor α of reinforcement cover c according to the selected theoretical models and two considered alternatives

7 Time-dependent credibility analysis of crack width model

The time-dependent credibility of the design value of crack width is based on a similar relationship as given in (9). The short-term and long-term effects of the imposed load and the time-effects of creep are taken into account. The probability that the random process $w\big(\boldsymbol{X}, \boldsymbol{J}(t)\big)$ exceeds in time t the calculated value of crack width $w(\boldsymbol{x}_k, t)$ determined in accordance with the relevant standards is expressed as

$$P_c = \mathrm{P}\left\{w\big(\boldsymbol{x}_k, t\big) - \xi_w \; w\big(\boldsymbol{X}, \boldsymbol{J}(t)\big) < 0\right\} \qquad (11)$$

where $\boldsymbol{J}(t)$ is the rectangular wave renewal vector process, represented here by the short-term and long-term components of the imposed load. An example of the time-dependent credibility of the design crack width for a slab depth of 0.25 m and a time period from 10 to 50 years according to ENV 1992-1-1 [4] is shown in Fig. 4 for three considered depths of the slab (0.19 m, 0.25 m and 0.29 m).

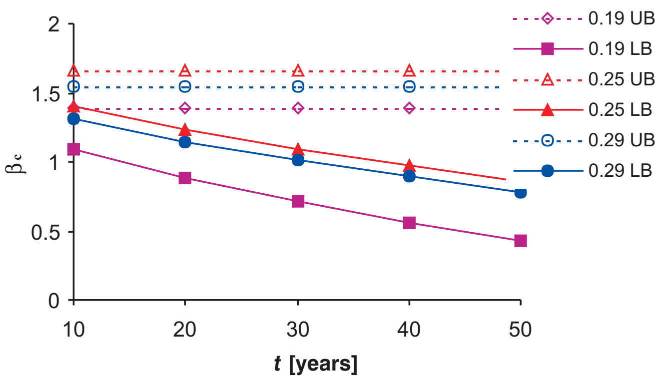

Fig. 4: Time-dependent credibility of the design crack width according to ENV 1992-1-1 for a time period from 10 to 50 years

8 Reliability analysis of reinforced concrete slab regarding crack width

The time-independent reliability analysis of the slab for the limit state of cracking, considering selected standards, deals with probability P_f of the random crack width $w(\boldsymbol{X})$ exceeding the required constraint w_{lim} expressed by

$$P_f = \mathrm{P}\left\{\xi_{lim} \; w_{lim} - \xi_w \; w(\boldsymbol{X}) < 0\right\} \qquad (12)$$

where $\boldsymbol{X}$ is a vector of basic variables and ξ_{lim} is the model uncertainty for the required crack width limit w_{lim} (it is considered $w_{lim} = 0.3$ mm for a quasi-static load combination, or for a serviceability load combination according to relevant national standards – the names of the standards are introduced in parentheses in Fig. 5).

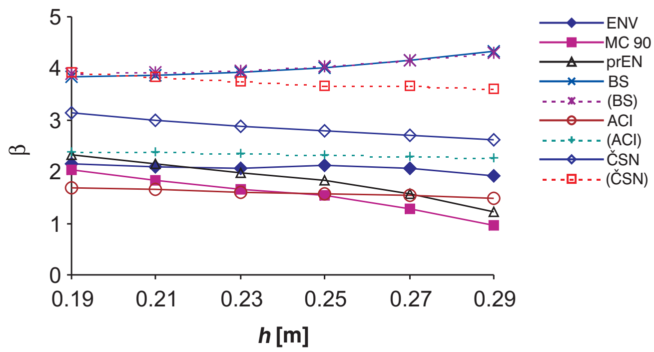

Fig. 5: Reliability index β of a slab of depth h for the limit state of cracking considering selected standards and two considered alternatives

The results of reliability analyses show that the reliability of the element depends on the theoretical model that is used. In most cases reliability index β is greater than the level of 1.5 recommended for serviceability limit states in prEN 1990 [14]. Fig. 5 shows that reliability index β is in a broad range from 1.5 to 4.9, and only in the case of thicker slabs does the index β decrease to value 1. The reliability of the slab is high according to the British standard (about 4.3) and Czech standard (about 3.8).

9 Conclusions

1. Deterministic methods of structural analysis commonly used for verification of structures do not enable objective evaluation of structural reliability.
2. It is shown that the theory of structural reliability and mathematical statistics enable comprehensive analysis of the reliability of a structure and assessment of the credibility of theoretical models.
3. A practical example of verification of a reinforced concrete slab with respect to the limit state of cracking shows that the same limit value of crack width is in practical application compared with different design values of crack

widths obtained on the basis of a broad range of normative recommendations.

4. The methods of structural reliability enable realistic analysis of concrete elements with respect to crack width. Reliability indices β_c assessed in the analysis of the credibility of the design crack width formulas and the reliability of a reinforced concrete slab with respect to limit crack width have a great scatter and are in some cases inadequate.
5. It is shown that the credibility of a theoretical model for crack width is independent of the previous design of a reinforced concrete slab for the ultimate limit states of Eurocodes or relevant national standards.
6. Analysis of the sensitivity factors α indicates the significant basic variables influencing the reliability index and the credibility of the design crack width: permanent and variable loads, thickness of the reinforcement cover, tensile strength of the concrete, influence of coefficients (expressing bond strength, duration of loading).
7. Our paper indicates that probabilistic methods can be used effectively for the development and calibration of new theoretical models applied in the design and verification of structures.

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