Contributions to the Study of Dynamic Absorbers, a Case Study

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Abstract

Dynamic absorbers are used to reduce torsional vibrations. This paper studies the effect of a dynamic absorber attached to a mechanical system formed of three reduced masses which are acted on by one, two or three order x harmonics of a disruptive force.

Keywords: dynamic absorber, torsional vibrations, reduced masses.

1 Introduction

This paper studies the effect of dynamic absorbers on reducing torsional vibrations.

Like the inertial effects, the forces and the torque produced by thermal processes in the cylinders of internal combustion engines produce forces and torsional moments that vary nonlinearly.

These forces, applied to the crankshaft and to the engine block, produce translational and rotational oscillations of the engine block and torsional oscillations in the crankshaft. They have to be reduced, because they cause noise and vibrations in an engine.

In order to calculate the torsional vibrations of a complex elastic system of this kind, the system must first be turned into a simpler equivalent dynamic system, formed of an elastic linear shaft of negligible mass loaded with circular reduced masses obtained by reducing the mobile gears.

This study focuses on three mechanical systems formed of three reduced masses, which receive a pendulum dynamic absorber. These masses are acted on by one, two or three order × harmonics, resulted from the decomposition of the Fourier series of the disruptive force with periodic variation. The dynamic absorber is attached at the end of the mechanical system.

2 A mechanical system acted on by one harmonic

The first case deals with a mechanical system formed of three reduced masses with a dynamic absorber attached according to Figure 1. The dynamic absorber is placed at the end of the mechanical system. The reduced mass m_3 is acted on by an order x harmonic of a disruptive force presenting a periodic variation marked with $P_x \cdot \cos(\Omega_x t - \varepsilon)$.

The dynamic absorber is replaced by an equivalent reduced mechanical system formed of the reduced mass m_4 and the segment of the reduced crankshaft with an elastic constant c_{34}.

The elastic constants of the segments of crankshafts between the two consecutive reduced masses and the mechanical axial moments of the reduced masses in relation to the symmetrical geometry axis of the shaft must be chosen in such a way that the kinetic energy and the potential energy of the real vibrating system (formed by the crankshaft and its mobile gears) is equal to the kinetic energy and the potential energy of the reduced vibrating system.

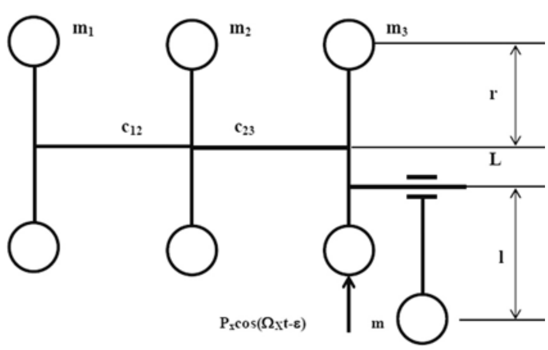

Figure 1: Mechanical system

This case starts from the differential equation system governing the torsional vibrations created by the mechanical system presented in Figure 2:

$$m_1 \frac{\mathrm{d}^2 a_1}{\mathrm{d}t^2} + c_{12}(a_1 - a_2) = 0$$

$$m_2 \frac{\mathrm{d}^2 a_2}{\mathrm{d}t^2} + c_{12}(a_2 - a_1) + c_{23}(a_2 - a_3) = 0$$

$$m_3\frac{\mathrm{d}^2a_3}{\mathrm{d}t^2} + c_{23}(a_3 - a_2) + c_{34}(a_3 - a_4) = P_x\cos(\Omega_x t - \varepsilon) \quad (1)$$
$$m_4\frac{\mathrm{d}^2a_4}{\mathrm{d}t^2} + c_{34}(a_4 - a_3) = 0$$

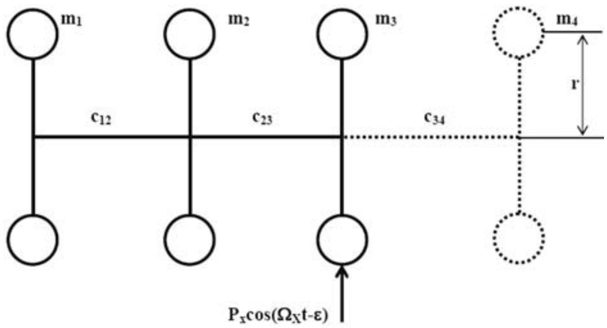

Figure 2: Equivalent mechanical system

The relative linear elongations (displacement) a_i measured on the reduced circle of radius r (of the vibrations of the four reduced masses) are expressed in relation to the linear amplitudes A_i:

$$a_i = A_i\cos(\Omega_x t - \varepsilon) \qquad (i = 1 \sim 4) \qquad (2)$$

In order for the mechanical system formed of the reduced mass m_4 and the shaft segment with elastic constant c_{34} to be dynamically equivalent to the dynamic absorber attached to the reduced mass m_3 (in other words for this mass m_3 to be subjected to the same torque as the dynamic absorber), it is necessary and sufficient that:

$$m_4 = m\frac{(L+l)^2}{r^2} \qquad c_{34} = m\frac{(L+l)^2}{r^2}\frac{L}{l}\omega_0^2 \qquad (3)$$

The square value of the order × harmonic is expressed by:

$$\Omega_x^2 = x^2\omega_0^2 \qquad (4)$$

where x represents the order of the harmonic and ω_0 the angular speed of the shaft.

Taking into consideration equations (2), (3), (4) and introducing the differential equation system (1), we obtain an algebraic system of equations. By solving this system of equations we get the four determinants. If the dynamic absorber is built in such a way that:

$$\frac{L}{l} = x^2 \qquad (5)$$

the expressions of the four determinants become:

$$\Delta_1 = -\frac{P_x}{m_3}\frac{c_{12}}{m_1}\frac{c_{23}}{m_2}\omega_0^2\left(x^2 - \frac{L}{l}\right)$$
$$\Delta_2 = \frac{P_x}{m_3}\frac{c_{23}}{m_2}\omega_0^2\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right)\left(x^2 - \frac{L}{l}\right)$$
$$\Delta_3 = -\frac{P_x}{m_3}\omega_0^2\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right)\cdot\left(x^2\omega_0^2 - \frac{c_{23}}{m_2} - \frac{c_{12}}{m_2}\right)\left(x^2 - \frac{L}{l}\right)$$
$$\Delta_4 = \frac{P_x}{m_3}\omega_0^2\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right)\cdot\left(x^2\omega_0^2 - \frac{c_{23}}{m_2} - \frac{c_{12}}{m_2}\right)\frac{L}{l}\omega_0^2 \qquad (6)$$

An analysis of the four determinants shows that the only mass that executes torsional vibrations is mass m_4, which is not part of the reduced crankshaft. The other reduced masses do not execute torsional vibrations. So:

$$A_1 = 0 \qquad A_2 = 0 \qquad A_3 = 0 \qquad A_4 \neq 0 \qquad (7)$$

3 A mechanical system acted on by two order × harmonics

This case deals with a mechanical system composed of three reduced masses, which receives a dynamic absorber placed at the end of the system presented in Figure 3. The reduced masses m_2, m_3 are acted on by two order × harmonics of the disruptive force presenting a periodic vibration marked with $P_x\cos(\Omega_x t - \varepsilon)$.

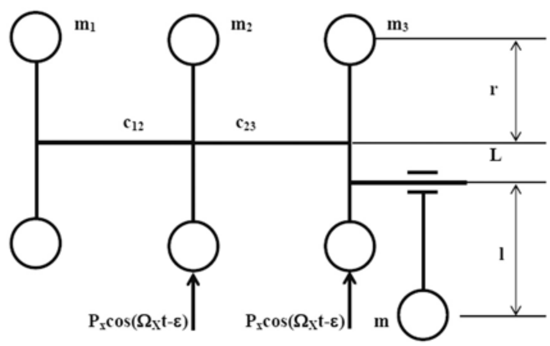

Figure 3: Mechanical system

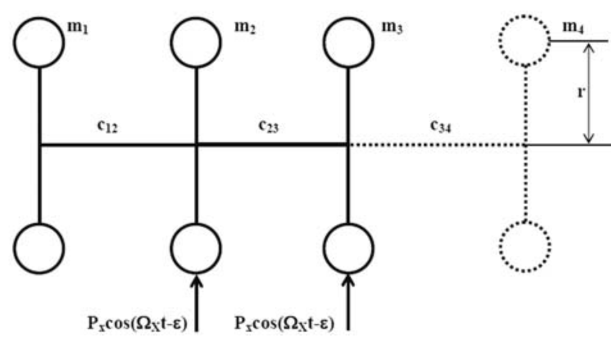

Figure 4: Equivalent mechanical system

The reduced mass m_4, together with the shaft segment with elastic constant c_{34}, forms a dynamic mechanical system equivalent to the dynamic absorber which applies the same torque to mass m_3 as the absorber (Figure 4). As in the previous case, the dynamic absorber is replaced by an equivalent dynamic system formed of the reduced mass m_4 and the reduced crankshaft segment having elastic constant c_{34} (Figure 4).

The study starts from the differential equation system that governs the torsion vibrations performed by the mechanical system represented in Figure 4:

$$\begin{aligned} m_1 \frac{\mathrm{d}^2 a_1}{\mathrm{d}t^2} + c_{12}(a_1 - a_2) &= 0 \\ m_2 \frac{\mathrm{d}^2 a_2}{\mathrm{d}t^2} + c_{12}(a_2 - a_1) + \\ c_{23}(a_2 - a_3) &= P_x \cos(\Omega_x t - \varepsilon) \\ m_3 \frac{\mathrm{d}^2 a_3}{\mathrm{d}t^2} + c_{23}(a_3 - a_2) + \\ c_{34}(a_3 - a_4) &= P_x \cos(\Omega_x t - \varepsilon) \quad (8) \\ m_4 \frac{\mathrm{d}^2 a_4}{\mathrm{d}t^2} + c_{34}(a_4 - a_3) &= 0 \end{aligned}$$

Relation (2) gives the elongation expressions a_i related to the amplitudes recorded on radius r on the reduction circles. Taking into account expressions (2), (3), (4) and introducing the differential system of equations (8) we obtain a system of algebraic equations. By solving this system, we obtain expressions for the five determinants.

$$\begin{aligned} \Delta &= \frac{1}{m_3} m \frac{(L+l)^2}{r^2} \left(\frac{L}{l}\omega_0^2\right)^2 \left[\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right) \cdot \left(x^2\omega_0^2 - \frac{c_{12}}{m_2} - \frac{c_{23}}{m_2}\right) - \frac{c_{12}}{m_1}\frac{c_{12}}{m_2}\right] \\ \Delta_1 &= -\frac{P_x}{m_2}\omega_0^4 \frac{c_{12}}{m_1}\frac{1}{m_3} m \frac{(L+l)^2}{r^2}\left(\frac{L}{l}\right)^2 \\ \Delta_2 &= -\frac{P_x}{m_2}\omega_0^4 \left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right)\frac{1}{m_3} m \frac{(L+l)^2}{r^2}\left(\frac{L}{l}\right)^2 \\ \Delta_3 &= 0 \\ \Delta_4 &= -\frac{P_x}{m_2}\frac{c_{23}}{m_3}\frac{L}{l}\omega_0^2\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right) + \frac{P_x}{m_3}\left[\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right)\left(x^2\omega_0^2 - \frac{c_{12}}{m_2} - \frac{c_{23}}{m_2}\right) - \frac{c_{12}}{m_1}\frac{c_{12}}{m_2}\right]\frac{L}{l}\omega_0^2 \quad (9) \end{aligned}$$

The analysis of the expressions (9) shows that the reduced masses m_1, m_2, and m_4 — born from a reduction operation — perform torsion vibrations. The only mass that does not perform any torsional vibrations is mass m_3, which is the mass that has the dynamic absorber attached.

$$A_1 \neq 0 \qquad A_2 \neq 0 \qquad A_3 = 0 \qquad A_4 \neq 0 \qquad (10)$$

4 The mechanical system acted on by three order × harmonics

This case investigates a mechanical system composed of three reduced masses which receive a dynamic absorber placed at the end of the system (Figure 5). The reduced masses m_1, m_2, m_3 are acted on by means of three order x harmonics of disruptive forces presenting a periodic variation marked with $P_x \cos(\Omega_x t - \varepsilon)$.

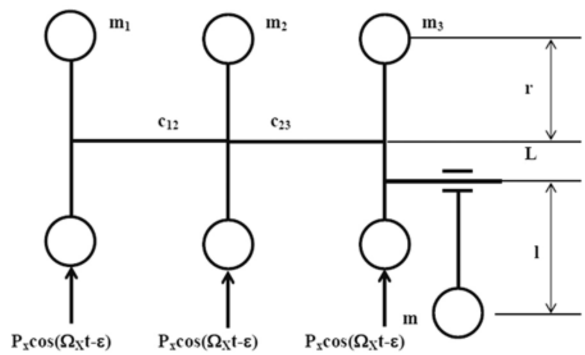

Figure 5: Mechanical system

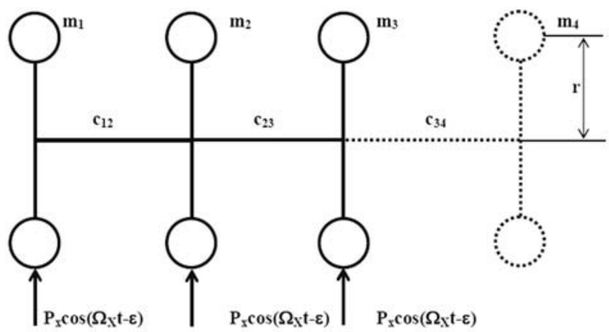

Figure 6: Equivalent mechanical system

Just as in the previous paragraph, the dynamic absorber is replaced by an equivalent dynamic system formed of the reduced mass m_4 and the reduced shaft segment having the elastic constant c_{34} (Figure 6). The differential equations governing the vibratory movements of the mechanical system represented in Figure 6 are:

$$\begin{aligned} m_1 \frac{\mathrm{d}^2 a_1}{\mathrm{d}t^2} + c_{12}(a_1 - a_2) &= P_x \cos(\Omega_x t - \varepsilon) \\ m_2 \frac{\mathrm{d}^2 a_2}{\mathrm{d}t^2} + c_{12}(a_2 - \\ a_1) + c_{23}(a_2 - a_3) &= P_x \cos(\Omega_x t - \varepsilon) \\ m_3 \frac{\mathrm{d}^2 a_3}{\mathrm{d}t^2} + c_{23}(a_3 - a_2) + \\ c_{34}(a_3 - a_4) &= P_x \cos(\Omega_x t - \varepsilon) \quad (11) \end{aligned}$$

$$m_4 \frac{\mathrm{d}^2 a_4}{\mathrm{d}t^2} + c_{34}(a_4 - a_3) = 0$$

The expressions of elongations a_i according to the amplitudes recorded on the radius r reduction circles are given by relation (2).

Taking into account expressions (2), (3), (4) and introducing the differential system of equations (11), we obtain an algebraic system of equations. Solving this system provides the four determinants.

$$\begin{aligned}
\Delta &= -\frac{1}{m_3} m \frac{(L+l)^2}{r^2} \left(\frac{L}{l}\omega_0^2\right)^2 \left[\left(x^2\omega_0^2 - \frac{c_{12}}{m_1}\right) \cdot \right. \\
&\quad \left. \left(x^2\omega_0^2 - \frac{c_{12}}{m_2} - \frac{c_{23}}{m_2}\right) - \frac{c_{12}}{m_1}\frac{c_{12}}{m_2}\right] \\
\Delta_1 &= \frac{P_x}{m_1}\left(x^2\omega_0^2 - \frac{c_{12}}{m_2} - \frac{c_{23}}{m_2}\right)\frac{1}{m_3} m \frac{(L+l)^2}{r^2} \cdot \\
&\quad \left(\frac{L}{l}\omega_0^2\right)^2 - \frac{P_x}{m_2}\frac{c_{12}}{m_1}\frac{1}{m_3} m \frac{(L+l)^2}{r^2}\left(\frac{L}{l}\omega_0^2\right)^2 \\
\Delta_2 &= -\frac{P_x}{m_1}\frac{c_{12}}{m_2}\frac{1}{m_3} m \frac{(L+l)^2}{r^2}\left(\frac{L}{l}\omega_0^2\right)^2 + \\
&\quad \frac{P_x}{m_2}\frac{1}{m_3} m \frac{(L+l)^2}{r^2}\left(\frac{L}{l}\omega_0^2\right)^2 \\
\Delta_3 &= 0
\end{aligned} \tag{12}$$

We observe that:

$$A_1 \neq 0 \qquad A_2 \neq 0 \qquad A_3 = 0 \tag{13}$$

This means that the only mass that does not execute torsional oscillations is mass m_3, which received a dynamic absorber. Since mass m_4 is not a result of reducing the crankshaft, it was no longer useful when calculating the amplitude vibration for this mass.

5 Conclusion

The three cases discussed here offer us the following information:

- In the case of the mechanical system formed of three reduced masses acted on by a single order x harmonic of the disruptive force, if the dynamic absorber is built according to relation (5), the three reduced masses m_1, m_2 and m_3 do not perform any torsional vibrations, irrespective of the value of the angular speed.
- In the case of the mechanical systems formed of three reduced masses acted on by two and three order x harmonics of the disruptive force, respectively, if the dynamic absorber is built according to relation (5), the only mass which does not execute torsional vibrations is the mass which receives the dynamic absorber.

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