Reliability Analysis of a Steel Frame

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A steel frame with haunches is designed according to Eurocodes. The frame is exposed to self-weight, snow, and wind actions. Lateral-torsional buckling appears to represent the most critical criterion, which is considered as a basis for the limit state function. In the reliability analysis, the probabilistic models proposed by the Joint Committee for Structural Safety (JCSS) are used for basic variables. The uncertainty model coefficients take into account the inaccuracy of the resistance model for the haunched girder and the inaccuracy of the action effect model. The time invariant reliability analysis is based on Turkstra's rule for combinations of snow and wind actions. The time variant analysis describes snow and wind actions by jump processes with intermittencies. Assuming a 50-year lifetime, the obtained values of the reliability index β *vary within the range from 3.95 up to 5.56. The cross-profile IPE 330 designed according to Eurocodes seems to be adequate. It appears that the time invariant reliability analysis based on Turkstra's rule provides considerably lower values of* β *than those obtained by the time variant analysis.*

Keywords: steel frame, lateral-torsional buckling, reliability, jump processes.

1 Introduction

Application of the partial factor method introduced in operational European standards for structural design often leads to unequal reliability of structures or structural members made of different building materials and exposed to different combinations of actions. Well-balanced structural reliability can be achieved using design procedures based on probabilistic methods. This approach to the verification of structural reliability is allowed in the fundamental European document on structural design EN 1990 Basis of Structural Design [1].

At present, the basic principles and data for the design and verification of structural members using probabilistic methods are partly provided in the technical literature and also in recent ISO and EN standards. Detailed guidelines can be found in the JCSS working materials [2]. It is expected that probabilistic design will become a practical design tool. Unfortunately, implementation of the design is limited by lack of required input data.

The reliability analysis presented in this paper provides reliability verification of a steel frame designed according to recommendations given in the Eurocodes [1,3]. The reliability index β, as a basic indicator of the level of reliability, is determined using both time invariant and time variant analysis provided by the software product COMREL [4]. The basic variables are described using probabilistic models recommended by JCSS [2]. The submitted analysis indicates possible procedures for implementing probabilistic methods of structural reliability in the design of civil engineering structures.

2 Deterministic design

2.1 Geometry

The portal frame analysed in this study is a double-pinned frame stiffened by haunches in the frame corners as indicated in Fig. 1. The span of the frame is 17.71 m. The height of the structure is 7.26 m. The slope of the roof is approximately 15°. The maximum loading width is 6.48 m. The cross-section of the frame consists of the rolled I-profile IPE 330. In the location of the haunches, a T-section of variable height (10–280 mm) is welded on it (see Fig. 1). The maximum section height is 610 mm in the frame corner. The lengths of the haunches are 2.0 and 2.8 m, respectively.

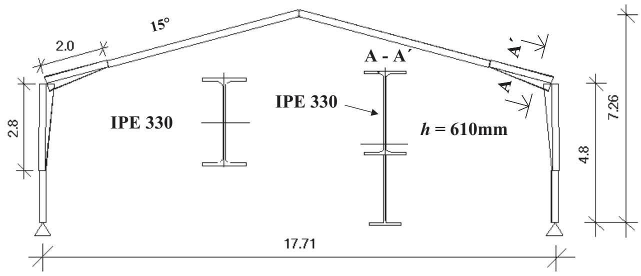

Fig. 1: Geometry of the frame [m]

2.2 Effects of actions

The frame is exposed to the self-weight of the load bearing girders and the roof, snow, and wind action. The effect of the imposed action and thermal actions is negligible. The action effects of the actions considered in the analysis consist of an axial force N and bending moment M. In the design calculation, the axial force and bending moment are represented by the design values N_d and M_d. The combination of actions is determined considering expression (6.10b), given in EN 1990 [1]. If the snow load is the leading variable action, then it follows that:

$$N_d = \xi\gamma_G\left(N_{frame,\,k} + N_{roof,\,k}\right) + \gamma_Q\left(N_{snow,\,k} + \Psi_{0,w}N_{wind,\,k}\right) \quad (1)$$

$$M_d = \xi\gamma_G\left(M_{frame,\,k} + M_{roof,\,k}\right) + \gamma_Q\left(M_{snow,\,k} + \Psi_{0,w}M_{wind,\,k}\right) \quad (2)$$

where $\xi = 0.85$ is the reduction factor for permanent actions, $\gamma_G = 1.35$ is the partial factor for permanent actions, $\gamma_Q = 1.5$ is the partial factor for variable actions, and $\psi_{0,w} = 0.6$ is the factor for the combination value of the wind action.

$N_{frame,k}$ is the characteristic value of the axial force due to the self-weight of the frame (the rolled sections) estimated as 0.49 kN/m. In the location of the haunches, it ranges from 0.49 to 0.76 kN/m. $N_{roof,k}$ is the characteristic value of the axial force due to the self-weight of the roof structure. The load, including the secondary longitudinal girders, is estimated as 0.15 kN/m². $N_{snow,k}$ is the characteristic value of the

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axial force due to snow action. The characteristic value of the snow load s_k determined according to [5] is given as:

$$s_k = \mu_1 C_e C_t s_{g,k} \tag{3}$$

where $\mu_1 = 0.8$ is the load shape coefficient considered for a uniform snow load covering a whole roof area and for a roof slope about 15°. Both the exposure coefficient C_e and the heat coefficient C_t are chosen equal to 1 and the characteristic value of the snow load on the ground at the weather station is taken as $s_{g,k} = 1.33$ kN/m^2 considering a given site locality (approximately corresponding to region III for the Czech Republic). $N_{wind,k}$ is the characteristic value of the axial force due to wind action. Following the recommendations provided in [6], the characteristic value of the wind pressure w_k is given as:

$$w_k = c_p q_p(z) \tag{4}$$

where c_p is the pressure coefficient dependent on the building geometry and the size of the loaded area (here, the loaded area is assumed to be larger than 10 m^2). It describes the outside pressure and suction combined with either the inside suction or the inside overpressure. In this case, more unfavourable effects are caused by a combination of outside pressure and inside suction. The peak velocity pressure can be written as:

$$q_p(z) = c_g(z)\, c_r(z)^2 \frac{1}{2} \rho\, v_b^2 \tag{5}$$

where $c_g(z) = 2.4$ m is the gust factor specified for the height of the structure $z \cong 7{,}5$ m and for the terrain category II – open terrain with isolated obstacles [6]. The roughness factor $c_r(z = 7{,}5 \text{ m}) = 0.95$ is also defined for the terrain category II. The air density ρ is taken as 1.25 kg/m^3. The reference wind speed v_b is 26 m/s. It results $q_p(z) = 0.92$ kN/m^2.

The design values of the bending moments are derived from the same assumptions as for the design values of the axial forces.

2.3 Structural analysis

The internal forces were determined using the deformation method. The structure has been modelled as a double-pinned frame. To model the real behaviour of the frame, the haunches of the girder were divided into 6 parts, each having a constant height corresponding to the middle cross-section of the relevant part. It appears that the shear does not affect the bending capacity and need not be taken into account. The structure is classified as a sway frame and consequently the sway moments caused by the wind action are increased by the factor $k = 1.28$.

The buckling length of the column with respect to axis y $L_y = 12.48$ m is taken as 2.6-multiple of the length of the column following the approximate procedure for sway frames shown in [3]. The buckling length L_z with respect to axis z is chosen as 2.2 m, which is the distance between the stays for lateral buckling restraint. As for the diaphragm beam, $L_y = 8.855$ m is half of the beam span. L_z is again 2.2 m.

Each of the cross-sections within the haunch is checked against buckling without lateral-torsional buckling and buckling with lateral-torsional buckling. The design criterion for buckling without lateral-torsional buckling seems to yield the most critical criterion for checking of the column. The most critical criterion for the diaphragm beam is the criterion for buckling with lateral-torsional buckling. It appears that the critical cross-sections within the column and diaphragm beam are just at the origin of the haunches.

The design criterion for buckling without lateral-torsional buckling is expressed as:

$$\frac{N_{Sd}}{\chi A \dfrac{f_{y,k}}{\gamma_{M1}}} + \frac{k_y C_{my} M_{Sd,y}}{W_{p1,y} \dfrac{f_{y,k}}{\gamma_{M1}}} \le 1 \tag{6}$$

where, in the critical cross-section of the column, $N_{Sd} = -115$ kN is the design value of the axial force due to the actions, $\chi = 0.63$ is the buckling coefficient (the lower of the values $\chi_y = 0.63$ and $\chi_z = 0.80$), A is the area of the relevant cross-section ($A_{IPE330} = 6261$ mm^2), $f_{yk} = 275$ MPa is the characteristic value of the yield strength of the steel S275, $\gamma_{M1} = 1.1$ is the partial factor for the material property, $k_y = 1.09$ is the moment amplification factor, $C_{my} = 0.95$ is the equivalent uniform moment factor, $M_{Sd,y} = -132$ kNm is the design value of the bending moment due to the actions and $W_{pl,y}$ is the plastic sectional modulus ($W_{pl,\,y,\,IPE330} = 804 \cdot 10^3$ mm^3).

The design criterion for buckling with lateral-torsional buckling is given as:

$$\frac{N_{Sd}}{\chi_z A \dfrac{f_{y,k}}{\gamma_{M1}}} + \frac{M_{Sd,y}}{\chi_{LT} W_{p1,y} \dfrac{f_{y,k}}{\gamma_{M1}}} \le 1 \tag{7}$$

where, in the critical cross-section of the beam, N_{Sd} is –91 kN, χ_z is 0.80, $M_{Sd,y}$ is –161 kN, and $\chi_{LT} = 0.89$ is the buckling coefficient of lateral-torsional buckling.

The Eurocodes [3] do not provide a procedure for determining the critical bending moment at the limit of the lateral-torsional buckling M_{cr} of the haunched girder, which is required for calculation of χ_{LT}. Therefore, the critical bending moment M_{cr} is approximately calculated neglecting the effect of the haunch. It is assumed that the I-section alone without the haunch resists lateral-torsional buckling.

Considering the criterion for buckling, the ratio between the design action effect and the design resistance for the critical cross-section of the column at the origin of the haunch is 0.8. For the critical cross-section of the beam at the origin of the haunch, the ratio is 0.97 taking into account the criterion for buckling with lateral-torsional buckling. Thus, this cross--section is also the most critical one within the whole structure and for this reason its reliability is verified in the following analysis.

3 Limit state function

As mentioned above, the reliability analysis concentrates on the critical cross-section of the beam at the origin of the haunch. The limit state function is derived from the design criterion for lateral-torsional buckling (7). In addition, the uncertainty model coefficients are used to take into account the inaccuracy of the resistance model for the haunched girder and the inaccuracy of the action effect model. The limit state function reads as:

$$g(X) = 1 - \left(\frac{\theta_{EN} |N_S|}{\theta_{RN} \chi_z A f_y} + \frac{\theta_{EM} M_{S,y}}{\theta_{RM} \chi_{LT} W_{p1,y}\, f_y} \right) \ge 0 \tag{8}$$

where θ_{EN} is the coefficient of the model uncertainties for axial force and θ_{EM} for bending moment, θ_{RN} is the coefficient of the model uncertainties for axial force resistance and θ_{RM} for bending moment resistance. Utilizing the results of the structural analysis, the internal forces in the critical cross-section can be simply written as:

$$N_S = -6.7\left(B g_{roof} + g_{frame}\right) - 6.47 Bs - 2.54 Bw \qquad (9)$$

$$M_{S,y} = 8.37\left(B g_{roof} + g_{frame}\right) + 8.09 Bs + 13.27 Bw \qquad (10)$$

where $B = 6.48$ m is the loading width, g_{roof} is the self-weight of the roof [kN/m^2], g_{frame} is the self-weight of the load bearing girders [kN/m], s is the snow load [kN/m^2] and w is the wind action [kN/m^2]. The limit state function given by equation (8) is applied in the following reliability analysis considering appropriate probabilistic models for the basic random variables described below.

4 Theoretical models for basic variables

4.1 Basic variables

Probabilistic models for basic variables are used in accordance with the models proposed by the Joint Committee for Structural Safety (JCSS). The sectional area A, the plastic sectional modulus $W_{pl,y}$, the loading width B and the span of the girder L are assumed to be deterministic values (D), while the others are considered as random variables. The statistical properties of the random variables are described by the normal distribution (N), lognormal distribution (LN) and the Gumbel distribution (G) indicated by the moment characteristics (the mean μ and standard deviation σ) [2,7] as listed in Tab. 1. The skewnesses α are implicitly given by the type of distributions as: $\alpha_N = 0$, $\alpha_{LN} = 3V_X + V_X^{\ 3}$, and $\alpha_G = 1.14$.

The statistical parameters used for the yield strength are estimated assuming:

$$\mu_{fy} = f_{y,\,k} + 2\sigma_{fy} \qquad (11)$$

Table 1: Statistical properties of basic variables

Var. types	Symbol X	Name of basic variable	Dist.	Dim.	Parameters				
					X_k	μ_X	μ_X/X_k	σ_X	V_X
MP*	f_y	Yield strength	LN	MPa	275	327	1.19	26.2	0.08
Geometric data	A	Sectional area	D	mm^2	6261	6261	1.00	-	-
	$W_{pl,y}$	Plastic sectional modulus	D	mm^3	804000	804000	1.00	-	-
	B	Loading width	D	m	6.48	6.48	1.00	-	-
	L	Girder span	D	m	nom	nom	1.00	-	-
	χ_z	Buckling coefficient	N	-	0.64	0.67	1.04	0.04	0.06
	χ_{LT}	Coefficient of lateral-torsional buckling	N	-	0.79	0.82	1.03	0.04	0.05
Model uncertainties	θ_{EN}	Axial force action effect	N	-	-	1	-	0.05	0.05
	θ_{EM}	Bending action effect	N	-	-	1	-	0.1	0.1
	θ_{RN}	Axial force resistance	N	-	-	1.1	-	0.07	0.07
	θ_{RM}	Bending resistance	N	-	-	1.1	-	0.07	0.07
Actions	g_{frame}	Self-weight due to girders	N	kN/m	0.49	0.49	1	0.03	0.05
	g_{roof}	Self-weight due to roof	N	kN/m^2	0.15	0.15	1	0.02	0.1
	s_{50}	50-year extremes of snow	G	kN/m^2	1.06	1.18	1.11	0.32	0.27
	s_1	Annual extremes of snow	G	kN/m^2	1.06	0.38	0.36	0.27	0.72
	w_{50}	50-year extremes of wind	G	kN/m^2	0.92	0.64	0.7	0.21	0.33
	w_1	Annual extremes of wind	G	kN/m^2	0.92	0.28	0.3	0.14	0.52

*MP = material properties

X_k is the characteristic value of the variable, μ_X is the mean, σ_X is the standard deviation and V_X is the coefficient of variation.

Table 2: Statistical properties of the basic variables used to derive coefficients χ_z and χ_{LT}

Var. types	Symbol X	Name of basic variable	Dist.	Dim.	Parameters				
					X_k	μ_X	μ_X/X_k	σ_X	V_X
MP	f_{yp}	Yield strength (S235)	LN	MPa	235	280	1.19	22.4	0.08
Coefficients	α	Imperfection coefficient	N	-	0.32	0.275	0.028	0.028	0.1
	C_1	Loading and end restraint factor	N	-	1.59	1.90	1.19	0.19	0.1
	C_2	Loading and end restraint factor	N	-	0.78	0.67	0.86	0.067	0.1

$$\sigma_{fy} = 0.08 \mu_{fy} \tag{12}$$

The parameters of the buckling coefficient χ_z are derived from the recommendation indicated in EN 1993 [3] taking into account the random variability of the yield strength f_y, comparative yield strength f_{yp} and imperfection coefficient α. Similarly, the statistical parameters of the coefficient of lateral-torsional buckling χ_{LT} are derived. In addition, the factors depending on loading and end restraint conditions C_1 and C_2 are also considered as random variables. The parameters applied in the analysis are listed in Tab. 2. The coefficients of the model uncertainties θ cover the imprecision and incompleteness of the theoretical models for the frame with the haunches. Their statistical parameters are assumed as in the JCSS Probabilistic Model Code [2]. The probabilistic models for the self-weight actions (g_{frame} and g_{roof}) are considered as in [7].

4.2 Snow load

As for the snow load s, the statistical parameters are derived considering equation (3). The characteristic value of the snow load on the ground $s_{g,k}$ is assumed to have the probability $p = 0.02$ to be exceeded by annual extremes. Assuming a Gumbel distribution and the coefficient of variation $V_{sg,1} = 0.7$ [8], the mean of the annual extremes $\mu_{sg,1} = 0.47$ kN/m^2 can be obtained from the following equation for a fractile of the Gumbel distribution:

$$\mu_{sg,1} = \frac{s_{g,k}}{1 - \left[0.45 + 0.78 \ln\left(-\ln(1-p)\right)\right] w_{sg,1}} \tag{13}$$

The standard deviation of the annual extremes is then $\sigma_{sg} = 0.33$ kN/m^2. For time invariant analysis, the parameters of the 50-year extremes must be determined. The standard deviation of the 50-year extremes is equal to the standard deviation of the annual extremes for the Gumbel distribution. The mean of the N-year extremes can be derived from the annual extreme parameters as:

$$\mu_{sg,N} = \mu_{sg,1} + 0.78 \ln(N) \sigma_{sg} \tag{14}$$

For $N = 50$, the mean of the 50-year extremes is $\mu_{sg,50} = 1.48$ kN/m^2. The statistical parameters of the other variables are used in accordance with JCSS Probabilistic Model Code [2].

The statistical parameters of the annual extremes and 50-year extremes of the snow load (denoted as s_1 and s_{50} in Table 1) result from equation (3) using the statistical models for random variables shown in Table 3.

4.3 Wind action

The statistical parameters of wind pressure w are determined assuming that:

$$w = c_p c_g c_r^2 m_q q_b \tag{15}$$

Table 3: Variables used in calculating the parameters of the snow load

$s = \mu_1 C_e C_t s_g$									
Var. types	Symbol X	Name of basic variable	Dist.	Dim.	Parameters				
					X_k	μ_X	μ_X/X_k	σ_X	V_X
Coef.	$\mu_1 C_e$	Shape and exposure coef.	N	-	0.8	0.8	1	0.12	0.15
	C_t	Heat coefficient	D	-	1	1	1	-	-
Actions	$s_{g,1}$	Annual extremes of snow load on ground	G	kN/m^2	1.33	0.47	0.35	0.33	0.7
	$s_{g,50}$	50-year extremes of snow load on ground	G	kN/m^2	1.33	1.48	1.11	0.33	0.22

Table 4: Variables used in calculating the parameters of the wind action

$w = c_p\, c_g\, c_r^2\, m_q\, q_b$									
Var. types	Symbol X	Name of basic variable	Dist.	Dim.	Parameters				
					X_k	μ_X	μ_X/X_k	σ_X	V_X
Coef.	c_p	Pressure coefficient	N	-	nom	nom	1	0.1nom	0.1
	c_g	Gust factor	N	-	2.4	2.4	1	0.24	0.1
	c_r^2	Roughness factor	N	-	0.91	0.73	0.8	0.073	0.1
	m_q	Model coefficient	N	-	1	0.8	0.8	0.16	0.2
Actions	$q_{b,1}$	Annual extremes of basic wind pressure	G	kN/m^2	0.42	0.20	0.44	0.085	0.43
	$q_{b,50}$	50-year extremes of basic wind pressure	G	kN/m^2	0.42	0.46	1.10	0.085	0.18

where m_q is the model coefficient describing the ratio between the expected and computed value of the basic wind pressure q_b, which can be written as:

$$q_b = \frac{1}{2}\rho v_b^2 \tag{16}$$

The characteristic value $q_b = 0.42$ kN/m^2 is defined to have the probability $p = 0.02$ to be exceeded by the annual extremes. The coefficient of variation of the annual extremes of the reference wind speed v_b is $V_{vb,1} = 0.2$ [8]. Supposing that the annual extremes of the reference wind speed can be modelled by a Gumbel distribution, the coefficient of variation of the annual extremes of the basic wind pressure $V_{qb,1} = 0.43$ results from:

$$V_{qb,1} = \frac{2V_{vb,1}\sqrt{1 + 1.14V_{vb,1}}}{1 + V_{vb,1}^2} \tag{17}$$

The mean of the annual extremes of the basic wind pressure $\mu_{qb,1} = 0.20$ kN/m^2, the mean of the 50-year extremes $\mu_{qb,50} = 0.46$ kN/m^2 and the standard deviation $\sigma_{qb} = 0.085$ kN/m^2 can be derived identically as for the statistical parameters of the extremes of the snow load on the ground (13,14). The statistical parameters of the other variables used in calculating the statistical parameters of the wind pressure w are taken in accordance with the JCSS Probabilistic Model Code [2] as listed in Table 4.

5 Reliability analysis

Climatic actions due to snow and wind are complex time-variant quantities that significantly complicate the reliability analysis. Two different approximations for describing them are considered in the following analysis. Firstly, Turkstra's rule is accepted in conjunction with time invariant analysis. Secondly, the Ferry Borges-Castanheta model (FBC) is applied together with time variant reliability analysis. The variable actions due to snow and wind are assumed to be uncorrelated.

The software product COMREL [4] has been applied in both types of analysis.

5.1 Time invariant analysis

In accordance with Turkstra's rule, the leading action is described by its lifetime (assumed as 50 years) extreme while the accompanying action is considered by its point-in-time value (approximated by annual extremes). In the following analysis, each climatic action, the snow and the wind action, is considered to be either a leading or an accompanying action.

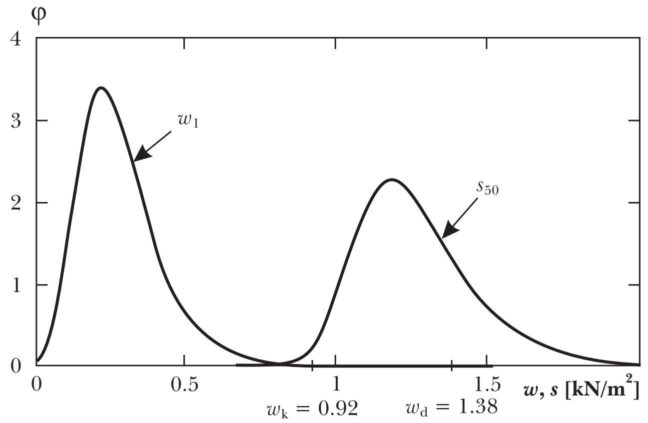

Fig. 2: The probability densities of the 50-year extremes of the snow load s_{50} and the annual extremes of wind pressure w_1

The probability densities of the 50-year extremes of the snow load s_{50} (considered as the leading variable action) and the annual extremes of the wind pressure w_1 are shown in Fig. 2. The characteristic value of the wind pressure w being the 98-percentage fractile of Gumbel's distribution is denoted as w_k, and w_d denotes the design value.

5.2 Time variant analysis

The time variant reliability analysis is based on the FBC model for the snow and wind actions. Both the climatic actions are described by jump processes without intermittencies (actions sometimes take a zero value), which approximate their real variation in time by rectangular wave renewal functions.

Each jump process with intermittencies is characterized by the jump rate λ (the average number of magnitude changes of the square waves in a reference time T_{ref}) and by the interarrival-duration intensity ρ (the product of the arrival rate λ and the mean duration with respect to a reference time T_{ref}).

As for the snow load, it is assumed that it takes its extreme five times a year. Considering the reference time T_{ref}= 1 year,

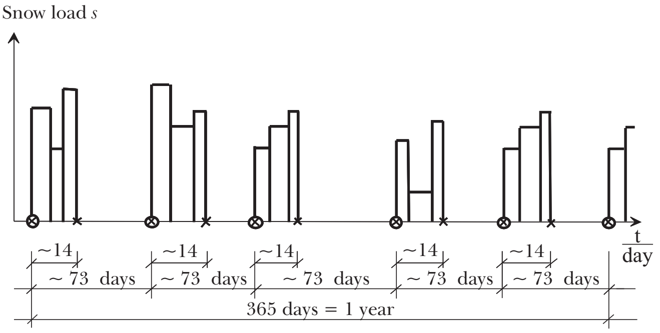

Fig. 3: The approximation of the snow load used in the time variant analysis

the arrival rate of on-times is therefore $\lambda_s = 5$. The mean duration (the time during which the structure is loaded by the extreme snow load) is supposed to be about 14 days. The interarrival-duration intensity is thus $\rho_s = 5 \times 14/365 = 0.19$.

The possible approximation of the snow load during the reference time T_{ref}= 1 year is shown in Fig. 3.

Windstorms are expected to appear ten times a year ($\lambda_w = 10$) and the mean duration of the storm is estimated as 8 hours. The interarrival-duration intensity is then
$\rho_w = 10 \times 8/(24 \times 365) = 0.009$.

5.3 Results of reliability analysis

According to the results of the time invariant analysis, the reliability of a structure of the IPE 300 profile seems to be rather low (β is less than 3), while the cross-profile IPE 330 seems to be acceptable. For the higher profile the resulting reliability index $\beta = 3.95$ corresponds well to the recommended value $\beta = 3.8$ [1], as shown in Table 5. Nevertheless, it should be mentioned that the time invariant analysis based on Turkstra's rule provides considerably lower values for reliability index β than those obtained by the time variant analysis.

The time variant analysis predicts the interval at which the reliability index β can be expected (a higher value of β corresponds to the lower bound of a failure probability while a lower value of β corresponds to the upper bound of a failure probability). The results obtained by the time variant analysis are more favourable and indicate that even the smaller profile

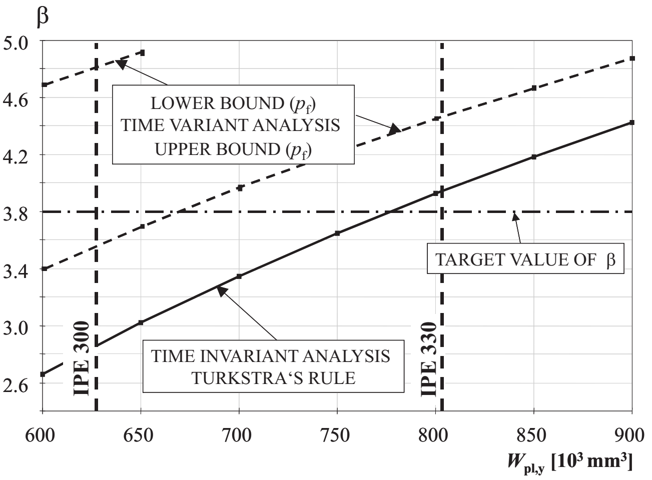

Fig. 4: Reliability index β as a function of the plastic sectional modulus $W_{pl,y}$

IPE 300 might be acceptable. The expected values of the reliability index β for IPE 300 are within the range from 3.57 (the upper bound) to 4.84 (the lower bound), for IPE 330 from 4.49 up to 5.56 as listed in Table 5. Fig. 4 shows the reliability index β determined by both the analyses as a function of the plastic sectional modulus $W_{pl,y}$.

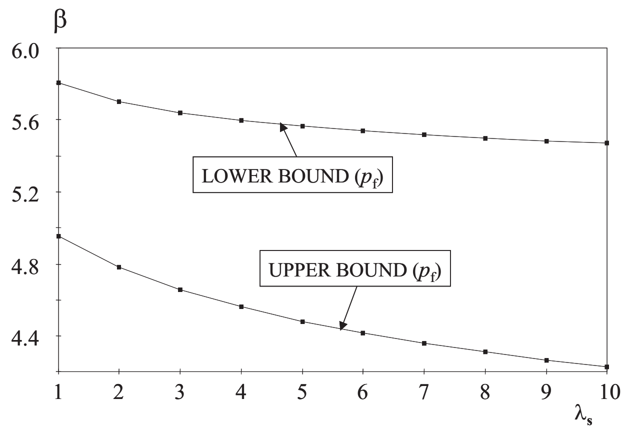

Fig. 5: Reliability index β as a function of the jump rate of the snow load λ_s

Table 5: Results of reliability analysis

Analysis	Used load models	Reliability index β	
		IPE 300	IPE 330
Time invariant	$s_{50} + w_1$	2.87	3.95
	$s_1 + w_{50}$	2.97	3.97
Time variant	Jump processes with intermittencies	3.57–4.84	4.49–5.56

5.4 *Effect of input data on resulting reliability index* β

The model parameters λ and ρ required for the time variant analysis are very difficult to specify. Nevertheless, parametric studies indicate that uncertainty in λ and ρ have an insignificant effect on the resulting reliability. For example, if the value of the jump rate of the snow load λ_s increases from 1 to 5 (i.e. if the snow load takes its extreme lasting 14 days five times a year, which is not real), the upper bound of β decreases approximately by 0.4 for the cross-profile IPE330, as shown in Fig. 5. Parametric studies of the jump rate of the wind action λ_w and of the interarrival-duration intensities of the two climatic actions ρ_s and ρ_w provide similar results.

5.5 *Probabilistic optimisation*

Probabilistic optimisation is based on minimisation of a simplified objective function expressed as the sum of the initial, marginal and expected malfunction cost. The decisive parameter is the sectional area A. The total cost C_{tot} can be expressed as:

$$C_{tot} = C_0 + C_m A + C_f p_f(A) \tag{18}$$

where C_0 denotes the initial cost independent of parameter A and failure probability p_f. The marginal cost is the product of the unit cost of the sectional area C_m and the sectional area A. The expected malfunction cost is the product of failure probability p_f and malfunction cost C_f when failure occurs. For probabilistic optimisation, equation (18) may be adapted as:

$$\frac{C_{tot} - C_0}{C_m} = C'_{tot} = A + \frac{C_f}{C_m} p_f(A) \tag{19}$$

The relative increment of the total cost C_{tot} is dependent only on the decisive parameter A and on the ratio C_f/C_m. Choosing various values of this ratio, different cross-sections seem to be adequate according to the results of the probabilistic optimisation shown in Fig. 6.

The arrows point to the minima of the relative increment C'_{tot} for assumed ratios C_f/C_m. The dot-and-dash curve shows the resulting reliability index β dependent on sectional area A assuming Turkstra's rule (the 50-year extremes of the snow load and the annual extremes of the wind action). The horizontal dashed line marks the target value of β ($\beta_t = 3.8$).

Obviously with the increasing cost ratio C_f/C_m the optimum cross-sectional area A increases. Profile IPE 330 seems to be optimal for $C_f/C_m = 5 \times 10^6$.

To get credible results of the optimisation, it is necessary to determine the values of C_m and C_f exactly.

6 Conclusions

The structural analysis of the frame shows that lateral-torsional buckling represents the most critical design criterion and indicates that the snow load is the leading variable action. Considering a 50-year lifetime, the reliability index β for IPE 330 varies within the range from 3.95 up to 5.56. According to the results of the reliability analysis, cross-profile IPE 330 designed using Eurocodes seems to be adequate.

The time invariant analysis based on Turkstra's rule provides considerably lower values of β than those obtained by the time variant analysis. It seems that Turkstra's rule leads to a rather conservative reliability level for a combination of variable actions having significant intermittencies. The great differences between the lower and upper bounds are most likely caused by the considerable intermittencies of the two variable actions. The model parameters required for time variant analysis are, however, very difficult to estimate. Nevertheless parametric studies indicate that this uncertainty has an insignificant effect on the resulting reliability.

Structural analysis of a beam with haunches exposed to lateral-torsional buckling is a very complicated task. It is foreseen that more precise results may be obtained by an analysis based on the model using the Finite Element Method.

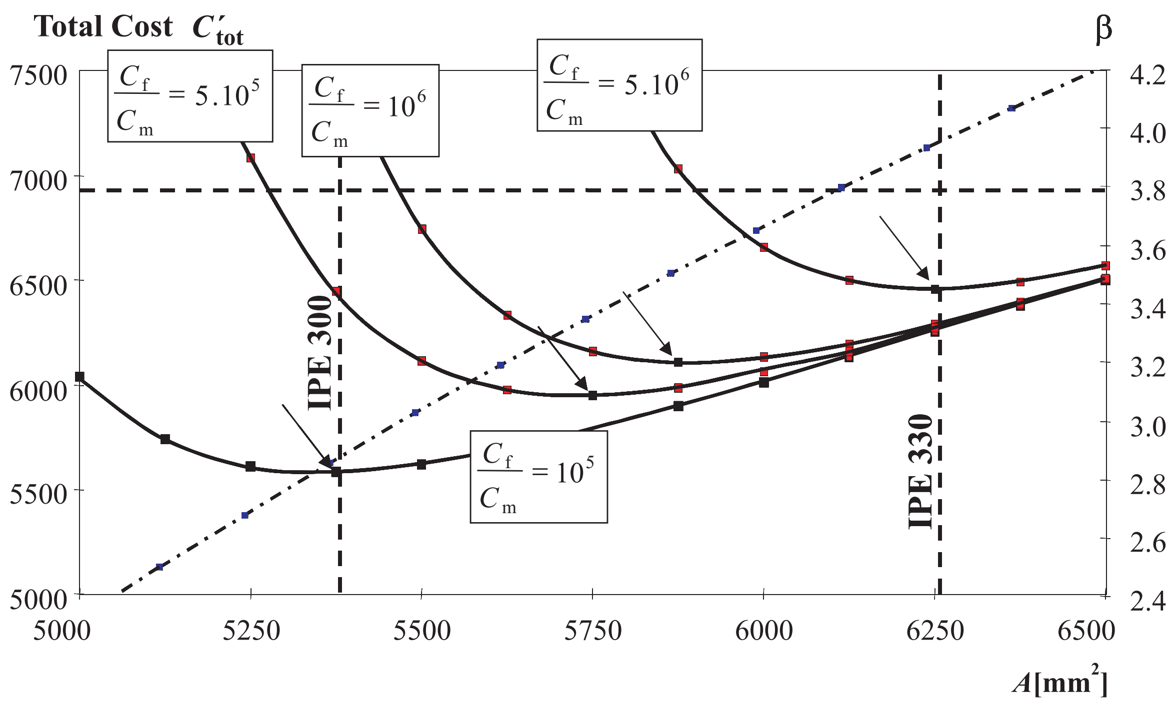

Fig. 6: Relative increment of total cost C_{tot} as a function of sectional area A using Turkstra's rule ($s_{50} + w_1$)

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