Dynamic Simulations in Cost and Time Estimation of the Construction Process

V. Beran, E. Hromada

This paper describes a model which is able to simulate the costs and the duration of construction for a building project. The model predicts the set of expected costs and the duration of the project depending on input parameters such as production rate, scope of the work, the time schedule, bonding conditions, maximum and minimum deviations from the scope of the work, and the production rate. Clients are able to make proper decisions concerning the time and cost schedules of their investments.

Keywords: simulation, time scheduling, cost scheduling, reliability and risk.

1 Introduction

A simulation of construction activities on the basis of production rate makes it possible to *monitor the reliability* of the expected time schedule and the total costs. The input parameters are production rate, scope of the work, the time schedule, bonding conditions, maximum and minimum deviations from the scope of the work, and the production rate.

The simulated model can be used at many levels of project management. Clients are able to make decisions about implementing their intentions; competitors can assess a bid price; building contractors can make a detailed calculation of the costs and the time schedule of construction activities, and they can optimize the construction process.

The simulation model stems form the research of Haas, Hajek [4] and then Beran, Dlask [1] and [2], which has been carried out at the Faculty of Civil Engineering, Czech Technical University in Prague during the last ten years.

2 Definition of the problem

Within the framework of initiating a simulation of a building project, it is necessary to define the problem as such. The application obtains input data by means of the *Module of input data*, which defines the particular construction activities, and the volume $\boldsymbol{Q}$ of these particular construction activities is expressed in physical or financial units, its production rate v and bonding conditions $\therefore \boldsymbol{D}_{\text{connection activities}}$ linking particular activities. $\boldsymbol{TAB}_{\text{project}}$ characterizes the calculation as a meta problem called *Dynamic progress chart* (flow-sheet). N generally characterizes the sequential networks N_i [2]. The set expression is as follows:

$$\boldsymbol{TAB}_{\text{project}} = \left\{ N_i \middle| \left[\boldsymbol{D} = f(\boldsymbol{Q}|risk, \boldsymbol{v}|risk, \therefore \boldsymbol{D}_{\text{connection activities}}) \right], i = 1, \ldots \right\}, \tag{1}$$

where i are partial processes and $\boldsymbol{D}$ is the set of activity durations, while *risk* influence is a conditioned externality (see Ta-

Table 1: Dynamic progress chart of a building project

Activity	Scope of work	Production speed	Time duration	Start	End	Day																								
						1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ground works	150	33	5	2	6		33	33	33	33	19																			
Water connection	191	23	10	8	17								23	22	22	21	20	19	19	18	17	10								
Sewerage connection	207	55	4	9	12									55	64	74	15													
Electricity connection	93	93	1	19	19																			93						
Bottom construction	456	36	13	13	25													36	36	37	37	37	38	38	38	39	39	40	40	1
Dumping place	50	50	1	4	4				50																					
Overhead construction	1010	42	14	26	39																									
Roof	308	48	5	41	45																									
Inside parget	66	32	3	47	49																									
Facade rendering	63	30	3	51	53																									
Completing works	40	7	6	50	55																									
Demands on resources through time Q'(t)						0	33	33	83	33	19	0	23	77	85	95	35	55	55	55	54	48	38	##	38	39	39	40	40	1
Total demands on resources through time Q(t)						0	33	66	149	182	200	200	224	301	386	481	516	571	626	680	735	782	820	951	989	1 028	1 068	1 107	1 147	1 148

Key:
- … Noncritical activity
- … Critical activity
- … Total reserve
- … Waiting for activity
- … Waiting for activity – on critical way

ble 1). The notation is supplement by conditionality of the breach of the assumed input parameters – the scope of the work and the production rate.

A practical solution of the calculation according to the dynamic progress chart (1) is based on inputting the work volume, the production rate and a time schedule of the particular activities. The time duration in the dynamic progress chart is calculated as the quotient of quantities $\boldsymbol{Q}$ and $\boldsymbol{v}$, or more precisely $D_i = Q_i / v_i$. The input data included in the *Module of input data* in the *Connection activities* sheet defines the bonding conditions among the particular production activities. The *Deviations of project parameters 1 and 2* sheet contain input data about the minimum / maximum deviations of the scope of the work and the production rate of the particular activities, based on the expected parameters of the building process [6].

3 Solution and an example

On the basis of the Excel VBA application, the algorithm enable us to calculate an instant dynamic progress chart of the building project, including a time schedule of resources in terms of expression (1), the dynamic progress chart is differences calculated on the basis of a common progress chart. The calculation is based on the production rates and the individual activities, which are described in columns *Start* and *End* (Tab. 1), which present the links between the individual activities. It practically represents the relations between the declared function $f(\boldsymbol{Q}|risk, \boldsymbol{v}|risk, \therefore \boldsymbol{D}_{\text{connection activities}})$ from expression (1) and the composition of the task as a consecutive process on the basis of the time duration of the individual processes N [$\boldsymbol{D}$] [1, 2, 5].

The dynamic progress chart creates a comprehensive methodically uniform model. The outputs of the model include information about the deadlines for the start and the end of the production activities, and information about cost schedules. The application creates a graphic visualization of the demand for resources in time (see Fig. 1).

The question of continuity of the project realization is interconnected with the cost-cutting management measures. The varying construction rate causes changes in construction costs. The flow of the construction costs is a significant indicator of the economy of the capital employment.

The calculation and software application described here can be used for evaluating of bid proposals for investment projects. The approach carries out a two-dimensional simulation. The project described in propositions *time* and *costs* will be marked as a predefined project. The discrete probabilistic variables (T; C) obtain the values (t_i; c_j). We mark it as follows: $\mathbf{P}(T = t_i; C = c_i) = p_{ij}$.

The SW application simulates the presumed development of the examined construction phase, the whole construction project or just a set of construction activities. We can identify the effects of changes and we can view management changes in the scope of particular jobs (construction activities) and the probability (reliability) of meeting the proposed (contracted) completion deadline t_{fin} and the proposed contracting cost limit c_{fin}. In general we are searching for acceptable

$$F(t_{\text{fin}}; c_{\text{fin}}) = \sum_t \sum_c \mathbf{P}(T < t_{\text{fin}}; C < c_{\text{fin}}) \tag{2}$$

for the selected project activities A_k

or their activity sets $A_k, A_l, \ldots, A_x$, functionality-designed into a network.

The results of particular simulations

$$\mathbf{P}(T;C) = \mathbf{P}(sim\boldsymbol{TAB}_{\text{project}}(T;C)) \tag{2a}$$

for the example described below are $T \in \langle 45; 89 \rangle$ and $C \in \langle 2411; 3131 \rangle$. Simulation data is continuously recorded on the basis of (1), [2]. The simulation is based on the time schedule given in Table 1. 50 000 simulations can be ranged into 30×30 categories. When a simulation is finished, the recorded data serves as a basis for a statistical analysis of the construction processes. The data file serves for the final analysis and the inter alia is a basis for modified 3D visualization, as shown in Fig. 2. The calculation of the expected or fixed probability starts, ends and reserves the results, as shown in Fig. 3 and Fig. 4.

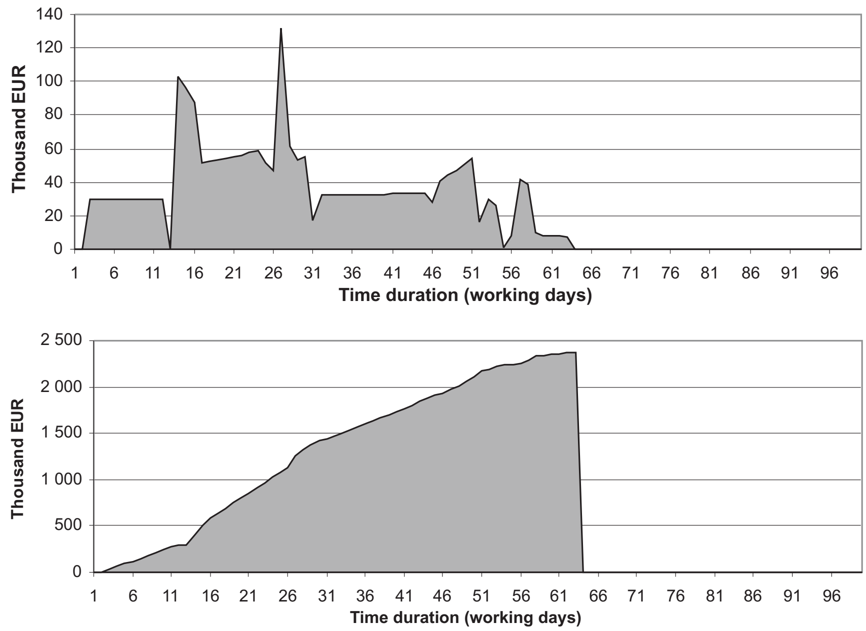

Fig. 1: Required cash flow of capital needed in the course of the construction period and the cumulative need for capital

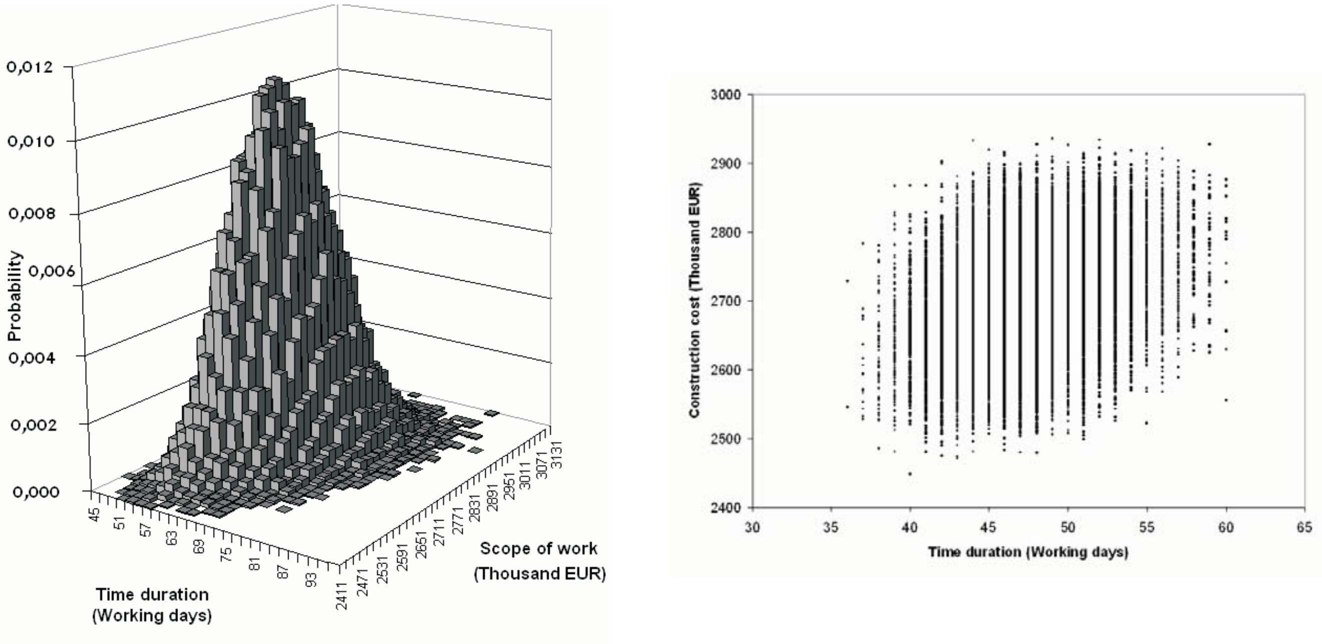

Fig. 2: Example of a 3D probability bar chart as an expression on the basis of (1)

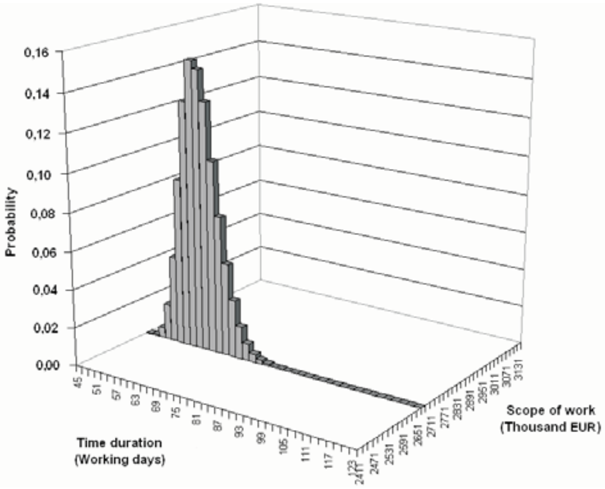

Fig. 3: Probability 3D bar chart for a construction project with fixed cost scope C

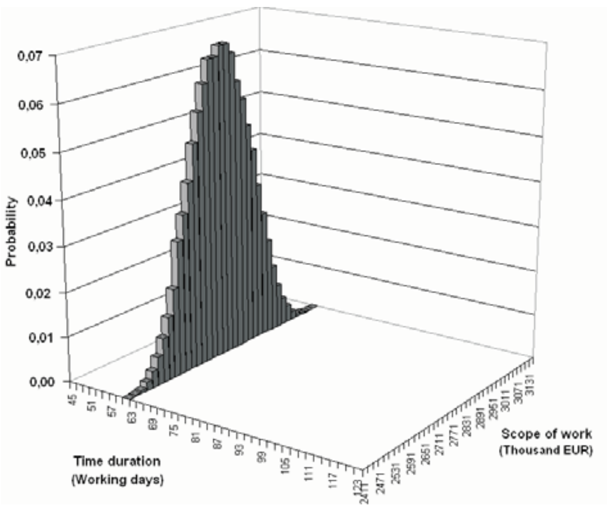

Fig. 4: Probability 3D bar chart for a construction project with fixed time duration T

Table 2 contains structural data of the comprehensive simulation example. The particular items are calculated as a construction bid proposal which is further described by a simulation study. This shows how practically competitive and realistic the supposed completion date and costs are.

$$\mathbf{P}(T_{(45,89)};C_{(2411,3131)}) = \mathbf{P}(sim\boldsymbol{TAB}_{\text{project}}(T;C)) =$$

$$\mathbf{P}(sim\{N_i \big| \big[\boldsymbol{D} = f(\boldsymbol{Q}|risk,\ \boldsymbol{v}|risk,\ \therefore \boldsymbol{D}_{\text{connection activities}})\big], \qquad (2b)$$

$$\text{for subprojects or subactivities } i = 1, \ldots\}).$$

The construction project [3] is proposed with the time schedule and the scope of the work as given in the 3D bar chart. The ellipse in Table 2 shows the shift of probability in time and costs. Using this approach, he results of the simulations can be specified more precisely. The occurrence frequencies of the particular scenarios of building project bids are comparable. The highest simulation frequency values in the 3D bar graph indicate the highest probabilities of potential success scenarios for the construction project. In the given case, the building project will be completed with satisfactory commercial probability, within the range of 57 to 59 days, and the construction cost will be in the range of 2 731 to 2 751 thousand EUR.

Within the framework a building project simulation, the calculation frequently reveals a unique regular solution. An example of a 3D probability bar chart with a unique regular solution is shown in Fig. 2.

In the case of complicated bonding conditions and other additional interdependences between the particular activities, the solution of the simulation may not be unique.

Fig. 5 and Fig. 5a present a building project in which the input parameters contain a specific interdependence within the first activity (ground works). Let us compare these results with Fig. 2. In the event that the first activity takes more than 25 days, the building ground machinery must without delay be removed to another major activity (another building project). This situation causes slippage of dates within the range

Table 2: The example of a 3D bar chart illustrating the result of 50 000 simulations

Classes	Scope of work (Thousand EUR)																																					Total
Time duration (day)	2411	2431	2451	2471	2491	2511	2531	2551	2571	2591	2611	2631	2651	2671	2691	2711	2731	2751	2771	2791	2811	2831	2851	2871	2891	2911	2931	2951	2971	2991	3011	3031	3051	3071	3091	3111	3131	
45					1			1						1			1			1																		5
47					2	4	1	1	5	1	2	1	1	2	2	1	1							2														26
49		1	2	4	6	10	6	14	17	17	19	19	25	18	20	21	14	8	13	7	5	5	3	2	1	1												258
51	1	3	5	8	19	23	33	42	42	68	68	71	74	69	81	67	41	54	60	30	31	19	10	7	9	2		1	2									940
53	2	3	6	10	24	41	64	81	101	138	136	157	180	170	191	158	145	122	94	113	84	86	56	43	30	16	8	8		2		1						2 270
55	1	5	2	13	34	53	77	110	135	201	224	268	313	317	336	336	307	287	267	253	205	177	127	127	75	56	34	23	10	6	5			2				4 386
57	1		8	20	26	63	87	111	182	240	299	351	373	446	450	454	478	477	458	383	341	286	263	205	141	111	87	52	26	23	9	3	2	1				6 457
59		1	2	12	28	46	87	110	156	245	298	342	421	433	480	535	543	552	542	469	402	397	351	204	250	174	129	92	58	31	17	10	6	2	1			7 516
61			4	9	21	33	50	98	140	212	257	313	319	410	491	514	485	522	515	520	480	434	382	298	252	197	141	87	60	28	22	10	3	1	2			7 310
63		1	1	8	10	23	47	56	93	137	178	230	279	355	378	465	447	445	462	480	415	383	348	315	278	228	170	112	83	53	32	15	2	2			1	6 532
65		1		6	8	10	22	40	56	106	137	130	210	259	251	308	341	364	351	374	353	334	290	277	245	168	154	97	65	41	22	13	12	3	1			5 049
67			1	3	3	4	12	20	32	58	69	97	155	139	194	224	219	233	270	304	262	255	232	208	169	161	106	83	51	29	25	11	5	2	1			3 637
69			1	2	1	2	6	7	22	34	39	62	74	96	106	114	156	170	171	179	181	169	186	158	115	107	96	52	54	31	23	8	3	2	1			2 428
71						2	2	6	4	17	13	25	24	44	52	74	81	101	119	113	118	127	104	115	91	84	58	54	29	22	12	5	6	2	3			1 507
73							1	4	1	7	9	13	24	23	29	37	36	59	70	50	70	65	60	65	49	53	34	35	22	16	13	3	2	2				852
75									1	2	2	2	7	11	14	12	20	25	25	33	39	36	38	44	32	28	29	20	8	11	2	1						442
77							1		2		1	1	1	5	4	5	6	16	14	13	15	12	17	23	14	15	11	11	14	5	1	5		1				213
79											1			2	1	1	7	2	5	5	5	9	10	6	7	12	8	4	3	5	5	3		1				102
81															2	1	1	1	2	4	2	4	2	4	5	3	9	3	2	2	1		2	1			1	52
83																		1	3				1	1	1		1	2			1							11
85																					1			2				2			1							6
87																																						0
89																												1										1
Total	5	15	32	95	183	314	496	701	989	1 483	1 752	2 082	2 480	2 800	3 082	3 327	3 329	3 439	3 441	3 331	3 009	2 798	2 480	2 196	1 764	1 416	1 075	739	487	305	191	88	43	22	9	0	2	50 000

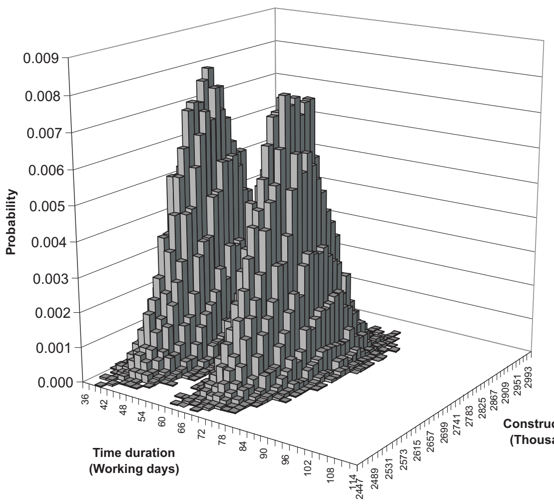

Fig. 5: Example of a 3D probability bar chart with heterogeneous solutions

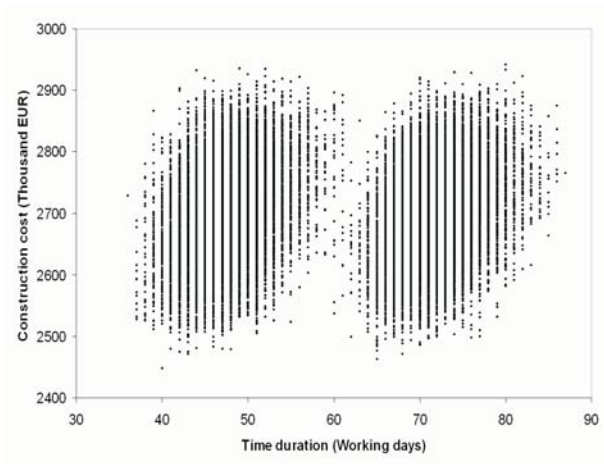

Fig. 5a: Example of a graph with interdependences between time duration and construction cost (frequencies)

of 21 days. After that period, the ground works can be resumed.

This specific condition leads to a heterogeneous solution of the simulation. It is difficult to find the solution of this building project using standard statistical methods. It is convenient to take advantage of visualization techniques and particular simulation calculations.

Important information regarding a proposal for a future project time schedule is specified by tests of the potential scenarios of the project development with current fixing of certain parameters of the building organizational model. Important information can be obtained about critical parameters of the planned project, for example by fixing the deviations of the scope of the work for particular activities, see $\boldsymbol{Q}$ in expression (1) [7].

It is common practice to present the probability of the total construction time of a building project without the cost viewpoint (Fig. 4). A better-expressed project cost is presented as a respected fixed value that will be stable and independent from the duration of the project. Addressing this notion, the proposed approach that simulates the interrelated *time* and *cost* values shown in Fig. 3, is more understandable and more comprehensive than the information shown in Fig. 4, where

$$\sum_{T} \mathbf{P}(a \le T \le b) = F(t_b) - F(t_a) \tag{3}$$

or than the calculation with a fixed scope of work

$$\sum_T \mathbf{P}(45 \le T \le 89) = \mathbf{P}(sim\boldsymbol{TAB}_{\text{project}}(T)). \tag{3a}$$

A similar situation arises if we fix the alternation of time schedules for the project. The scope of the work given as C is specified as

$$\sum_C \mathbf{P}(x \le T \le y) = F(c_y) - F(c_x) \tag{3b}$$

for the data simulated in Table 2 and demonstrated in Fig. 4.

$$\sum_C \mathbf{P}(2411 \le C \le 3131) = \mathbf{P}(sim\boldsymbol{TAB}_{\text{project}}(C)), \tag{3c}$$

in the calculation with a fixed time ratio.

Fig. 4 shows the changes in the project cost with fixed duration of the observed project.

The expected time duration of the total construction project is given by its mean value

$$\mathbf{E}\left[T_{\text{project}} \middle| C = \text{const.}\right] = \sum_T t_i \, P(T = t_i) = \bar{t} \tag{4}$$

In the same way, we can quantify the expected scope of the work on the total construction project by its mean value

$$\mathbf{E}\left[C_{\text{project}} \middle| T = \text{const.}\right] = \sum_C c_i \, P(C = c_i) = \bar{c} \tag{5}$$

4 The search for reliable construction cost and time duration

The simulation model is able to calculate the adequate construction costs and the time duration of a project on the basis of the input probability level. The reciprocal view aims to find out the adequate level of probability for construction cost and activity durations.

There are two ways to calculate an adequate level of probability. The first way consists in fixing one variable parameter and investigating the changes in the remaining parameter. The second way involves a simultaneous investigation of the deviations of both parameters.

The approach used in this paper is based on expression (1) and Table 3 (discrete probability density table), and it enables us according to the data in Table 2 to calculate the level of probability as a cumulative density function

$$F(T;C) = \sum_T \sum_C P(T = t_i; C = c_i), \tag{6}$$

where t_i and c_i run through the set of all possible values of T and C and

$$\sum_T \sum_C P(T = t_i; C = c_i) = 1. \tag{6a}$$

In the course of a closer investigation of the results of particular simulations, the dependence between the level of probability and the construction cost and time duration was found. The following figures show the bilateral interactions of these project parameters.

5 Conclusion

The model described here enables us to predict the expected costs and the duration schedule of a project depending on input parameters such as the production rate, the scope of the work, the time schedule, bonding conditions, maximum and minimum deviations from the scope of the work, and the production rate. The results present a useful risk evaluation for projects or project activities.

Table 3: Example of calculating the level of Discrete Probability Distribution (DPD) for 50 000 simulations

Time duration (day) / Classes	Scope of work (Thousand EUR): 2411	2431	2451	2471	2491	2511	2531	2551	2571	2591	2611	2631	2651	2671	2691	2711	2731	2751	2771	2791	2811	2831	2851	2871	2891	2911	2931	2951	2971	2991	3011	3031	3051	3071	3091	3111	3131	Total
45					1			1						1			1			1																		5
47					2	4	1	1	5	1	2	1	1	2	2	1	1							2														26
49		1	2	4	6	10	6	14	17	17	19	19	25	18	20	21	14	8	13	7	5	5	3	2	1	1												258
51	1	3	5	8	19	23	33	42	42	68	68	71	74	69	81	67	41	54	60	30	31	19	10	7	9	2		1	2									940
53	2	3	6	10	24	41	64	81	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	158	145	122	94	113	84	86	56	43	30	16	8	8		2		1						2 270
55	1	5	2	13	34	53	77	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	307	287	267	253	205	177	127	127	75	56	34	23	10	6	5			2					4 386
57	1		8	20	26	63	87	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	478	477	458	383	341	286	263	[illegible]	141	111	87	52	26	23	9	3	2	1					6 457
59		1	2	12	28	46	87	110	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	480	535	543	552	542	469	402	397	351	294	250	174	129	92	58	31	17	10	6	2	1			7 516
61			4	9	21	33	50	98	140	212	257	313	319	410	491	514	485	522	515	520	480	434	382	298	252	197	141	87	60	28	22	10	3	1	2			7 310
63		1	1	8	10	23	47	56	93	137	178	230	279	355	378	465	447	445	462	480	415	383	348	315	278	228	170	112	83	53	32	15	2	2			1	6 532
65		1		6	8	10	22	40	56	106	137	130	210	259	251	308	341	364	351	374	353	334	290	277	245	168	154	97	65	41	22	13	12	3	1			5 049
67			1	3	3	4	12	20	32	58	69	97	155	139	194	224	219	233	270	304	262	255	232	208	169	161	106	83	51	29	25	11	5	2	1			3 637
69			1	2	1	2	6	7	22	34	39	62	74	96	106	114	156	170	171	179	181	169	186	158	115	107	96	52	54	31	23	8	3	2	1			2 428
71						2	2	6	4	17	13	25	24	44	52	74	81	101	119	113	118	127	104	115	91	84	58	54	29	22	12	5	6	2	3			1 507
73							1	4	1	7	9	13	24	23	29	37	36	59	70	50	70	65	60	65	49	53	34	35	22	16	13	3	2	2				852
75									1	2	2	2	7	11	14	12	20	25	25	33	39	36	38	44	32	28	29	20	8	11	2	1						442
77							1		2		1	1	1	5	4	5	6	16	14	13	15	12	17	23	14	15	11	11	14	5	1	5		1				213
79											1			2	1	1	7	2	5	5	5	9	10	6	7	12	8	4	3	5	5	3		1				102
81															2	1	1	1	2	4	2	4	2	4	5	3	9	3	2	2	1		2	1			1	52
83																		1	3				1	1	1		1	2			1							11
85																					1			2				2			1							6
87																																						0
89																												1										1
Total	5	15	32	95	183	314	496	701	989	1 483	1 752	2 082	2 480	2 800	3 082	3 327	3 329	3 439	3 441	3 331	3 009	2 798	2 480	2 196	1 764	1 416	1 075	739	487	305	191	88	43	22	9	0	2	50 000

Level of probability: 0,7476

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