

Actual Optical and Thermal Performance of Photovoltaic Modules

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Field testing is costly, time-consuming and depends heavily on prevailing weather conditions. Adequate security and weather protection must also be provided at the test site. Delays can be caused due to bad weather and system failures. To overcome these problems, a photovoltaic array simulation may be used. In any simulation scheme involving photovoltaic systems, one important choice is the selection of a mathematical model.

In the literature several approaches to the problem have been made. Most procedures designed for this purpose are based on analytical descriptions of the physical mechanisms inside the solar cell that can be represented by a circuit diagram with discrete components, like a two-exponential model. Such simulators have some merits. However, their limited flexibility in readily simulating the influence of solar radiation, temperature and various array parameters is a serious drawback that has been noted. To get more accurate results in predicting the actual performance of photovoltaic modules, the parameters influencing incoming (optical parameters) and outgoing power flow (electrical and thermal parameters) were investigated by simulation and by some verifying experiments, to get a closer insight into the response behavior of this element, and to estimate the overall performance as well as optimization of the parameters.

Keywords: solar radiation, reflection losses, radiation shape factor, radiative surface area, temperature distribution, emissivity, isotropic model, Hay and Klucher's anisotropic models.

1 Introduction

Design and performance evaluation of photovoltaic systems usually involve an estimation of irradiation incident on the photovoltaic module plane. For design purposes only global horizontal irradiation, published by meteorological institutes, is usually available. Using irradiation models, we can calculate the gain in irradiation on a tilted plane with respect to horizontal irradiation. Concerning a part of this task, most textbooks [4] recommend that the diffuse component be treated as if it were isotropically emanating from the sky vault. However, theoretical as well as experimental results have shown that this simplifying assumption is generally far from reality. Dave, J., 1977 examined the validity of this isotropic distribution approximation for a sun-facing flat surface located at the bottom of plane parallel models of non-absorbing homogeneous atmospheres. Even for such idealized and somewhat unrealistic models, he showed that the use of an isotropic distribution approximation results in systematic underestimation of the diffuse energy contribution to the sun-facing surfaces. That study demonstrated the need to test this approximation for more realistic, atmospheric conditions. Thus it appears that sky radiance should be treated as anisotropic, particularly because of the strong forward scattering effect of aerosols [2, 9, 14, 16].

In the first part of this study, some results concerning the accuracy of models to estimate irradiance on inclined planes are tested. The models chosen for discussion here are for arbitrary sky conditions and are supposed to be applicable anywhere in the world. These methods begin with measured hourly global and diffuse radiation received on a horizontal surface. These quantities are then transposed onto an inclined plane by a mathematical procedure. The accuracies of the models are then compared on the basis of statistical error tests, and the most accurate model is recommended. The analyses are also extended to include the optical processes occurring inside the encapsulation solar module.

Solar concentrators are frequently used to increase the specific power and decrease the space of radiation degrada-

tion of solar cell parameters. On the other hand, at high light concentration coefficients, a large amount of thermal energy evolves in solar cells. If cooling is not provided, the cell working temperature increases. This affects the carrier concentration and light absorption process, causing a reduction in open circuit voltage, a slight increase in short circuit current, a reduction of the fill factor, and degradation of the cell power output. The degradation rate of cell output is approximately double for each 10 °C increase in temperature [7, 22]. Therefore, the modules and solar array must take full advantage of radiative, conductive and convective cooling and absorb the minimum of unused radiation. Hence, there is a need for an exact technique to calculate accurately and efficiently the temperature distribution of photovoltaic solar modules, from which we can adjust safe and proper operation at maximum ratings. The computer code was written in Fortran, a programming language quite flexible and suitable for organizing the screen with multiple windows and for drawing curves and histograms. The program structure consists of a main executable file, which has several external units to perform the different calculations; program maintenance is therefore quite easy, as is the implementation of new models or extension to a new configuration. The derived equations are solved using an explicit technique and iteration processes [18, 20]. The models and simulation programs developed here allow us to predict the thermal performance, and they are promising tools for evaluating new photovoltaic power plants with the aim of increasing efficiency. Three main criteria are considered in evaluating the models. The first is versatility: What range of photovoltaic modules and systems can the model handle? Accuracy is the second issue: What is the physical basis and how closely can the model replicate manufacturers' data? The final issue is computational speed. This criterion is of secondary importance for this work.

2 Apparatus and measurements

Total radiation was monitored with a high precision pyranometer, which is sensitive in the wavelength range from 300 to 3000 nm. Sky diffuse radiation was measured by

a pyranometer equipped with a special shading device to exclude direct radiation from the sun. All data sets were subjected to various quality control tests. Three types of data checks were performed to identify missing data, data that clearly violates physical limits, and extreme data. Hours when the data was known to be "bad" or "missing" were omitted. Second, any hour with an observation that violated a physical limit or conservation principle was eliminated from the data set, including: reported hours with a diffuse fraction greater than 1, or beam radiation exceeding the extraterrestrial beam radiation. To eliminate the uncertainty associated with radiation measurements at large incidence angles, hours with a zenith angle larger than 80° were eliminated. The final data set was constructed from the measured data that passed all of the quality control checks.

3 Optical performance of photovoltaic modules

In order to make a precise representation of the actual optical conditions in the module, a model for an encapsulating cell was developed which determines the insolation reaching the cell from sun and sky irradiance. This was done by modeling the optical processes occurring outside and inside the encapsulation. Most collecting devices associated with solar energy systems are tilted at some angle with respect to the horizontal. Due to the lack of measured tilted surface solar radiation data, models are employed to estimate the radiation incident on a collector surface from measured horizontal radiation. In the present work, methods for calculating radiation on a tilted surface from horizontal data are evaluated.

3.1 Evaluation of irradiation on a tilted south-oriented surface

Consider a plane inclined at an angle β from the horizontal position. For the time being, we assume that the surface is inclined in such a way that it faces the sun; that is, direct radiation is not striking the back of this plane. Total radiation arriving on an inclined surface is composed of three components: beam radiation, sky diffuse radiation and radiation reflected from the ground. The models discussed here all share the same formulations for the beam and ground reflected components. Hourly irradiations with a solar elevation angle $\gamma > 8^\circ$ are tested only in order to avoid errors resulting from dividing small numbers. It is useful to write down the hourly formulation of these components.

3.1.1 Beam radiation incident on an inclined surface

For a surface oriented in any direction with respect to the meridian, the trigonometric relation for the incidence angle, θ_I can be written in the following form:

$$\begin{aligned} \cos \theta_I = & (\sin \phi \cos \beta - \cos \phi \sin \beta \cos \gamma) \sin \delta + \\ & + (\cos \phi \cos \beta + \sin \phi \sin \beta \cos \gamma) \cos \delta \cos \omega + \\ & + \cos \delta \sin \beta \sin \gamma \sin \omega \end{aligned} \quad (1)$$

where the meanings of the different symbols are as given in the nomenclature list. There are several commonly occurring cases for which Equation 1 is simplified. For a fixed surface sloped toward the south or north, that is, with a surface azimuth angle γ of 0° or 180° (a very common situation for fixed flat plate collectors), the last term drops out.

$$\begin{aligned} \cos \theta_I = & (\sin \phi \cos \beta - \cos \phi \sin \beta) \sin \delta + \\ & + (\cos \phi \cos \beta + \sin \phi \sin \beta) \cos \delta \cos \omega = \\ & = \sin \delta \sin(\phi - \beta) + \cos \delta \cos \omega \cos(\phi - \beta). \end{aligned} \quad (2)$$

For horizontal surfaces, the angle of incidence, θ_I is the zenith angle of the sun, θ_Z . Its value must be between 0° and 90° when the sun is above the horizon. For this situation $\beta = 0$, and Equation 1 becomes:

$$\cos \theta_Z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta. \quad (3)$$

The geometric factor R_b , the ratio of beam radiation on the tilted surface, $I_{b,T}$ to that on a horizontal surface at any time, can be calculated exactly by appropriate use of Equation 1.

The ratio $I_{b,T}/I_{b,H}$ is given by:

$$R_b = \frac{\cos \theta_I}{\cos \theta_Z} = \frac{\sin \delta \sin(\phi - \beta) + \cos \delta \cos \omega \cos(\phi - \beta)}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta}. \quad (4)$$

The hourly beam radiation received on an inclined surface can be expressed as:

$$I_{b,T} = I_{b,H} \cdot R_b = (G_H - I_{d,H}) \cdot R_b. \quad (5)$$

It should be noted that at the grazing angles (just at sunrise or at sunset) R_b can change rapidly and may approach infinity or zero because both the numerator and the denominator are small numbers. This depends on slope, latitude and date.

3.1.2 Sky diffuse radiation incident on an inclined surface

Diffuse irradiance is difficult to determine accurately with the simple parameterization methods that were used to calculate direct normal irradiance in the previous section, since its spatial distribution is generally unknown and time dependent. Three diffuse subcomponents are used to approximate the anisotropic behavior of diffuse radiation. The first is an isotropic part received uniformly from the entire sky dome. The second is circumsolar diffuse resulting from forward scattering of solar radiation and concentrated in the part of the sky around the sun. The third, referred to as horizon brightening, is concentrated near the horizon, and is most pronounced in clear skies. Several models have been proposed to estimate the diffuse radiation on a tilted surface (not all of which account for these three diffuse subcomponents).

The **Isotropic model** [12] is the simplest of the tilted surface models. In this model the intensity of sky diffuse radiation is assumed uniform over the sky dome, i.e., it is independent of the azimuth and zenith angles. It approximates the completely overcast sky condition. The formula for the hourly sky diffuse radiation incident on an inclined plane is given by the product of the hourly diffuse radiation incident on a horizontal surface and the configuration factor from the surface to the sky, $(1 + \cos \beta)/2$.

$$I_{d,T} = I_{d,H} \frac{1 + \cos \beta}{2}. \quad (6)$$

Under completely cloudy skies, the isotropic model becomes a good approximation. As skies become clearer, the validity of the isotropic sky model deteriorates due to the presence of circumsolar and horizon brightening anisotropic effects.

Klucher, T., 1979 model: This model is based on a study of clear sky conditions by Temps, R. and Coulson, K., 1977. Their model was modified by Klucher, who incorporated

conditions of cloudy skies. It takes into account the increase in the diffuse radiation in the vicinity of the sun (circumsolar radiation) and that near the horizon (horizon brightening). Klucher's formulation of the hourly sky diffuse radiation incident on an inclined surface is:

$$I_{d,T} = I_{d,H} \left[\left(\frac{1 + \cos \beta}{2} \right) \left(1 + F_1 \sin^3 \left(\frac{\beta}{2} \right) \right) \left(1 + F_1 \cos^2 \theta_Z \sin^3 \theta_Z \right) \right]. \quad (7)$$

Where F_1 is the modulating function given by:

$$F_1 = 1 - \left(\frac{I_{d,H}}{G_H} \right)^2. \quad (8)$$

When the skies are completely overcast, $F_1 = 0$, Klucher's model reverts to the isotropic model.

Hay, J. and Davies, J., 1980 developed a model to predict tilted surface diffuse radiation, which accounts for both circumsolar and isotropic diffuse radiation. Realizing that the anisotropic behavior of circumsolar diffuse radiation becomes more pronounced under clear sky conditions, Hay, J. and Davies, J., 1980 defined an "anisotropic index, A_1 " to weight the circumsolar, $I_{T, \text{cir}}$ and isotropic, $I_{T, \text{iso}}$ radiation components. The anisotropy index defines a portion of the diffuse radiation to be treated as circumsolar with the remaining portion to be considered isotropic. The circumsolar diffuse radiation is projected onto the tilted surface in the same fashion as beam radiation.

$$A_1 = I_{b,H} / I_o \quad (9)$$

$$I_{T, \text{cir}} = I_{d,H} \cdot A_1 \cdot R_b. \quad (10)$$

The remaining diffuse radiation is treated as isotropic diffuse.

$$I_{T, \text{iso}} = I_{d,H} (1 - A_1) \frac{1 + \cos \beta}{2}. \quad (11)$$

The total diffuse radiation on a tilted surface is the sum of (10) and (11).

$$I_{d,T} = I_{d,H} \left[(1 - A_1) \left(\frac{1 + \cos \beta}{2} \right) + A_1 \cdot R_b \right]. \quad (12)$$

Under clear skies, the anisotropy index will be high and the circumsolar diffuse is weighted more heavily than the isotropic diffuse. Under cloudy skies, the anisotropy index goes to zero and all diffuse is treated as isotropic. The Hay

model does not account for horizon brightening radiation incident on south facing surfaces.

3.1.3 Ground reflected radiation incident on an inclined surface

In order to correctly evaluate the diffuse radiation incident on an inclined surface of any orientation and of any tilt angle, we need to know separately the sky diffuse radiation as well as the ground reflected radiation. In this section we are going to focus on ground reflected radiation, which is also significant and which can sometimes reach values of the order of 100 W/m^2 for a vertical plane. It has already been previously found, but based on restricted data, that using an albedo value measured on site and considered as constant leads to satisfactory results, Ineichen et al., 1987.

$$I_{g,H} = \rho(\text{site}) \cdot G_H. \quad (13)$$

A common method for calculating the ground reflected radiation incident on a tilted surface is to assume that the foreground in the collector field of view is a diffuse reflector and that the horizon is unobstructed. Other authors have proposed anisotropic ground reflectance models [6, 8], but lack of experimental data has hampered their validation. Therefore, the ground reflected radiation is assumed to be diffuse. Under the isotropic condition, the ground reflected radiation incident on the inclined surface is given by the above quantity multiplied by the configuration factor from the ground to the inclined surface. Thus:

$$I_{g,T} = \frac{1}{2} G_H \rho (1 - \cos \beta). \quad (14)$$

3.1.4 Model evaluation

Three irradiance models are considered in this work: the isotropic model, Hay's and Klucher's anisotropic models. For each model, measured diffuse and horizontal global values were used to calculate the irradiance on surfaces tilted at 30° , 60° and 90° above the horizon. The results were compared with the irradiances monitored and presented in terms of usual statistics: mean bias error (MBE) and root mean square error (RMSE). The results for south facing surfaces are given in Table 1 and Table 2. It is noted that the RMSEs for all three models increase as the slope of the collector increases, but remain in a domain of errors for which these relations can

Table 1: Root mean square errors [%] for global radiation received on inclined surfaces

Month	Isotropic Model RMSE			Hay's Model RMSE			Klucher's Model RMSE		
	30°	60°	90°	30°	60°	90°	30°	60°	90°
January	18.0	22.0	22	8.0	11.0	10.0	8.0	11.0	13
February	16.0	21.0	22	7.5	11.0	12.0	7.0	11.0	13
March	9.0	12.0	12	4.0	6.0	8.5	3.0	5.0	8
April	6.0	14.0	8	4.0	12.0	8.0	3.0	12.0	8
May	3.5	5.5	12	3.0	5.5	12.0	3.0	5.0	8
June	4.0	9.0	25	3.0	9.0	21.0	4.5	8.0	24
July	3.5	6.0	19	2.0	5.0	13.0	4.0	6.5	18
August	3.0	7.0	12	2.0	5.0	10.0	3.0	5.0	12
September	11.0	15.0	17	6.5	9.0	12.0	4.5	7.0	10
October	13.0	16.0	17	6.5	7.0	9.0	5.0	6.5	9
November	24.0	31.0	36	13.0	8.0	20.0	10.0	14.0	19
December	18.0	23.0	26	9.0	13.0	14.0	9.0	11.0	13

Table 2: Mean bias errors [%] for global radiation received on inclined surfaces

Month	Isotropic Model MBE			Hay's Model MBE			Klucher's Model MBE		
	30 °	60 °	90 °	30 °	60 °	90 °	30 °	60 °	90 °
January	-8.3	-10.3	-9.2	-2.2	-1.6	-1.2	-3.3	-3.9	-4.2
February	-8.2	-11.0	-11.7	-2.8	-3.3	-3.3	-2.8	-4.7	-5.5
March	-5.5	-6.1	-4.4	-1.7	-0.6	1.0	-0.2	-0.1	1.7
April	-3.3	-5.3	-1.8	-1.1	-2.5	-0.6	-0.2	-0.2	4.5
May	-1.7	-2.2	-2.4	-1.2	-2.2	-5.5	1.6	2.2	4.8
June	-0.6	-1.1	1.2	-0.5	-1.9	-4.4	2.2	3.0	10.0
July	-0.8	-0.6	1.1	-0.4	-1.1	-3.9	2.2	4.7	8.9
August	-2.5	-2.8	-2.2	-1.1	-0.8	-2.2	1.9	2.2	5.5
September	-4.4	-4.4	-3.6	-2.2	-1.1	-0.6	0.1	1.1	3.2
October	-5.0	-8.9	-9.4	-3.3	-2.8	-2.8	-2.2	-2.8	1.9
November	-8.3	-9.4	-10.0	-4.4	-2.6	-2.2	-3.3	-2.9	0.8
December	-6.7	-7.8	-7.2	-1.1	-0.2	0.0	-2.1	-0.3	-0.5

be applied with good accuracy. Inspecting the results, it is apparent that the models agree quite well with each other during the summer months. They deviate from each other in the winter months, when the effect of the difference in the diffuse radiation parameterization is at its maximum. The RMSE results indicate that the anisotropic models (Hay and Klucher) show similar performance on an overall basis (within 5 %) but the isotropic model exhibits much larger error. The MBE results show that the isotropic model substantially underpredicts the irradiance incident on an inclined surface, and the Klucher model considerably overpredicts irradiance incident on an inclined surface on an overall basis. Our findings confirm the observation that the Klucher model describes the irradiance on inclined planes more accurately than the isotropic model or Hay's model.

Due to lack of measured tilted surface solar radiation data, Klucher's model is employed to estimate the radiation incident on a collector surface from measured horizontal radiation. The results of these calculations are plotted against the angle of tilt for summer, winter and all year round intended use. A summary of the effects of tilt angle on solar irradiation for south facing surfaces is shown in Fig. 1.

3.2 Optical parameters of the encapsulation module

Reflection losses produced in photovoltaic modules operating in real conditions in comparison with standard ones are commonly neglected or simplified to a value that does not take into account the module tilt angle. In some applications of interest such as building integration of photovoltaic modules, architectural criteria or other limitations frequently force modules to have tilt angles far from their optimal values. As a consequence, optical losses increase to non-negligible quantities [17].

The different encapsulation layers cause multiple reflections inside and among the slabs (see scheme in Fig. 2). The

Fig. 1: Annual variation in estimating average daily radiation for a flat plate collector facing south for a latitude of 50°, and ground reflectance of 0.2

Fig. 2: Ray tracing through the module layers

abbreviation EVA stands for ethylene-vinyl-acetate, which serves as a mechanical, thermal and optical interface between the solar cells and the glass cover. ARC stands for the anti-reflective coating of the solar cell. It consists of TiO_2 or another optical material to bridge over the refractive indices of silicon and the front cover. Refractive indices and transmissivity coefficients are given in Table 3.

Table 3: Refractive indices and transmissivity coefficient for the wavelength of $\lambda = 800$ nm [5]

Materials	Refractive index	Thickness	Transmissivity coefficient	
			Normal incidence	Incidence at 60°
Glass	1.526	3.2 mm	0.951	0.941
EVA	1.450	0.6 mm	0.992	0.990
ARC	1.700	50–60 nm	1.000	1.000
Silicon	3.690	0.5 mm	–	–

Light rays are modeled as objects having the properties of electromagnetic waves [1, 15]. Each ray can exist either as incident or as one of its reflected, refracted, absorbed and transmitted components, produced by the respective optical process (i.e., reflection, refraction, etc.). At the interfaces, a matrix method derived from electromagnetic theory is used to determine the probability of a photon being reflected, transmitted or absorbed in the interface layers. The calculation technique is based on the model of [19, 23], which calculates the radiation transfer properties of multi-layer structures. Their model was modified to generate an equivalent application in the domain of photovoltaic solar cell simulation.

3.2.1 Radiation transmission through a photovoltaic module

When light is incident upon the interface between two optically different materials, in general part of the light is reflected and part transmitted. If the medium on what is nominally chosen to be the outside (o) of the interface has a complex refractive index n_o and the medium on the inside (i) has a refractive index, n_i then, for a given wavelength, Snell's law states that:

$$\theta_o = \arcsin[(n_i/n_o) \sin \theta_i]. \quad (15)$$

For an incident wave of amplitude E_o^+ traveling across an interface from outside to inside (the superscript denotes the direction of propagation), the amplitude reflection coefficient (r) and amplitude transmission coefficient (t) are defined as:

$$r_{oi} = E_o^-/E_o^+; \quad t_{oi} = E_i^+/E_o^+. \quad (16)$$

Similar expressions hold for waves traveling in the opposite direction, for which it can be shown that:

$$r_{io} = -r_{oi}; \quad t_{oi} t_{io} - r_{oi} r_{io} = 1. \quad (17)$$

An alternative approach is to calculate the transmission through the multi-layer using transfer matrices. Each matrix, in an ordered sequence of matrix multiplications, represents one of two possible transformations of the radiation fields. The transformations are due to:

- Transmission across an interface (interface matrix),
- Transmission through a layer (layer matrix).

The **interface matrix**: The definition of the interface (M) follows from consideration of the boundary conditions at an interface. Let E_o^+ and E_i^+ represent the amplitudes of the net electric field propagation towards, and away from, an interface in the direction from the outside to the inside medium (Fig. 3). E_o^- and E_i^- are the corresponding net electric fields traveling in the reverse direction. E_i^+ is then composed of the transmitted component of E_o^+ and the reflected component of E_i^- , E_o^- consists of the transmitted component of E_i^- and the reflected component of E_o^+ . Equations (16) and (17) enable this to be expressed mathematically as:

$$E_i^+ = t_{oi} E_o^+ + r_{io} E_i^- \quad (18)$$

$$E_o^- = t_{io} E_i^- + r_{oi} E_o^+$$

or

$$\begin{bmatrix} E_o^+ \\ E_o^- \end{bmatrix} = \begin{bmatrix} 1 & r_{oi} \\ t_{oi} & t_{oi} \\ r_{oi} & 1 \\ t_{oi} & t_{oi} \end{bmatrix} \cdot \begin{bmatrix} E_i^+ \\ E_i^- \end{bmatrix} = M_{oi}^{(E)} \cdot \begin{bmatrix} E_i^+ \\ E_i^- \end{bmatrix}. \quad (19)$$

Fig. 3: The electric fields at an interface

When transmission across an interface is considered in terms of intensities, the following matrix equation is obtained:

$$\begin{bmatrix} I_o^+ \\ I_o^- \end{bmatrix} = \begin{bmatrix} 1 & \Re_{oi} \\ \tau_{oi} & \tau_{oi} \\ \Re_{oi} & 1 \\ \tau_{oi} & \tau_{oi} \end{bmatrix} \cdot \begin{bmatrix} I_i^+ \\ I_i^- \end{bmatrix} = M_{oi}^{(I)} \cdot \begin{bmatrix} I_i^+ \\ I_i^- \end{bmatrix}. \quad (20)$$

The **layer matrix**: The layer matrix (N) describes the attenuation of a wave as it traverses the medium within a layer. When a wave propagates in the positive direction between the interfaces of a thin layer, its initial amplitude $E_f^+(1)$ within the layer is attenuated to, say, $E_f^+(2)$ on reaching the second interface (Fig. 4). A wave of initial amplitude $E_f^-(2)$ traveling in the opposite direction through the layer is reduced to $E_f^-(1)$. If the layer has thickness d then from the definition of an attenuation factor,

$$E_f^+(2) = \Delta_f(d) E_f^+(1) \quad (21)$$

$$E_f^-(2) = \Delta_f(-d) E_f^-(1)$$

or

$$\begin{bmatrix} E_f^+(1) \\ E_f^-(1) \end{bmatrix} = \begin{bmatrix} 1/\Delta_f(d) & 0 \\ 0 & \Delta_f(d) \end{bmatrix} \cdot \begin{bmatrix} E_f^+(2) \\ E_f^-(2) \end{bmatrix} = N_f^{(E)} \cdot \begin{bmatrix} E_f^+(2) \\ E_f^-(2) \end{bmatrix}. \quad (22)$$

Matrix multiplication: The following equation represents the transformation of the electric fields by a thin layer

Fig. 4: Transmission through a thin absorbing layer

(f) sandwiched between a pair of outer (o) and inner (i) media:

$$\begin{bmatrix} E_o^+ \\ E_o^- \end{bmatrix} = M_{of}^{(E)} \cdot N_f^{(E)} \cdot M_{fi}^{(E)} \cdot \begin{bmatrix} E_i^+ \\ E_i^- \end{bmatrix}. \quad (23)$$

This involves the multiplication of three matrices, the first and third being interface materials, the second the layer matrix. A system is composed of a greater number of thin layers, say layers 1, 2, 3, ..., q sandwiched in the transformation.

$$\begin{bmatrix} E_o^+ \\ E_o^- \end{bmatrix} = M_{o1}^{(E)} \cdot N_1^{(E)} \cdot M_{12}^{(E)} \cdot N_2^{(E)} \cdot \dots \\ \dots \cdot N_{q-1}^{(E)} \cdot M_{q-1,q}^{(E)} \cdot N_q^{(E)} \cdot M_{qi}^{(E)} \begin{bmatrix} E_i^+ \\ E_i^- \end{bmatrix}. \quad (24)$$

Equation (24) can be used to obtain the total amplitude reflection and transmission coefficients, \mathfrak{R}_{oi} and τ_{oi} of the multiple thin layer structure. This follows by considering the surrounding media to be of infinite thickness, so that there is no net reflected wave in medium (i) incident upon the inner interface (i.e., $E_i^- = 0$). By setting $E_o^+ = 1$ Equation (24) becomes:

$$\begin{bmatrix} 1 \\ \mathfrak{R}_{oi} \end{bmatrix} = M_{o1}^{(E)} \cdot N_1^{(E)} \cdot M_{12}^{(E)} \cdot N_2^{(E)} \cdot \dots \\ \dots \cdot N_{q-1}^{(E)} \cdot M_{q-1,q}^{(E)} \cdot N_q^{(E)} \cdot M_{qi}^{(E)} \begin{bmatrix} \tau_{oi} \\ 0 \end{bmatrix} \quad (25)$$

which can be solved for \mathfrak{R}_{oi} and τ_{oi} .

3.2.2 Absorbed solar radiation

The prediction of collector performance requires information on the solar energy absorbed by the collector absorber plate. The solar energy incident on a tilted collector can be found by the methods of Section 3.1. This incident radiation has three different spatial distributions: beam radiation, diffuse radiation, and ground reflected radiation, and each must be treated separately. Using Klucher's formulation, Equation (7) can be modified to give the absorbed radiation, S , by multiplying each term by the appropriate transmittance-absorptance product:

$$S = (\tau\alpha)_b(G_H - I_{d,H})R_b + (\tau\alpha)_d \cdot I_{d,H} \\ \cdot \left[\frac{1 + \cos\beta}{2} \left(1 + F_1 \sin^3 \frac{\beta}{2} \right) \left(1 + F_1 \cos^2\theta \sin^3\theta_Z \right) \right] + \\ + \frac{1}{2}(\tau\alpha)_g G_{HP}(1 - \cos\beta). \quad (26)$$

The subscripts b , d , and g represent beam, diffuse, and ground, respectively. Equation (27) can be used to find the proper absorptance:

$$\alpha = 1 - \tau_a \\ \tau_a = \frac{I_{\text{transmitted}}}{I_{\text{incident}}} = \exp\left(-\frac{K \cdot L}{\cos\theta_o}\right) \quad (27)$$

where K is the proportionality constant, the extinction coefficient, which is assumed to be a constant in the solar spectrum. For glass, the value of K varies from approximately 4 m^{-1} for "water white" glass (which appears white when viewed on the edge) to approximately 32 m^{-1} for poor (greenish cast of edge) glass. L is the path length in the medium.

3.2.3 Efficiency limits of photovoltaic modules

The analytical results above were applied to analyze the optical behavior of an encapsulated photovoltaic solar cell. Photovoltaic modules have an angle of incidence AOI dependent optical behavior that can be measured and used to improve the analysis of array performance. Like absolute air mass, solar angle of incidence is time of day dependent. Its effect on the short circuit current of a photovoltaic module results from two causes.

The first is familiar to solar enthusiasts as the "cosine effect". The "cosine effect" is independent of the module design, and is only geometry related. Due to the cosine effect, the short circuit current from a module varies directly with the cosine of the AOI. For example, at AOI = 60° the cosine effect reduces the short circuit current by one half compared to the normal incidence condition.

The second way in which the short circuit current is affected by the AOI is dependent on the module design. The optical characteristics of the module materials located between the sun and the solar cells cause the effect. Above 45° incidence angle the reflectivity starts to increase, and reaches 20% losses at 75° (see Fig. 5). The influence of a module's optical characteristics on its performance is shown in Fig. 6.

Fig. 5: Calculated relative transmission, reflection, and absorption of radiation for glass cover

4.1 Problem formulation

The solution starts by assuming that the photovoltaic solar module is composed of three layers. For the purposes of the 3D-thermal model the solar module is divided into a number of meshes, each mesh having six faces, (XY = Top, YX = Bottom, YZ = Front, ZY = Back, XZ = Right and ZX = Left), i, j, k are the coordinates of the meshes. The computer code was written in Fortran. Simulation outputs allow a complete evaluation of the temperature distribution inside the photovoltaic module and can be of great interest both for system designers and for researchers. The technique used in this code involves calculating the temperature by an explicit method using iteration processes. The study is confined to a single silicon photovoltaic module of the following dimensions: thickness of silicon 0.5 mm, sandwiched between a glass layer which is taken as 3 mm and Mylar 0.5 mm, giving a total photovoltaic module thickness of 4 mm. Both length and width are taken as 40 mm. Also, it is assumed that all the heat is absorbed inside the silicon wafer homogeneously. The results are shown in Figs. 7 and 8, which show the temperature distribution along the XY-Plane at the mid Z-axis of the silicon wafer; and the temperature distribution along the XZ-Plane at mid Y-axis.

Fig. 6: Influence of angle of incidence on cell parameters

4 Thermal performance of a photovoltaic module

Solar radiation irradiating at the surface of the solar module is absorbed partially by various materials of the module. Almost all radiation is absorbed in the silicon. Only a very small part is absorbed in other materials of the solar module. This will produce heat inside the solar cell. There are heat loss mechanisms from the front and rear surface of the module: free convection, wind convection and radiative heat loss. Inside cell heat is transmitted by conduction. The equations describing the heat flows within the cell are given below:

Radiation heat transfer equation: The total radiative heat flow from the module, Q_{rad} [W] to the environment comprises the components of the radiation exchange of the front and rear sides of the module with the sky and the ground. These components are also functions of the module surface temperature, $T_{(i,j,k)}$, ambient air temperature, T_{∞} and the relevant emissivity ε also; these components are determined by the so-called radiation shape factor, F_s the radiative surface area of the module, A_m and the Stefan-Boltzmann constant δ . In general this can be expressed as [11]:

$$Q_{\text{rad}} = F_s \varepsilon \delta A_m \left(T_{(i,j,k)}^4 - T_{\infty}^4 \right). \quad (28)$$

Convection heat transfer equation: Convective heat flows, Q_{conv} cannot be treated in a closed mathematical model and, therefore have to be computed by iteration or approximation. The convective heat transfer coefficient h_{∞} [W/m²K] is a bulky function of the actual air temperature and module surface temperature, air viscosity, thermal conductivity and heat capacity, and finally wind speed. The models used are based on the following equation:

$$Q_{\text{conv}} = h_{\infty} A_c \left(T_{(i,j,k)} - T_{\infty} \right). \quad (29)$$

Conduction heat transfer equation: can be given as:

$$Q_{\text{cond}} = \frac{K_{\text{th}} A_m}{h} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right). \quad (30)$$

Fig. 7: Temperature distribution along the XY plane at mid Z-axis

Fig. 8: Temperature distribution along the XZ plane at mid Y-axis

For the meshes at the corners of the cellFor $i = 1, j = 1, k = 1$

$$\begin{aligned}
Q_{(i,j,k)} = & F_s \delta \varepsilon_{xz} A_{xz} \left(T_{(i,j,k)}^4 - T_{\infty z}^4 \right) + F_s \delta \varepsilon_{xy} A_{xy} \left(T_{(i,j,k)}^4 - T_{\infty y}^4 \right) + F_s \delta \varepsilon_{yz} A_{yz} \left(T_{(i,j,k)}^4 - T_{\infty z}^4 \right) + h_{\infty z} A_{xz} \left(T_{(i,j,k)} - T_{\infty z} \right) + \\
& + h_{\infty y} A_{xy} \left(T_{(i,j,k)} - T_{\infty y} \right) + h_{\infty z} A_{yz} \left(T_{(i,j,k)} - T_{\infty z} \right) + K_{thm} \frac{A_{zx}}{h_y} \left(T_{(i,j,k)} - T_{(i,j+1,k)} \right) + \\
& + K_{thm} \frac{A_{yx}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k+1)} \right) + K_{thm} \frac{A_{zy}}{h_x} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right)
\end{aligned} \quad (31)$$

For the meshes at the edges of the cellFor $i = 1, k = 1, j = 2, \dots, (n_y - 1)$

$$\begin{aligned}
Q_{(i,j,k)} = & F_s \delta \varepsilon_{yz} A_{yz} \left(T_{(i,j,k)}^4 - T_{\infty z}^4 \right) + h_{\infty z} A_{yz} \left(T_{(i,j,k)} - T_{\infty z} \right) + F_s \delta \varepsilon_{xy} A_{xy} \left(T_{(i,j,k)}^4 - T_{\infty y}^4 \right) + h_{\infty y} A_{xy} \left(T_{(i,j,k)} - T_{\infty y} \right) + \\
& + K_{thm} \frac{A_{yx}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k+1)} \right) + K_{thm} \frac{A_{zy}}{h_x} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right) + K_{thm} \frac{A_{xz}}{h_y} \left(T_{(i,j,k)} - T_{(i,j+1,k)} \right) + \\
& + K_{thm} \frac{A_{zx}}{h_y} \left(T_{(i,j,k)} - T_{(i,j-1,k)} \right)
\end{aligned} \quad (32)$$

For the meshes inside the cell bodyFor $i = 2, \dots, (n_x - 1), j = 2, \dots, (n_y - 1), k = 2, \dots, (n_z - 1)$

$$\begin{aligned}
Q_{(i,j,k)} = & K_{thm} \frac{A_{xy}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k-1)} \right) + K_{thm} \frac{A_{yx}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k+1)} \right) + K_{thm} \frac{A_{zy}}{h_x} \left(T_{(i,j,k)} - T_{(i-1,j,k)} \right) + \\
& + K_{thm} \frac{A_{yz}}{h_x} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right) + K_{thm} \frac{A_{xz}}{h_y} \left(T_{(i,j,k)} - T_{(i,j-1,k)} \right) + K_{thm} \frac{A_{zx}}{h_y} \left(T_{(i,j,k)} - T_{(i,j+1,k)} \right)
\end{aligned} \quad (33)$$

For the meshes at the surfaceFor $i = 2, \dots, (n_x - 1), j = 2, \dots, (n_y - 1), k = 1$

$$\begin{aligned}
Q_{(i,j,k)} = & F_s \delta \varepsilon_{xy} A_{xy} \left(T_{(i,j,k)}^4 - T_{\infty y}^4 \right) + h_{\infty y} A_{xy} \left(T_{(i,j,k)} - T_{\infty y} \right) + K_{thm} \frac{A_{yx}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k+1)} \right) + \\
& + K_{thm} \frac{A_{yz}}{h_x} \left(T_{(i,j,k)} - T_{(i-1,j,k)} \right) + K_{thm} \frac{A_{zy}}{h_x} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right) + K_{thm} \frac{A_{zx}}{h_y} \left(T_{(i,j,k)} - T_{(i,j-1,k)} \right) + \\
& + K_{thm} \frac{A_{xz}}{h_y} \left(T_{(i,j,k)} - T_{(i,j+1,k)} \right)
\end{aligned} \quad (34)$$

For the meshes at the gap

$$\begin{aligned}
Q_{(i,j,k)} = & K_{thm} \frac{A_{yz}}{h_x} \left(T_{(i,j,k)} - T_{(i-1,j,k)} \right) + K_{thm} \frac{A_{zy}}{h_x} \left(T_{(i,j,k)} - T_{(i+1,j,k)} \right) + \\
& + K_{thm} \frac{A_{xz}}{h_y} \left(T_{(i,j,k)} - T_{(i,j+1,k)} \right) + K_{thm} \frac{A_{zx}}{h_y} \left(T_{(i,j,k)} - T_{(i,j-1,k)} \right) + \\
& + K_{thm} \frac{A_{xy}}{h_z} \left(T_{(i,j,k)} - T_{(i,j,k-1)} \right) + h_{gap} A_{yx} \left(T_{(i,j,k)} - T_{(i,j,k+1)} \right)
\end{aligned} \quad (35)$$

Where:

h_x = width of mesh in x, h_y = width of mesh in y, h_z = width of mesh in z, $A_{xz} = h_x h_z$, $A_{xy} = h_x h_y$, $A_{yz} = h_y h_z$, m = material number, n_x = total No. of meshes in x-direction, n_y = total No. of meshes in y-direction and n_z = total No. of meshes in z-direction.

4.2 Temperature dependence of solar module parameters

Fig. 9 shows the dependence of I_{sc} , V_{oc} , FF , and P_m on temperature. As the cell temperature increases, the short circuit current, I_{sc} somewhat increases, and the maximum power, P_m the open circuit voltage, V_{oc} and the fill factor, FF decrease. The short circuit current increase with temperature is attributed to light absorption variation. Under the conditions considered, the solar absorption edges shift due to a decrease

in the crystal forbidden gap width, and the number of pairs generated in the bulk increases. Alternatively, as the temperature increases, the intrinsic light absorption factor changes and the short circuit current increases. To explain the variation of V_{oc} with temperature, the effects of all voltage controlling parameters (mobility, lifetime, energy band gap and absorption coefficient) have to be included in the analysis. In the general case, open circuit voltage is determined by the relation $V_{oc} \approx \ln(I_{sc}/I_d)$, by which V_{oc} must grow as I_{sc} increases. Depending on the current flowing mechanism, the

Fig. 9: Variation of key parameters of a silicon solar cell at high temperatures

value of I_d is proportional to n_i^2 . As the temperature increases n_i grows exponentially and causes exponential growth of I_d . Moreover, the crystal forbidden gap narrowing with temperature increase leads to an increase in the dark current, I_d and minimizes the positive effect of the absorption factor. As a result, V_{oc} decreases. The maximum power across the optimal load and the efficiency depend strongly on temperature; the shape of this dependence is close to linear.

Conclusions

A simple and accurate close-form solution for the optical and thermal behaviors of a solar cell array were investigated by simulation and by some verifying experiments, in order to obtain more accurate results in predicting the actual performance of photovoltaic modules, without resorting to a lengthy, time-consuming iterative solution which has to be repeated for any change in the parameters of the array.

Nomenclature

G_H	Hourly global radiation incident on a horizontal surface, kWh/m ² /day
G_T	Hourly global radiation incident on an inclined surface, kWh/m ² /day
$I_{b,H}$	Hourly beam radiation incident on an inclined surface, kWh/m ² /day
$I_{b,T}$	Hourly beam radiation incident on a horizontal surface, kWh/m ² /day
$I_{d,H}$	Hourly sky diffuse radiation incident on a horizontal surface, kWh/m ² /day
$I_{d,T}$	Hourly sky diffuse radiation incident on an inclined surface, kWh/m ² /day
I_o	Extraterrestrial hourly radiation incident on a horizontal surface, kWh/m ² /day
$I_{g,H}$	Hourly ground reflected radiation incident on a horizontal surface
$I_{g,T}$	Hourly ground radiation incident on an inclined surface, kWh/m ² /day
$I_{T,cir}$	Circumsolar radiation component
$I_{T,iso}$	Isotropic radiation component

n	Refractive index
A_1	Anisotropic index
R_b	The ratio of beam radiation on the tilted surface to that on a horizontal surface
r	Reflection of incident
β	Surface slope, degrees
δ	Declination, degrees
ϕ	Latitude, degrees
ρ	Ground albedo
θ_1	Incidence angle, the angle between beam radiation and the vertical, degrees
θ_o	Angles of refraction
ω	Sunrise hour angle, degrees
γ	Surface azimuth angle, degrees
E_o^-, E_i^+	Amplitudes of the reflected and transmitted waves
\mathfrak{R}	Reflectance
Q_{rad}	Total radiative heat flow from the module, W
Q_{conv}	Convective heat flow, W
Q_{cond}	Quantity of heat transfer by conduction from one mesh to another, W
K_{th}	Thermal conductivity of different materials of solar cell, W/mK
$T_{(i,j,k)}$	Module surface temperature, °C
T_∞	Ambient temperature, °C
ε	Emissivity
F_s	Radiation shape factor
A_m	Radiative surface area of the module
A_c	The surface area of the cell, m ²

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