

Post-breakage Tensile and Bending Response of Laminated Glass

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Laminated glass, composed by glass plies sandwiching polymeric interlayers, can provide a safe post-glass breakage response, in compliance with the *fail-safe* approach used in the structural design. In fact, when glass breaks, shards remain attached to the polymer, preventing danger from falling materials and imparting a "tension stiffening" effect to the interlayer, so that the broken panel maintains a certain residual load-bearing capacity. Here, a homogenized approach is presented to describe the mechanical properties of broken heat-treated laminated glass under tensile stresses. The model accounts for the stress diffusion from the delaminated zones, where shards are bridged by the interlayer-ligament only, to the regions where glass is bonded to the interlayer. The model provides a simple but accurate estimate of the effective tensile properties of the cracked laminate. Here, the influence of the interlayer thickness, the size of the glass shards and the glass-polymer delamination on the post-critical response is accurately investigated, and analytical results are compared with numerical ones. The obtained expression for the tensile modulus is used to predict, in more general terms, the response of cracked laminated glass under in-plane and out-of-plane bending. In both cases, a key point is the correct evaluation of the tension stiffening in the polymeric interlayer due to the adhesion with the glass shards.

Keywords: Laminated glass, Post-breakage response, Delamination, Tension stiffening, Effective bending stiffness.

1. Introduction

In the structural design of glass, it is mandatory to use the *fail-safe* approach, typical in aircraft design, according to which the failure of one or more components in extreme situations shall not compromise the overall stability. In particular, it is necessary to verify that, in case of partial or total fragmentation of glass, sufficient stiffness and strength are maintained so to withstand at least the permanent loads as well as a fraction of the live loads. Laminated glass, composed by glass plies sandwiching polymeric interlayer sheets, can provide, when properly designed, a safe post-glass breakage response. Indeed, when glass fails, the interlayer prevents sharp pieces from spreading and the assembly maintains a certain "consistency" that prevents detachment from fixings. The flexibility of the interlayer and its capability of adhesion with the shards, make laminated glass an effective material for very special purposes, i.e., to mitigate the effects of blast loading on buildings or to construct structural diaphragms able to counterbalance the earthquake-induced accelerations.

One of the most relevant aspects that influence the post-breakage response is the partial delamination between glass and polymer, which depends upon the glass-interlayer adhesion properties and allows for the polymer stretching (Delincé *et al.* 2008). Other important factors are the interlayer stiffness, ruled by polymer type, temperature and characteristic duration of the action, and the size and shape of the glass fragments. The latter depends upon the thermal treatments made to increase the bending strength of glass. In thermally-toughened glass, the breakage pattern is characterized by small blunt particles, whereas in heat-strengthened glass, for which the cooling is more gradual, the shards are larger. In both cases, fractures develop almost instantaneously in the whole element because of the sudden conversion of elastic strain energy into fracture energy. Remarkably, the adherence with the glass fragments produces the *tension stiffening* of the polymeric film, which otherwise would present negligible mechanical performance.

Many theoretical, numerical and experimental studies have regarded the pre-glass breakage response of laminated glass, but the modelling of the post-breakage response has received limited attention. Experimental activities have been conducted on broken laminated glass under in-plane and out-of-plane bending. The results are usually interpreted with numerical simulations that model the single glass shard, but analyses of this type are usually computationally difficult due to the high number of glass fragments. A simple analytical approach for the *tensile response* of broken laminated glass, innovative to our knowledge, has been very recently presented in (Galuppi and Royer Carfagni 2016). Here, this approach is revisited so to investigate in detail the influence of various aspects, such as the thermal treatment performed on glass (determining the size of glass fragments), the glass-interlayer delamination length, and the interlayer thickness.

The theoretical results are used to predict, in more general terms, the response of cracked laminated glass under *in-plane* and *out-of-plane bending*. In particular, as discussed in (Galuppi and Royer Carfagni 2018) the response under in-plane bending may be modelled as the bending of a bimodulus element, with the Young's modulus of glass in

compression, and the *effective* modulus of the interlayer in tension. The out-of-plane bending presents somehow an analogy with the mechanics of reinforced concrete: the bending moment is equilibrated by the compression stress, due to contact of the glass shards, and the tensile stress in the polymeric interlayer, which plays the role of reinforcement, tension-stiffened by the adhesion with the glass shards. Starting from the theoretical approach, we propose compact formulas to calculate the *effective bending stiffness* of laminated glass element in the post-breakage phase. In the aforementioned analysis, a key point is the correct evaluation of the tension stiffening in the interlayer due to the adhesion with the glass shards; otherwise, the polymer alone would be too compliant to provide any noteworthy contribution for the stiffness of the assembly.

2. Tensile response of broken laminated glass

In the post-glass-breakage phase, thermally-toughened and heat-strengthened laminated glass are characterized by a mosaic texture of the shards, approximatively uniform, with fragments of comparable size, depending upon the type of thermal treatment. Hence, with the aim of defining an effective stiffness of the damaged element, it can be regarded as a composite formed by the interlayer to which randomly distributed glass shards remain adherent, with homogeneously distributed fragmentation. Partial delamination usually occurs at the glass-polymer interface. Since the Young's modulus of glass is 3 or 4 orders of magnitude higher than that of the interlayer, the glass fragments can be considered rigid with respect to the interlayer, and therefore subjected to null strain. Hence, one can consider the *isolated interlayer* for which a uniform tensile stress, assumed as the reference state, is perturbed by the contact with the rigid fragments.

2.1. The model problem

The considered reference geometry is a broken laminated glass element composed by two equally-thick glass plies, bonded by a polymeric interlayer of thickness t and Young's modulus E_p , subjected to uniaxial tensile loading in x direction, as shown in Fig. 1a. The glass plies are damaged by symmetric cracks approximatively parallel to the (y, z) and (x, z) planes. This is a simplified view of the real problem, because the fragment shapes are in general random and the cracks are unlike to be symmetric on the two plies. However, several studies (see, for example, (Biolzi *et al.* 2016)) have confirmed that the stiffness recorded in the symmetric-crack case should be considered a *lower bound* value for the effective stiffness of a cracked element.

Clearly, the cracks parallel to the direction of loading (i.e., parallel to the (x, z) plane in Fig. 1a) are subject to null or negative opening stress (due to the lateral contraction of the inner layer), so that they do not influence the tensile response. Therefore, one can consider only the effects of the cracks orthogonal to the tensile stress and the problem may be reduced to a plane problem in the (x, z) reference system, i.e., either plane stress or plane strain conditions.

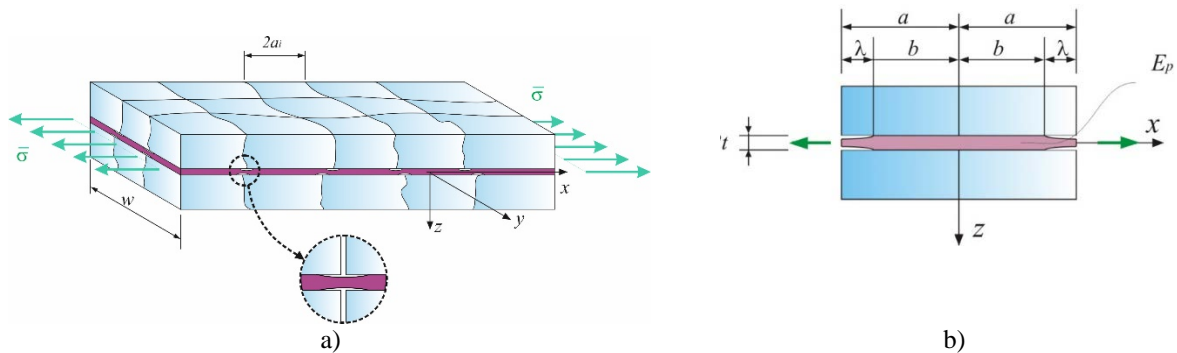


Fig. 1 a) Schematic view of a cracked laminated glass element under uniaxial tension and b) elementary portion comprised between two crack planes.

As shown in Fig. 1b, the representative element is the region comprised between two consecutive cracks, of length $2a$. The same figure also evidences that each glass fragment is assumed to be symmetrically detached from the interlayer at its ends for a length λ and, hence, remains bonded in the central zone of length $2b=2(a-\lambda)$.

2.2. Stress state in the interlayer

Since the strain in the glass plies is supposed to be negligible, the tensile response is governed by the deformation of the interlayer only. The reference state for the interlayer is the one in which it is uniformly stressed along x under the uniaxial tensile stress $\bar{\sigma}$. The actual stress can be written as the sum of the reference state and of a perturbation stress field due to the tension stiffening of the shards. Under this assumption, a simple equilibrated elastic solution, determined by means of energy minimization methods, is proposed in (Galuppi and Royer Carfagni 2016). In such a solution, the stress is constant in-the-thickness of the interlayer, but can vary along x .

The obtained generalized plane stress distribution is characterized by a *stress diffusion phenomenon*, from the external detached zones towards the bonded region. In particular, the mean axial stress in the interlayer turns out to be constant in the external detached zones, where the presence of the glass fragments does not lead to stress perturbations, and decreases starting from the boundary and going to the center of the bonded region, with a slope that depends upon the interlayer thickness and its Poisson's ratio. This result, which is due to the fact that the stress is transferred to the glass shards through the bonding, has been discussed in detail in (Galuppi and Royer Carfagni 2016). The stress diffusion phenomenon has been confirmed by FEM analyses, as qualitatively shown by Fig. 2.

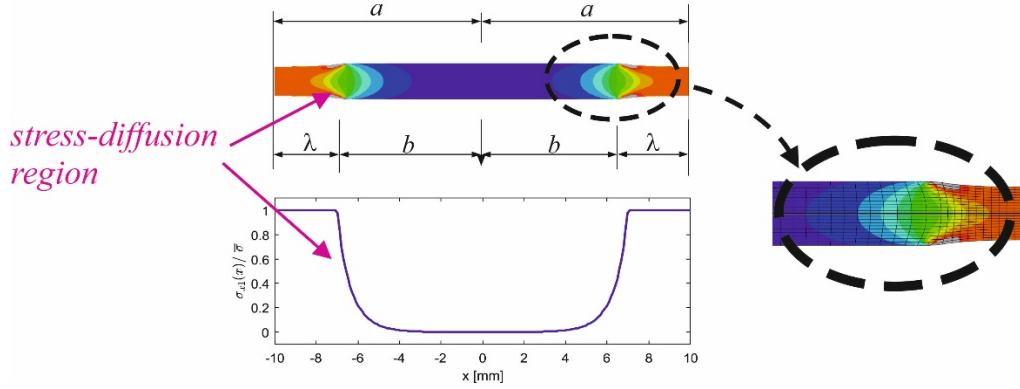


Fig. 2 Qualitative comparison between the numerically evaluated axial stress (with evidence of the stress diffusion inside the bonded zone) and the analytically evaluated mean axial stress.

As it is evident in the magnified detail in Fig. 2, the axial stress is not constant in the thickness of the interlayer, and it tends to flow also through the internal fibers of the interlayer. However, the approximate solution here proposed *accurately captures the average stress at each cross section* of the interlayer, as demonstrated by comparison with numerical results recorded in (Galuppi and Royer Carfagni 2016).

2.3. Effective tensile stiffness

Starting from the so obtained stress field, energy theorems of elasticity theory are used in (Galuppi and Royer Carfagni 2016) to find a lower bound on the *effective stiffness* of the cracked laminate under tension, defined as the stiffness of a homogeneous body presenting the same tensile properties in terms of elongation. A *lower bound* for the effective Young's modulus E_{eq} of the cracked laminated glass element may be written in the form

$$E_{eq} \geq E_p \frac{a}{\lambda} \chi(a, \lambda, t), \quad (1)$$

where E_p represents the modulus of the interlayer only, and $\chi(a, \lambda, t)$ is a non-dimensional quantity, depending on the detachment length, the glass fragment size and the interlayer thickness. This quantity takes different expressions for plane stress and plane stress conditions. Accurate charts for the determination of such a coefficient are proposed in (Galuppi and Royer Carfagni 2018). In the case of laminated glass element with different characteristic fragment sizes in different regions (e.g., depending upon the distance from the crack origin), the effective moduli of the different regions may be evaluated via eq. (1), with different values of a . In (Galuppi and Royer Carfagni 2017), this approach has been extended to the study of the response of broken laminated glass plates under equi-biaxial state of stress.

Remarkably, to predict the gross load-displacement response of the cracked laminated glass elements, the progressive detachment of the interlayer from the glass shards, and the consequent redistribution of the axial stress, should be considered. Furthermore, the elastic modulus of the viscoelastic polymeric interlayer is strongly affected by the load duration and the temperature. Consequently, the response of the cracked laminated glass beam, and consequently its effective tensile modulus E_{eq} , is highly time-dependent.

It should be mentioned that, to our knowledge, the only other formulation available in the literature (Bennison and Stelzer, 2009) defines the equivalent Young's modulus of cracked laminated glass in the form

$$E_{eq} = E_p \frac{a}{\lambda}. \quad (2)$$

The underlying hypothesis of this approach is that the stress and strain state are uniaxial, and that only the detached zones of the inner layer contribute to the elongation of the element, while the contact with the rigid fragments prevents the straining in the bonded zones. In other words, it completely neglects the stress diffusion phenomenon.

3. Parametric analyses and comparison with numerical results

The simple approach of (2) neglects the influence on the effective stiffness of the glass shards and of the interlayer thickness, and provides values of E_{eq} correlated only with the percentage of detached zone $\xi = \lambda/a$. In the sequel, the influence of these factors will be evaluated in detail, and the results obtained with the expression (1) for the proposed model will be compared with those from numerical analyses performed with the code Abaqus, with reference to a cracked composite element like that of Fig. 1b. In the numerical experiments, the equivalent axial modulus is evaluated as the ratio between the applied axial stress and the average strain. The case of *plane strain* condition, representative of the cracked laminated panel shown in Fig. 1, where displacement in the z direction are prevented by the presence of the adjacent glass fragments, will be considered.

3.1. Influence of the glass fragment length

Let us consider, first, the influence of the glass fragment length on the residual stiffness of laminated glass after breakage. Fig. 3 shows the comparison of the results in terms of effective elastic modulus E_{eq} , normalized by the interlayer modulus E_p . The values obtained with the proposed model, *via* eq. (1), are juxtaposed with those from numerical simulations as a function of $\xi = \lambda/a$, for different values of the fragment size (ranging from 10 mm to 200 mm) and interlayer thickness $t = 0.76$ mm. Observe that by adopting the elementary model of eq. (2), one obtains a straight line, also indicated in the graphs, because E_{eq} would be independent of the glass fragment length, but only related with the percentage ξ of detachment. Remarkably, the elementary approach is not on the safe side. On the other hand, our proposed model provides quite an accurate lower bound for the effective Young's modulus: for standard geometric parameters, the mean error is lower than 4%.

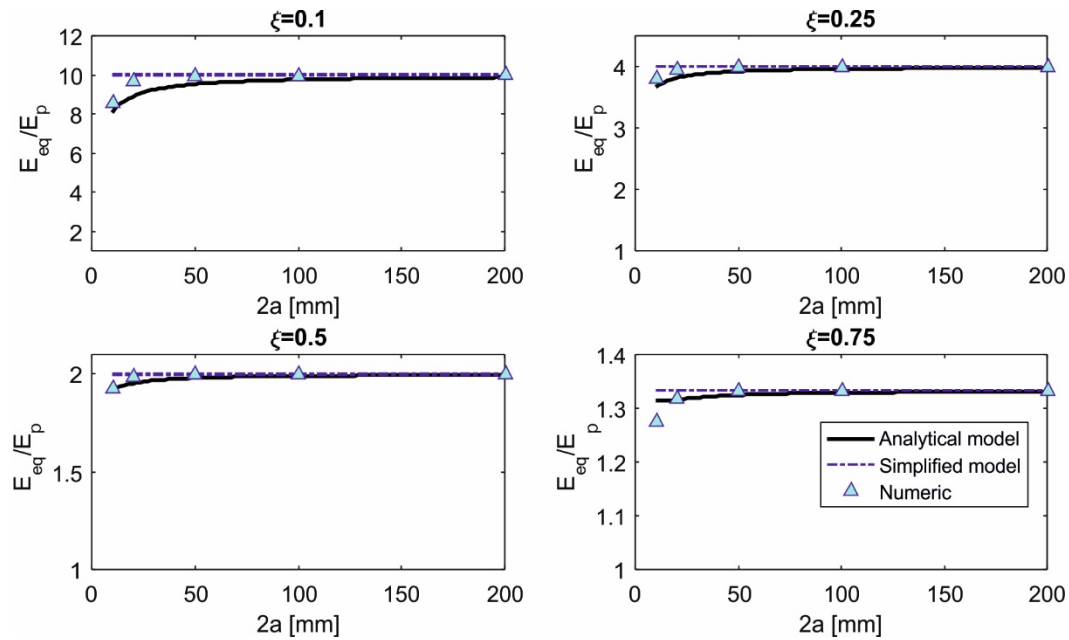


Fig. 3 Ratio E_{eq} / E_p as a function of the fragment length $2a$, for interlayer thickness 0.76 mm and for different values of $\xi = \lambda/a$. Comparison between analytical and numerical results.

Observe that the effective stiffness of cracked laminated glass is considerably higher than that of the isolated interlayer. This is due to the stiffening effect of the external rigid fragments, which provokes the stress diffusion phenomenon described in the previous Section. For a given value of ξ , the effective modulus is lower for low values of a , because the stress diffusion phenomena become more relevant for small sizes of the glass-interlayer bonded region. For high values of the glass fragment length and for low values of ξ (i.e., for long bonded region), the effect of the stress diffusion phenomenon becomes negligible and the effective Young's modulus tends to be that of eq. (2). Furthermore, it may be observed that the analytic prediction for the lower bound of E_{eq} results to be higher than the numerical calculation for high values of ξ and low values of a . This is because, when the bond length is very small, the two stress-diffusion zones tend to merge, so that part of the axial load directly flows through the interlayer, with no need of being transferred to the adherent glass fragments. The proposed analytical model, based upon the constant-in-the-thickness axial stress approximation, cannot correctly capture this effect, but apart from this, it seems to be very reliable.

Fig. 4 is the analogue of Fig. 3, for higher interlayer thickness (1.52 mm). It may be observed that, even if the qualitative behavior is very similar to that recorded previously, the effective stiffness, for given values of a and ξ , is lower in the latter case.

Post-breakage Tensile and Bending Response of Laminated Glass

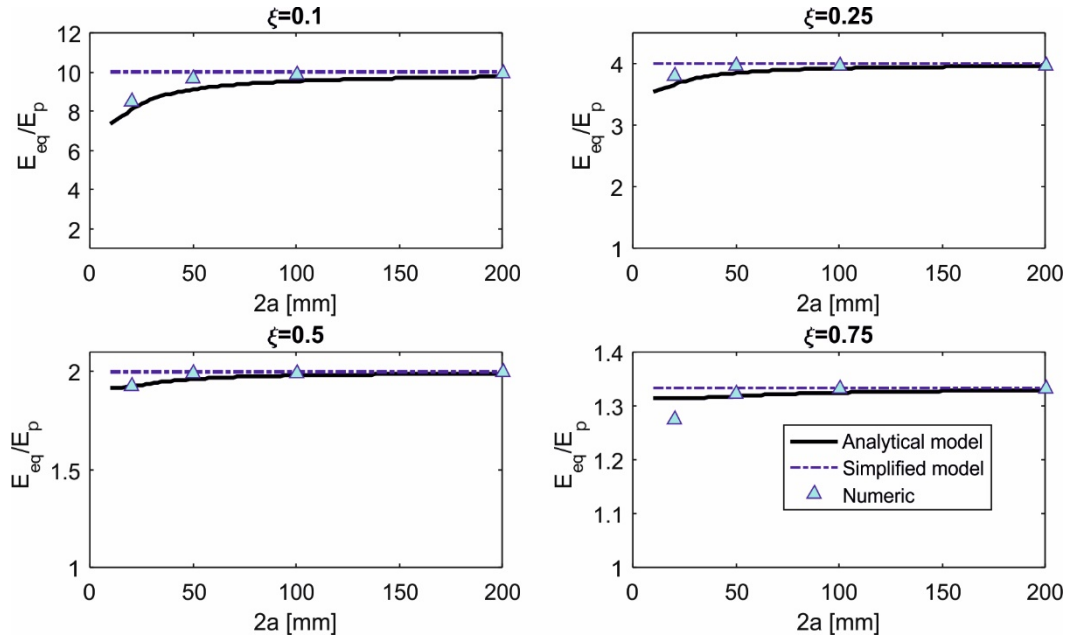


Fig. 4 Ratio E_{eq}/E_p , as a function of the fragment length $2a$, for interlayer thickness of 1.52 mm, for different values of $\xi = \lambda/a$. Comparison between analytical and numerical results.

This is because, for high values of interlayer thickness, the stress-diffusion phenomenon becomes more relevant since the stress tends to flow through the central fibers. Consequently, the dependence of the effective modulus on the glass fragment length is more marked. This fact will be discussed more in detail in Sect. 3.3.

3.2. Influence of the delamination length

Fig. 5 and Fig. 6 show the effective elastic modulus E_{eq} normalized by the interlayer elastic modulus E_p as a function of the percentage of detached interlayer, for different values of the fragment length and for interlayer thickness of 0.76 mm and 1.52 mm, respectively.

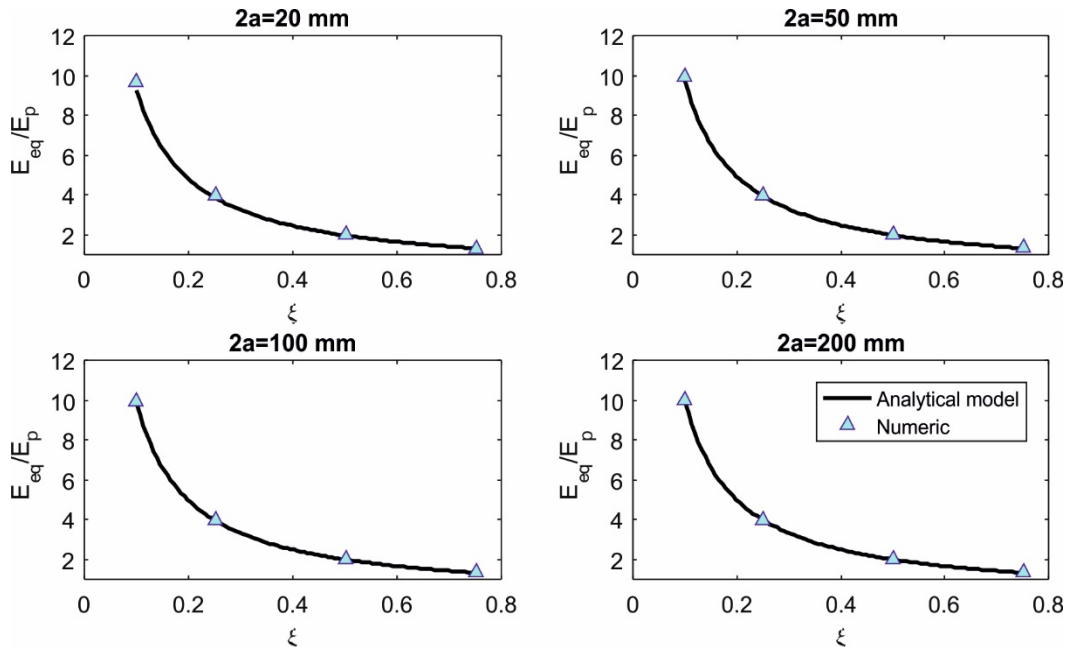


Fig. 5 Ratio E_{eq}/E_p , as a function of the percentage of detached interlayer $\xi = \lambda/a$, for different values of the fragment length $2a$ and for interlayer thickness of 0.76 mm. Comparison between analytical and numerical results.

These graphs confirm the strong influence of the local delamination on the post-critical response of laminated glass. For high values of ξ (i.e., for almost complete debonding), part of the axial load directly flows through the interlayer and, consequently, the effective stiffness tends to that of the interlayer only ($E_{eq}/E_p \rightarrow 1$). Moreover, notice that the influence of the detached zone length is higher when glass is fragmented into small pieces, i.e., for low values of a .

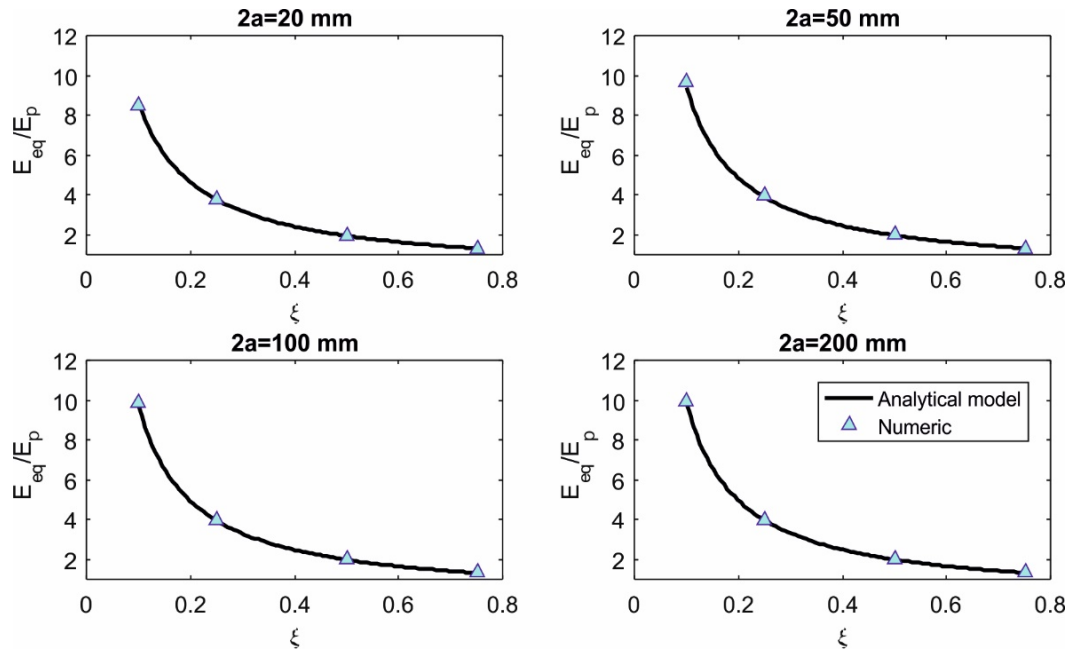


Fig. 6 Ratio E_{eq}/E_p , as a function of the percentage of detached interlayer $\xi = \lambda/a$, for different values of the fragment length $2a$ and for interlayer thickness of 1.52 mm. Comparison between analytical and numerical results.

By comparing Fig. 5 and Fig. 6, it is confirmed that, for given value of fragment size and debonding length, the stress diffusion phenomena are more relevant for high values of interlayer thickness, and, hence, the effective stiffness is lower in this case.

3.3. Influence of the interlayer thickness

Fig. 7 shows the effective modulus E_{eq} , normalized by the interlayer modulus E_p , as a function of the interlayer thickness, for different values of the glass fragment length and of the delamination length. Analytic (eq. (1)) and numerical results are here compared. Since polymeric sheets are usually provided by the manufacturer with thickness of 0.38 mm, 0.76 mm or 1.52 mm, that can also be stacked to form thicker interlayers, here interlayer thicknesses from 0.38 mm to 3.04 mm are considered.

The obtained results confirm the influence of the interlayer thickness on the post-critical response of laminated glass, in particular for glass fragmented in “small” shards. More precisely, the higher is the interlayer thickness, the more pronounced are the stress diffusion phenomena and the “flowing” of stresses through the inner region of the interlayer, and, consequently, the lower is the correspondent effective stiffness.

Post-breakage Tensile and Bending Response of Laminated Glass

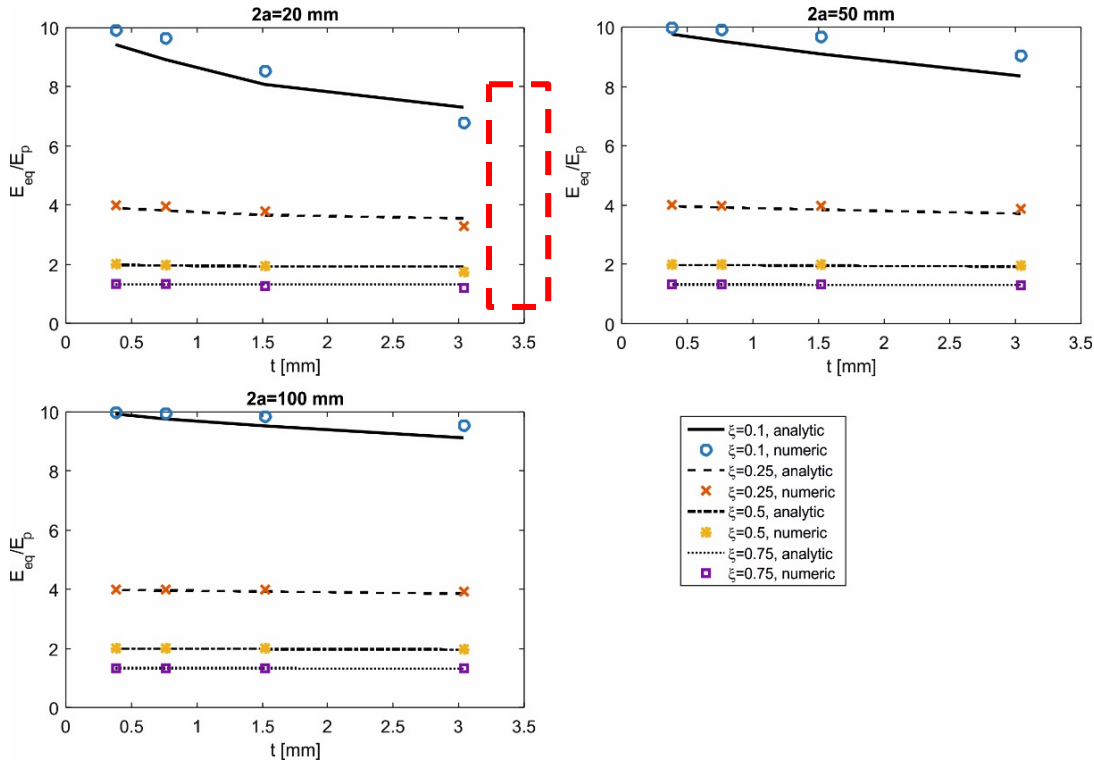


Fig. 7 Ratio E_{eq} / E_p , as a function of the interlayer thickness, for different values of percentage of detached interlayer $\xi = \lambda/a$ of the fragment length $2a$.

Notice that the analytical estimations loose accuracy for high values of t and low values of a (see highlighted region in Fig. 7), i.e., whenever the interlayer thickness is of the order of its length. In this case, the constant-in-the-thickness axial stress approximation is not precise, and the analytical model cannot provide an accurate lower bound for the effective stiffness.

4. Bending response of broken laminated glass

The approach described in the previous Sections may be extended to evaluate the in-plane and out-of-plane response of completely cracked laminated glass under the most general loading conditions (Galuppi and Royer Carfagni 2018). For in-plane bending, the state of stress is composed by compression, due to direct contact of the shards, and tension, transmitted by the polymer tension-stiffened by the shards. For out-of-plane bending, we follow the description in (Kott and Vogel 2004), according to which three distinct stages can be recognized while increasing the bending load: *I*) both glass plies are sound, the interlayer constrains the shear sliding of the glass plies and the laminate glass element behaves as a sandwich structure (Galuppi and Royer Carfagni 2012); *II*) one of the glass plies breaks and the load is carried by the ply remaining sound; *III*) both the glass plies break, but the element maintains a residual post-breakage load-bearing capacity and stiffness because the bending moment is balanced by the compression stresses transferred between the glass shards and the tensile stresses sustained by the interlayer (Delincé *et al.* 2008).

In general, the post-glass-breakage response is strongly influenced by the interlayer material. As discussed in the sequel, Ionoplastic interlayers, stiffer and less sensitive to viscosity than PVB, can strongly improve the post-glass breakage performance.

4.1. Response under in-plane bending

The response of a laminated glass panel under in-plane bending, in case of complete fragmentation of both the glass plies, may be modelled by considering that the glass fragments can transfer compressive stress by direct contact. On the other hand, the tensile stresses are carried by the interlayer, stiffened by the adherent glass fragments, whose effective modulus may be estimated by means of eq. (1). The membrane (in-plane) bending of an element of width w composed by two glass plies of thickness h and an interlayer of thickness t is thus regarded as the bending of a bimodulus material, having tensile stiffness tE_{eq} and compressive stiffness $2hE_G$, where E_G denotes the compressive Young's modulus of sound glass, as qualitatively shown in Fig. 8.

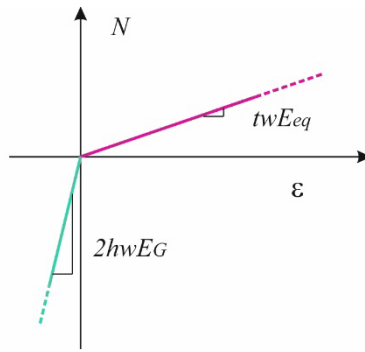


Fig. 8 Qualitative plot of the axial load – axial deformation relation of cracked laminated glass.

As discussed in (Galuppi and Royer Carfagni 2018), since the axial stress distribution is bilinear (see Fig. 9), the effective in-plane bending stiffness reads

$$(EI)_{PB,m} = \frac{2hw^3}{3} [E_G k^3 + E_{eq} (1-k)^3 t / (2h)], \quad (3)$$

where the nondimensional parameter k , determining the position of the neutral axis as shown by Fig. 9, may be evaluated from the axial equilibrium of the element. For standard material and geometric parameters, k is of the order of 3% of the beam height. Consequently, the contribution of the compressive zone is quite low and the effective post-breakage bending stiffness is very low if compared to the pre-breakage one.

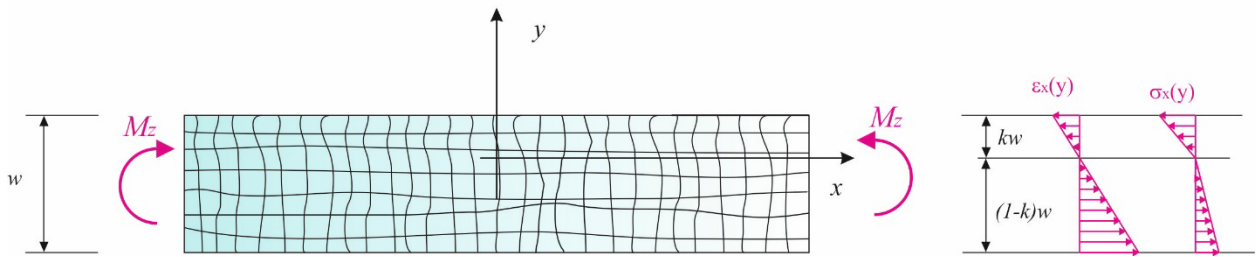
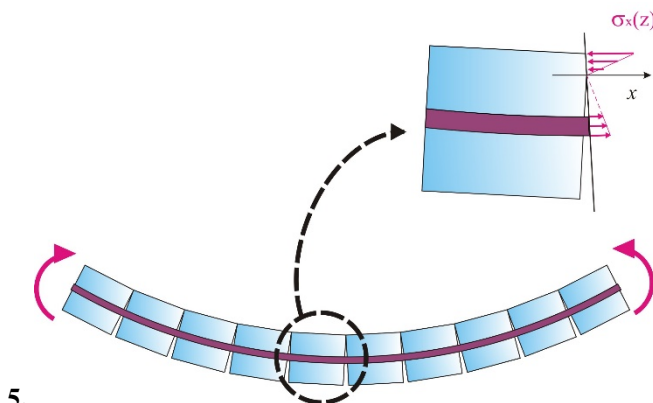


Fig. 9 Cracked laminated glass beam under in-plane bending: a) schematics and b) qualitative distribution of the axial strain and stress.

Comparisons with experimental results recorded in (Galuppi and Royer Carfagni 2018) have demonstrated that the proposed model should give a quite accurate estimation of the post-glass-breakage in-plane stiffness.

4.2. Response under out-of-plane bending

For what concerns the *out-of-plane bending response* of broken laminated glass, there is weak analogy with the mechanics of reinforced concrete, where the glass fragments play the role of the concrete (no-tension material), while the interlayer corresponds to the steel reinforcement, as schematically shown in Fig. 10. However, there is also a strong difference, because the polymer is much more compliant than a steel reinforcement, so that the tension stiffening due to the adhesion with the glass represents the most important phenomenon to consider.



5.

Fig. 10 Schematics of the out-of-plane bending of cracked laminated glass element, with qualitative distribution of the axial stress.

Since the interlayer is stiffened by the glass fragments, its tensile modulus should be assumed to be E_{eq} , as defined *per* eq. (1). Simple considerations based upon the analogy with a bi-material beam, supposing that the cross sections remain plane, allow to obtain for the post-breakage out-of-plane stiffness the expression

$$(EI)_{PB;p} = \frac{hw^3}{3} \left[E_G k^3 + E_{eq} \left(\left(\frac{2t}{h} + 1 - k \right)^3 - (1 - k)^3 \right) \right], \quad (4)$$

where, again, k identifies the position of the neutral axis. This value that is found by requiring that the resultant of the axial stress is null.

As discussed in (Galuppi and Royer Carfagni 2018), glass elements laminated with Ionoplastic interlayers present a noteworthy residual stiffness after glass breakage, of the order of almost 10%, while PVB interlayers provide a very poor performance (the ratio between post- and pre-glass breakage bending stiffness is less than 10^{-3}). Observe that, in general, broken laminated glass elements remains in use for a short period of time (i.e., the time required for replacement), and, hence, a lifetime of the order of days may be assumed in the post-failure phase. It may be verified that Ionoplastic interlayers are stiff enough to maintain considerable stiffness for such a time, while elements laminated with standard PVB (Bennison, 2009) exhibit a sudden stiffness decay even after one hour due to viscosity. Comparisons with experimental results (Galuppi and Royer Carfagni 2018) have demonstrated that the proposed model in general provides a quite accurate estimation of the post-glass breakage bending stiffness. A proper evaluation of the tension stiffening of the interlayer in the post critical phases, i.e., of the effective Young's modulus E_{eq} , is clearly at the basis for a correct evaluation of the post-critical bending response of the laminated glass element.

6. Conclusions

The consequences associated with the loss of stiffness and strength after glass breakage should be carefully considered when the function of the element is that of carrying dead loads, e.g., snow loads or people, as it is the case for roofs or floors. In this case, it is necessary to consider the possibility of breakage since the design stage, and to require that the element can withstand the design loads when one or more glass plies are broken. This is a crucial aspect in the *fail-safe* approach to the structural design of glass, as prescribed by modern Codes.

In order to facilitate the structural calculations it is useful to define the *effective* mechanical properties of broken laminated glass elements. In (Galuppi and Royer Carfagni 2016), an innovative model for the evaluation of the post-glass breakage tensile stiffness of laminated glass is proposed, accounting for the glass-interlayer local delamination and based upon a proper evaluation of the stress diffusion phenomena inside the bonded region. The strength of the proposed method consists in the possibility of accounting for the stress diffusion phenomena in simple mathematical terms, allowing to reach a simple but accurate estimate of the effective stiffness of the cracked laminate. Here, an accurate evaluation of the influence of several geometric parameters, such as the glass fragment length, the amount of delamination and the interlayer thickness, has been made. Comparison with numerical results confirm the accuracy of the proposed approach. For standard geometric parameters, the mean error is less than 4%.

The so obtained effective tensile stiffness may be used as the basis for a proper evaluation of the bending response of broken laminated glass elements. The mathematical treatment for the *in-plane* and *out-of-plane* response of broken laminated glass elements is in analogy with the bending of a beam made of a bimodulus material and the mechanics of reinforced concrete. This provides compact formulae for the evaluation of the bending stiffness. In all cases, a proper evaluation of the tension stiffening of the interlayer in the post critical phases is the basis for a correct design of laminated glass elements and structures. The residual stiffness after glass breakage depends upon the interlayer properties, the geometry and the ratio between interlayer thickness and glass thickness. In general, lamination with Ionoplastic interlayers enhances the post-breakage stiffness with respect to PVB interlayers, especially if one takes into account the time-dependent response. In any case, the post-critical stiffness and strength are always much lower than those of the sound element.

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