

The Influence of Spillback Modelling when Assessing Consequences of Blockings in a Road Network

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Robustness of a network is a main objective for road network managers these days, and has therefore become an important study area for transportation scientists. This article discusses one specific aspect in assessing road network robustness: the consequences of the closure of a link. These spillback effects have been examined in a dedicated traffic simulator in which the representation of spillback can be switched on and off. The impacts are studied in a simulation study of a road network of a regional size in which sequentially links are blocked. Two scenarios for route choice are considered: a fixed route choice based on a daily congestion pattern and a route choice adapted to the actual congestion caused by the closure. The study has also shown the influence of information which makes travellers adapt their routes. Road network robustness and characteristics of vulnerable links are evaluated for both spillback and non-spillback cases. It is found that a valid spillback modelling is a prerequisite for correctly analysing the robustness of the network as a whole, as well as for correctly indicating the locations in the network where a closure causes the largest delays. Furthermore, without simulating spillback, it is not possible to identify correctly the most vulnerable links for the network performance.

Keywords: robustness; vulnerable links; spillback; blocking back

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1. Introduction

Reliable road networks are valued by both travellers and network authorities, as shown for instance by Bogers et al. (2005) and Liu et al. (2004). From the perspective of the traveller, Bates et al. (2001) found, for example, that one minute reduction of standard deviation of travel time and two minutes of actual travel time are equally valued. Bogers and van Zuylen (2004) showed that drivers avoid routes that are on the average the quickest but have a probability of exceptional high travel times. Robustness of networks is the ability of a network to cope with variations in demand or network capacity without much influence on travel times. This network property is a corner stone for travel time reliability. The mentioned variations can be caused by normal daily fluctuations in demand and supply as empirically shown by Tu et al. (2005). Another cause for this variation is the closure of a link by an incident or road maintenance. This is not part of the normal daily fluctuations and reduces the capacity.

There are two analyses that are considered in this article. The first one is the reduction of performance of a network caused by an incident. That is, we only consider the impacts on network robustness caused by a fluctuation in the supply side. The different risks on incidents on different links are not considered in this article. The second one is the location where a link closure has the biggest impact. We take a look at both these analyses.

Research projects assessing road network robustness use different traffic simulation models. The models differ, among others, in the way the spatial dynamics of traffic flow and congestion are described. Due to the complexity of the network and traffic flow modelling (and thus computation time), sometimes spillback, i.e. congestion propagation to a more upstream link, is not modelled (for instance, Kraan et al. (2008)). This article compares the simulation of link network robustness for scenarios with and without spillback modelled in order to assess the need of proper spillback modelling. The method we use works as follows: we sequentially block all links, one at a time, and compare the results in network performance. Furthermore, we analyze the influence of route advice: a distinction is made for cases in which road users adapt their route choice to the new situation with a blocked link and congestion and a situation in which they take their usual routes.

It turns out that different modelling assumptions on spillback models affect the results of assessment of the consequences of link blockings. The results of the assessment can be either a value for network vulnerability or a ordering of most important links. The aim of this article is to show the effects of spillback modelling assumption on the outcomes of a study.

It is found that the robustness assessment differs considerably for different approaches. Therefore spillback should be well modelled in robustness studies.

2. State of the Art

A considerable part of delays is caused by incidents. Kwon et al. (2006) show that this is around 25% for the USA; a similar number is found for the Netherlands in "Inventarisatie Beleidseffecten Incidentmanagement" (2007). In this article, we discuss the consequences of an incident in detail. One could separate the search for important (sometimes called vulnerable) links in four categories, which do have overlaps.

Looking at past research on network vulnerability, four categories can be identified:

- Flow characteristics of network links
- Single assignment (user-equilibrium)
- Analytical game-theoretical approaches

- Dynamic simulation of all possible closures

The first approach is the one taken by, amongst others, Tampère et al. (2007). Their paper presents criteria to find important link. The method accounts for the traffic dynamics. For example, the time until a backwards growing traffic jam reaches a junction is computed. A short time to reach a junction and a high flow on the upstream link are two of their criteria. There are more papers which present indicators (for example, volume over capacity ratio) in an equilibrium-assigned network to find the most important links. Knoop et al. (2007b) review criteria presented in three articles and conclude that these link-based indicators cannot predict the consequence of a link closure.

The second approach is the one proposed by Murray-Tuite and Mahmassani (2004). It starts with an equilibrium assignment, just as Tampère et al. (2007). This approach adds a rerouting for traffic over blocked links. Sequentially links are blocked and the flow on the blocked links is reassigned. The travel times depend on the flows on the links. Kraan et al. (2008) propose a scheme which identifies important links in different stages. The first two steps are similar to the method proposed by Murray-Tuite and Mahmassani (2004). In the final step, links that are potentially most important are ordered using a more accurate model. Kraan et al. (2008) use this approach to overcome the computational effort of doing a separate simulation for each blocked link. In addition to the effect of incidents, they also estimate the probability of a closure and compute the vulnerability as probability times effect, in which the effect is the total delay.

The third category is introduced by Bell (2000) and Bell and Cassir (2002). It is an analytical, game theoretical approach. Rather than using the statistic probabilities on an incident, they calculate the maximum disruption of one incident, given that the place is unknown. Mathematically, it is described as an "evil entity" that wants to destroy a network and is given the possibility to destroy a limited number of links. In the second approach, there is just one rerouting step. In this approach, the travellers will account for failing routes when making their route choice. It is iteratively calculated which routes are optimal for the users (users will take that route) and what will be the worst place for a link closure (travellers will count on that situation). Those links that harm the network performance most if they are blocked are called important. One could make various assumptions for the route choice that is made. Where Bell (2000) and Bell and Cassir (2002) propose a strict risk-averse route choice, Nagae and Akamatsu (2007) relax this assumption and propose a distribution of risk-averseness. It holds for all of these contributions that the lower the effect of the dropout of links is, the more robust one can consider the network. In this approach, results are based on an analytical approach and a mathematical framework is set; in both articles by Bell (2000) and Bell and Cassir (2002), a simple network is used as test case.

The fourth approach, described by Knoop et al. (2005), takes the computational demanding route of computing a dynamic traffic assignment for each possible closure. It identifies important links in a real-size network by running a full dynamic simulation for each possible case, i.e. each place of a closure. It then tries to deduct the properties of the most important links. The study underlines the importance of spillback effects in busy areas. Jenelius et al. (2006) also takes the approach of a full computation. Since they apply the method to an area in East Sweden with less traffic and less congestion (roads with 450 vehicles per day summed over both directions), there is no need for a simulation which accounts for spillback.

The majority of the articles on important links do not take spillback into account. Earlier, we presented a study to see the importance of spillback for finding the vulnerability of a link Knoop et al. (2007a). In that study, the route choice was taken fixed. In this article, we show that also if the route choice varies over time, spillback effects are important for the robustness and the identification of the important links. This article focuses mainly on the importance of spillback on the identification of the most important links in a real-world network.

3. Research approach

The research approach that is taken in this article is outlined in Figure 1.

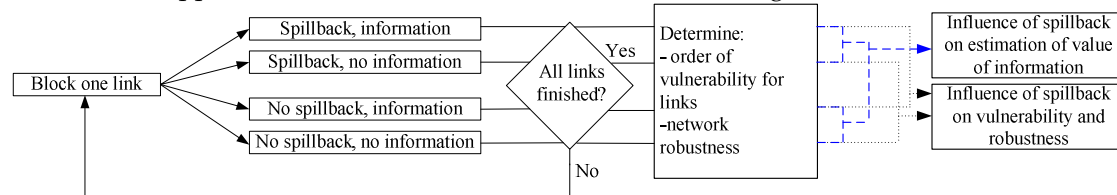


Figure 1. Outline of the research approach

We will simulate the traffic flow with all different blockages. Two different traffic flow models will be used. They are identical, except for the modeling of spillback. In one of the models, there is spillback, in the other, there is not. They are described in detail in section 3.1. Regarding route choice, we consider two options. In the first option, the route choice is adapted to the recurrent congestion, but travelers will never deviate from their standard routes (even though there is an unexpected queue or a closure). In the second option, they will be informed about the traffic states in the network every 15 minutes and adapt their routes accordingly. A detailed description of the route choice models is given in section 3.2.

One could combine the results pair-wise. One pair could be the scenario with spillback and with information and the scenario without spillback and with information. Comparing these will show whether it is needed to model spillback. This is also found in the following pair: the model with spillback and without information and the model without spillback without information. Comparing the result with spillback with information and the result with spillback without information gives the value of informing the travelers. This value can be computed from a non-spillback model. This gives a pair of a model result without spillback and with information and a result without spillback without information. The value of the information predicted by this non-spillback simulator can be compared by the value of the information with spillback modeled.

3.1 Traffic Flow Modelling

The best way to compare the results of the vulnerability of links in a spillback and in a non-spillback simulator is to use one simulator that can run simulations both with and without spillback. In this way, there are no differences in systematic errors. Because we are examining effects of the location of queues, we want to have a model with a reasonable queue dynamics. As far as we know, there is no such a model on the market in which spillback can be switched off. Therefore, we developed a macroscopic model in which spillback can be switched on and off. The section below briefly states the working of the model.

We use the continuum LWR-model proposed by Lighthill and Whitham (1955) and Richards (1956) that we solve with a Godunov scheme (Godunov, 1959). Lebacque (1996) showed how this is used for traffic flows, yielding a deterministic continuum traffic flow simulation model.

When a queue occurs, the queue may grow further upstream than the end of a link. Furthermore it can, depending on traffic conditions, dissolve from the head, while the tail of the queue still moves upstream. The traffic dynamics for a road stretch with a temporal bottleneck – which is typical for an incident – is plotted in Figure 2a. The queuing area is shaded in the top figure. In the lower figure, the number of cars in the queue is plotted. This is all present in a LWR model.

We choose a LWR model exactly for these properties: the queue dynamics are realistic, but it is not unnecessary complicated. For instance, second order effects (e.g., from synchronized flow to

wide moving jams) have some minor influences, see Ngoduy (2006). We will neglect these in the remainder of this article.

When changing it to a non-spillback model, the flow at the upstream link is (by definition) not influenced by the queues on the (downstream) link. In our representation of a non-spillback model, the queue will grow upstream but does not cross the link border. This is achieved as follows: we choose the same LWR-representation of the traffic flow, but now the inflow in the most upstream cell of a link is not influenced by the density, but only determined by the (static) link capacity. In this way, the traffic dynamics are the same as the in the model with spillback modelled and the only difference is the spillback. Consequently, the density in this cell can reach very (unrealistically) high values. The queue dynamics of this model are plotted in Figure 2b. In the upper figure, the space-time diagram of the queue is plotted; in the lower figure the number of vehicles in the queue is plotted.

This study compares the situation with queues without spillback (as in Figure 2b) with the situation of full spillback (Figure 2a).

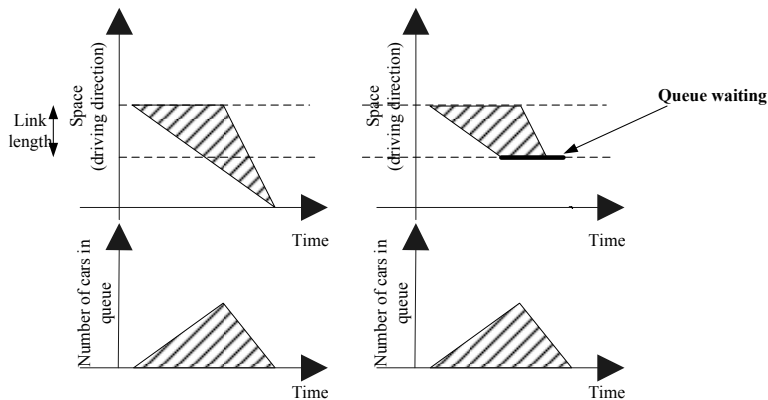


Figure 2. Congestion dynamics in a space-time plane. From left to right: a) realistic queue development, including spillback, b) implementation of a non-spillback model

3.2 Route Choice Modelling

We consider two route choice possibilities. The first possibility is that the road users choose the routes that are fastest without an incident and do not deviate from it in case an incident occurs. That is, if their route turns out to be congested or blocked, they will have to wait in the queue. The second possibility is that when a road is blocked, travellers will adapt their routes according to the new situation. This implies that travellers are somehow correctly informed about prevailing traffic condition.

3.2.1 Fixed routes

In this scenario, we assume that travellers use fixed routes to reach their destination. The routes should represent the everyday choice of the travellers. An equilibrium assignment would be suitable, but in our model, it would be too time consuming to compute. Instead, we choose to assign routes to travellers according to the fastest routes at the moment. The route choice model can also be found in Figure 3.

Routes are always determined for a 15-minute time interval. At the start of the next time interval, again the fastest routes are computed and vehicles are assigned to these new fastest routes. So we use a stochastic traffic assignment in which the (dynamically derived) traffic times of the previous period are used. This is repeated at the end of each time period. Half of the travellers

are assigned to the new route, whereas the other half holds the route in the previous period. Note that these routes are still based on the everyday travel times without a link closure.

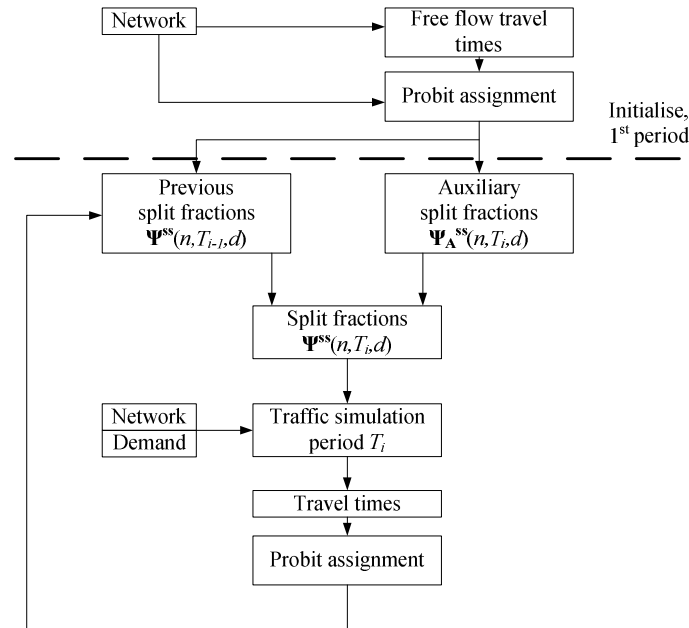


Figure 3. The route choice module

The stochastic routes are found using a probit assignment. This means that for each node, we draw 20 random sets of draws of the link travel time. In each of these 20 sets, we determine the fastest route from that node to each destination. These 20 routes to destination d lead over a few exit links from the node. The destination specific split fraction Ψ is chosen proportional to the number of paths over the exit link from node n . We assume that the perceived link travel times are normally distributed with a standard deviation of 10% of the average travel time. This is the outcome of a basic calibration described in section: "Case study description".

The routes are stored as split fractions, which differ per node, destination and time interval. This split fraction $\Psi(n, T, d)$ means for instance that in time T at node n of all travellers heading towards d 50% goes straight on, 30% turns left, and 20% turns right.

After a vehicle has set off for a certain route, at the next node it reaches, it will follow the routes found for that node. This can be another route than the route it was heading for. Even with 20 routes in the probit assignment, more than 20 routes are used from an origin to a destination.

In this scenario, the split factors are fixed and do not change because of the changing traffic operations. The resulting route set is referred to as $\pi^*(G, ss)$, in which G is the network these routes are based on. A network with link b blocked is denoted as G_b .

The network flows and therefore also the delays D can be different for the scenario in which spillback is modelled and the scenario in which no spillback is modelled, so it depends also on the simulation of spillback, ss .

The performance of the network in this scenario can now mathematically be expressed as:

$$D(\pi^*(G, ss), G_b, ss) \quad (1)$$

The most important link b^* is the link for which the network performance is lowest if it is blocked.

$$b^* = \operatorname{argmax}_b \left(D \left(\pi^*(G, ss), G_b, ss \right) \right). \quad (2)$$

In this notation it is assumed that a lower D equals a better performance, which is the case if delay is chosen as indicator. If another performance indicator D were chosen for which a higher value would mean a better performance (e.g., total arrivals), b^* could be found by minimising D .

3.2.2 Route choice with information provision

The basic principles for the route choice are the same as in the previous situation. We still refer to Figure 3. The travellers' choices are still modelled by the same model. The difference is that the network which is put into the route choice module now is the network with the closure on link b . Consequently, there are also extra queues (caused by this blockings) which are now simulated. The road users adapt their behaviour to information of the new situation with the closure. Therefore, they will also change their routes during the simulated time due to congestion.

Just as in the scenario with fixed routes, routes are chosen based on expected travel time. Routes are updated every time period of 15 minutes based on the congestion, including the congestion caused by the incident. Not all travellers have access to information and some will be unwilling to change their route. Therefore, only a part of the travellers will be assigned to a new route; the other part will choose the old route for the coming period. The path set found in this case, is called $\pi^*(G_b)$. It depends on the blocked link b .

As network flows will differ for scenarios with and without spillback, the network performance can also differ dependent on the simulation of spillback.

For this scenario, the network performance function that needs to be evaluated is

$$D \left(\pi^*(G_b), G_b, ss \right). \quad (3)$$

This function is calculated for each choice of blocked link b . The most important link b^* is:

$$b^* = \operatorname{argmax}_b \left(D \left(\pi^*(G_b, ss), G_b, ss \right) \right). \quad (4)$$

This can be translated in terms of a mathematical game between the road users and an evil entity wanting to harm the network performance most. In this game, this link b^* from equation (4) would be the Stackelberg optimum (see Fudenberg and Tirole (1991)) for an evil entity to block if he was given the opportunity to block one link (and given the users' response). In case of the fixed routes, the travellers do not change routes and they therefore are no players; hence, there is no mathematical game any more and the resulting link b^* from equation (2) is not a Stackelberg optimum.

4. Case study description

We perform a case study on a regional size network with both motorways and underlying roads. A morning peak period from 6.30 to 9.30 is simulated. 468 links with different link properties (capacity, speed limit) and link connections give insight to which properties are relevant for the network impact of a link being blocked. The network we used is the ring road around the city of Rotterdam (around 600.000 inhabitants). A map of the area is given in Figure 4. In the peak period the network it is rather busy. The model is qualitatively calibrated for the normal situation: the capacities, demands and the perception error in route choice are chosen to match the daily congestion. Especially near the motorway junctions, some queues develop, but none

with a length of more than a few kilometres (less than the distance to the next motorway junction).

There are two approaches for the route choice modelling and two approaches for the traffic flow modelling. The combination gives four different possibilities:

- no spillback modelled, fixed route choice;
- no spillback modelled, route choice with information provision;
- spillback modelled, fixed route choice;
- spillback modelled, route choice with information provision.

In the case study, we compute the consequences of the closure of each link in the network in each of these 4 scenarios. There are 4 different scenarios for which each of the links is sequentially blocked. Each scenario gives 468 total performances, one for each of the blocked links.

The route choice with information provision requires the part of the travellers adapting their routes to be set. If the fraction of people that take a different route is too small, the effect of the information provision cannot be seen. On the other hand, a very large share is unrealistic, see for instance Bogers et al. (2005) which states that users, even when informed, will also stick to their original routes. Furthermore, it needs to be considered that not everyone will get the information. An examination of the most important links would require a careful calibration of the route adaptation of travellers for a specific network. Since the aim of this article is to illustrate the effects of spillback, it suffices to choose an arbitrary level for the part of travellers adapting their route choice. In this article, we use a level of 50% of travellers adapting their routes and 50% of the travellers keeping their routes fixed.

In this article, we choose the *total* or *collective delays* as the main performance indicator. This is in line with the route choice which is also only based on travel time. Based on the demand and the free flow travel times, the free flow arrival pattern can be constructed. Any delay in the arrival pattern contributes to the total delay.



Figure 4. The case study area, the ring road around Rotterdam, left to right around 25 km

5. Case study results

Figure 5 shows the performance of the network in the different cases. The x-axis shows the delay without spillback modelled and on the y-axis shows the delay with spillback modelled. For each

closure, we find a delay with and without spillback modelled. This is represented in the graph. This is done for the case with and without path update. We find the delay to be higher if spillback is included: all points lay higher than the line $x = y$, as we would expect.

One dot indicates one specific blocked link. We fitted a linear relationship for both the rerouting and for the fixed route case. The correlation shows how well a simulation without spillback can foretell the impact of the closure of a link. For this purpose, we fitted the relationship:

$$D_{\text{spillback}} = \alpha + \beta D_{\text{no spillback}} \quad (5)$$

and found parameters in Table 1.

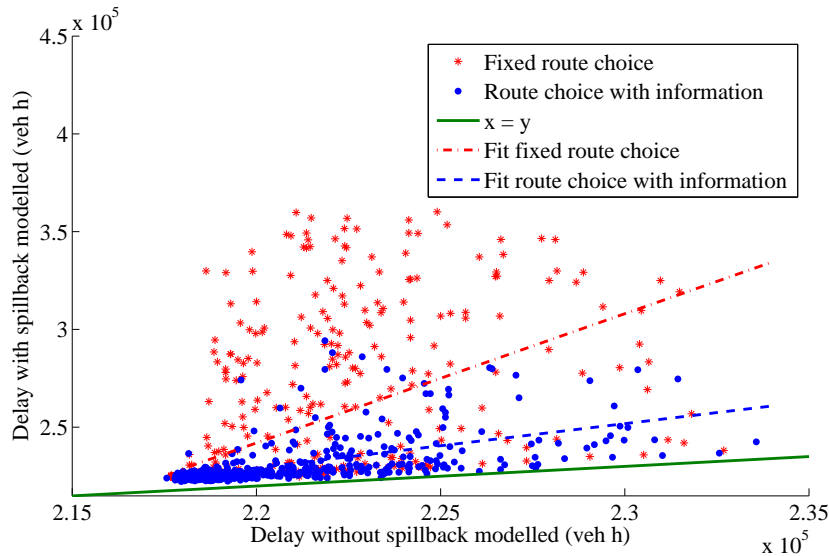


Figure 5. Comparison of the impacts of link closure in scenarios between spillback simulations and non-spillback simulations.

Table 1. The fit parameters for the relationship between the non-spillback performance and the spillback performance

	Fixed routes scenario	Adaptive routes scenario
α	-1.2 (± 0.2) E+6 veh h	-2.7 (± 0.7) E+06 veh h
β	6.6 \pm 1	2.3 \pm 0.3
R^2	0.32	0.33

The regression lines are plotted as dotted lines in Figure 5. The correlation coefficient R^2 indicates how much of the variance in performance reduction in a spillback case can be explained by the variation in arrivals in the corresponding scenario without spillback. We see that this value is low, around 33%, which can be seen by the large scatter of points in the figure. An important conclusion therefore is that *a static, non-spillback simulation cannot be used to identify the links in a road network having the largest impact when struck by an incident.*

Furthermore, the factor β is larger than 1. It indicates how much larger the consequences are when spillback is modelled compared to the consequences without spillback modelled. It makes sense that this is higher for the scenario with the fixed paths: in this case, the consequences are underestimated by a factor 6.6 if one uses a non-spillback model compared to a spillback model. In the case of adaptive paths, the underestimation is limited to a factor of 2.3, which is still very substantial.

Figure 6 shows the impact of closure for the individual links and where these links are located. If a link is red, the impact of closure that link is large. Figure 5 showed that the magnitude of variance of the performance reductions differ among the different scenarios. Therefore, the colour scales in subplots in Figure 6 are not the same. Figure 6 shows the same area as Figure 4.

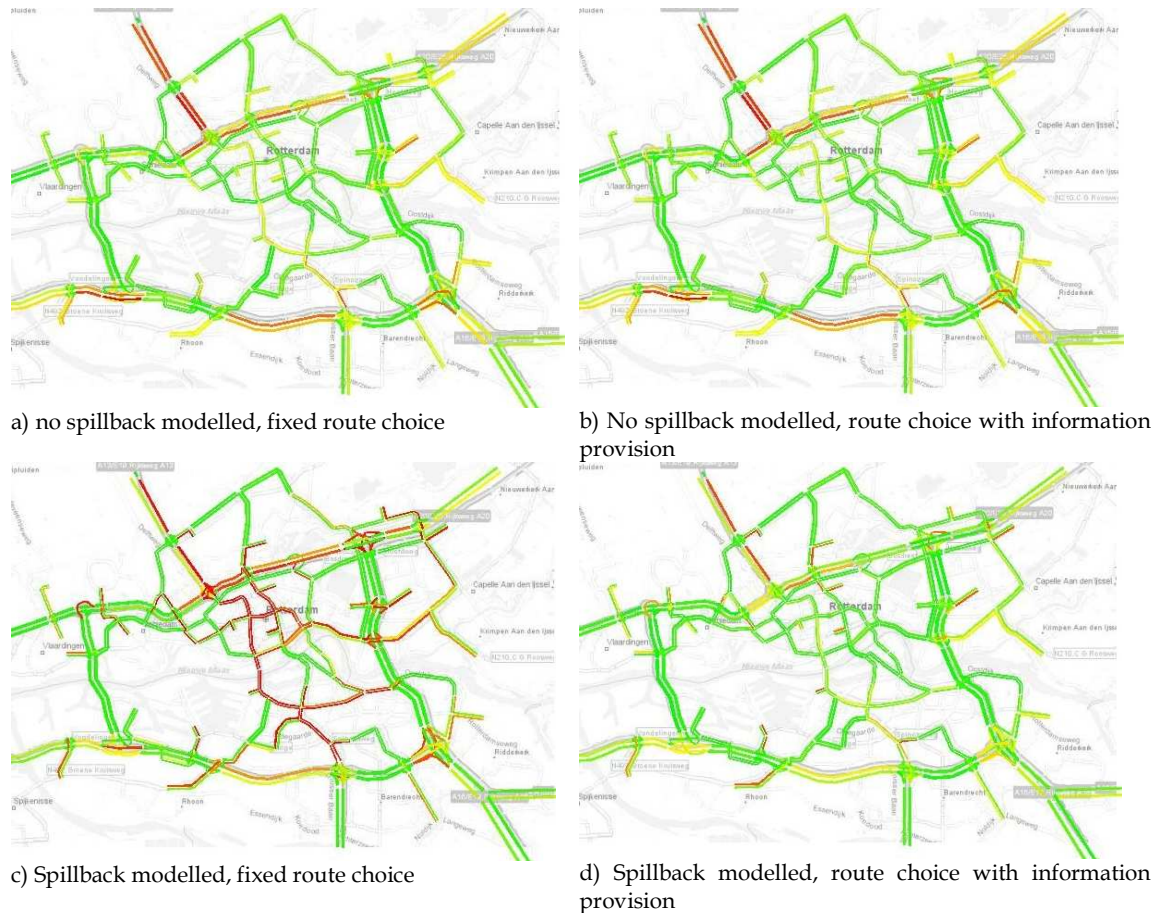


Figure 6. The impact of closure a link. The colour of a link indicates its impact: delays are large when there is a closure on a red link; delays are minor if there is a closure on a green link.

We see that in cases without spillback modelled, the motorways are particularly important for the network performance (the motorways are coloured red). If one of these links is blocked, the performance reduces most. This makes sense, since the motorways are usually the roads that are used most.

Once spillback is included, the motorways appear to be less critical (compared to the average of all links). When travellers are not informed about the routes, the urban roads are important (in case of realistic spillback modelling). Since it is at maximum two lanes wide, the urban link is completely blocked. Even in low intensity traffic, the queue builds up. As the tail propagates through the network, many links are blocked.

When travellers are informed, they will quite early already be rerouted, since the queue starts building up immediately. The most important parts in the network in this scenario are not the urban links (as it was without information); the destination links are now important since people cannot exit since there is no alternative for the exit links, see Li (2008).

To show how much performance can be gained by providing information, we computed the relative advantage A of updating the paths:

$$A(b, ss) = \frac{D(\pi^*(G, ss), G_b, ss) - D(\pi^*(G_b, ss), G_b, ss)}{D(\pi^*(G, ss), G_b, ss)}. \quad (6)$$

Each blocked link b leads to an advantage. The 468 numbers are ordered and plotted below in Figure 7. So without spillback modelled, this information improves the network performance by at maximum a few percent. When spillback is modelled, the provision of information can improve the network performance much more (in a third of the cases over 10%).

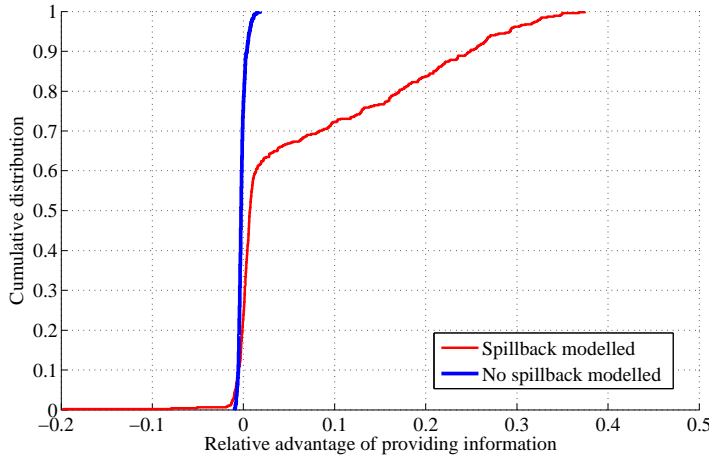


Figure 7. Relative decrease of delay when providing information compared to fixed paths; cumulative distribution for blocking a randomly chosen link

In many cases (i.e., for a large share of the 468 possible locations of an incident), the advantage of route information is in the spillback scenario much larger than in the non-spillback scenario. That could also be derived from Figure 5, which shows a big performance decrease for the spillback scenario without rerouting. If there is rerouting, the performance reduction is much less. The new advices lead people around the bottleneck and hence reduces the delays, but, more important, also reduces the secondary delays (i.e., the delays induced by spillback) considerably.

We also investigated the (relative) performance of the network if a link is blocked. For all 468 possible blocking locations (links), we compute the relative impact of the blocking of a link:

$$I(\pi, b, ss) = \frac{D(\pi^*, G_b, ss)}{D(\pi^*(G, ss), G, ss)} \quad (7)$$

This can be computed for 4 scenarios. Whether spillback is modelled or not influences the numerator and the denominator (ss). The adaptation of paths influences π^* in the numerator: it either becomes $\pi^*(G_b, ss)$ when adapted, or $\pi^*(G, ss)$ when not adapted.

The distribution for the impacts I is plotted in Figure 8. The figure shows how well the network performs compared to the case in which no link is blocked.

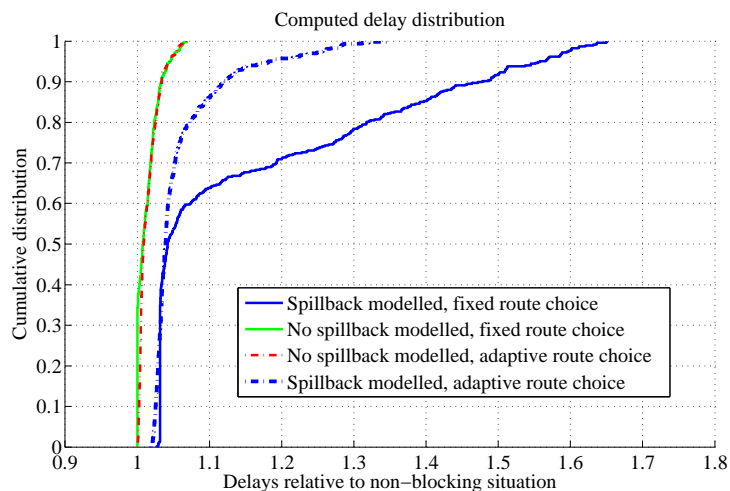


Figure 8. The distribution of the impact I when blocking a randomly chosen link

From this analysis we conclude that the closure of a link appears of less importance if spillback is not simulated. For both the case with and without rerouting, the robustness is about the same. In none of the cases, the delay increases more than a few percent. If spillback is taken into account, there are more links causing a large performance drop. If paths are updated, the travel time increases by at most 36%, the point where the line “spillback, adapted routes” reaches 1. With fixed routes, the travel times can increase by more than 60% compared to a non-blocking scenario. So, robustness is overestimated if it is assessed by a non-spillback simulator and robustness can be increased by giving proper route information.

6. Conclusions and further research

We simulated a morning traffic flow on a real, regional sized, network for which sequentially one of the links was blocked. The traffic simulator had the possibility to simulate both fixed and adaptive choices, and situations with and without spillback: paths could be adapted to the situation or not and, independently, spillback could be switched on and off. This yields four scenarios which have been considered in the simulation study.

An important result is that the links that are considered to be important in terms of their impact on network performance reduction when being blocked differ substantially per scenario. We found that motorways appear to be the most important if spillback is not taken into account. When considering congestion spillback, the impact of a link closure depends on the information given to the drivers, which links are most important. Without dynamic route information, the urban links in the city cause many problems if being blocked; a blocking leads to a total grid lock. If people are informed, the most important links are the links for which there is no route alternative, i.e. the destination links.

The main conclusion of this research is that the links in a network that will cause a major disruption in the network flow operations *cannot be identified by a non-spillback simulator*. Hence, the results of previous studies that have been undertaken using models not incorporating spillback have very limited validity. Only a third of the variations of impact of link blocking in realistic spillback simulations can be derived by performing a simplified, non-spillback simulation. Modelling spillback is also important in assessing robustness of a network. Without spillback being modelled, impact of the closure of one link is much less and therefore the network is considered more robust in a non-spillback simulation program.

Finally, when a non-spillback simulator is used, the advantage of giving route information is highly underestimated. With spillback being modelled, in around 50% of the link closure locations, route information can increase network performance considerably. This shows that for realistic situations, where spillback will indeed occur, network robustness can be increased substantially by informing road users properly.

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