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Finite Subgroups of Units in Integral Group Rings [1]

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Abstract: This is a short survey in which some questions related to the Zassenhaus Conjecture on finite subgroups in integral group rings are discussed. The bibliography is incomplete but gives a possibility to surch for more references and to find out the details of the subject development.

Key words: group rings, torsion units, Zassenhaus Conjecture.

Let $\mathcal{U}(\mathbb{Z}G)$ denote the group of units of the integral group ring of a finite group G, and set $\mathcal{U}_1(\mathbb{Z}G) = \{u \in \mathcal{U}(\mathbb{Z}G) \mid \varepsilon(u) = 1\}$, where $\varepsilon : \mathbb{Z}G \to \mathbb{Z}$ denotes the augmentation map. We say that two subgroups H_1 and H_2 of $\mathcal{U}_1(\mathbb{Z}G)$ are rationally conjugate if there exists a unit $\alpha \in \mathcal{U}(\mathbf{Q}G)$ so that $\alpha^{-1}H_1\alpha = H_2$. We shall write $H_1 \sim H_2$ in $\mathbf{Q}G$, refering to this fact.

For a finite group G it was conjectured by H.J. Zassenhaus (see [58, Chapter 5]) that

$$\textbf{(ZC)} \quad H \subset \mathcal{U}_1(\mathbb{Z}G), |H| < \infty \Longrightarrow H \sim G_o \subset G \text{ in } \mathbf{Q}G.$$

We denote by $Aut_N(\mathbb{Z}G)$ the augmentation preserving automorphisms of $\mathbb{Z}G$ and by $T\mathcal{U}_1(\mathbb{Z}G)$ the set of torsion units of $\mathcal{U}_1(\mathbb{Z}G)$. The following three particular cases of **(ZC)** has been proposed:

$$\textbf{(ZC1)} \quad u \in T\mathcal{U}_1(\mathbb{Z}G) \Longrightarrow u \sim g \in G \text{ in } \mathbf{Q}G.$$

$$\textbf{(ZC2)} \quad H \subset \mathcal{U}_1(\mathbb{Z}G), \mathbb{Z}H = \mathbb{Z}G \Longrightarrow H \sim G \text{ in } \mathbf{Q}G.$$

$$\textbf{(Aut)} \quad \theta \in Aut_N(\mathbb{Z}G) \Longrightarrow \theta(g) = \alpha^{-1}g^{\varphi}\alpha (g \in G)$$
$$\text{for some } \varphi \in Aut(G) \text{ and } \alpha \in \mathcal{U}(\mathbf{Q}G).$$

The second and third cases are intimately connected to the well-known isomorphism conjecture **(Iso)** for integral group rings (see [58, Chapter 5]), namely

$$\textbf{(Aut)}+\textbf{(Iso)} \Longleftrightarrow \textbf{(ZC2)}.$$

The first result on the Zassenhaus Conjecture was published in 1972 by G.L. Peterson [46]: he proved **(Aut)** for the symmetric group S_n (see also [47]). Several extensions of this result to wreath products of S_n with given groups have been obtained by A. Giambruno, M. Parmenter, S.K. Sehgal and A. Valenti ([17],

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[18], [19], [44], [60]). The investigation of **(ZC1)** began by considering finite metacyclic groups and nilpotent of class 2 groups in a series of papers (see [4], [49], [50]). Now we know that **(ZC)** is true for split metacyclic groups with semidirect factors having relatively prime orders (see [48], [61]) and **(ZC1)** holds for some metabelian groups (see [38], [40], [59] and [61]). In the eghties K.W. Roggenkamp and L. Scott constructed a finite metabelian supersolvable group which does not satisfy **(Aut)** (see [52, chapter IX] and [33]). The case of nilpotent groups is more lucky. In 1987 K.W. Roggenkamp and L. Scott published a proof of **(ZC2)** for finite nilpotent groups [54], which was followed by a strong result of A. Weiss on p-adic integral representations of finite p-groups [62]. Weiss's "strong p-group theorem" has important applications. It was used by A. Weiss to prove **(ZC)** for finite nilpotent groups [63] and to get as a corollary the following

Theorem 1 ([62]) *Let G be a finite p-group and $\mathbb{Z}_p$ be the p-adic integers. Then any finie p-subgroup H of $U_1(\mathbb{Z}_pG)$ is conjugate in $U_1(\mathbb{Z}_pG)$ to a subgroup of G.*

A particular case of this result when $|H| = |G|$ has been proved in [54]. Note that the Weiss's arguments are true over more general rings of coefficients (see [58, Appendix] and [51]).

A useful application of Weiss's "strong p-group theorem" is the following:

Theorem 2 ([58, Theorem 41.12]) *Suppose that P is a normal Sylow p-subgroup of a finite group G. Then any p-subgroup of $U_1(\mathbb{Z}G)$ is rationally conjugate to a subgroup of P.*

Note that **(ZC1)** has been proved for some particular alternating and symmetric goups (see [16], [37], [39]) and **(ZC2)** has been obtained for various finite simple groups [5] (see also Example 3.5 in [52, Chapter X]). Analogies of the Zassenhaus conjecture and of **(Iso)** for integral alternative loop rings have been established by E. Goodaire and C. Polcino Milies in [20], [21], [22] (see also [2], [3] and [34]).

As the Zassenhaus conjecture is false, it is natural to look at weaker versions of it on one hand (see [7, Chapter 2, §2], [9], [12], [15], [24], [25], [26], [29], [30], [36], [41], [52], [58]) and, on the other, to restrict it to some "well-behaved" subgroups. Note that seemingly no counterexample is known to **(ZC1)**. Looking at some results mentioned above and their proofs one can get an impression that p-subgroups in $U_1(\mathbb{Z}G)$ should be considered. Therefore, we proposed (see [13]) the following conjecture:

(p-ZC) Let G be a finite group and H be a finite p-subgroup of $U_1(\mathbb{Z}G)$. Then, there exists a unit $u \in \mathbf{Q}G$ such that $u^{-1}Hu \subset G$.

In [13] this conjecture has been proved for finite nilpotent-by-nilpotent groups, for finite solvable groups in which any Sylow subgroup is either abelian or generalized quaternion, for solvable Frobenius groups and for solvable groups whose orders are not divisible by the fourth power of any prime. In particular, the conjecture is true for finite metabelian groups and for finite supersolvable groups. As an intermediate step, **(ZC)** has been proved in [13] for A_4, S_4 and the Binary Octahedral group. Theorem 2 is essentially used in the proofs of these results. Another useful tool is the next theorem.

Let N be a normal subgroup of a group G, set $\overline{G} = G/N$ and let $\psi : \mathbb{Z}G \to \mathbb{Z}(G/N)$ be the natural map.

Theorem 3 ([13, Theorem 2.2]) *Let H be a finite subgroup of $U_1(\mathbb{Z}G)$ such that $(|H|, |N|) = 1$ and let G_0 be a subgroup of G with $(|G_0|, |N|) = 1$. Then, H is rationally conjugate to G_0 if and only if $\varphi(H)$ is conjugate to $\varphi(G_0)$ in $\mathbf{Q}\overline{G}$.*

In [14] the following result has been obtained:

Theorem 4 ([14]) *Let G be a finite Frobenius group. Then*
(i) G satisfies **(p-ZC)** *if $p > 2$.*
(ii) G satisfies **(2-ZC)** *if S_5 is not a homomorphic immage of G.*

In the proof of this theorem A_5, $SL(2,5)$ and S_5 appear naturally, and **(ZC)** has been established for them [14].

Note that **(p-ZC3)** implies Problem 32 of [58] (see [13, Proposition 2.12]). On the other hand, if **(p-ZC3)** is true for a group G, then any Sylow p-subgroup P of $U_1(\mathbb{Z}G)$ is rationally conjugate to a subgroup of G. However, we should know more about the order of P. Let $|G| = p^n m$, where p does not divide m. By [58, Lemma 37.3], $|P|$ divides p^n. It follows from a result of A. Zimmermann [64] that $|P|$ is not necessarily equal p^n. We propose the following

Problem 1. Characterize the finite groups G such that the order of any Sylow p-subgroup of $U_1(\mathbb{Z}G)$ equals p^n.

For a finite p-group G Theorem 1 implies that the order of every Sylow p-subgroup of $U_1(\mathbb{Z}_p G)$ is equal $|G|$. We ask the following

Problem 2. Let G be a finite group and $|G| = p^n m$, where p does not divide m. Is it true that the order of every Sylow p-subgroup of $U_1(\mathbb{Z}_p G)$ equals p^n?

Conjugation of those Sylow p-subgroups which can be imbedded in group bases was investigated in [29], [30] together with weaker versions of **(Aut)** which imply **(Iso)** (see also [52]). Using a result announced by

L. Scott it was proved in [52, Chapter XII, §2] that if H is a group basis of the integral group ring of a finite solvable group G then every Sylow p-subgroup of H is rationally conjugate to a subgroup of G (see also [30]). A nontrivial group basis H of $\mathbb{Z}G$ isomorphic to G is constructed in [8] for a finite nonabelian nonhamiltonian 2-group G. It was conjectured that H and G are not conjugate in $U_1(\mathbb{Z}G)$. The conjecture has been verified with additional restrictions on G [8]. More information around Zassenhaus conjectures and the isomorphism problem for group rings can be found in [1], [6], [7], [27], [31], [52], [45], [56], [57], [58] (see also [5], [23], [28], [31], [32], [43], [55]).

Finite subgroups of units (torsion units) in integral group rings of infinite groups with relation to the Zassenhaus conjecture were studied in [9], [12], [15], [25], [26] , [35], [36], [41], (see also [58, Chapter 6]). In [9], [12], [15], [25], [26], [41], trace properties of torsion units of integral group rings were considered whereas conjugation of a torsion unit of $U_1(\mathbb{Z}G)$ to an element of G was investigated in [9, III], [35] and [36]. In [36] a weaker version of the Zassenhaus conjecture has been proved for torsion units of integral group rings of some free products of groups (the conjugation was taken in an overring of the rational group algebra). "Conjugationlike" embeddings of finite subgroups of units of $U_1(\mathbb{Z}G)$ into G for polycyclic groups G which admit "Hirsch induction" wer

e studied in [41]. The trace of a torsion matrix unit $U \in GL_n(\mathbb{Z}G)$ and conjugation of U to a diagonal matrix of the form $diag(g_1, ..., g_n)(g_i \in G)$ were considered in [9] (see also [58, Chapter 6]).

Now we discuss an alternative way to weaken **(ZC1)** and **(Aut)** which was introduced in [15]. For an element $\alpha = \sum\limits_{g\in G} \alpha(g)g \in \mathbb{Z}G$ we put $\tilde{\alpha}(g) = \sum\limits_{h\in C_g} \alpha(h)$ where C_g is the conjugacy class of $g \in G$. It is well-known that for a finite group G **(ZC1)** is equivalent to the following "unique trace property" (see [40] or [58, Corollary 41.6]):

(UTP) for every $u \in U_1(\mathbb{Z}G)$ there exits a $g_0 \in G$, unique up to conjugacy in G, such that $\tilde{u}(g_o) \neq 0$.

Note that **(UTP)** has been established for several classes of infinite groups [9], [26].

Let G be an arbitrary group (finite or infinite), $g \in G$ and $[g, G]$ be the group generated by all commutators $[g, t] = g^{-1}t^{-1}gt$. Taking the coset $g[g, G]$ instead of the conjugacy class $\{ g[g, t] : t \in G\}$ we put

$$L_g = g[g, G] \setminus \bigcup t[t, G],$$

where the union is taken over all proper subsets $t[t, G]$ of $g[g, G]$.

It is easy to see that the L_g's give a partition of G into a disjoint union of G-invariant subsets. We call them the C-classes. Looking at the partial sums $\overline{u}(g) = \sum_{t \in L_g} u(t)$ we propose (see [15]) the following

Conjecture 1. For any torsion unit $u \in U_1(\mathbb{Z}G)$ there exits a $t \in G$ such that $\overline{u}(t) = 1$ and $\overline{u}(g) = 0$ for all $g \overline{\in} L_t$.

In [15] the following result has been obtained:

Theorem 5 ([15, Theorem 1.3]) *Conjecture 1 is true for abelian-by-polycyclic groups and for metabelian groups.*

In particular, the conjecture holds for finite solvable groups. Note that for metabelian groups Conjecture 1 implies Bovdi's conjecture [15, Corollary 1.4]. For more information on Bovdi's conjecture on generalized traces of torsion units see [11] (or [12]), [24], [25] and [41].

It was shown in [15] that for an arbitrary group G Conjecture 1 follows from the following

Conjecture 2. Let A and B be such normal subgroups of G that $A \cap B = 1$. Then the subgroup

$$[1 + \Delta(G, A)\Delta(G, B)] \bigcap U_1(\mathbb{Z}G)$$

is torsion-free.

Using a result from [10, Lemma 3.3] and the Jategaonkar-Roseblade Theorem [45, p.556] Conjecture 2 has been proved in [15] for polycyclic-by-finite groups and for metabelian groups.

The same idea of replacing the conjugacy classes by the C-classes gives us a weaker version of **(Aut)** which we shall describe now.

Let G be a finite group and $g \in G$. The sum $\overline{C_g} = \sum_{h \in C_g} h$ (where C_g, as above, is the conjugacy class of $g \in G$) is called the class sum of $\mathbb{Z}G$ corresponding to C_g. It is well-known (see [58, Proposition 43.1]) for a finite group G that **(Aut)** is equivalent to the following property:

For any augmentation preserving automorphism θ of $\mathbb{Z}G$ there exists an automorphism
φ of G such that $(\varphi^{-1}\theta)\overline{C_g} = \overline{C_g}$ for all $g \in G$.

Here φ is naturally extended to $\mathbb{Z}G$.

Put $\overline{L_g} = \sum_{h \in L_g} h$. Replacing the conjugacy classes by the L_g's we get the following weaker version of **(Aut)**:

Conjecture 3. For any normalized automorphism θ of $\mathbb{Z}G$ there exists an automorphism φ of G such that $(\varphi^{-1}\theta)\overline{L_g} = \overline{L_g}$ for all $g \in G$.

It is easy to see that if a normalized automorphism of $\mathbb{Z}G$ fixes the sums $\overline{L_g}$ then for all normal subgroups H of G it fixes also the sums $\sum_{h \in H} h$. For a finite group G Lemma 1.1 from [52, Chapter X] implies now that if Conjecture 2 is true for every group G_0 such that $\mathbb{Z}G_0 = \mathbb{Z}G$ or $\mathbb{Z}G_0 = \mathbb{Z}[G \times G]$ then **(Iso)** is true for G. Thus, a result similar to Theorem 5 would be very wellcome and we propose the following problem:

Problem 2. Prove Conjecture 3 for finite solvable groups.

It follows from the previous remarks that the positive solution of Problem 2 for finite solvable groups would imply **(Iso)** for them.

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