Scale and Externalities in an Evolutionary Game Model

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RESUMO

Este artigo discute o processo de seleção de escalas de produção em um modelo de jogo evolucionário no qual os requisitos de racionalidade são muito limitados. Supõe-se, apenas, que os agentes seguem um processo simples de imitação. Examina-se, então, o processo de seleção de equilíbrios sob duas hipóteses: a presença ou não de externalidades associadas a específicas escalas de produção. Em ambos os casos, discutem-se a viabilidade de equilíbrios com heterogeneidade de escalas, a possibilidade de lock-in e as implicações de política econômica.

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competição evolucionária, retornos de escala e seleção, racionalidade limitada

ABSTRACT

This paper discusses the process of production scale selection in an evolutionary game model for which the rationality requirements are very limited. It is merely assumed that agents follow a simple process of imitation. The process of equilibrium selection is then examined in light of two hypotheses: either externalities associated with specific production scales are present or they are not. The feasibility of equilibria with heterogeneous technologies, the possibility of lock-in and the policy implications are discussed for both cases.

KEY WORDS

evolutionary competition, returns of scale and selection, bounded rationality

JEL Classification

L1, L2, D2.

INTRODUCTION

The literature on evolutionary economics in recent years shows that increasing returns and bounded rationality drastically alter the theory of the firm.[1] In particular, seminal work by Arthur (1994) and Witt (1993) emphasizes that it is not enough to identify the equilibria associated with different hypotheses about production scale. Given the possibility of multiple equilibria and path dependence, it is necessary to investigate the equilibrium selection process itself in a given market.

Bounded rationality may take various forms, such as imitation, learning, artificially intelligent agents, etc. Different ways of formalizing the decision-making process presumably produce different results. This paper discusses scale selection in an evolutionary game model that entails minimal rationality requirements. The assumption on which the model is based is that given the impossibility of knowing the future, firms follow a simple process of imitation formalized by means of a replicator dynamics. Selection processes are therefore investigated in accordance with two alternative hypotheses on scale returns: either positive externalities associated with specific production scales are present or they are not. The possibility of lock-in and the optimality of market solutions are examined for both cases.

Three types of increasing returns are normally distinguished in the literature:[2] static internal economies associated with the scale of the firm; static external economies, which depend on market size; and dynamic economies, which derive from accumulated experience and may be internal or external. Generally speaking, these three types of return are built into models as arguments for unit cost functions. This paper uses a different treatment. Instead of considering unit cost as a synthesis of technology and behavioral rules, the production function arguments used here are the scale of the firm and the number of firms that adopt this scale.

The paper is organized as follows. The next section focuses on firms and short-term equilibrium. Section 2 discusses the environment and the proc-

1 See for example GROENEWEGEN & VROMEN (1997). METCALFE (1994) provides many references on the subject. A summary of evolutionary theory can be found in WITT (1992)

2 See METCALFE (1994: 337).

ess whereby firms choose production scale. Section 3 analyzes the model according to the hypothesis that externalities are absent, and section 4 introduces externalities. Section 5 discusses the implications of model optimality, and the last section presents conclusions.

1. *FIRMS AND SHORT-TERM EQUILIBRIUM*

In this article, we assume the existence of a specific type of externality. We suppose that there are two populations of firms. Each one of them uses a production scale, and the increase in the number of firms that adopt a particular scale raises the productivity of all the firms in this population. This type of externalities is merely an assumption. Notwithstanding, this procedure can be justified with the help of tacit information concept. This concept is summed up with precision by Dietrich (1997, p. 83):

> *Tacit information can only effectively be acquired while undertaking an activity, with emphasis on learning by doing. An important characteristic is that such information can only be understood in the context of particular actions and may be shared to a significant degree by individuals who have a common (organizational) experience. Hence the acquisition of tacit information requires the development of particular skills and expertise* (NELSON & WINTER, 1982) *with the complexity this involves.*

As in the Marshallian tradition,[3] we consider that production scale can be conceptualized as a synthesis of the organizational aspects of a firm. Of

3 The relationships between scale and organization constitute the main object of chapters VII to XII of the book IV of the *Principals*. For Marshall, different production scales normally correspond to different ways of organizing productive activities. Internal economies are defined as those dependent on the resources of individual firms, on their organization and efficiency. External economies are those dependent on the general organization of the industry as a whole. Thus, although Marshall himself argues in the beginning of the Book IV (1909, p. 139) that the organization should be reckoned as a "distinct agent of production", the scale of production (of a firm or of a market) is conceived as a synthesis of the organizational aspects of a firm (internalities) or a market (externalities). For an account of Marshallian theory of firm, see ARIDA (1983) and PRENDERGAST (1992, 1993). See SCHERER (1980, chapter 4) for a discussion of the sources of and limits to economies on scale and its relations with industrial organization.

course, this is surely a very strong simplification, but it is useful to keep the model tractable. From this point of view, different production scales normally correspond to different ways of organizing productive activities. Such activities comprise not only aspects that are typically internal to the firm - such as the division of labor, management methods etc. - but also elements shared by similar firms, such as a supplier network, etc. Furthermore, the mobility of labor among similar firms tends to disseminate and generalize their internal practices.

Firms that adopt the same scale therefore have similar organizational experiences. According to this hypothesis, it can be conjectured that the more firms adopt a specific scale, the more likely they will presumably be to enhance the productive activities pertaining to this scale. In other words, the various activities involved may benefit from the tacit information associated with the organizational context. Thus for each firm these benefits acquire the status of an externality which depends on the number of firms opting for the same scale. It can therefore be assumed that when a firm changes its production scale, productivity is affected not only by the internal scale returns but also by this type of externality.

Consider a market in which firms can choose between two pure strategies corresponding to different production scales: K_i, $i = 1, 2$. Let capital be the only production factor and the depreciation rate be equal to one. In each period the quantity produced is sold at the price for which demand equals supply, characterizing short-term equilibrium. Next, firms decide what scale to adopt in the ensuing period based on a process of imitation, as discussed in the next section.

Suppose there can be positive externalities associated with the capital added by scale. The increase in the number of firms that adopt a particular scale therefore raises the productivity of all the firms in this group. Let the production function for the firm representing group i be as follows:

$$y_i = n_i^{\gamma} k_i^{\alpha} \tag{1}$$

where n_i is the number of firms that adopt scale i.

If $\alpha = 1$, technology is subject to constant internal scale returns; $0 < \alpha < 1$ and $\alpha > 1$ correspond to the cases of diminishing and increasing internal returns respectively. If $\gamma > 0$, there are positive externalities; if $\gamma = 0$, there are no externalities.

Let $\lambda_{i,j}$ be the relationship between scales i and j:

$$\lambda_{i,j} = \frac{k_j}{k_i}. \tag{2}$$

Suppose $k_2 > k_1$, which means by way of (1) that $\lambda_{1,2} > 1$. For the sake of notational simplicity, let $\lambda_{1,2}$ be designated merely by λ. By defining $n = n_1 + n_2$ and $v = n_1/n$, it is therefore possible to express the productions functions in terms of the capital of firms 1:

$$y_1 = (nv)^{\gamma} k_1^{\alpha}; \qquad y_2 = (n(1-v))^{\gamma} (\lambda k_1)^{\alpha}. \tag{3}$$

The quantity supplied to the market, y, is obtained by adding the various groups:

$$y = n_1 y_1 + n_2 y_2 = (nv)^{1+\gamma} k_1^{\alpha} \left[v^{1+\gamma} + (1-v)^{1+\gamma} \lambda^{\alpha} \right]. \tag{4}$$

The profit of firms in group 1 is:

$$\pi_1 = p(nv)^{\gamma} k_1^{\alpha} - qk_1, \tag{5}$$

where p is the price of the product and q is the cost of a unit of capital multiplied by the gross rate of interest.

Let there be the following inverse demand function:

$$p = \frac{1}{y}. \tag{6}$$

Replacement of the profit function gives the pay-off corresponding to strategy k_1:

$$\pi_1 = \frac{(nv)^{\gamma}}{n^{1+\gamma}\left(v^{1+\gamma}+(1-v)^{1+\gamma}\lambda^{\alpha}\right)} - qk_1, \tag{7}$$

which can be simplified as,

$$\pi_1 = \frac{1}{nv\left(1+\left(\frac{1-v}{v}\right)^{1+\gamma}\lambda^{\alpha}\right)} - qk_1. \tag{8}$$

Symmetrically, for strategy k_2 we have:

$$\pi_2 = \frac{1}{n(1-v)\left(1+\left(\frac{v}{1-v}\right)^{1+\gamma}\lambda^{-\alpha}\right)} - q\lambda k_1. \tag{9}$$

2. THE ENVIRONMENT AND THE SELECTION PROCESS

An environment of bounded rationality prevails in the marketplace. Firms ignore demand and the scale of their competitors. A firm may compare its pay-offs in the present with those of a randomly chosen competitor. This is the only information available to guide decisions as to the scale to be chosen for the next period. Under these hypotheses, it is convenient to assume that firms are aware of the environment in which they operate. In other words, they know that a comparison of effective profits is only an indication of expected pay-offs in the following period.

Consider first the comparison between the pay-offs effected by firms. For the sake of simplicity, let the total number of firms be normalized[4] : $n = 1$. Because there are only two scales, the number of firms opting for strategy k_1 is v and the number choosing k_2 is $(1 - v)$. The probability that a firm k_1 compares its pay-off with that of a firm k_2 is $(1 - v)$. If $\pi_2 > \pi_1$, the number of firms of type k_1 disposed to change strategy is $v\ (1 - v)$. Evidently under this hypothesis with regard to profit, no firm k_2 is disposed to change scale.

In the rest of the text, you formulate them they are presented in the not normalized version.

Now admit that $\pi_2 < \pi_1$. In this case, the only firms that may consider changing strategy are those of type k_2. The probability that a firm k_2 will compare its profit with that of a firm k_1 is v. Hence the number of firms disposed in principle to change strategy is $(1 - v)\ v$. If $\pi_1 = \pi_2$, no firm reviews its strategy.

The process of comparison described above should not be considered sufficient to determine the number of firms that change strategy. Indeed, because firms are aware that effective pay-offs do not necessarily correspond to future pay-offs, it is reasonable to suppose that for a given difference in pay-offs only some firms will change strategy. Others may deem the difference insufficient. It therefore seems appropriate to suppose that the number of firms that decide to change strategy is a positive function of the magnitude of the difference in profit.

Assuming continuous time, the considerations set out above can be represented by the following formula:[5]

$$\dot{v} = -v(1-v)\max\{\pi_2[v]-\pi_1[v],0\}-(1-v)v\min\{0,\pi_2[v]-\pi_1[v]\}. \quad (10)$$

4 We adopted this normalization to facilitate the exposition of the dynamics replicator. In the remainder of the article, we use the non-normalized version.

5 For the sake of notational simplicity, v is used instead of $v(t)$.

Hence,

$$\dot{v} = -v(1-v)\left(\pi_2[v]-\pi_1[v]\right). \tag{11}$$

The above differential equation is evidently an adaptation of the replicator dynamics used in biology to formalize Darwinian natural selection processes. The social interpretation of the replicator is similar but not identical to that of Nachbar (1990).[6] According to this author, once the population that might potentially change strategy has been identified, the percentage of agents that effectively change strategy depends on the cost of change, which follows a specific probability distribution. Hence he obtains a more general formula than the above (i.e. applicable to a game with more than two strategies). In the present case, we opted to replace the idea of the probabilistic cost of change with the simpler hypothesis that the number of firms that change strategy is proportional to the difference in profit, since this hypothesis simplifies the presentation and reflects the supposition that firms are aware of the bounded rationality environment.

Let us now discuss the state space of the model. If there are no externalities, $\gamma = 0$, and the difference in pay offs, $\phi[v]=\pi_2-\pi_1$, is:

$$\phi[v]=\frac{1-\lambda^{\alpha}}{n\left(v\left(\lambda^{\alpha}-1\right)-\lambda^{\alpha}\right)}-qk_1(\lambda-1). \tag{12}$$

Then,

$$\phi[0]=\frac{1-\lambda^{-\alpha}}{n}-qk_1(\lambda-1), \tag{13}$$

6 See VEGA REDONDO (1996, p. 89-90).

and

$$\phi[1] = -\frac{1-\lambda^{\alpha}}{n} - qk_1(\lambda - 1). \tag{14}$$

Of course, $\lim_{v \to 0^+} \phi[v] = \phi[0]$ and $\lim_{v \to 1^-} \phi[v] = \phi[1]$.

But, if $\gamma > 0$ the function $\phi[.]$ is not defined when $v = 0$ or $v = 1$. Limits exist, nevertheless:

$$\lim_{v \to 0^+} \phi[v] = \frac{1}{n} - qk_1(\lambda - 1), \tag{15}$$

$$\lim_{v \to 1^-} \phi[v] = -\frac{1}{n} - qk_1(\lambda - 1). \tag{16}$$

Hence the function can be redefined as below (where it should be noted that $\gamma = 0$ and $\gamma = 1$ yield two different limits) and the state space is the unit closed interval [0, 1].

$$\phi[v] = \begin{cases} \pi_2 - \pi_1, & \textit{if } v \in (0,1); \\ \lim_{v \to 0^+} (\pi_2 - \pi_1), & \textit{if } v = 0; \\ \lim_{v \to 1^-} (\pi_2 - \pi_1), & \textit{if } v = 1. \end{cases} \tag{17}$$

The dynamic behavior of the system can therefore described by the following non-linear differential equation:

$$\dot{v} = -v(1-v)\phi[v] \tag{18}$$

An examination of the above equation permits identification of stationary points $v = 0$, $v = 1$ and any $v \in (0, 1)$ such that $\phi[v] = 0$. Moreover, because $v(1 - v)$ is always positive in the specified domain, the sign of the differential equation is governed by the function $\phi[v]$.

3. *ABSENCE OF EXTERNALITIES*

If there are no externalities, $\phi[v]$ is strictly increasing, since, from (12):

$$\left.\frac{\partial \phi}{\partial v}\right|_{\gamma=0} = \frac{\left(\lambda^{\alpha}-1\right)^{2}}{n\left(v+\lambda^{\alpha}(1-v)\right)^{2}} > 0. \tag{19}$$

The above result allow cases of the dominant strategy to be identified, as set out in the proposition below.

Proposition 1: *Let externalities be absent, i. e., $\gamma = 0$. Then, precisely one asymptotically stable fixed point exists, and any trajectory starting in the interior of the state space, converges to this asymptotically stable fixed point.*

Proff: If there exists $\bar{v} \in [0,1]$ such that $\phi[\bar{v}] = 0$, then $h(\bar{v}) = -\bar{v}(1-\bar{v})\phi[\bar{v}] = 0$. If $\bar{v} \in \{0,1\}$, $h[\bar{v}] = 0$, whatever the value of $\phi[\bar{v}]$. Moreover, in any case, $0 < v < \bar{v}$ implies $\phi[v] < 0$ and $h[v] > 0$, whereas $\bar{v} < v < 0$ implies $\phi[v] > 0$ and $h[v] < 0$. ∎

Let us interpret this proposition. Because $\phi\ [v]$ is increasing, if $\phi\ [0] \geq 0$ the adoption of scale k_2 generates more profit than k_1, whatever the percentage of firms that choose one or the other scale. This is therefore a dominant strategy in the context of evolutionary games. Hence firms will tend to adopt this scale. Thus whatever the initial condition in the interval (0,1) the market converges to v equals 0. Analogously, if $\phi\ [1] \geq 0$ strategy k_1 is dominant and $v > 0$, and scale k_1 is eventually chosen by all firms. The mixed-strategy equilibrium – which in the context of evolutionary games means two scales coexist in the marketplace – is possible, as set out in the proposition. When $\gamma = 0$, the solution to $\phi\ [v] = 0$ is:

$$\overline{v} = \frac{1-\lambda^{\alpha} + n\,qk_1\left(\lambda^{1+\alpha} - \lambda^{\alpha}\right)}{n\,qk_1\,(\lambda-1)\left(\lambda^{\alpha}-1\right)}. \tag{20}$$

A mixed-strategy solution exists if $0 < \overline{v} < 1$, i.e. if:

$$\frac{1-\lambda^{-\alpha}}{n(\lambda-1)} < qk_1 < -\frac{1-\lambda^{\alpha}}{n(\lambda-1)}. \tag{21}$$

These inequalities are possible (they permit positive cost of capital) since $\lambda > 1$, $\alpha > 0$ and $n > 0$.

If the parameters are such that (21) is satisfied, there will be three equilibria: two pure-strategy equilibria and one mixed-strategy equilibrium, the first two being unstable and the last globally stable. Thus among the infinite possible initial conditions, only two generate pure-strategy equilibria. All the others lead to a mixed-strategy equilibrium.[7] This justifies the affirmation that in the present case unstable pure-strategy equilibria are not relevant.

7 In other words, if the distribution is continuous, then the probability that the initial conditions will be such that they result in pure-strategy equilibria is equal to zero

Substitution of the mixed-strategy equilibrium in the profit function gives:

$$\pi_1 = \frac{qk_1\left(\lambda - \lambda^{\alpha}\right)}{\lambda^{\alpha} - 1}. \tag{22}$$

The sign of π_i depends on the parameter α. It is null, negative or positive according to whether α is equal to 1, greater or smaller than 1. Thus if there are constant internal scale returns, the mixed-strategy equilibrium entails zero profit; if internal returns are increasing, equilibrium corresponds to negative profit and the reverse is true if returns are diminishing.

The results presented above are traditional. It is worth recalling their interpretation. If internal returns are constant, the adoption of a larger scale merely leads to a linear increase in the results of the smaller scale. Whatever the result (profit or loss), it will be greater if the scale is larger. Thus the only possibility of equal profits is zero profits. If scale returns are increasing, the larger scale always presents lower unit costs. If the smaller scale generates profit, the same will occur more significantly for the larger scale, so that the profits generated by both scales will never be equal and concomitantly positive or null. However, when both strategies generate a loss, it is possible that the advantage associated with the unit cost of the larger scale will be offset by lower production volume under the smaller scale, thus enabling both to generate the same total loss. The reverse evidently occurs in the case of diminishing returns, which explains why profits can only be equal if positive.

The existence of increasing internal scale returns and bounded rationality therefore entails a situation in which firms are locked into an adverse equilibrium for both strategies. This situation (like diminishing returns) is evidently incompatible with long-term full equilibrium defined as a situation of extraordinary profits equal to zero. Long-term full equilibrium may be obtained through a reduction in the number of firms (n) or in the cost (q). Either of these factors reduces losses at both scales but relatively favors scale k_2, which has lower unit costs. This is the same as saying that v diminishes with the reduction in n or q, which is guaranteed since:

$$\frac{\partial \overline{v}}{\partial n} = \frac{1}{n^2 q k_1 (\lambda - 1)}, \ \frac{1}{n q^2 k_1 (\lambda - 1)} > 0. \tag{23}$$

Thus the market should converge to v and π_2 equal to zero, i.e. only the larger scale should be adopted at the end of a hypothetical period of long-term adjustment. Similar arguments justify defining v equal to one as being the long-term equilibrium when returns are diminishing. It must be noted, however, that in an evolutionary context convergence to long-term equilibrium would be justified only if it were obtained as a result of the market game itself. This is not the case in the model. We therefore present this here as a finding derived from "comparative statistics" but leave it aside in the rest of the analysis.

4. EXTERNALITIES

In the presence of externalities, $\gamma > 0$ and market behavior changes drastically. A mixed strategy equilibrium exists. In fact, the introduction of externalities preserves the continuity of function $\phi\ [v]$. From (15) and (16), $\phi\ [0] > \phi\ [1]$. Therefore, the existence of mixed-strategy equilibrium is guaranteed (a sufficient condition, not a necessary one) if $\phi\ [0] > 0$ and $\phi\ [1] < 0$, i.e. if:

$$-\frac{1}{n(\lambda - 1)} < q k_1 < \frac{1}{n(\lambda - 1)}. \tag{24}$$

However, $\phi[.]$ does not present sufficient properties to guarantee equilibrium unicity of a mixed strategy equilibrium. In other words, without establishing ad hoc hypotheses about the parameters it is impossible, for example, to guarantee that $\phi(.)$ increases or decreases monotonously. This evidently opens up the possibility of multiple equilibria and different dynamic properties. In order to limit the possibilities of the model for the sake of convenience, a constraint on the admissible equilibria can be established, similar to that effected by Debreu in the context of general equilibrium: let v be defined as regular equilibrium if and only if $\phi[\bar{v}]=0$ and $\partial\phi[\bar{v}]/\partial v \neq 0$. Let V be the set of equilibria in the market. If every $v \in V$ is a regular equilibrium, then the market is regular. Of course, a regular fixed point need not be unique. But, if this regular mixed strategy equilibrium is the unique one, the dynamic properties of the model can be summarized in the following proposition.

Proposition 2: *In the presence of externalities, if the regular fixed point is the unique fixed point in the interior of the state space, then it is unstable.*

Proof. Let $\bar{v}$ be the unique regular fixed point in the interior of the state space and $v(0)\in(0,1)$. Because $\phi[v]$ is continuous and ϕ [0] > ϕ [1], the hypotheses of unicity and regularity implies $\partial\phi[\bar{v}]/\partial v<0$ and $\partial h[\bar{v}]/\partial v>0$. Thus, if $v(0)>\bar{v}$ then $\phi[v]<0$ and $\dot{v}>0$, hence $v(t)\rightarrow 1$ when $t\rightarrow\infty$; if $v(0)<\bar{v}$, then $\phi[v]>0$ and $\dot{v}<0$, hence $v(t)\rightarrow 0$ when $t\rightarrow\infty$. ■

To facilitate interpretation of proposition 2, consider the following example. Let k_1 and n be equal to 1. Assume constant internal scale returns and the following values for the other parameters: $\{\lambda, q\} = \{3/2, 2/3\}$. If there are no externalities, there is no mixed-strategy equilibrium, since:

$$qk_1=\frac{1-\lambda^{-\alpha}}{n(\lambda-1)}=\frac{2}{3}; \tag{25}$$

that is, the value of capital is exactly equal to the lower limit of the condition expressed in (21). Hence strategy k_2 is strictly dominant, i.e. whatever the initial condition in (0,1) the economy tends to v = 0. The pay-off functions are presented in Figure 1.

Now admit as the only difference with the previous case that externalities are present, γ = 1/10. A mixed-strategy equilibrium is possible, since the condition expressed in (7) is satisfied; the equation ϕ [v] = 0 has a single real root, $v \cong 0.71$. Figure 2 shows the pay-off functions and ϕ [v].

The presence of a relatively bounded externality not only makes scale heterogeneity feasible but also makes this equilibrium globally unstable. Thus the probability that the market will present a mixed-strategy equilibrium is limited. Any disturbance of this equilibrium drives the market toward one of the extreme solutions. Moreover, the market path depends crucially on the initial condition characterizing path dependence or, in Sargent's terms (1993, p. 112), "history dependence", since the initial conditions take on an importance that does not disappear over time.

Now suppose that there are increasing internal scale returns, $\alpha = 12/10$. In the absence of externalities, strategy k_2 obviously remains dominant. If externalities are present, there continues to be a mixed-strategy equilibrium, $v \cong 0.87$, and the same kind of comment can be made, i.e. history is relevant. Figure 2 shows the functions.

Lastly, it bears repeating that externalities opens up the possibility of multiple mixed strategy equilibria. For example, a set of parameters $\{\gamma, \alpha, \lambda, q, k, n\} = \{1/10, 1, 2, 33/50, 1, 1\}$ generates the function $\phi\,[v]$ presented in Figure 4. It can be seen that there are three normal mixed-strategy equilibria, the first and last of which are unstable while the second is stable.

5. *WELFARE ANALYSIS*

In the evolutionary context of the model, it may be asked whether the market game leads to an optimal situation, when there are mixed-strategy equilibria. This situation can be defined as one in which resources are utilized efficiently in the sense that reallocation of resources among firms does not enable production to be increased. In order to verify the properties of productive efficiency, it is convenient to compare the equilibria of the market game with those that would be implemented by a central planner. First, recall that because of the stability properties identified in propositions 1 and 2, the relevant market equilibria are those pertaining to the mixed strategy in the absence of externalities, and those pertaining to the pure strategy in the presence of externalities.

Imagine a central planner whose goal is to maximize output for a given resource, k. When choosing whether to allocate capital to either of two groups of firms, the planner decides how many firms will adopt one or the other scale. The planner's program can be defined as follows:

$$\max_{n_1,n_2}: \quad y = \left(n_1^{1+\gamma} + n_2^{1+\gamma}\lambda^{\alpha}\right)k_1^{\alpha}, \tag{26}$$

$$s.to: \quad k_1\left(n_1 + n_2\lambda\right) \leq k, \tag{27}$$

$$n_1, n_2 \geq 0. \tag{28}$$

Note that there is no limit on the aggregate number of firms. Thus n is variable since in accordance with constraint (27), allocating all the capital to technology k_2 entails a smaller number of firms than opting for k_1.

The objective function is increasing, convex and, if $\gamma > 0$, strictly convex. Hence it suffices to compare the values of the function at points $(n_1, 0)$ and $(0, n_2)$, provided n_1 and n_2 satisfy (27) with an equal sign, to identify the conditions that determine the extreme values.

If n_2 is equal to zero, the maximum value of n_1 is $\hat{n}_1 = k / k_1$. Substitution in the objective function gives:

$$y\left[\frac{k}{k_1}, 0\right] = k^{1+\gamma} k_1^{\alpha-\gamma-1}. \tag{29}$$

The same procedure for $(0, n_2)$ results in:

$$y\left[0, \frac{k}{\lambda k_1}\right] = k^{1+\gamma} \lambda^{\alpha-\gamma-1} k_1^{\alpha-\gamma-1}. \tag{30}$$

Therefore,

$$y\left[0,\frac{k}{\lambda k_1}\right]-y\left[\frac{k}{k_1},0\right]=k^{1+\gamma}k_1^{\alpha-\gamma-1}\left(\lambda^{\alpha-\gamma-1}-1\right). \tag{31}$$

Identification of the maxima depends on the sign of $\left(\lambda^{\alpha-\gamma-1}-\lambda^{0}\right)$, i.e. there are three possibilities to consider depending on the relation between α and $1+\gamma$. Let us first analyze the various possibilities under the hypothesis of positive externalities.

If $\alpha > 1 + \gamma$, the central planner will unequivocally choose to allocate all the available capital to technology k_2. The economic interpretation is direct. The effect of increasing scale returns exceeds the effect of externalities. Hence it is best to opt for the larger scale, even though the number of firms is smaller. This provides for the possibility that the market solution will not be Pareto-optimal, since depending on the initial condition the market may converge to k_1. If $\alpha < 1 + \gamma$, scale gains are insufficient to offset the externalities and the central planner unequivocally opts for technology k_1. Once again, the market solution may be inferior to the planner's, since the market may converge to k_2. Thus in the evolutionary context both cases justify the traditional proposition that externalities do not entail optimality.

However, if $\alpha = 1 + \gamma$, increasing scale gains exactly offset the increment in output that would be obtained in $\left(\hat{n}_1,0\right)$ owing to the externalities. The planner is indifferent between points $\left(\hat{n}_1,0\right)$ and $\left(0,\hat{n}_2\right)$ but prefers them to an intermediate solution owing to strict convexity. Because a mixed-strategy market equilibrium is improbable, the game leads to an efficient solution. Curiously, increasing returns and externalities, which are normally characterized as market failures, offset each other so as to result in an optimal equilibrium.

Now consider the case of an absence of externalities. With $\gamma = 0$, the three possibilities presented above correspond to the hypotheses of increasing, decreasing and constant scale returns respectively. In cases of increasing and decreasing returns, the planner chooses the pure-strategy equilibria. Because the market tends toward a mixed-strategy equilibrium, the market solution is inferior to that of the central planner.

If scale returns are constant, the objective function of the central planner is linear and assumes constant values on the boundary determined by the constraint. Any distribution of capital between the two technologies is indifferent for the central planner. Hence the market solution is efficient.

CONCLUSION

In an environment of hyper-rationality and absence of externalities, the final equilibrium of a market does not depend on the initial conditions and is efficient. This traditional result of neoclassical economic theory indicates that government interference to stimulate the adoption of specific production scales is unnecessary or even counter-productive unless it is motivated by distribution issues or due to the presence of externalities.

The model presented in this paper is designed to investigate the question of externalities in a dynamic context. The first point made is that if there are no externalities the market may present a globally stable mixed-strategy equilibrium. Scale heterogeneity is therefore possible regardless of whatever hypothesis is raised about internal scale returns. However, if the zero-profit condition is imposed, a mixed-strategy equilibrium is compatible only with constant scale returns. Hence even in conditions of bounded rationality formalized by the replicator dynamics, the market-game model reproduces the traditional results.

However, in the presence of bounded rationality and externalities associated with production scale, a different situation results. In this case if there is a mixed-strategy equilibrium it is globally unstable. The final market equilibrium therefore depends crucially on the initial conditions. The market may possibly converge toward inferior equilibria.

Thus while in the traditional theory the relationship between externalities and inefficiency is direct, in the model presented here it is mediated by history. The model exemplifies, in a highly simplified context, the dependence between the development of a competitive industry and the historical conditions of its birth. Industries born with small scales and a large number of producers may tend to remain in this condition as a result of the competitive process itself. The transition to more productive scales will then require government interference.

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FIGURE 1

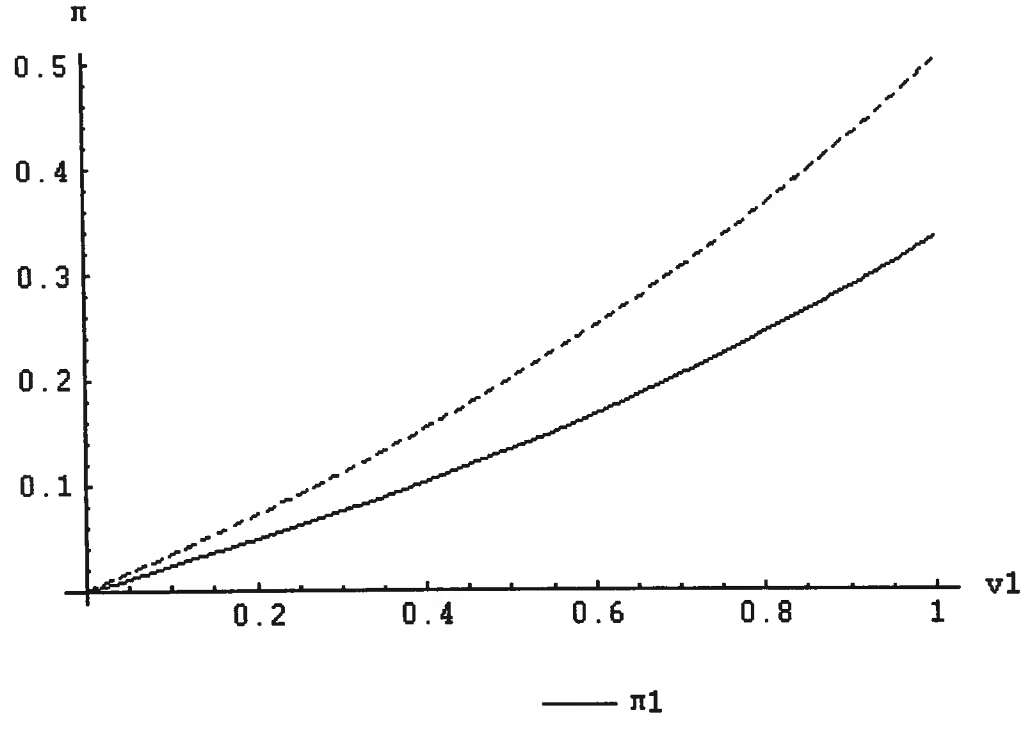


FIGURE 2

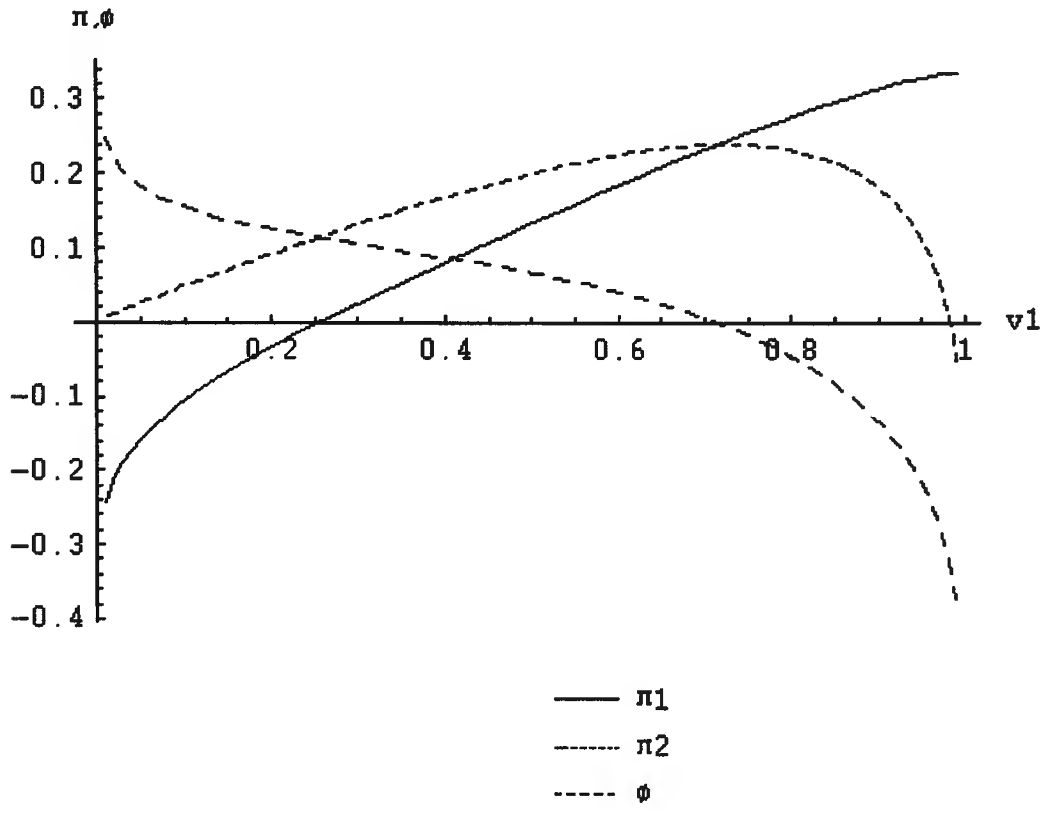


FIGURE 3

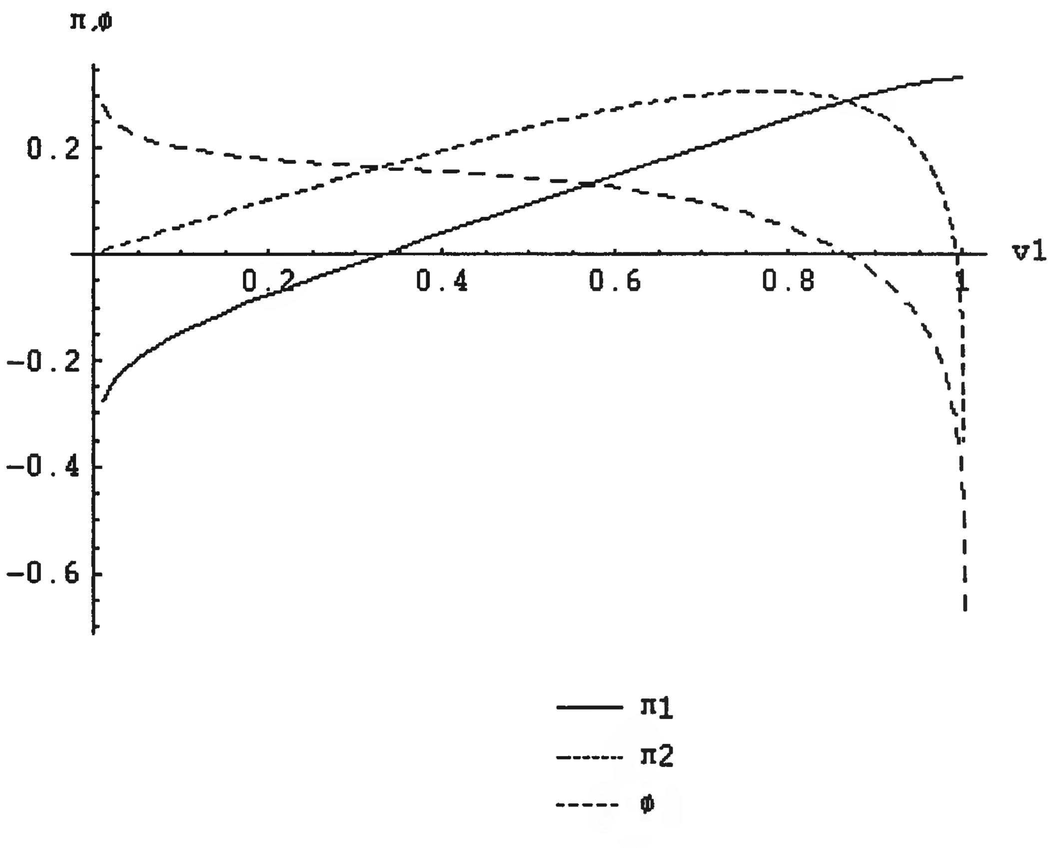
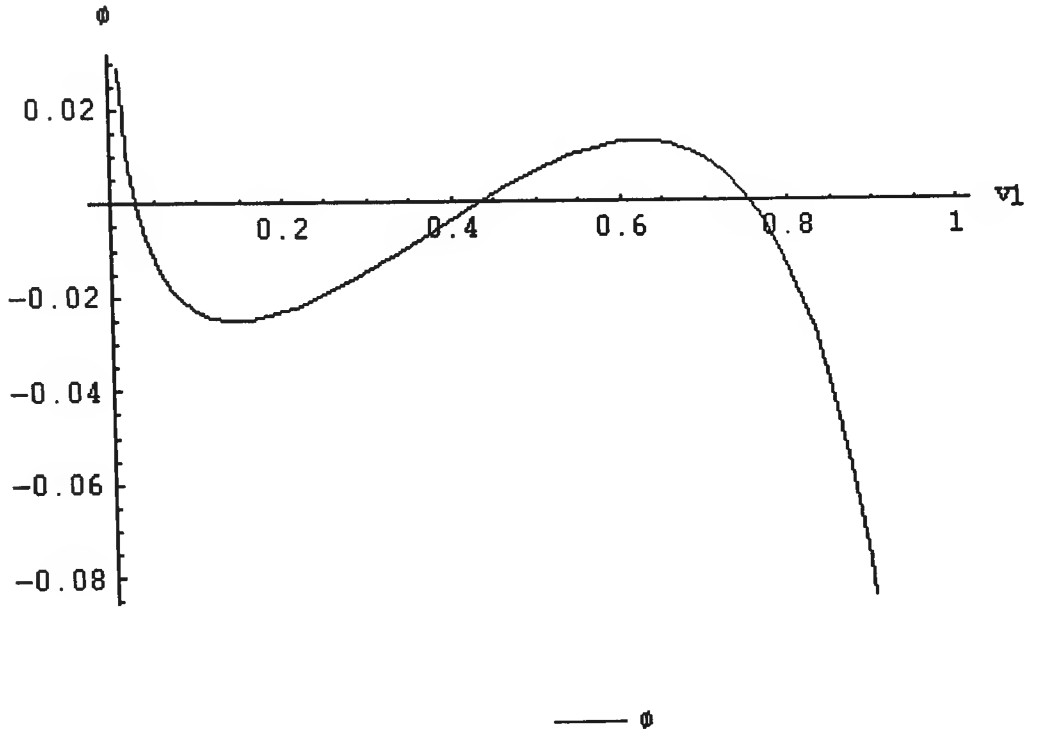


FIGURE 4



For valuable comments the authors thank Samuel Pessoa and two anonymous referees. The usual disclaimer applies.

(Recebido em novembro de 2000. Aceito para publicação em fevereiro de 2001.)