Forecasting Brazilian Output and its Turning Points in the Presence of Breaks: A Comparison of Linear and Nonlinear Models

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RESUMO

Este artigo compara as habilidades preditivas de modelos lineares e não-lineares, com quebras estruturais, nas previsões da taxa de crescimento do PIB real do Brasil. Os modelos com mudanças de regime markovianas, propostos por Hamilton (1989) e generalizados por Lam (1990), são estimados para dados trimestrais de 1975:1 a 2000:2. Os modelos são estimados permitindo quebras estruturais durante os planos Collor. As probabilidades de recessão dos modelos são utilizadas para analisar o ciclo de negócios brasileiro. A capacidade de previsão da taxa de crescimento do PIB fora e dentro da amostra desses modelos é comparada com modelos lineares e com uma regra não-parametrizada. Os resultados indicam que os modelos não-lineares são os que apresentam o melhor desempenho preditivo quando comparados com modelos lineares. A inclusão de quebras estruturais é importante para a representação do ciclo de negócios no Brasil, além de levar a um desempenho de previsão consideravelmente melhor do que os modelos sem intervenção, dentro e fora de amostra.

PALAVRAS-CHAVE

previsão, ciclo de negócios, não-lineares, quebra estrutural, mudança markoviana

ABSTRACT

This paper compares the forecasting performance of linear and nonlinear models under the presence of structural breaks for the Brazilian real GDP growth. The Markov switching models proposed by Hamilton (1989) and its generalized version by Lam (1990) are applied to quarterly GDP from 1975:1 to 2000:2 allowing for breaks at the Collor Plans. The probabilities of recessions are used to analyze the Brazilian business cycle. The in-sample and out-of-sample forecasting ability of growth rates of GDP of each model is compared with linear specifications and with a non-parametric rule. We find that the nonlinear models display a better forecasting performance than linear models. The specifications with the presence of structural breaks are important in obtaining a representation of the Brazilian business cycle and their inclusion improves considerably the models forecasting performance within and out-of-sample.

KEY WORDS

forecast, business cycle, nonlinearities, structural breaks, Markov switching

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INTRODUCTION

The increasing global economic integration and intense volatility in emerging market economies in recent years have re-emphasized the importance of forecasting fundamentals in developing countries, and in particular, gauging the potential of future economic recessions. Recently, the currency crisis in Argentina has raised strong interest in the potential economic vulnerability of neighboring countries, especially of its main trading partner, Brazil.

Nevertheless, the task of forecasting emerging market economies has proven to be a special difficult one, given the great instability in these economies. In particular, models that do not take into account changes in the dynamics of these economies in form of structural breaks may perform poorly out-of-sample. This paper examines the performance of several models in forecasting Brazilian output when structural breaks are explicitly taken into account. First, we examine whether nonlinear time series models produce short run and long run forecasts that improve upon linear models. Second, we compare whether there are gains in endogenously modeling structural breaks to produce out-of-sample forecasts. We conduct an examination of various forecast horizons at one through eight-quarter ahead for the rate of growth of real Brazilian GDP. The predictions are based on recursively estimating the models using data revised solely through the date of each forecast.

Linear models have been widely applied in earlier forecasting literature. However, these models have been used to generate forecasts of the growth rate of output rather than forecasts of nonlinear events such as a turning point, that is, the beginning or end of an economic recession. Generally the filters used to extract turning point forecasts from a linear model require the use of ex-post data. This paper uses two classes of Markov switching models, which directly provide current turning point forecasts in addition to predictions of GDP growth.

Recently, a number of studies has examined the forecasting performance of nonlinear and linear models, including Weigand and Gershenfeld (1994), Hess and Iwata (1997), Stock and Watson (1998), and Camacho and Perez-Quiros (2000), among others. These authors detect nonlinearities in several macroeconomic time series with conflicting results with respect to the models' forecasting performance. As Camacho and Perez-Quiros (2000) conclude for the U.S. economy, we find that nonlinear switching specifications that take into account structural breaks in the Brazilian economy yield better forecasts than linear models of GDP growth, especially at longer horizons. In addition, nonlinear models replicate more accurately Brazilian business cycle features.

We compare our results with a non-parametric rule to determine turning points developed by Bry-Boschan (BB 1971). We find that the several estimated Markov switching models with breaks yield closer turning points to each other and to the ones obtained from BB routine than the models without intervention. In fact, models without intervention yield several extra recessions, indicating that the introduction of intervention improves somewhat the models' forecasting performance.

The remainder of this paper is organized as follows. The forecasting models are presented in section 1. The algorithm used to estimate the Markov switching models and their differences are described in the Appendix. Section 2 examines the major structural break in the Brazilian economy due to Collor stabilization Plan implemented in 1990-1992. The results are presented and discussed in section 3, and conclusions are summarized in the last section.

1. THE MODELS AND THE ESTIMATION METHODS

1.1 Hamilton's Markov Switching Model (MS)

Hamilton (1989) models the log of GDP, y_t, as divided into a trend, n_t, and a gaussian cyclical component, z_t:

$$y_t = n_t + z_t \tag{1}$$

$$n_t = n_{t-1} + \alpha_0(1-S_t) + \alpha_1 S_t \tag{2}$$

$$\phi(L)(1-L)z_t = \varepsilon_t \tag{3}$$

where $\varepsilon_t \sim$ iid $N(0, \sigma^2)$, ε_t is independent on $n_{t+k}\ \forall k$, and S_t is a latent first-order Markov chain. The drift switches between two states: it takes the value of α_0 when the economy is in an expansion ($s_t = 0$) and α_1 when the economy is in a recession ($s_t = 1$). The changes in regimes are ruled by the transition probabilities $p_{ij} = prob[s_t = j | s_{t-1} = i]$ where $\sum_{j=0}^{1} p_{ij} = 1$, i, j = 0,1.

In this model, both n_t and z_t display unit roots and the roots of $\phi(L) = 0$ lie outside the unity circle. Hence, the cyclical component follows a zero mean ARIMA(r, 1, 0) process:

$$z_t - z_{t-1} = \phi_1(z_{t-1} - z_{t-2}) + \phi_2(z_{t-2} - z_{t-3}) + \ldots + \phi_r(z_{t-r} - z_{t-r-1}) + \varepsilon_t \tag{4}$$

Taking the first difference of (1) we get:

$$\Delta y_t = \mu_{st} + \phi_1(z_{t-1} - z_{t-2}) + \phi_2(z_{t-2} - z_{t-3}) + \ldots + \phi_r(z_{t-r} - z_{t-r-1}) + \varepsilon_t \qquad (5)$$

where $\Delta = 1\text{-}L$. and $\mu_{st} = \alpha_0(1\text{-}S_t) + \alpha_1 S_t$.

1.2 Lam's Markov Switching Model (MSG)

Lam (1990) suggests a modification of Hamilton's model that has important implications for the characterization of output trend and cycle. In particular, Lam decomposes the log of GDP into a trend n_t and a cyclical component z_t, where only the trend displays a unit root:

$$y_t = n_t + z_t \qquad (6)$$

$$n_t = n_{t-1} + \alpha_0(1\text{-}S_t) + \alpha_1 S_t \qquad (7)$$

That is, the autoregressive process z_t is now given by:

$$\phi(L)z_t = \varepsilon_t \qquad (8)$$

where $\varepsilon_t \sim$ iid $N(0, \sigma^2)$. Taking the first difference of (6) we get:

$$\Delta y_t = \mu_{st} + z_t - z_{t-1} \qquad (9)$$

where $\mu_{st} = \alpha_0(1\text{-}S_t) + \alpha_1 S_t$. This model allows for both temporary and permanent shocks – the roots of $\phi(L)=0$ are outside the unity circle, which implies that z_t can be interpreted as the transitory deviations of y_t from its long run trend n_t. Therefore, this model can capture both short run pulse breaks and long run level breaks in the trend of Brazilian GDP. On the other hand, since in Hamilton's model both the cyclical component and the trend present unit roots, all shocks to output are permanent.

Both models require different nonlinear filters to be estimated. A detailed description of Hamilton and Lam filter can be found in Hamilton (1989) and in Lam (1990), respectively. The filter used to estimate Lam's model involves substantial more computation than Hamilton's algorithm for two reasons. First, in the calculation of the error the states for each observation include all the history of the Markov process, which is treated as an additional variable. Second, the initial value of the autoregressive component is treated as an additional free parameter to be estimated. The Appendix presents a brief description of both filters.

2. STRUCTURAL BREAKS AND INTERVENTION

Markov switching models have been extensively used to represent cyclical changes or structural breaks in the economy. Hamilton (1989) applied this model to the quarterly change in the log of U.S. real GNP from 1952:2 to 1984:4, assuming that the cyclical component follows an AR(4) process. The estimated Markov states obtained were closely associated with the U.S. expansions and recessions as dated by the NBER.

More recently, McConnell and Perez-Quiros (2000) have found evidence of a structural break in the volatility of U.S. GDP growth towards stabilization in the first quarter of 1984. They show that one implication of the break is that the smoothed probabilities miss the 1990 U.S. recession when more recent data are used. There are different ways to handle the problem of structural breaks. McConnell and Perez-Quiros suggest augmenting Hamilton's model by allowing the residual variance to switch between two regimes, and letting the mean growth rate vary depending on the state of the variance.[1] The resulting estimated smoothed probabilities of the augmented model capture the 1990-1991 recession. Notice that Hamilton's model decomposes the log of GDP into the sum of a trend and a cycle, each of which presents unit roots processes that are not identifiable from each other. Thus, in the presence of a structural break, both terms capture jointly the business cycle component and the volatility break. McConnell and Perez-Quiros' model identifies breaks in the variance from breaks in the mean by allowing each to follow different and dependent Markov processes. Thus, while the Markov chain for the variance captures the break in 1984, the Markov states for the mean capture the business cycle component for the full sample.[2]

Lima and Domingues (2000) model the change in the log of Brazilian GDP as a hidden Markov chain with an AR(4) component. Alternatively, Chauvet (2002a and 2002b) model the change in the log of Brazilian and U.S. GDP, respectively, as a hidden Markov chain with no autoregressive component. This specification captures business cycle features of these economies regardless of the presence of structural breaks in the mean or variance of output. Several authors such as McConnell and Perez-Quiros (2000), Harding and Pagan (2001) or Albert and Chib (1993), among others, have found that the GDP growth in the U.S. and other countries is better modeled as a low autoregressive process. In particular, Albert and Chib use Bayesian methods to estimate Hamilton's model and find that the best specification for changes in GDP is an AR(0) process, as the autoregressive coefficients are not statistically significant. This

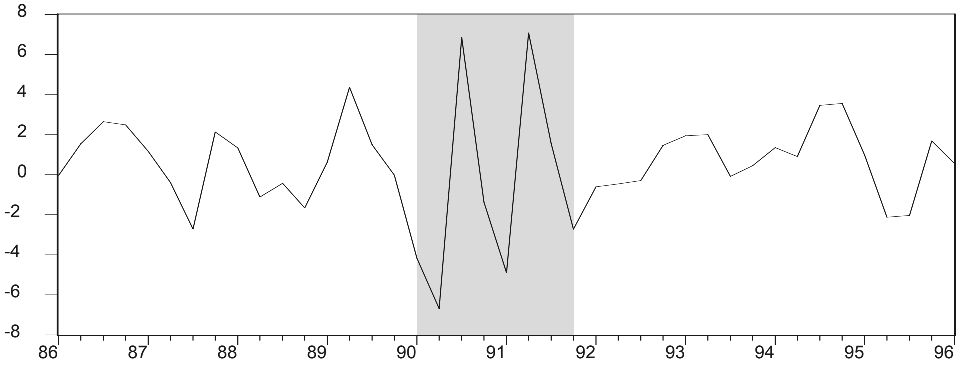
1 This amounts in estimating four mean growth rates: low growth under high and low volatility states, and high growth under high and low volatility states.

2 The smoothed probabilities obtained from a model with switching variance and constant mean captures the break in 1984, while a model with switching mean and constant variance captures the business cycle phases up to the breakpoint only (see McConnell and Perez-Quiros, 2000).

finding is perhaps due to the presence of structural breaks in the stochastic process of GDP.

The Brazilian economy also displays several structural breaks. In particular, the series of stabilization plans and changes in policy regime in the last two decades resulted in several breaks in the Brazilian GDP, especially in the early 1990s due to the Collor Plan. Figure 1 shows the Brazilian GDP[3] around the period of implementation of the Collor Stabilization Plan. As it can be observed, the economy faced a period of large swings for 5 quarters. Upon introduction of the Plan in the second quarter of 1990, GDP decreased at a quarterly average rate of –6.7%. In the third quarter GDP experienced an abrupt increase of 6.8%, only to fall again in the two following quarters by 1.4% and 4.9%, respectively. In the second quarter of 1992 the economy again underwent a large growth rate of 7.1%.

FIGURE 1 – BRAZILIAN GDP GROWTH AND THE COLLOR PLAN



These large pulse-breaks in the Brazilian economy cause estimation problems for standard Markov switching models, and the optimization routines frequently converges to a local maximum.[4] If the number of autoregressive terms is not enough, or if they do not display a unit root, then the models and probabilities capture solely the pulse breaks due to the Collor Plan. For example, when the MS specification with AR(1) or AR(2) components (MS-AR(1) or MS-AR(2)) and the MSG specification with different autoregressive components (from MSG-AR(1) to MSG-AR(5)) are ap-

3 The data on real Brazilian GDP were seasonally adjusted using the X-12 method. The series was obtained from IPEA database and is reported in Table 15.

4 The estimation procedure was as follows: first, the MS model was estimated considering an AR(0). Second, the MLE parameters from this model were used to initialize the estimation of the MS-AR(1). Next, the MLE parameters of the MS-AR(1) were used to initialize the MS-AR(2) and so on. The MLE parameters of the MS models were then used to initialize the MSG model.

plied to real Brazilian GDP growth, the filtered and smoothed probabilities of recessions (state 1) increase only around observations between 1990:I to 1991:II (Collor I and Collor II Plans), as illustrated in Figures 3 and 4 (for MS-AR(2) and MSG-AR(3), respectively). That is, without intervention both models capture solely the abrupt pulse breaks experienced by the Brazilian economy during the Collor Plans instead of cyclical economic expansions and contractions.

We estimate, several autoregressive specifications of MS and MSG models without intervention. The models are estimated allowing both mean and variance to switch regimes. However, the specifications allowing only the mean to switch between states do not converge. Overall, the estimates from Lam's model are more stable as the number of lags whereas Hamilton's model present instability with respect to the parameters as the number of lags increase. [5]

Using the likelihood ratio test we find that the best specifications without intervention are an AR(4) process for the MS model (MS-AR(4)) and an AR(2) process for the MSG model (MSG-AR(2)). We have also tested the out-of-sample forecasting performance of several Markov switching models with autoregressive components, comparing them with linear models and with the MS-AR(0) model. The MS-AR(4) gives the best short-run forecasts (1 to 2 steps ahead). The linear AR(3) model does better than the other models for longer forecasts.

Models With Intervention

We introduce interventions in the models for two reasons. First, the Collor Plan has engendered strong real effects in the economy, which influence the specification of the MS and MSG models. In particular, when the models are estimated without intervention there is a tendency for the filtered probabilities to concentrate around this period.[6] Second, without explicitly modeling the breaks, the MSG model does not capture the Brazilian business cycle. As it will be shown, interventions yield estimated probabilities that characterize recessions and expansions rather than solely the Collor Plan, and increase the forecasting ability of the MS and MSG models.

We estimate the models under several alternative interventions in the 1990:1-1991:2 period in order to overcome the problem of structural breaks, such as specifications in which the drift parameters are allowed to take different values during Collor I and II stabilization plans. We also estimate the model treating the observations of Collor

5 For both models, the likelihood function increases as the probability of recessions converges to a very small value, capturing the break related to the Collor Plans instead of expansions and recessions in the Brazilian output.

6 This is the case for the MS-AR(1) and MS-AR(2) models and all estimated MSG specifications.

I and II plans as outliers. We report the results for only the two interventions that were successful in characterizing the Brazilian business cycle.[7] The first intervention is modeled as the sum of an additional parameter δ_i during the Collor Plan (Intervention Type 1):

$$\mu_{st} = \mu_0(1\text{-}S_t) + \mu_1 S_t + \delta_I \qquad \text{for } i = 1990{:}1, \ldots, 1991{:}2$$

$$\mu_{st} = \mu_0(1\text{-}S_t) + \mu_1 S_t \qquad \text{otherwise}$$

The second intervention considers the period of the Collor Plans (90.1 to 91.2) as outliers (Intervention Type 2). One advantage of this method is that the intervention capturing the break is not restricted to be present only in the trend component.

3. RESULTS

There is no convergence problem for the models with intervention types 1 and 2 and the regime switching parameters are significant at all levels. Compared with the alternative specifications, these interventions are the ones that yield the most reasonable results. The results for the best models are discussed below.

3.1 Results for Selected Models

Based on the likelihood ratio test, Theil-U statistic, and the filtered probabilities, the models that present the best fit to the Brazilian business cycle are the MS-AR(2) and MSG-AR(2) with interventions of type 1 and 2. Table 1 shows the results for the MS and MSG models with intervention of type 1, while Table 2 reports the results with intervention type 2. Since the results are similar for both interventions, we choose to report the ones for intervention type 2.

The estimated parameters from both models are very similar and the sample identifies two significant states for the Brazilian economy. Table 3 shows a summary of these results. The MS-AR(2) model estimates that the economy grows at a negative average rate of around 1.4% per quarter (-5.6% a year) during recessions (state 1) and an average rate of 1.6% per quarter (6.4% a year) during expansions (state 0). For the MSG-AR(2) model the economy grows at an average negative rate of around 1.5% per quarter (-6% a year) during recessions and at a rate of 1.7% per quarter (6.8% a year) during expansions. In general, recessions in Brazil last a short time, averaging between

7 The results for the other interventions are available from the authors upon request.

2 and 3 quarters for both models, while expansions last twice as long. In particular, the MS model estimates that periods of positive growth last on average between 6 and 7 quarters (p_{00}=0.85), while for the MSG model the duration of expansions is around 4 or 5 quarters (p_{00}=0.77). Therefore, these models predict that the length of the Brazilian business cycle is between 2 and 3 years. This short duration of the Brazilian business cycle is a consequence of the economic instability and turbulence due to the hyperinflationary process in the 1980s and the implementation of several stabilization plans in the last two decades. These results are very similar to those obtained for Brazil in Chauvet (2002a) and Mejia-Reyes (1999). In addition, Mejia-Reyes finds that several other Latin American countries present these same business cycle features.

The filtered and smoothed probabilities for the selected models are plotted in Figures 5 to 8. The shaded areas correspond to periods of recessions in Brazil, which were dated according to the following criteria:

Definition 1: A business cycle *peak* is said to occur in month *t+1* if the economy was in an expansion in month *t* and:

a) Rule 1: $P(S_{t+1} = 1) \geq 0.5$ or

b) Rule 2: $P(S_{t+i} = 1) \geq 0.5$ for i = 1 and 2.

Definition 2: A business cycle *trough* is said to occur in month *t+1* if the economy was in a recession in month *t* and:

a) Rule 1: $P(S_{t+1} = 1) < 0.5$ or

b) Rule 2: $P(S_{t+i} = 1) < 0.5$ for i = 1 and 2.

Several results stand out from the probability inferences. First, the filtered and smoothed probabilities are very similar, which points out to the stability of the recursive one-step-ahead estimation (filtered probabilities) compared to the estimation using the whole sample (smoothed probabilities). Second, the probabilities from the MS and the MSG models are also very similar, capturing the same features and phases of the Brazilian business cycles.

Using rule 1 to date business cycles described above, the Brazilian economy experienced ten downturns between 1980 and 2000. However, some of these contractions were very short-lived, lasting only one quarter (e.g.: the low growth phase in 1984 and the expansion in 1998). If we consider recessions as periods of negative growth with a minimum duration of 6 months (rule 2), the downturns in 1982-83, 1983-84 would be considered as one longer recession rather than a double dip. This is also the case for the downturns in 1997-1998. Under rule 2 for dating business cycle phases, the Brazilian economy experienced eight recessions in the last two decades according to model MS-AR(2) and nine recessions according to MSG-AR(2) (Table 6). These results are corroborated by the findings in Mejia-Reyes (1999)[8] and Chauvet (2002a).

We compare our results with a non-parametric rule developed by Bry and Boschan's (BB 1971). The BB procedure can be applied to a single seasonally adjusted monthly time series. It entails the extraction of points identified as local maxima/minima satisfying the following criteria: a) extreme values are identified and discarded; b) the minimum phase duration is 5 months; c) the minimum cycle duration is 15 months; d) if flat or double turning points are found in the period, the last turning point is selected.

We have followed Monch and Uhlig (2005)'s modification of the original BB routine[9] with a criterion for amplitude/phase length such that it eliminates business cycle expansions that are short and flat, and some of the restrictive symmetries imposed across recession and expansion phases.[10]

We apply BB algorithm to the monthly GDP series from Table 16.[11] The recession dates obtained from the smoothed probabilities of the Markov switching models applied to quarterly GDP and from BB routine are reported in Tables 5 and 6.

8 The results are consistent with the ones obtained by this author up to the last year of its estimation for Brazil (1995).

9 Originally, the BB routine consists of: 1) elimination of extreme values; 2) determination of cycles in 12-month moving average with identification of points higher or lower than 5 months on either side; and selection of highest multiple peaks to warranty alternation of turns; 3) determination of corresponding turns in a Spencer curve with identification of points higher or lower than 5 months of selected turns in the 12-term moving average; forcing minimum cycle duration of 15 months by eliminating lower peaks and higer troughs of shorter cycles; 4) determination of turning point in a short-term moving average depending on months of cyclical dominance; with identification of highest or lowest value within 5 months of the selected turn in the Spencer curve; 5) determination of turning points in the original series: identification of the highest or lowest value within 4 months, or the months of cyclical dominance, whichever is larger, of the selected turn in the short-term moving average; elimination of turning points within six months of beginning and end of series; elimination of troughs/peaks at both ends of series that are lower/higher than values closer to the end; elimination of cycles whose duration is less than 15 months; elimination of phases whose duration is less than 5 months; 6) The final turning points are then found.

10 Other modifications followed by these authors include determination of cycles with moving average of 9 months instead to avoid cycles too long, and setting the months of cycle dominance to 3 for determining turning points in the non-smoothed series. We use Uhlig's Matlab toolkit aviable in the site http://www.wiwi.hu-berlin.de/wplo/.

11 We use a monthly GDP series present in Table 16 for dating with Bry-Bochan procedure. Details on the construction of the data are available from the authors upon request.

3.2 Comparison Between the MS and MSG Models

The MSG-AR(3) model nests the models selected as presenting the best fit to the Brazilian business cycle, the MS-AR(2) and the MSG-AR(2). The likelihood ratio used to test the MSG-AR(2) model against the MSG-AR(3) model has a standard asymptotic distribution, $\chi^2(1)$, and can be easily calculated using the likelihood values presented in Table 2. Given the likelihood ratio value of 2.584, we cannot reject that the MSG-AR(2) model fits the data better than the MSG-AR(3) model. If we can reject the MS-AR(2) model compared to the MSG-AR(3) model than by transition we could conclude that the MSG-AR(2) model fits the data better than the MS-AR(2) model. However, the likelihood ratio of this last test does not have a standard distribution and we report below Monte Carlo simulations used to implement the test.

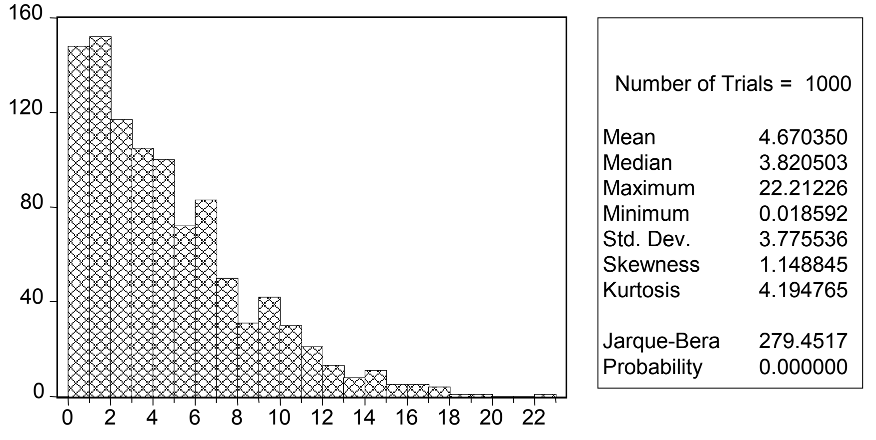
We have generated 1000 trials simulating the MS-AR(2) model under intervention type 2 – each with the same number of observations as our sample size. For each trial both models (MS-AR(2) and MSG-AR(3)) were estimated and the likelihood ratio statistic was computed. Figure 2 below shows the histogram of the likelihood ratio statistic obtained for these 1000 trials. The null hypothesis of the test is the MS-AR(2) estimated under intervention type 2, and the alternative hypothesis is the MSG-AR(3) specification.

In the Monte Carlo simulations the likelihood ratio statistic computed at each trial is less or equal to 11.94 for 95% of the trials, whereas the estimated likelihood ratio computed using the likelihood values of Table 2 is equal to 16.53. The results indicate that the null is rejected at a level of significance smaller than 5%.[12] Therefore, we can conclude that the MSG-AR(3) model fits the data better.

We also test the MS-AR(0) model against the MSG-AR(3) model. The likelihood ratio statistic of the test has a standard asymptotic distribution, $\chi^2(4)$, and can be computed using the likelihood values presented in Table 2. The estimated likelihood ratio statistic is equal to 22.082. Therefore, the MS-AR(0) specification is rejected at a level of significance smaller than 1%.

12 Note that the MSG-AR(3) model has two more parameters than the MS-AR(2) model. If we were to apply the standard critical value it would have been equal to 5.99 (c2(2)) instead of 11.94.

FIGURE 2 – HISTOGRAM OF THE LIKELIHOOD RATIO(NULL:MS-AR(2), ALTERNATIVE:MSG-AR(3))



3.3 Average Out-of-Sample Forecasting Performance

This section compares the out-of-sample forecasting performance of several Markov switching models with autoregressive components with linear models and the MS-AR(0) model. Two linear models for changes in GDP were estimated for comparison with the Markov switching models: an AR(3) and an ARMA(1,1) model.[13] All models were estimated from 1976:2 up to 1992:1, and then recursively re-estimated for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2 to generate the out-of-sample forecasts. Appendix B shows how these forecasts were calculated.

Results

We use as a statistic to compare any two models the mean squared forecast error (MSE) of one of the models divided by the MSE of the other model. We also report standard errors for these relative MSEs.[14] The standard errors are heteroskedastic and autocorrelated consistent (HAC) robust and were estimated using a Bartlett kernel

13 The identification of the ARMA model was implemented using AIC and SBC criteria. In addition, given that structural breaks generally lead to serial correlation in the residuals, Durbin-Watson test was used to test whether the residuals of the selected model are white noise. The identification was implemented considering or not dummies for the period between 1990.1 a 1991.2.

14 The standard errors were calculated using the Gauss routine made available by Mark W. Watson in his web site http://www.wws.princeton.edu/~mwatson/

with the number of lags, for each step-ahead, equal to the number of computed forecast errors.[15]

Table 7 shows the root mean squared forecast error (RMSE) of the linear AR(3) model and the relative MSE (to the AR(3) model) of several Markov switching models, with interventions type 1 and 2, for forecasts from 1 to 8 quarters ahead. The model with the smallest relative MSE, for forecasts from 2 to 7 quarters ahead and for both types of intervention is the MS-AR(2). Almost all the relative MSEs of the MS-AR(2) model are smaller than one with the exception of the 8-quarter-ahead forecast. Nevertheless, they are significantly smaller than one only for intervention type 2 and for forecasts from 4 to 6 quarters ahead. The ARMA(1,1) model beats the AR(3) model for forecasts from 1 to 2 steps-ahead. The "No Change" model, where the forecast of GDP growth is constant end equal to zero, has the worst forecasting ability for all steps-ahead

Table 8 compares the same models with the ARMA(1,1) model. It shows that the relative MSEs of the MS-AR(2) model are smaller than one for forecasts from 3 steps-ahead and on. Nevertheless, they are significantly smaller than one for forecasts 4 and 6 steps-ahead and for intervention type 2. The AR(3) model forecasts significantly better than the ARMA(1,1) only 4 quarters ahead and for both types of intervention.

Table 9 reports the MSE of the models relative to the MSE of the MS-AR(0) model. It shows that the MS-AR(2) model has a relative MSE significantly smaller than one for almost all steps-ahead and for both types of intervention. The same is true for the AR(3) and ARMA(1,1) models for short run forecasts, 1 to 2 quarters ahead.

Linear Versus Nonlinear Models

For one-quarter-ahead forecast, the ARMA (1,1) model presents the lowest relative MSE. On the other hand, the Markov switching models present the best forecasting performance for 2-quarter-ahead and on. In particular, the MS-AR(2) is the best in forecasting 2 to 7 quarter-ahead. Thus, for forecasts of the annual growth of real GDP, the MS-AR(2) model is the one with the most accurate prediction in this out-of-sample forecasting test.

Intervention Versus Non-intervention

Tables 10 and 11 show the relative out-of-sample performance of several Markov switching models, for both types of intervention, compared to their counterparts without

15 See West (1996) for an asymptotic justification for the procedure adopted to calculate the standard errors for recursively estimated models.

intervention. Table 10 shows the results for Hamilton's models (MS-AR(0), MS-AR(2) and MS-AR(4)) and Table 11 for Lam's models (MSG-AR(1), MSG-AR(2) and MSG-AR(3)). Most of the relative MSEs are smaller than one indicating that the interventions have improved the models' forecasting ability. The MSG the MS-AR(2) models exhibit the smallest relative MSE overall. This is not surprising given that the probability of recession from these models without intervention concentrate around the period of the Collor plans. Nevertheless, because the standard errors are relatively high for most models, the relative MSEs are in general not significantly smaller than one. However, the greatest advantage of introducing interventions is that they characterize the Brazilian business cycle without loss of forecasting ability.

These findings corroborate the evidence obtained by several authors in that modeling nonlinearities underlying GDP growth improves its forecasting performance. This is particularly true for the case of Markov switching models that take into account abrupt changes and asymmetries of business cycle phases.

Recent Forecast Performance

As an illustration of the recent performance in forecasting GDP growth, a second out-of-sample test was performed. The models were estimated from 1976:2 up to 2000:2, and then the parameters were used to predict the annual rate of growth of GDP from 2000:3 to 2001:4. Table 14 reports the out-of-sample forecasts of the annual rate of growth of real GDP for 2000:3-2001:4. As it can be observed, the MS-AR(2) and the AR(3) models in this period yield the closest forecast of changes in GDP compared to the alternative models. The best overall model, for intervention type 2, is the MS-AR(2).

3.4 Out-of-Sample Turning Point Forecasting Performance

This section compares the out-of-sample turning point forecasts of several Markov-switching models. The out-of-sample forecast is obtained by recursively re-estimating the model parameters – with the exception of the parameters that enters the numerical optimization routine which were estimated with data from the beginning of the sample until the second semester of 2000 – and computing sequentially the one and two quarter-ahead forecasts of the recession probabilities, from the last quarter of 1994 until the end of the sample. The peaks are then dated following the criteria in definition 1 – rule 2, which take into account a minimum phase duration of two quarters, as described in section 3.1. That is, at each sample point starting in the first quarter of 1995 the model signals the beginning of a recession (a peak) if the probabilities of recession are equal or greater than 50% for both one and two-step ahead forecasts, that is, $E_t[P(S_{t+1}=1)] \geq 0.5$ and $E_t [P(S_{t+2}=1)] \geq 0.5$).

We compare our results with the peak dating obtained by the Bry and Boschan's algorithm (BB) from monthly GDP, as explained in section 3.1. We compare the methods in two ways: 1) when the model probabilities do not signal a peak, using the procedure in step 1, but the BB algorithm does; 2) when the model probabilities detect a peak but the BB algorithm does not. The results are summarized in Tables 12 and 13. Notice that the Markov switching models are being evaluated out-of-sample, whereas the BB routing uses the full sample, relying on ex-post data. However, the results can serve as a base for comparison of forecast performances of different models.

From 1994 until the end of the sample, BB signals a total of 4 recessions. Table 12 compares BB results to those of the Markov switching models. With the exception of the MS-AR(4), without intervention, none of the models captures any recession signalized by BB. On the other hand, the MS-AR(4) model identifies 22 extra peaks that the other models and the BB algorithm don't. Thus, models without intervention tend to differ from BB, and MS models with intervention, in that they signal several extra recessions. However, models with intervention tend to not signal recessions.

Table 4 contains the unconditional probabilities of states 1 and 2. It can be verified there that models without intervention have a considerably higher and unrealistic unconditional probability of recession, generating a large forecast bias towards wrongly signaling peaks. The results presented in Tables 12 do not allow us to come to a definite conclusion when comparing the forecasting ability of the different models. So we decided to construct another measure of forecast accuracy, reported in Table 13, computing deviations of probability of recession forecasts. The deviations are based on the definition of peaks described in **definition 1- rule 2** of section 3.1. The new statistic is constructed as follows:

If a *peak* is not detected by the model probabilities at period t but it is by BB, then the deviation is equal to $\sum_{j=1}^{2}(0.5 - p(t+j))$, where p(t+j) is the j-step-ahead probability of recession forecast at period t. Otherwise, if the peak is detected the deviation is equal to zero.

If a *peak* is detected in the model probabilities at period t and it is not by BB, then the deviation is equal to $\sum_{j=1}^{2}(p(t+j) - 0.5)$, where p(t+j) is the j-step-ahead probability of recession forecast at period t. Otherwise, if the *peak* is not detected the deviation is equal to zero.

Table 13 shows that models without intervention have larger deviations from BB and from other models due to the detection of extra *peaks*. However, due to the abnormally large unconditional probability of recession, they have smaller deviations at recessions signalized by the BB algorithm. Adding up all deviations, models with intervention yield better forecasts of the future state of the economy than models without intervention. Thus, the introduction of interventions seems to improve somewhat model's forecasting performance.

The model with the closest result to BB routine is the MSG-AR(2) with intervention. This model is also the one that best fits the data in-sample, as reported in section 3.2.

CONCLUSIONS

This paper fits Hamilton and Lam's models to quarterly Brazilian GDP series for the period from 1975:1 to 2000:2, allowing for breaks at the Collor Plans. We find that the Hamilton's Markov switching model and Lam's model both following an AR(2) process (MS-AR(2) and MSG-AR(2)) present the best fit to the Brazilian business cycle under the two different types of interventions considered.

The sample identifies two significant states for the Brazilian economy, representing recession and expansion phases. For both models, the economy grows at a negative rate of around 1.4-1.5% per quarter during recessions in state 1, and at a rate of 1.6-1.7% per quarter during expansions. In general, recessions in Brazil last a short time, averaging between 2 and 3 quarters for both models. Expansions last twice as long. In particular, the Markov switching models estimate that periods of expansion in Brazil last on average between 4 and 7 quarters.

We compare the out-of-sample performance of several Markov switching models to linear models such as ARMA(1,1) and AR(3) models. The models were recursively reestimated from 1992:1 until the last quarter of the sample to generate out-of-sample forecasts. Overall, the MS-AR(2) model displays the best forecasting performance especially at longer horizons, with the smallest relative MSE for two to seven quarters ahead. This finding corroborates the evidence obtained by several authors that modeling nonlinearities underlying changes in GDP growth improves forecasting performance. This is particularly true for the case of Markov switching models that take into account asymmetries of business cycle phases.

We also compare the out-of-sample performance of several Markov switching models estimated under two types of intervention with their counterparts without intervention. The results indicate that the interventions improve considerably the models' forecasting ability. Overall, the MSG models and the MS-AR(2) model yield the smallest relative MSE. The greatest advantage of introducing interventions is that they better characterize the Brazilian business cycle without loss of forecasting ability.

We compare our results with the peak dates obtained applying Bry and Boschan's algorithm to monthly GDP. The models with intervention yield closer turning points to each other and to the ones from BB routine than the models without intervention. In fact, the models without intervention tend to signal many extra recessions, and display a significantly higher unconditional probability of recession. Thus, the introduction of interventions improves somewhat the models' forecasting performance.

Finally, as an illustration of the recent performance in forecasting GDP growth, the models were estimated from 1976:2 up to 2000:2, and then used to predict the annual rate of growth of GDP from 2000:3 to 2001:4. Once again, we find that the best overall model was the MS-AR(2) model with intervention type 2.

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TABLE 1 – HAMILTON'S MODEL (MS) AND LAM'S MODELS (MSG) UNDER DIFFERENT SPECIFICATIONS AND INTERVENTION TYPE 1

	Hamilton's Model (MS)										Lam's Model (MSG)					
	AR(0)	AR(0)	AR(1)	AR(0)	AR(1)	AR(2)	AR(0)	AR(1)	AR(2)	AR(3)	AR(1)	AR(1)	AR(2)	AR(1)	AR(2)	AR(3)
Num. Obs.	101	100	100	99	99	99	98	98	98	98	100	99	99	98	98	98
LOG(L(?))	271.113	268.444	268.447	265.130	265.139	269.050	261.864	261.915	265.775	266.401	274.177	272.058	277.074	270.144	275.604	277.066
P_{00}	0.875	0.876	0.877	0.874	0.875	0.853	0.874	0.876	0.854	0.851	0.853	0.853	0.835	0.830	0.833	0.834
	(0.060)	(0.060)	(0.061)	(0.062)	(0.064)	(0.047)	(0.063)	(0.068)	(0.048)	(0.052)	(0.044)	(0.044)	(0.045)	(0.048)	(0.045)	(0.045)
P_{11}	0.503	0.500	0.498	0.499	0.496	0.571	0.498	0.486	0.570	0.521	0.567	0.572	0.552	0.540	0.552	0.545
	(0.143)	(0.145)	(0.147)	(0.145)	(0.151)	(0.107)	(0.147)	(0.159)	(0.109)	(0.122)	(0.109)	(0.107)	(0.102)	(0.108)	(0.102)	(0.101)
μ_0	0.015	0.015	0.015	0.015	0.015	0.017	0.015	0.014	0.017	0.016	0.017	0.016	0.017	0.016	0.017	0.017
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.003)	(0.001)	(0.002)	(0.0005)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0004)
μ_1	-0.016	-0.016	-0.016	-0.016	-0.016	-0.014	-0.016	-0.016	-0.014	-0.015	-0.015	-0.015	-0.015	-0.015	-0.015	-0.015
	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.002)	(0.005)	(0.005)	(0.002)	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
σ_0	0.013	0.013	0.013	0.013	0.013	0.011	0.013	0.013	0.011	0.011	0.010	0.010	0.009	0.010	0.009	0.009
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
ϕ_1	-	-	0.012	-	0.021	-0.201	-	0.053	-0.184	0.140	0.424	0.391	0.555	0.405	0.595	0.603
			(0.110)		(0.176)	(0.147)		(0.172)	(0.165)	(0.233)	(0.107)	(0.107)	(0.108)	(0.099)	(0.116)	(0.106)
ϕ_2	-	-	-	-	-	-0.456	-	-	-0.457	-0.358	-	-	-0.337	-	-0.327	-0.437
						(0.127)			(0.133)	(0.142)			(0.090)		(0.092)	(0.108)
ϕ_3	-	-	-	-	-	-	-	-	-	0.260	-	-	-	-	-	0.189
										(0.166)						(0.102)
Intervention 1	-0.042	-0.042	-0.041	-0.042	-0.041	-0.050	-0.041	-0.041	-0.049	-0.049	-0.046	-0.045	-0.041	-0.045	-0.041	-0.043
	(0.013)	(0.013)	(0.013)	(0.013)	(0.014)	(0.012)	(0.013)	(0.014)	(0.012)	(0.012)	(0.011)	(0.010)	(0.009)	(0.010)	(0.009)	(0.009)
Intervention 2	-0.100	-0.100	-0.100	-0.100	-0.100	-0.109	-0.100	-0.099	-0.109	-0.107	-0.103	-0.102	-0.106	-0.102	-0.106	-0.107
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.010)	(0.011)	(0.010)	(0.010)
Intervention 3	0.059	0.059	0.059	0.059	0.059	0.061	0.059	0.060	0.061	0.063	0.057	0.057	0.054	0.057	0.054	0.057
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.012)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)
Intervention 4	-0.035	-0.034	-0.034	-0.034	-0.034	-0.032	-0.034	-0.034	-0.032	-0.032	-0.037	-0.036	-0.038	-0.036	-0.038	-0.034
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.012)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)
Intervention 5	-0.059	-0.059	-0.059	-0.059	-0.059	-0.064	-0.059	-0.059	-0.064	-0.063	-0.061	-0.061	-0.061	-0.061	-0.061	-0.061
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.010)	(0.011)	(0.010)	(0.010)
Intervention 6	0.047	0.048	0.048	0.048	0.048	0.040	0.048	0.048	0.040	0.042	0.045	0.046	0.048	0.046	0.048	0.045
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.011)	(0.012)	(0.010)	(0.010)	(0.009)	(0.010)	(0.009)	(0.009)
Z_0	-	-	-	-	-	-	-	-	-	-	-0.033	-0.035	-0.063	-0.068	-0.063	-0.066
											(0.006)	(0.006)	(0.004)	(0.006)	(0.004)	(0.005)
Theil-U	0.525	0.525	0.460	0.525	0.458	0.428	0.523	0.455	0.426	0.407	0.515	0.512	0.497	0.510	0.499	0.488

Note: standard deviation in parenthesis.

TABLE 2 – HAMILTON'S MODEL (MS) AND LAM'S MODELS (MSG) UNDER DIFFERENT SPECIFICATIONS AND INTERVENTION TYPE 2

	Hamilton's Model (MS)										Lam's Model (MSG)					
	AR(0)	AR(0)	AR(1)	AR(0)	AR(1)	AR(2)	AR(0)	AR(1)	AR(2)	AR(3)	AR(1)	AR(1)	AR(2)	AR(1)	AR(2)	AR(3)
Num. Obs.	**101**	**100**	**100**	**99**	**99**	**99**	**98**	**98**	**98**	**98**	**100**	**99**	**99**	**98**	**98**	**98**
LOG(L(θ))	**251.295**	**248.634**	**248.660**	**245.374**	**245.415**	**248.116**	**242.165**	**242.275**	**244.941**	**245.864**	**251.327**	**249.011**	**253.387**	**247.737**	**251.914**	**253.206**
P_{00}	**0.864**	**0.865**	**0.868**	**0.862**	**0.865**	**0.849**	**0.862**	**0.863**	**0.848**	**0.846**	**0.779**	**0.777**	**0.768**	**0.770**	**0.767**	**0.769**
	(0.069)	(0.070)	(0.074)	(0.073)	(0.078)	(0.054)	(0.075)	(0.087)	(0.055)	(0.074)	(0.040)	(0.038)	(0.034)	(0.040)	(0.034)	(0.034)
P_{11}	**0.502**	**0.498**	**0.491**	**0.498**	**0.489**	**0.554**	**0.496**	**0.477**	**0.551**	**0.495**	**0.567**	**0.584**	**0.596**	**0.570**	**0.592**	**0.594**
	(0.150)	(0.152)	(0.160)	(0.152)	(0.162)	(0.116)	(0.154)	(0.171)	(0.118)	(0.137)	(0.123)	(0.109)	(0.088)	(0.113)	(0.088)	(0.090)
$\mu 0$	**0.015**	**0.015**	**0.014**	**0.015**	**0.014**	**0.016**	**0.015**	**0.014**	**0.016**	**0.015**	**0.017**	**0.017**	**0.017**	**0.017**	**0.017**	**0.017**
	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.001)	(0.0005)	(0.0004)	(0.0006)	(0.0004)	(0.0005)
$\mu 1$	**-0.015**	**-0.015**	**-0.016**	**-0.015**	**-0.015**	**-0.014**	**-0.015**	**-0.015**	**-0.014**	**-0.016**	**-0.015**	**-0.014**	**-0.015**	**-0.015**	**-0.015**	**-0.015**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.003)	(0.005)	(0.006)	(0.003)	(0.005)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma 0$	**0.013**	**0.013**	**0.013**	**0.013**	**0.013**	**0.012**	**0.013**	**0.014**	**0.012**	**0.012**	**0.010**	**0.010**	**0.009**	**0.010**	**0.009**	**0.009**
	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\phi 1$	-	-	**0.036**	-	**0.047**	**-0.123**	-	**0.080**	**-0.098**	**0.193**	**0.432**	**0.398**	**0.546**	**0.410**	**0.582**	**0.596**
			(0.158)		(0.165)	(0.164)		(0.174)	(0.175)	(0.232)	(0.116)	(0.123)	(0.112)	(0.109)	(0.120)	(0.113)
$\phi 2$	-	-	-	-	-	**-0.383**	-	-	**-0.384**	**-0.303**	-	-	**-0.321**	-	**-0.309**	**-0.420**
						(0.143)			(0.141)	(0.169)			(0.100)		(0.100)	(0.115)
$\phi 3$	-	-	-	-	-	-	-	-	-	**0.275**	-	-	-	-	-	**0.185**
										(0.172)						(0.101)
Z_0	-	-	-	-	-	-	-	-	-	-	**-0.064**	**-0.066**	**-0.063**	**-0.067**	**-0.063**	**-0.065**
											(0.006)	(0.006)	(0.004)	(0.006)	(0.004)	(0.005)
Theil-U	**0.628**	**0.627**	**0.687**	**0.624**	**0.684**	**0.633**	**0.622**	**0.679**	**0.629**	**0.594**	**0.752**	**0.749**	**0.729**	**0.749**	**0.732**	**0.717**

Note: standard deviation in parenthesis.

TABLE 3 – BUSINESS CYCLE FEATURES FOR SELECTED MODELS

		Type 1		Type 2		
		MS-AR(2)	MSG-AR(2)	MS-AR(0)	MS-AR(2)	MSG-AR(2)
Recession	Mean Growth rate	-1.4%	-1.5%	-1.5%	-1.4%	-1.5%
	Duration in quarters	2-3	2-3	1-2	2-3	2-3
Expansion	Mean Growth rate	1.7%	1.7%	1.5%	1.6%	1.7%
	Duration in quarters	6-7	6-7	7-8	6-7	4-5

TABLE 4 – STEADY-STATE PROBABILITIES OF EXPANSION AND RECESSION

		$P(S_t=0)$ Expansion	$P(S_t=1)$ Recession
No Intervention	MS-AR(0)	0.28	0.72
	MS-AR(2)	0.93	0.07
	MS-AR(4)	0.4	0.6
	MSG-AR(2)	0.94	0.06
Intervention	MS-AR(0)	0.79	0.21
	MS-AR(2)	0.75	0.25
	MS-AR(4)	0.71	0.29
	MSG-AR(2)	0.63	0.37

TABLE 5 – DATING GROWTH CYCLE TURNING POINTS (MODELS WITHOUT INTERVENTION) – QUARTERLY FREQUENCY: 1975:I –2003:IV

Bry-Boschan		MS-AR(0)		MS-AR(2)		MS-AR(4)		MSG-AR(2)	
Peaks	Troughs	Peaks	Troughs	Peaks	Troughs	Peaks	Troughs	Peaks	Troughs
80QIII	81QIII	81QI	83QI	-	-	80QIV	81QIV	-	-
82QII	83QI	-	-	-	-	82QIV	83QIII	-	-
-	-	-	-	-	-	87QII	87QIII	-	-
88QI	88QIV	-	-	-	-	88QII	88QIV	-	-
89QIII	90QII	89QII	92QI	90QII	91QIII	90QI	91QI	90QII	91QIII
91QIII	92QIII	-	-	-	-	91QIV	92QIII	-	-
95QI	95QIII	95QII	95QIII	-	-	95QII	95QIII	-	-
97QIV	98QIV	-	-	-	-	98QI	99QI	-	-
01QI	02QII	01QI	01QIV	-	-	01QI	02QII	-	-
02QIV	03QII	03QI	03QII	-	-	03QI	03QII	-	-

Note: The Bry-Boschan procedure uses monthly GDP series presented in Table 16 and the Markov switching models use quarterly. GDP series presented in Table 15. The recession dates of the Markov switching models.were obtained from the smoothed probabilities.

TABLE 6 – DATING OF GROWTH CYCLE TURNING POINTS (MODELS WITH INTERVENTION –TYPE 2) – QUARTERLY FREQUENCY: 1975:I –2003:IV

Bry-Boschan		MS-AR(0)		MS-AR(2)		MS-AR(4)		MSG-AR(2)	
Peaks	Troughs	Peaks	Troughs	Peaks	Troughs	Peaks	Troughs	Peaks	Troughs
80QIII	81QIII	81QI	81QIV	80QIV	81QIV	80QIV	81QIV	80QIV	81QIV
82QII	83QI	82QIV	83QI	82QIV	83QI	-	-	82QIII	83QI
-	-	-	-	87QII	87QIII	87QII	87QIII	-	-
88QI	88QIV	88QII	88QIV	88QII	88QIV	88QII	88QIV	88QII	88QIV
89QIII	90QII	Interv.	Interv.	Interv.	Interv.	Interv.	Interv.	Interv.	Interv.
91QIII	92QIII	91QIV	92QIII	91QIV	92QIII	91QIV	92QIII	91QIV	92QIII
95QI	95QIII	95QII	95QIII	95QII	95QIII	95QII	95QIII	95QII	95QIII
97QIV	98QIV	-	-	-	-	-	-	97QIV	98QI
-	-	-	-	-	-	-	-	98QIV	99QI
01QI	02QII	01QII	01QIV	01QII	01QIV	-	-	01QII	01QIV
02QIV	03QII	03QI	03QII	03QI	03QII	03QI	03QII	02QIV	03QII

Note: The Bry-Boschan procedure uses monthly GDP series presented in Table 16 and the Markov switching models use quarterly. GDP series presented in Table 15. The recession dates of the Markov switching models were obtained from the smoothed probabilities

TABLE 7 – LINEAR AND NONLINEAR MODELS: OUT-OF-SAMPLE FORECASTING PERFORMANCE – MSE OF EACH MODEL RELATIVE TO THE MSE OF THE AR(3) MODEL

		Linear AR(3)	No Change		ARMA (1,1)		MS-AR(0)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
							Intervention type 1							
Steps-ahead	1	0.01573	1.952	(0.412)	0.923	(0.033)	1.186	(0.070)	0.955	(0.108)	1.042	(0.126)	0.963	(0.125)
	2	0.01644	2.916	(1.476)	0.923	(0.035)	1.142	(0.043)	0.922	(0.056)	0.915	(0.089)	0.829	(0.100)
	3	0.01676	1.862	(0.733)	1.014	(0.016)	1.004	(0.011)	0.980	(0.017)	1.033	(0.012)	1.002	(0.009)
	4	0.01686	1.919	(0.391)	1.034	(0.011)	1.019	(0.017)	0.974	(0.025)	1.009	(0.006)	0.994	(0.009)
	5	0.01701	2.566	(0.820)	1.010	(0.011)	1.016	(0.014)	0.991	(0.030)	1.020	(0.007)	1.028	(0.012)
	6	0.01709	1.910	(0.440)	0.992	(0.010)	1.012	(0.012)	0.980	(0.020)	1.017	(0.009)	1.031	(0.015)
	7	0.01700	1.288	(0.220)	0.993	(0.008)	1.006	(0.019)	0.971	(0.029)	1.023	(0.009)	1.023	(0.009)
	8	0.01745	2.259	(0.506)	1.003	(0.007)	1.008	(0.018)	1.000	(0.022)	0.999	(0.009)	1.001	(0.010)
							Intervention type 2							
Steps-ahead	1	0.01574	1.949	(0.422)	0.926	(0.018)	1.173	(0.070)	0.954	(0.084)	1.051	(0.060)	0.986	(0.064)
	2	0.01636	2.946	(1.527)	0.934	(0.015)	1.141	(0.061)	0.929	(0.037)	0.951	(0.059)	0.911	(0.071)
	3	0.01677	1.860	(0.752)	1.013	(0.032)	1.000	(0.022)	0.978	(0.022)	1.064	(0.074)	1.041	(0.066)
	4	0.01694	1.903	(0.385)	1.025	(0.011)	1.008	(0.004)	0.971	(0.014)	1.078	(0.079)	1.037	(0.059)
	5	0.01711	2.536	(0.801)	0.998	(0.006)	0.999	(0.002)	0.980	(0.017)	1.027	(0.050)	1.014	(0.040)
	6	0.01719	1.889	(0.431)	0.982	(0.008)	0.995	(0.002)	0.978	(0.010)	1.007	(0.045)	1.017	(0.049)
	7	0.01704	1.282	(0.214)	0.989	(0.006)	0.997	(0.002)	0.977	(0.013)	1.030	(0.061)	1.060	(0.079)
	8	0.01745	2.259	(0.507)	1.003	(0.008)	1.004	(0.002)	1.001	(0.004)	0.982	(0.041)	1.003	(0.047)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No Change" (martingale) model forecast a constant rate of growth for GDP equal to zero. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the AR(3) model. The standard errors, shown in parentheses, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 8 – LINEAR AND NONLINEAR MODELS: OUT-OF-SAMPLE FORECASTING PERFORMANCE – MSE OF EACH MODEL RELATIVE TO THE MSE OF THE ARMA(1,1) MODEL

		ARMA(1,1)	No Change		Linear AR(3)		MS-AR(0)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
Intervention type 1														
Steps-ahead	1	0.01511	2.115	(0.515)	1.083	(0.038)	1.285	(0.101)	1.034	(0.099)	1.129	(0.118)	1.043	(0.115)
	2	0.01579	3.160	(1.770)	1.084	(0.041)	1.237	(0.087)	1.000	(0.030)	0.992	(0.071)	0.898	(0.082)
	3	0.01688	1.836	(0.702)	0.986	(0.015)	0.990	(0.018)	0.966	(0.013)	1.018	(0.022)	0.988	(0.020)
	4	0.01715	1.856	(0.361)	0.967	(0.011)	0.986	(0.014)	0.942	(0.027)	0.976	(0.009)	0.962	(0.009)
	5	0.01710	2.540	(0.803)	0.990	(0.010)	1.005	(0.009)	0.981	(0.022)	1.010	(0.010)	1.018	(0.011)
	6	0.01703	1.925	(0.454)	1.008	(0.010)	1.020	(0.010)	0.988	(0.012)	1.025	(0.016)	1.039	(0.022)
	7	0.01694	1.297	(0.219)	1.007	(0.008)	1.013	(0.011)	0.978	(0.022)	1.030	(0.012)	1.030	(0.012)
	8	0.01747	2.253	(0.504)	0.997	(0.006)	1.005	(0.012)	0.997	(0.017)	0.997	(0.005)	0.998	(0.005)
Intervention type 2														
Steps-ahead	1	0.01514	2.105	(0.509)	1.080	(0.022)	1.267	(0.095)	1.030	(0.086)	1.135	(0.058)	1.064	(0.060)
	2	0.01581	3.152	(1.761)	1.070	(0.018)	1.221	(0.083)	0.995	(0.029)	1.018	(0.064)	0.975	(0.080)
	3	0.01687	1.837	(0.703)	0.988	(0.032)	0.988	(0.016)	0.966	(0.015)	1.050	(0.049)	1.028	(0.042)
	4	0.01714	1.857	(0.361)	0.976	(0.011)	0.984	(0.009)	0.947	(0.023)	1.052	(0.066)	1.012	(0.047)
	5	0.01710	2.541	(0.803)	1.002	(0.006)	1.001	(0.006)	0.982	(0.017)	1.029	(0.051)	1.016	(0.040)
	6	0.01703	1.924	(0.453)	1.019	(0.008)	1.013	(0.008)	0.997	(0.007)	1.026	(0.048)	1.036	(0.053)
	7	0.01695	1.297	(0.219)	1.011	(0.006)	1.009	(0.005)	0.988	(0.016)	1.042	(0.058)	1.072	(0.077)
	8	0.01747	2.253	(0.504)	0.997	(0.008)	1.001	(0.006)	0.998	(0.012)	0.979	(0.034)	1.000	(0.040)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No Change" (martingale) model forecast a constant rate of growth for GDP equal to zero. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the ARMA(1,1) model. The standard errors, shown in parentheses, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 9 – LINEAR AND NONLINEAR MODELS: OUT-OF-SAMPLE FORECASTING PERFORMANCE – MSE OF EACH MODEL RELATIVE TO THE MSE OF THE MS-AR(0) MODEL

		MS-AR(0)	No Change		Linear AR(3)		ARMA (1,1)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
							Intervention type 1							
Steps-ahead	1	0.01713	1.646	(0.257)	0.843	(0.049)	0.778	(0.061)	0.805	(0.082)	0.879	(0.097)	0.812	(0.101)
	2	0.01757	2.554	(1.101)	0.876	(0.033)	0.808	(0.057)	0.808	(0.069)	0.802	(0.096)	0.726	(0.105)
	3	0.01679	1.854	(0.731)	0.996	(0.011)	1.010	(0.018)	0.976	(0.014)	1.028	(0.016)	0.998	(0.012)
	4	0.01702	1.883	(0.376)	0.981	(0.016)	1.015	(0.014)	0.956	(0.016)	0.990	(0.017)	0.976	(0.015)
	5	0.01714	2.527	(0.796)	0.985	(0.014)	0.995	(0.008)	0.976	(0.018)	1.005	(0.012)	1.012	(0.012)
	6	0.01719	1.888	(0.430)	0.988	(0.011)	0.981	(0.009)	0.969	(0.013)	1.005	(0.011)	1.019	(0.017)
	7	0.01705	1.281	(0.210)	0.994	(0.018)	0.988	(0.011)	0.965	(0.014)	1.018	(0.020)	1.017	(0.018)
	8	0.01752	2.241	(0.499)	0.992	(0.017)	0.995	(0.012)	0.992	(0.005)	0.991	(0.010)	0.993	(0.008)
							Intervention type 2							
Steps-ahead	1	0.01704	1.662	(0.263)	0.853	(0.051)	0.790	(0.059)	0.813	(0.077)	0.896	(0.075)	0.840	(0.082)
	2	0.01747	2.583	(1.128)	0.877	(0.047)	0.819	(0.056)	0.815	(0.071)	0.834	(0.076)	0.799	(0.082)
	3	0.01677	1.859	(0.735)	1.000	(0.022)	1.012	(0.017)	0.978	(0.010)	1.063	(0.057)	1.041	(0.049)
	4	0.01700	1.888	(0.377)	0.992	(0.004)	1.017	(0.009)	0.963	(0.015)	1.070	(0.077)	1.029	(0.057)
	5	0.01710	2.539	(0.803)	1.001	(0.002)	0.999	(0.006)	0.981	(0.018)	1.028	(0.050)	1.015	(0.040)
	6	0.01714	1.899	(0.435)	1.006	(0.002)	0.987	(0.008)	0.984	(0.011)	1.013	(0.045)	1.022	(0.049)
	7	0.01702	1.286	(0.214)	1.003	(0.002)	0.992	(0.005)	0.980	(0.012)	1.033	(0.061)	1.063	(0.080)
	8	0.01748	2.250	(0.503)	0.996	(0.002)	0.999	(0.006)	0.997	(0.006)	0.978	(0.039)	0.999	(0.045)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No Change" (martingale) model forecast a constant rate of growth for GDP equal to zero. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the MS-AR(0) model. The standard errors, shown in parentheses, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 10 - HAMILTON' S MODEL WITH AND WITHOUT INTERVENTION: OUT-OF-SAMPLE FORECASTING PERFORMANCE – MSE OF THE MODEL WITH INTERVENTION RELATIVE TO THE MSE OF THE MODEL WITHOUT INTERVENTION

		MS-AR(0)					MS-AR(2)					MS-AR(4)				
		No intervention	Intervention				No intervention	Intervention				No intervention	Intervention			
			Type 1		Type 2			Type 1		Type 2			Type 1		Type 2	
		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE	
Steps-ahead	1	0.0172	0.995	(0.025)	0.985	(0.020)	0.0159	0.938	(0.092)	0.939	(0.080)	0.0153	1.113	(0.120)	1.061	(0.122)
	2	0.0180	0.951	(0.025)	0.941	(0.027)	0.0164	0.927	(0.040)	0.925	(0.039)	0.0157	1.073	(0.022)	1.065	(0.020)
	3	0.0174	0.936	(0.036)	0.934	(0.035)	0.0169	0.961	(0.023)	0.959	(0.018)	0.0175	0.953	(0.050)	0.940	(0.060)
	4	0.0177	0.930	(0.040)	0.928	(0.036)	0.0171	0.949	(0.040)	0.953	(0.036)	0.0181	0.915	(0.070)	0.921	(0.062)
	5	0.0176	0.947	(0.034)	0.943	(0.031)	0.0171	0.976	(0.038)	0.977	(0.033)	0.0175	1.012	(0.031)	1.004	(0.025)
	6	0.0171	1.006	(0.020)	1.000	(0.017)	0.0170	0.987	(0.022)	0.996	(0.018)	0.0178	0.977	(0.028)	0.953	(0.038)
	7	0.0172	0.983	(0.039)	0.979	(0.034)	0.0170	0.969	(0.039)	0.979	(0.033)	0.0179	0.957	(0.038)	0.945	(0.038)
	8	0.0181	0.940	(0.060)	0.936	(0.056)	0.0175	0.995	(0.033)	0.997	(0.029)	0.0177	0.970	(0.029)	0.959	(0.031)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No Change" (martingale) model forecast a constant rate of growth for GDP equal to zero. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the same model without intervention. The standard errors, shown in parentheses, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 11 – LAM´S MODEL WITH AND WITHOUT INTERVENTION: OUT-OF-SAMPLE FORECASTING PERFORMANCE – MSE OF THE MODEL WITH INTERVENTION RELATIVE TO THE MSE OF THE MODEL WITHOUT INTERVENTION

		MSG-AR(1)					MSG-AR(2)					MSG-AR(3)				
		No intervention	Intervention				No intervention	Intervention				No intervention	Intervention			
			Type 1		Type 2			Type 1		Type 2			Type 1		Type 2	
		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE	
Steps-ahead	1	0.0166	1.003	(0.161)	1.079	(0.066)	0.017	0.944	(0.142)	0.954	(0.089)	0.016	0.922	(0.142)	0.944	(0.085)
	2	0.0168	1.003	(0.064)	0.994	(0.032)	0.017	0.829	(0.124)	0.853	(0.084)	0.017	0.786	(0.129)	0.855	(0.071)
	3	0.0172	0.977	(0.051)	0.976	(0.028)	0.018	0.916	(0.078)	0.944	(0.043)	0.018	0.884	(0.072)	0.920	(0.039)
	4	0.0173	0.978	(0.074)	0.982	(0.042)	0.018	0.922	(0.093)	0.994	(0.046)	0.018	0.905	(0.086)	0.952	(0.048)
	5	0.0173	0.987	(0.061)	0.976	(0.041)	0.017	0.991	(0.063)	1.009	(0.027)	0.017	0.988	(0.061)	0.986	(0.031)
	6	0.0173	0.986	(0.044)	0.984	(0.019)	0.018	0.967	(0.047)	0.968	(0.020)	0.017	0.986	(0.043)	0.983	(0.013)
	7	0.0175	0.962	(0.070)	0.995	(0.017)	0.018	0.947	(0.072)	0.958	(0.034)	0.018	0.951	(0.071)	0.990	(0.017)
	8	0.0178	0.969	(0.072)	1.003	(0.021)	0.018	0.967	(0.070)	0.950	(0.044)	0.018	0.964	(0.071)	0.966	(0.034)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No Change" (martingale) model forecast a constant rate of growth for GDP equal to zero. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the same model without intervention. The standard errors, shown in parentheses, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 12 – TURNING POINTS FORECASTING PERFORMANCE (SUM OF DIFFERENCES)

	MS-AR(0) without intervention	MS-AR(0) with intervention	MSG-AR(2) with intervention	MS-AR(2) with intervention	MS-AR(4) with- out intervention	MS-AR(4) with intervention
Number of Peaks Not Detected by Models but Detected by BB	4	4	4	4	0	4
Number of Peaks Detected by Models but not Detected by BB	5	0	0	0	22	0
Total Sum of Differences	9	4	4	4	22	4

Note: The Bry Boschan algorithm detects 4 peaks during the covered period (from 1995.I to 2003.IV).

TABLE 13 – TURNING POINTS FORECASTING PERFORMANCE (SUM OF ABSOLUTE DEVIATIONS)

Type	MS-AR(0) with- out intervention	MS-AR(0) with intervention	MSG-AR(2) with intervention	MS-AR(2) with intervention	MS-AR(4) with- out intervention	MS-AR(4) with intervention
Peak Not Detected by Models but Detected by BB	2.65	2.70	1.79	2.49	0.00	2.30
Peak Detected by Models but not Detected by BB	1.90	0.00	0.81	0.25	7.67	0.41
Total Sum of Deviations	4.55	2.70	2.60	2.74	7.67	2.71

Note: The Bry Boschan algorithm detects 4 peaks during the covered period (from 1995.I to 2003.IV).

TABLE 14 – OUT-OF-SAMPLE FORECASTING PERFORMANCE (2000:3 - 2001:4)

			AR(3)	ARMA(1,1)	MS-AR(0)	MS-AR(2)	MS-AR(4)	MSG-AR(1)	MSG-AR(2)	MSG-AR(3)
		AR(3) RMSE	**MSE of the Model Relative to the MSE of the AR(3) Model**							
No Intervention		0.03013	-	1.04738	1.006	1.03045	1.11523	1.01906	1.04587	1.04721
Intervention	Type 1	0.03022	-	1.03407	0.870	0.95607	1.06238	1.03477	1.19631	1.12761
	Type 2	0.03031	-	1.03226	1.021	0.95640	1.03538	1.03333	1.18076	1.12016
		ARMA(1,1) RMSE	**MSE of the Model Relative to the MSE of the ARMA(1,1) Model**							
No Intervention		0.03084	0.95476	-	0.960	0.98383	1.06478	0.97296	0.99856	0.99984
Intervention	Type 1	0.03073	0.96705	-	0.842	0.92456	1.02738	1.00067	1.15689	1.09045
	Type 2	0.03080	0.96875	-	0.989	0.92651	1.00303	1.00104	1.14386	1.08516
		MS-AR(0) RMSE	**MSE of the Model Relative to the MSE of the MS-AR(0) Model**							
No Intervention		0.03022	0.99409	1.04119	-	1.02436	1.10864	1.01303	1.03969	1.04102
Intervention	Type 1	0.02819	1.14916	1.18832	-	1.09867	1.22085	1.18911	1.37475	1.29580
	Type 2	0.03062	0.97983	1.01144	-	0.93712	1.01451	1.01249	1.15695	1.09757

TABLE 15 – BRAZILIAN QUARTERLY REAL GDP

	GDP Index				Seasonally Adjusted GDP Index			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1975	56.787	64.705	63.008	62.329	59.723	61.493	62.260	63.435
1976	63.574	70.248	69.003	67.759	66.839	66.802	68.148	68.940
1977	66.930	75.828	72.170	71.190	70.222	72.245	71.233	72.395
1978	68.966	77.563	77.487	76.394	72.325	74.066	76.422	77.645
1979	74.245	82.880	81.598	80.956	77.795	79.344	80.313	82.257
1980	81.082	87.657	86.827	84.192	84.919	84.062	85.313	85.595
1981	80.977	85.348	81.539	77.232	84.912	81.993	79.911	78.518
1982	77.256	85.975	84.637	79.832	80.876	82.743	82.838	81.054
1983	74.828	82.507	81.690	78.957	78.486	79.599	79.812	79.992
1984	77.935	86.282	85.909	84.694	81.965	83.419	83.798	85.558
1985	83.251	91.076	93.835	93.089	87.677	88.270	91.355	93.874
1986	89.245	98.020	101.486	99.861	94.335	95.141	98.565	100.594
1987	96.359	104.822	102.110	99.812	102.075	101.805	98.743	100.712
1988	96.474	104.507	104.557	97.619	102.467	101.335	100.708	98.815
1989	93.737	108.126	110.049	104.183	99.904	104.594	105.687	105.719
1990	96.181	98.081	105.870	98.044	102.900	94.492	101.729	99.736
1991	89.375	105.629	106.607	98.389	95.395	101.528	102.723	100.355
1992	94.101	103.706	101.491	97.624	99.973	99.339	98.416	99.781
1993	96.864	109.072	105.726	102.014	102.171	104.298	103.169	104.225
1994	101.786	113.110	112.373	112.159	106.962	108.063	110.218	114.430
1995	112.430	118.861	113.268	112.252	117.945	113.684	111.211	114.309
1996	109.976	121.541	121.607	118.251	115.639	116.293	119.304	120.263
1997	112.863	126.953	125.191	121.022	118.980	121.436	122.597	123.010
1998	115.162	128.082	125.734	118.945	121.640	122.347	122.937	120.950
1999	114.752	128.668	125.366	123.049	121.445	122.780	122.443	125.269
2000	121.153	134.096	134.669	132.747	128.569	127.856	131.141	134.869
2001	134.612	141.834	137.395	131.292	141.461	135.226	133.835	133.325
2002	130.014	136.798	139.762	136.379	134.576	133.607	137.560	138.131
2003	132.437	135.335	137.684	136.297	136.553	132.751	135.502	137.861

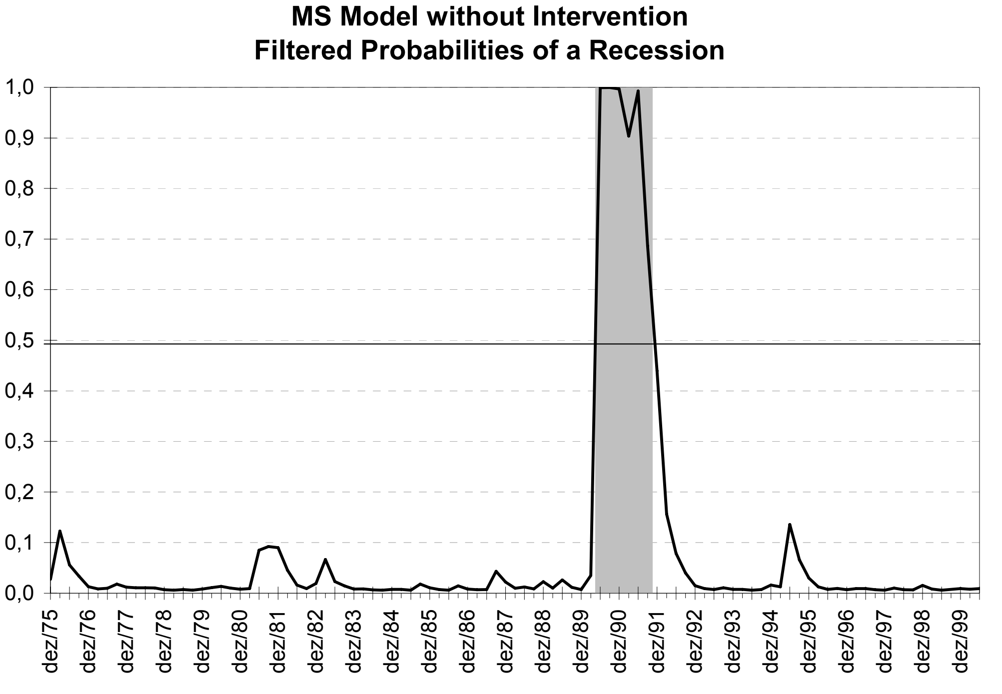
Note: Fixed Base (1980) GDP. The GDP was seasonally adjusted using the X12-ARIMA software.
Source: IPEADATA (1975:1 - 1979:4); IBGE (1980:1 to 1997:3); IPEA (1997:4-2001:1), elaborated with basic data (complete data set) provided by IBGE and following IBGE 's Fixed Base methodology; IPEA (2001:2-2001:4), elaborated with basic data provided by IBGE (with an almost complete data set) and following IBGE 's Fixed Base methodology; IBGE (2002:1-2003:4), using IBGE 's currently adopted index which is not Fixed Base.

TABLE 16 – BRAZILIAN MONTHLY REAL GDP (SEASONALLY ADJUSTED INDEX)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1975	59.699	60.173	59.308	62.338	60.975	61.180	62.556	61.329	62.909	62.974	63.345	63.999
1976	65.153	68.721	66.659	66.472	66.655	67.292	68.249	67.953	68.256	68.595	68.723	69.518
1977	68.978	70.081	71.621	71.533	72.862	72.357	70.863	71.550	71.302	71.094	72.867	73.239
1978	72.172	71.997	72.822	74.275	73.897	74.043	75.304	77.460	76.519	77.638	77.555	77.757
1979	78.570	77.462	77.370	78.797	79.837	79.416	80.287	81.746	78.922	82.979	82.006	81.803
1980	84.271	85.550	84.953	83.696	84.416	84.091	84.972	83.567	87.417	86.244	85.605	84.952
1981	85.575	86.632	82.547	82.294	80.863	82.842	82.482	78.521	78.747	78.346	78.250	78.975
1982	79.657	80.039	82.951	83.408	82.069	82.770	82.316	82.970	83.246	80.570	81.253	81.356
1983	78.301	77.658	79.516	78.958	80.283	79.573	78.781	80.827	79.845	78.499	80.583	80.911
1984	80.441	83.770	81.703	81.754	83.882	84.641	84.226	84.502	82.685	85.484	85.546	85.656
1985	88.181	86.784	88.082	86.973	88.654	89.207	91.664	91.203	91.230	93.826	93.260	94.531
1986	97.064	94.351	91.614	95.763	94.994	94.697	97.722	97.185	100.833	101.972	99.769	100.014
1987	102.012	102.827	101.418	102.831	101.557	101.063	98.791	97.452	100.052	99.716	101.452	100.911
1988	100.028	101.785	105.637	101.311	100.387	102.336	99.941	101.561	100.695	95.816	98.690	101.864
1989	99.608	97.630	102.533	102.968	104.605	106.245	105.156	106.984	105.001	105.339	106.325	105.376
1990	105.876	103.480	99.439	88.038	98.304	97.173	100.356	103.470	101.408	101.600	100.390	97.071
1991	98.488	94.238	93.645	101.211	102.100	101.282	103.697	103.277	101.203	103.089	100.076	97.694
1992	98.988	101.976	99.304	99.295	98.462	100.223	99.038	97.172	98.976	98.477	100.026	100.580
1993	100.496	101.065	105.507	104.065	104.477	104.252	102.978	103.167	103.237	102.260	104.138	105.958
1994	106.395	105.275	109.909	106.779	109.052	108.231	106.418	111.672	112.369	111.057	114.397	117.476
1995	117.320	116.794	120.535	115.151	112.643	113.108	109.801	112.227	111.396	113.266	115.089	114.347
1996	115.926	117.198	114.487	116.487	117.888	114.397	119.923	119.181	118.497	120.709	119.997	119.898
1997	119.850	119.903	117.722	123.230	120.537	120.563	121.731	121.223	124.446	126.202	120.737	122.068
1998	121.579	120.866	122.771	122.046	122.674	122.489	123.717	121.832	122.493	121.275	121.503	119.568
1999	120.814	120.379	123.048	123.230	122.684	122.598	120.526	123.295	122.576	122.867	124.500	125.358
2000	124.700	133.233	124.341	125.096	127.217	128.083	130.895	132.151	127.122	133.319	133.072	134.869
2001	138.167	138.916	143.790	137.992	134.938	129.394	134.145	134.929	129.112	133.209	132.839	130.620
2002	132.522	133.245	130.763	130.379	131.788	131.509	134.678	134.613	136.031	137.836	135.121	134.047
2003	136.514	133.987	131.855	133.114	130.369	127.673	134.526	132.055	132.677	135.792	135.833	134.585

Source: 1975:1 - 1979:12: IPEADATA (quarterly data (Table 15)) and interpolation (Harvey's methodology); 1980:1-2001:12: monthly Fixed Base index elaborated by IPEA using data provided by IBGE and the same methodology adopted by IBGE for its quarterly Fixed Base index; 2002:1 - 2003:12: IBGE's currently adopted quarterly index (Table (15)), which is not Fixed Base, and Harvey's interpolation.methodology.

FIGURE 3 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MS-AR(2) MODEL WITHOUT INTERVENTION



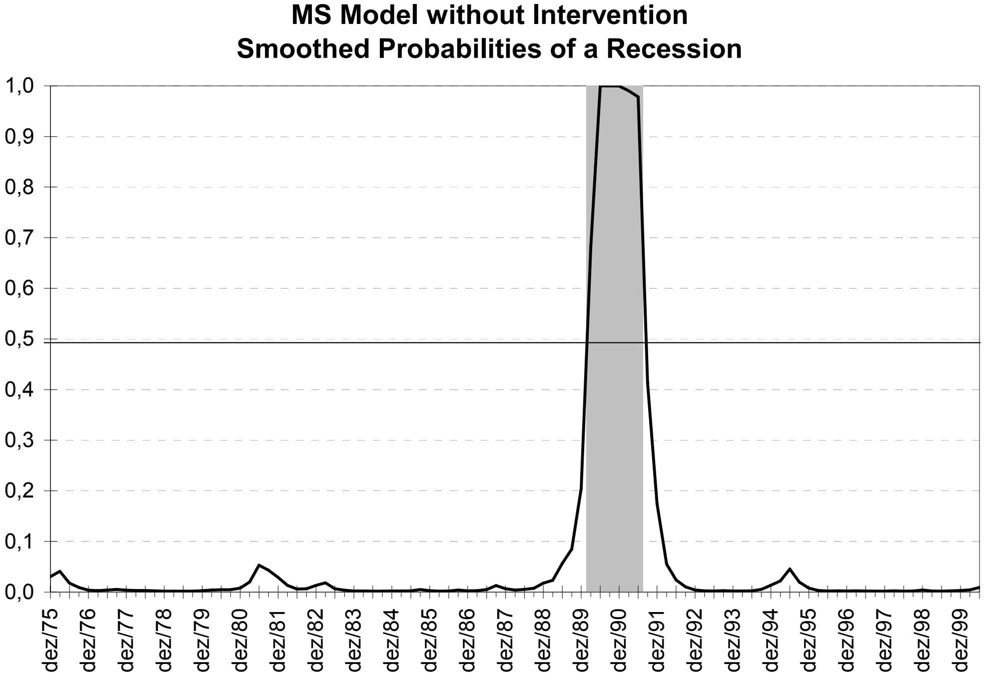
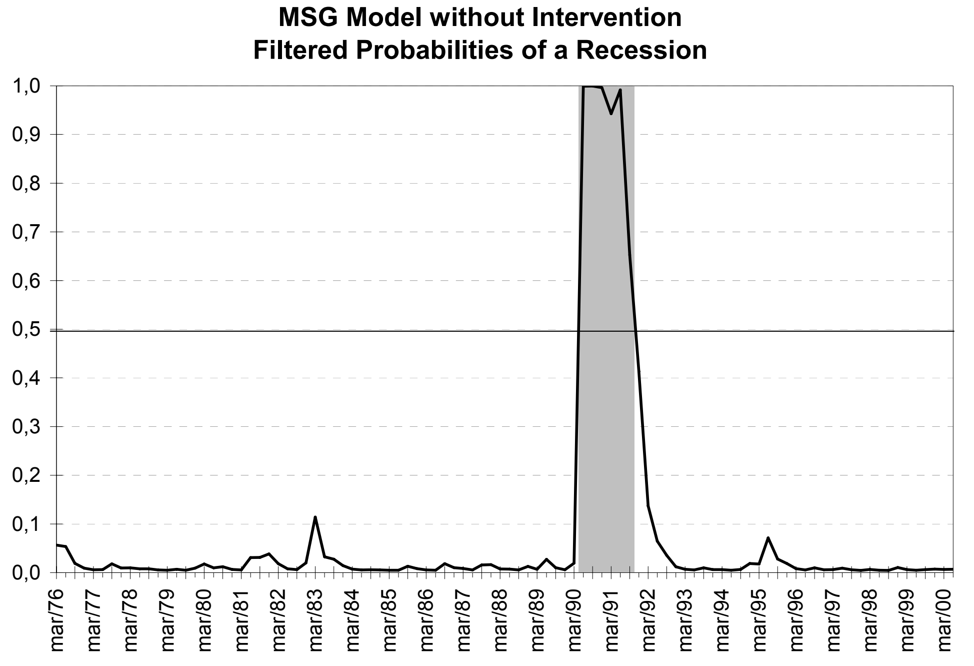


FIGURE 4 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MSG-AR(3) MODEL WITHOUT INTERVENTION



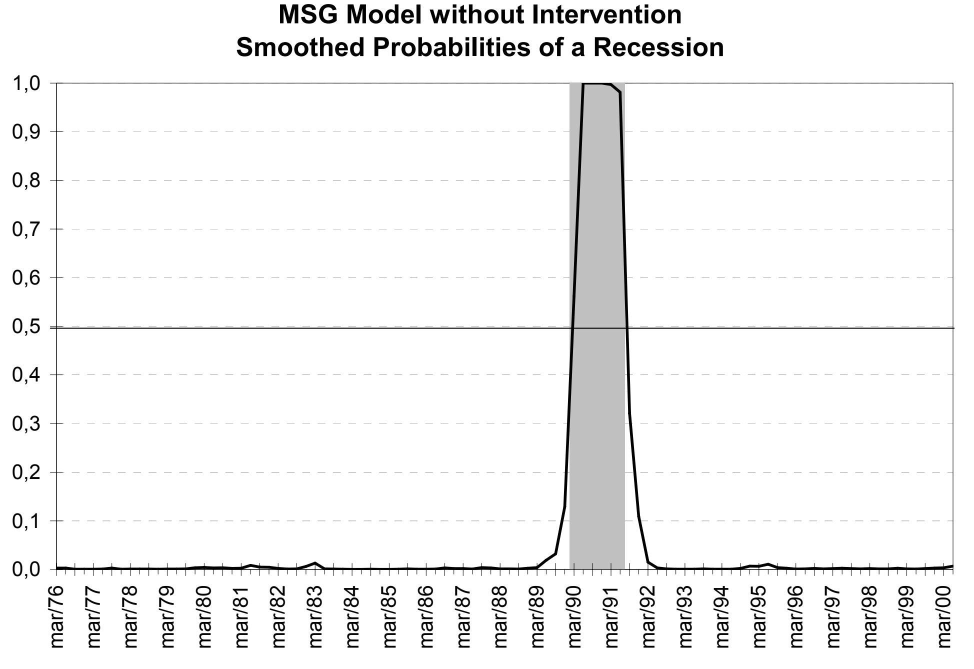
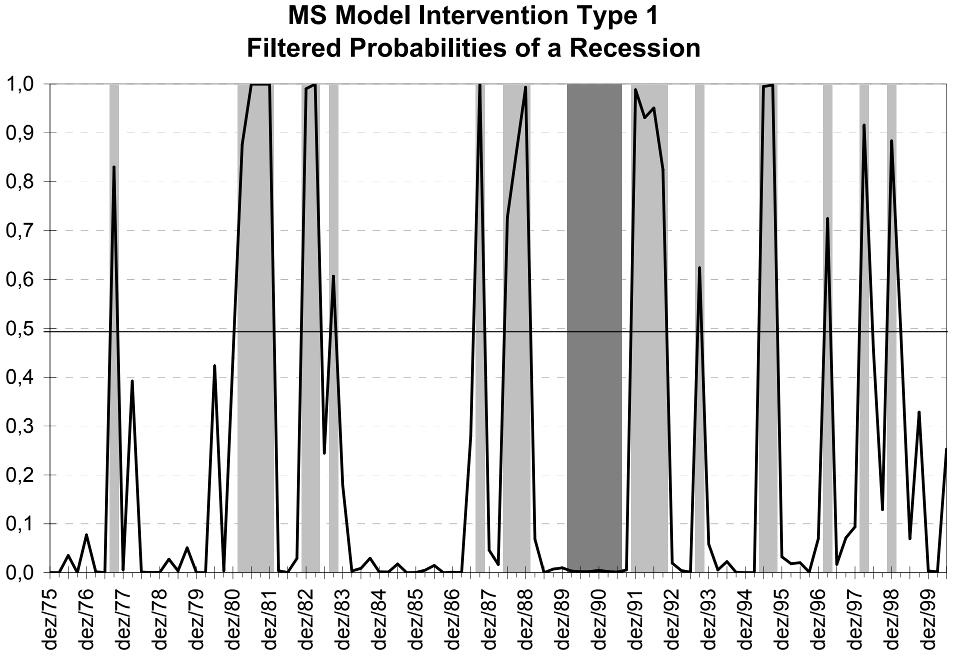
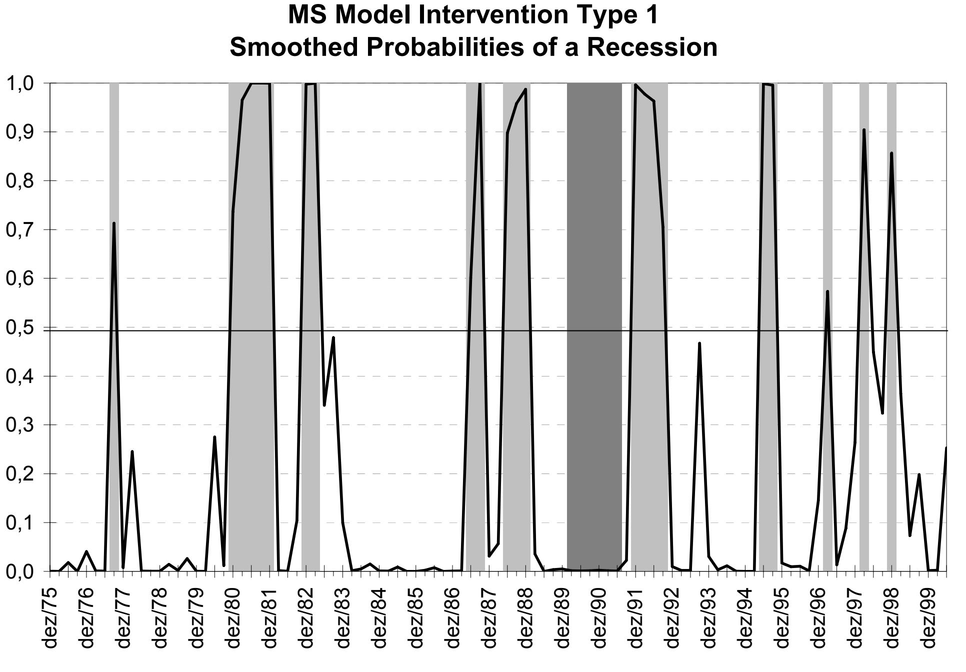


FIGURE 5 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MS AR(2) MODEL (INTERVENTION TYPE 1)

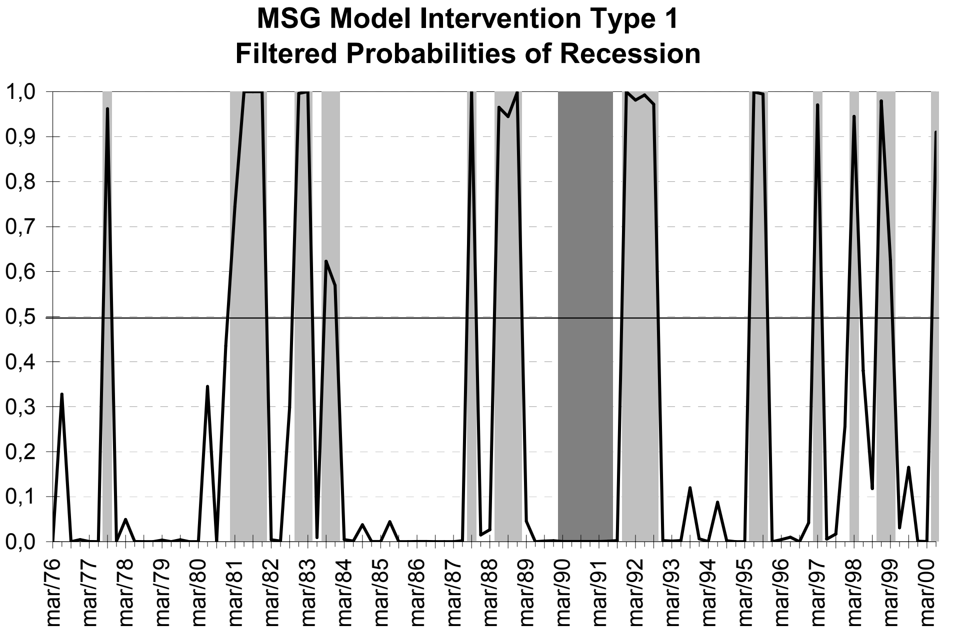


Note: The darker shaded area represents the period of intervention.

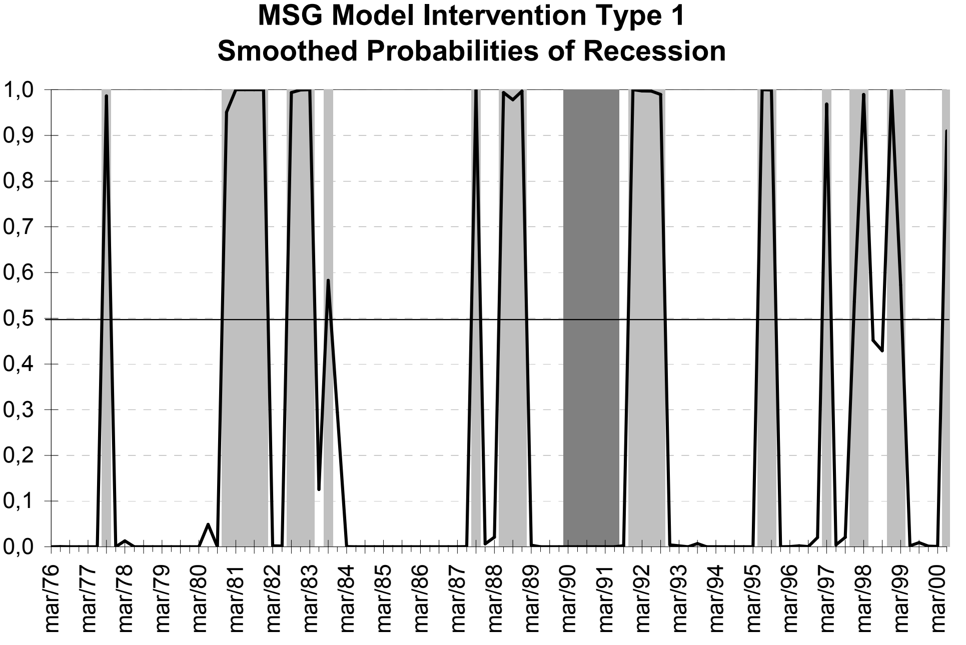


Note: The darker shaded area represents the period of intervention.

FIGURE 6 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MSG AR(2) MODEL WITH INTERVENTION TYPE 1

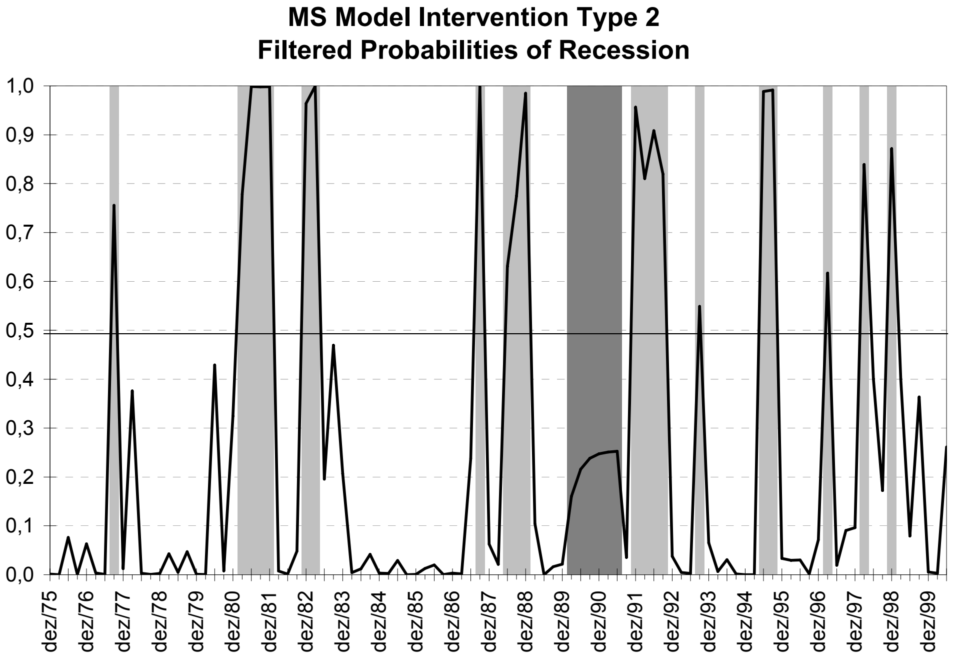


Note: The darker shaded area represents the period of intervention.

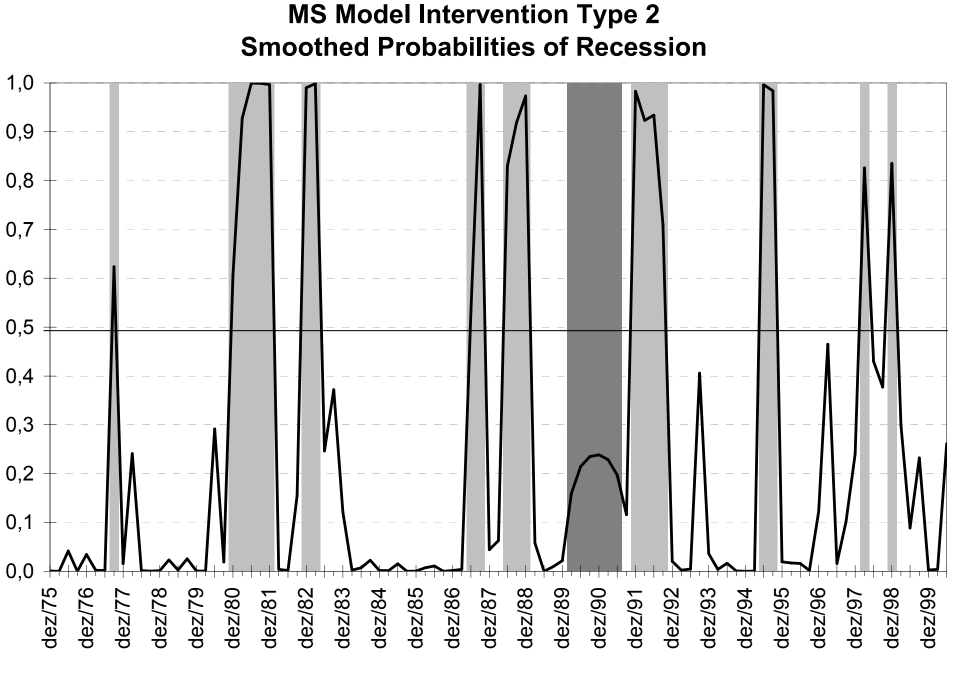


Note: The darker shaded area represents the period of intervention.

FIGURE 7 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MS AR(2) MODEL INTERVENTION TYPE 2

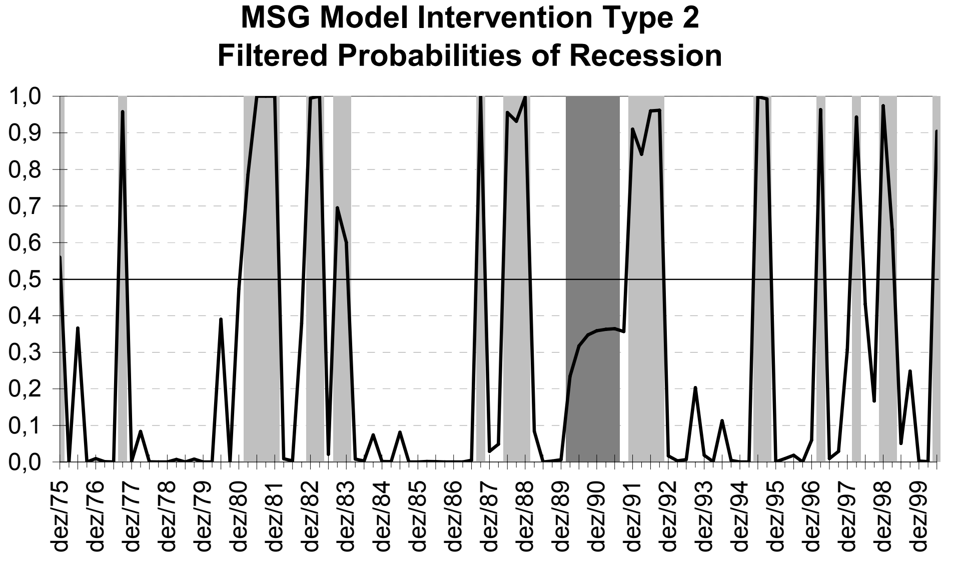


Note: The darker shaded area represents the period of intervention.

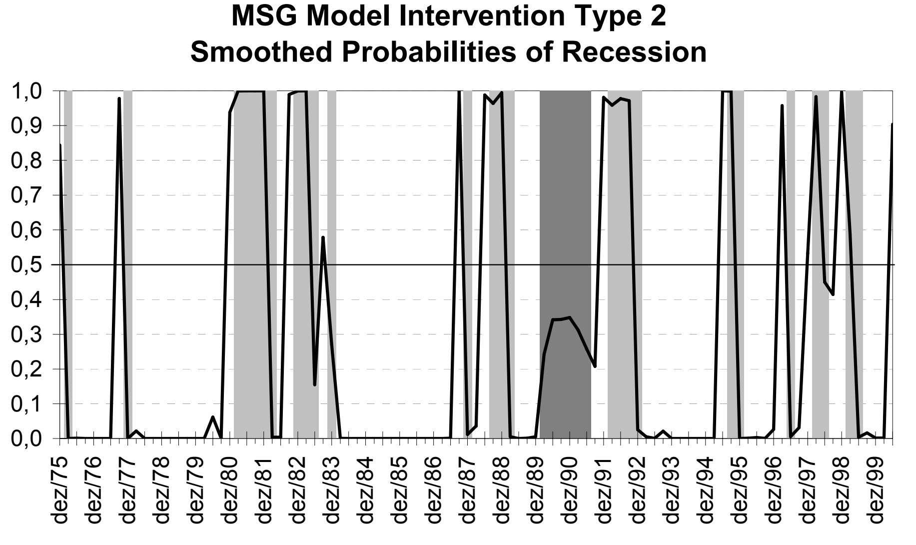


Note: The darker shaded area represents the period of intervention.

FIGURE 8 – FILTERED AND SMOOTHED PROBABILITIES OF RECESSIONS: MSG MODEL AR(2) WITH INTERVENTION TYPE 2



Note: The darker shaded area represents the period of intervention.



Note: The darker shaded area represents the period of intervention.

APPENDIX A

Hamilton's Filter

Hamilton's nonlinear filter uses as input the ergodic and transition probabilities:

$$\text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}) = p_{ij} \sum_{h=0}^{1} \text{Prob}(S_{t-2} = h, S_{t-1} = i \mid I_{t-1}). \quad (10)$$

From these joint conditional probabilities, the density of Δy_t conditional on S_{t-1}, S_t, and I_{t-1} is:

$$f(\Delta y_t | S_{t-1} = i, S_t = j, I_{t-1}) = [(2\pi)^{-k/2} \,|\, Q_t^{(i,j)} \,|^{-1/2} \exp(-\frac{1}{2} N_{t|t-1}^{(i,j)'} Q_t^{(i,j)^{-1}} N_{t|t-1}^{(i,j)}) . \quad (11)$$

The joint probability density of states and observations is then calculated by multiplying each element of (10) by the corresponding element of (11):

$$F(\Delta y_t, S_{t-1} = i, S_t = j | I_{t-1}) = f(\Delta y_t | S_{t-1} = i, S_t = j, I_{t-1}) \,\text{Prob}(S_{t-1} = i, S_t = j | I_{t-1}) \quad (12)$$

The probability density of Δy_t given I_{t-1} is:

$$F(\Delta y_t | I_{t-1}) = \sum_{j=0}^{1} \sum_{i=0}^{1} f(\Delta y_t, S_{t-1} = i, S_t = j | I_{t-1}). \quad (13)$$

The joint probability density of states is calculated by dividing each element of (12) by the corresponding element of (13):

$$\text{Prob}(S_{t-1} = i, S_t = j \mid I_t) = f(\Delta y_t, S_{t-1} = i, S_t = j \mid I_{t-1}) \,/\, f(\Delta y_t \mid I_{t-1}) \quad (14)$$

Finally, summing over the states in (14), we obtain the filtered probabilities of recessions and expansions:

$$\text{Prob}(S_t = j \mid I_t) = \sum_{i=0}^{1} \text{Prob}(S_{t-1} = i, S_t = j \mid I_t). \quad (15)$$

The first-order assumption of the Markov chain implies that all relevant information for predicting future states is included in the current state. Thus, Δy_t depends only on the current and r most recent values of s_t, on r lags of Δy_t, and on a vector of parameters θ:

$$p(\Delta y_t | s_t, s_{t-1}, \ldots, \Delta y_{t-1}, \Delta y_{t-2}, \ldots; \theta) = p(\Delta y_t \mid s_t, s_{t-1}, \ldots, s_{t-r}, \Delta y_{t-1}, \Delta y_{t-2}, \ldots, \Delta y_{t-r}; \theta).$$

Lam's Filter

The first step of the algorithm is initialized with the distribution of the states in this period conditional on information in the previous periods. From this, the distribution of the states is generated, for the following period, using the Markov process. Thus, the first step calculates:

1st Step:

$$P[S_t=1, S_{t-1}=s_{t-1},\ldots, S_{t-r+1}=s_{t-r+1}, \sum_{i=1}^{t} S_i = x \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots]=$$

$$P[S_t=1 \mid S_{t-1}=s_{t-1}] \text{ x } \sum_{S_{t-r}=0}^{1} P[S_{t-1}=s_{t-1},\ldots, S_{t-r+1}=s_{t-r+1}, \sum_{i=1}^{t-1} S_i = x-1 \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots] \quad (16)$$

and

$$P[S_t=0, S_{t-1}=s_{t-1},\ldots, S_{t-r+1}=s_{t-r+1}, \sum_{i=1}^{t} S_i = x \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots]=$$

$$P[S_t=0 \mid S_{t-1}=s_{t-1}] \text{ x } \sum_{S_{t-r}=0}^{1} P[S_{t-1}=s_{t-1},\ldots, S_{t-r+1}=s_{t-r+1}, \sum_{i=1}^{t-1} S_i = x-1 \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots] \quad (17)$$

where $\sum_{i=1}^{t} S_i = x$ is the sum of the past states up to period t.

2nd Step:

The second step, which uses the result from the first step as input, computes the joint distribution of the current observation and of the states:

$$f(\Delta y_t, S_t, S_{t-1},\ldots, S_{t-r+1}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) = \quad (18)$$

$$f(\Delta y_t \mid S_t, S_{t-1},\ldots, S_{t-r+1}, \sum_{i=1}^{t} S_i, \Delta y_{t-1}, \Delta y_{t-2},\ldots) P[S_t=s_t,\ldots, S_{t-r+1}=s_{t-r+1}, \sum_{i=1}^{t-1} S_i = x \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots]$$

and

$$f(\Delta y_t, S_t, S_{t-1},\ldots, S_{t-r+1}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) = \quad (19)$$

$$\left(1/\sqrt{2\pi\sigma}\right)\cdot \exp\left\{(1-(2\sigma^2))\cdot(1-\phi_1 L-\phi_2 L^2 -5 \; -\phi_r L^r)\text{x}\left[\Delta y_t + \sum_{i=1}^{t-1}\Delta y_i - \alpha_0 t\right] + \right.$$

$$\left. +(1-\phi_1-\phi_2-5 \; -\phi_r)z_0 - \alpha_1(1-\phi_1-\phi_2-5 \; -\phi_r)\sum_{i=1}^{r} S_i + \alpha_1 \sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right) S_{t-j+1}\right\}^2$$

3rd Step:

In the third step, the joint distribution obtained above is used to compute the likelihood of the observation conditional to its past:

$$f(\Delta y_t, S_t, S_{t-1},\ldots, S_{t-r+1}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) = \tag{20}$$

$$= \sum_{s_t=0}^{1} 5 \sum_{s_{t-r+1}=0}^{1} \sum_{x=0}^{t} f(y_t, S_t = s_t,\ldots, S_{t-r}=s_{t-r}, \sum_{i=1}^{t} S_i = x \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) =$$

4th Step:

In the fourth step, the algorithm uses the result from the second and third steps to calculate the distribution of the states conditional on the current information:

$$P[S_t=s_t,\ldots, S_{t-r}=s_{t-r}, \sum_{i=1}^{t-1} S_i = x \mid \Delta y_t, \Delta y_{t-1},\ldots] =$$

$$= f(y_t, S_t, S_{t-1},\ldots, S_{t-r}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) / f(\Delta y_t \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) \tag{21}$$

Through these four steps the algorithm generates the conditional likelihood value to each observation (3rd step) and the distribution of the states (from the 4th step), which is then used to initialize again the algorithm for the following observation. The algorithm is repeated for all observations, and the conditional likelihood function is obtained from the sum of its value for each observation:

$$L[\Delta y_T, \Delta y_{T-1}, \Delta y_{T-2},\ldots, \Delta y_1] = \sum_{t=1}^{T} \log\, f(\Delta y_t \mid \Delta y_{t-1}, \Delta y_{t-2}\ldots, \Delta y_1) \tag{22}$$

Since the second step requires data from r previous periods, the algorithm is initialized in the observation r+1. For the first step, the probabilities below are required, which are obtained from their non-conditional counterparts.

$$P[S_r=s_r,\ldots, S_1=s_1, \sum_{i=1}^{t-1} S_i = x \mid \Delta y_r,\ldots] \tag{23}$$

The filter used to estimate Lam's model involves substantial more computation than Hamilton's algorithm for two reasons. First, in the calculation of the error, the states for each observation include all the history of the Markov process, which is treated as an additional variable. Second, the initial value of the autoregressive component is treated as an additional free parameter to be estimated. These two components are represented in the third and second terms of equation (24), respectively. When α_0 and α_1 are independent from t, the computation of the error E is:

$$E = (1-\phi_1 L - \phi_2 L^2 - 5 \ -\phi_r L^r)\left[\sum_{i=1}^{t} \Delta y_i - \alpha_0 t\right] + (1-\phi_1-\phi_2-5 \ -\phi_r)z_0 \qquad (24)$$

$$-\alpha_1(1-\phi_1-\phi_2-5 \ -\phi_r)\sum_{i=1}^{t} S_i \ -\alpha_1 \sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right) S_{t-j+1}$$

When dummies are introduced in Lam's model, the parameters α_0 e α_1 depend on t and the error is then calculated as:

$$E = (1-\phi_1 L - \phi_2 L^2 - 5 \ -\phi_r L^r)\left[\sum_{i=1}^{t} \Delta y_i - \alpha_0^t t\right] + (1-\phi_1-\phi_2-5 \ -\phi_r)z_0 \qquad (25)$$

$$-(1-\phi_1-\phi_2-5 \ -\phi_r)\sum_{i=1}^{t} S_i \alpha_1^i - \sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right) S_{t-j+1}\alpha_1^{t-j+1}$$

APPENDIX B

One-step-ahead Predictions

As an illustration of the procedure, the predicted one-step ahead mean for the MS AR(2) at the first forecast date T+1 = 1992:2 is given by:

$$\Delta\hat{y}_{t+1} \mid I_t = \hat{\mu}_{t+1} + \phi_1(\Delta y_t - \mu_t) + \phi_2(\Delta y_{t-1} - \mu_{t-1})$$

where $\hat{\mu}_{t+i} = \alpha_0 \hat{P}(S_{t+i} = 0) + \alpha_1 \hat{P}(S_{t+i} = 1)$ are the estimated drifts for each state. The estimated probabilities are obtained from the filtered probabilities and from the transition matrix. For example, the one-step-ahead predicted probability of a recession is given by:

$$\hat{P}(S_{t+1} = 0) = P(S_t = 0)p_{00} + P(S_t = 1)p_{10}$$

where $P(S_t = i)$ for i = 0,1 are the ergodic probabilities. At time T+2 = 1992:3, a new observation of Δy_t is considered, and the models are re-estimated to obtain the parameters and filtered probability. This procedure is repeated for each subsequent observation up to T = 2000:3 in order to obtain the recursive one-step-ahead forecasts of the filtered probability and the forecasts the Brazilian GDP growth.

Two-step-ahead Predictions

A similar procedure is used to obtain two-step-ahead prediction of the mean and filtered probabilities of a recession at the first forecast date, which are now given by:

$$\Delta\hat{y}_{t+2} \mid I_t = \hat{\mu}_{t+2} + \phi_1(\Delta\hat{y}_{t+1} - \hat{\mu}_{t+1}) + \phi_2(\Delta y_t - \mu_t)$$

$$\hat{P}(S_{t+1} = 0) = P(S_t = 0)(p_{00}p_{00} + p_{01}p_{10}) + P(S_t = 1)(p_{10}p_{00} + p_{11}p_{10})$$

Three- steps- ahead and on Predictions

$$\Delta\hat{y}_{t+h} \mid I_t = \hat{\mu}_{t+h} + \phi_1(\Delta\hat{y}_{t+h-1} - \hat{\mu}_{t+h-1}) + \phi_2(\Delta\hat{y}_{t+h-2} - \hat{\mu}_{t+h-2}) \quad \forall h > 2$$

$$\hat{P}(S_{t+h} = 0 \mid I_t) = P^h \hat{P}(S_t = 0 \mid I_t)$$

where P is the transition probability matrix with elements p_{ij} = pr[s_t = j| s_{t-1} = i], i denotes the i^{th} column and j the j^{th} row. Each column of P sums to one, so that 1_2' P = 1_2', where 1_2 is a column vector of ones. For h-step ahead there are 2^h possible cases for the probabilities, which are computed directly from Hamilton's filter.