

In Memory of Vladimir Shelkovich (1949–2013)

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Vladimir Shelkovich was born in spring 1949 in Rostov-on-Don, a Southern Russian city. After graduation from the Rostov University, he had been working for some time in Rostov and then, in 1980, moved to Leningrad (now Saint Petersburg) and began to work at the St. Petersburg State University of Architecture and Civil Engineering, where he had been working till the last days of his life.

I met Vladimir in the second half of 1980s. At that time, I was already a rather well-known scientist in Russia and a coauthor of several books, and hence, at the very beginning, our relations resembled the relations between a teacher and a pupil, but even at that initial stage of our acquaintance, the influence of Vladimir's ideas was significant. At that time, he was studying the algebraic constructions in the theory of generalized functions on the basis of harmonic approximations. It was clear from the very beginning that this work would not find wide application, but the idea of constructive approach to studying the algebraic structures related to generalized functions was itself very close to the field of my research.

In a sense, our common results were based on these ideas but the harmonic approximations were replaced by smooth approximations (arbitrary or special, which were determined by some equations).

From the very beginning of our acquaintance, Vladimir was my closest friend, and his outstanding talents very soon allowed him to become my full-right coauthor.

As a result of our common work (Georgii Omel'yanov joined us later), we formulated a new approach for constructing the limit problems describing the propagation and (which is most important!) interaction of singularities of solutions to nonlinear partial differential equations. We called this approach the weak asymptotics method. There are rather many methods for constructing approximate analytic (asymptotic) solutions of PDE. All of them are based on the construction of functions depending on a small parameter and satisfying the problem conditions (the equation, initial data,

etc.) not exactly but with an arbitrary, usually polynomial, accuracy in a small parameter. The accuracy (smallness) of the discrepancy is understood in the sense of the maximum of the absolute value. In the weak asymptotics method, we proceed in the same way but the discrepancy smallness is understood in a weak sense.

There is an analogy between the weak asymptotics method, the Maslov–Whitham method for constructing asymptotic solutions with localized fast variation, and the method for constructing solutions of nonlinear equations developed by J.-F. Colombeau.

In the framework of the weak asymptotics method, the delta-shock wave solutions were defined for different systems of conservation laws and the superposition laws were obtained for such solutions. It should be noted that construction of the solutions describing the interaction of nonlinear waves with localized fast variation is a characteristic feature of the weak asymptotics method. Such constructions are impossible in the framework of the Maslov–Whitham method, and in the framework of the J.-F. Colombeau theory, such a construction is technically possible but has not been performed.

In the last years of his life, we did not write joint papers but were close friends. Vladimir was one of two or three people who knew the details of my life and I also knew almost everything about him. His untimely demise was absolutely unexpected for me, because I was sure that Vladimir was much younger. On the whole, it was very easy to deal with him, although our temperaments were quite different.

Thinking about our meetings and conversations, I recall his decency, sympathy, extreme sensitiveness to justice, and humor. He could not ignore a sick animal, was always attentive and caring to people. Vladimir’s attitude to his wife Zhenya was very touching, and they brought up their beautiful son Mikhail. Vladimir was a many-sided personality, and his deep knowledge in many areas, even in Hinduism and Buddhism, was fantastic, he also knew Sanskrit (and had been teaching it for a short time !). Vladimir wrote and published a philological paper about Dante’s poem “The Divine Comedy”, and was writing a paper about J. Joyce’s roman “Ulysses”.

His bright life came abruptly to an end, but the blessed memory of Vladimir Shelkovich will forever remain in the hearts of his students, friends, and colleagues.

Mathematical publications of Vladimir M. Shelkovich :

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In the last years of his life, we did not write joint papers but were close friends. Note that, in these last years, Vladimir’s scientific interests broadened to the field of p -adic analysis, and he obtained many interesting results there.

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Mathematical publications of Vladimir M. Shelkovich

References

- [1] Khrennikov, A. Y.; Kosyak, A. V.; Shelkovich, V. M., “Wavelet analysis on adeles and pseudo-differential operators,” *J. Fourier Anal. Appl.* 18 (2012), no. 6, 1215–1264.
- [2] Kosyak, A. V.; Khrennikov, A. Yu.; Shelkovich, V. M., “Pseudodifferential operators on adèles and wavelet bases,” *Dokl. Akad. Nauk* 444 (2012), no. 3, 253–257 (Russian) ; translation in *Dokl. Math.* 85 (2012), no. 3, 358–362.
- [3] Kosyak, F.V.; Khrennikov, A. Yu.; Shelkovich, V. M., “Wavelet bases on adeles,” *Dokl. Akad. Nauk* 442 (2012), no. 4, 446–450 (Russian); translation in *Dokl. Math.* 85 (2012), no. 1, 75–79.
- [4] Shelkovich, V. M., “Propagation of delta-shocks in one system of conservation laws,” *Rend. Semin. Mat. Univ. Politec. Torino* 69 (2011), no. 4, 393–402.
- [5] Shelkovich, V. M., “ p -adic evolution pseudodifferential equations and p -adic wavelets,” *Izv. Ross. Akad. Nauk Ser. Mat.* 75 (2011), no. 6, 163–194 (Russian); translation in *Izv. Math.* 75 (2011), no. 6, 1249–1278.

- [6] Nilsson, B.; Shelkovich, V. M. Mass, “Momentum and energy conservation laws in zero-pressure gas dynamics and delta-shocks,” *Appl. Anal.* 90 (2011), no. 11, 1677–1689.
- [7] Nilsson, B.; Rozanova, O. S.; Shelkovich, V. M., “Mass, momentum and energy conservation laws in zero-pressure gas dynamics and δ -shocks: II,” *Appl. Anal.* 90 (2011), no. 5, 831–842.
- [8] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., “The Cauchy problems for evolutionary pseudo-differential equations over p -adic field and the wavelet theory,” *J. Math. Anal. Appl.* 375 (2011), no. 1, 82–98.
- [9] Shelkovich, V. M., “Singular solutions to systems of conservation laws and their algebraic aspects. Linear and non-linear theory of generalized functions and its applications,” *Banach Center Publ.*, 88, Polish Acad. Sci. Inst. Math., Warsaw, 2010, pp. 251–266.
- [10] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., “Theory of p -adic distributions: linear and nonlinear models,” *London Mathematical Society Lecture Note Series*, 370. Cambridge University Press, Cambridge, 2010.
- [11] Khrennikov, A. Yu.; Shelkovich, V. M., “Non-Haar p -adic wavelets and their application to pseudo-differential operators and equations,” *Appl. Comput. Harmon. Anal.* 28 (2010), no. 1, 1–23.
- [12] Shelkovich, V. M., “Transport of mass, momentum and energy in zero-pressure gas dynamics. Hyperbolic problems: theory, numerics and applications,” *Proc. Sympos. Appl. Math.*, 67, Part 2, Amer. Math. Soc., Providence, RI, 2009, pp. 929–938.
- [13] Khrennikov, A. Yu.; Shelkovich, V. M.; Skopina, M., “ p -Adic orthogonal wavelet bases. p -Adic Numbers,” *Ultrametric Anal. Appl.* 1 (2009), no. 2, 145–156.
- [14] Khrennikov, A. Yu.; Shelkovich, V. M., “An infinite family of p -adic non-Haar wavelet bases and pseudo-differential operators. p -Adic Numbers,” *Ultrametric Anal. Appl.* 1 (2009), no. 3, 204–216.
- [15] Khrennikov, A. Yu.; Shelkovich, V. M.; Skopina, M., “ p -Adic refinable functions and MRA-based wavelets,” *J. Approx. Theory* 161 (2009), no. 1, 226–238.
- [16] Shelkovich, Vladimir; Skopina, Maria, “ p -Adic Haar multiresolution analysis and pseudo-differential operators,” *J. Fourier Anal. Appl.* 15 (2009), no. 3, 366–393.
- [17] Khrennikov, A. Yu.; Shelkovich, V. M., “Asymptotical behavior of one class of p -adic singular Fourier integrals,” *J. Math. Anal. Appl.* 350 (2009), no. 1, 170–183.
- [18] Shelkovich, V. M., “On delta-shocks and singular shocks. Hyperbolic problems: theory, numerics, applications,” Springer, Berlin, 2008, pp. 971–979.
- [19] Shelkovich, V. M., “Singular solutions of δ - and δ' -shock wave type of systems of conservation laws, and transport and concentration processes,” *Uspekhi Mat. Nauk* 63 (2008), no. 3(381), 73–146 (Russian); translation in *Russian Math. Surveys* 63 (2008), no. 3, 473–546.
- [20] Khrennikov, A. Yu.; Shelkovich, V. M., “Non-Haar p -adic wavelets and pseudo-differential operators,” *Dokl. Akad. Nauk* 418 (2008), no. 2, 167–170 (Russian); translation in *Dokl. Math.* 77 (2008), no. 1, 42–45.
- [21] Shelkovich, V. M., “Associated and quasi associated homogeneous distributions (generalized functions),” *J. Math. Anal. Appl.* 338 (2008), no. 1, 48–70.
- [22] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., “ p -Adic semilinear evolution pseudodifferential equations in Lizorkin spaces,” *Dokl. Akad. Nauk* 415 (2007), no. 3, 295–299 (Russian); translation in *Dokl. Math.* 76 (2007), no. 1, 539–543.

- [23] Albeverio, S.; Shelkovich, V. M., "On the delta-shock front problem. Analytical approaches to multidimensional balance laws," Nova Sci. Publ., New York, 2006, pp. 45–87.
- [24] Panov, E. Yu.; Shelkovich, V. M., " δ -Shock waves as a new type of singular solution of hyperbolic systems of conservation laws," Dokl. Akad. Nauk 407 (2006), no. 5, 595–599 (Russian).
- [25] Shelkovich, V. M., "The Rankine–Hugoniot conditions and balance laws for δ -shock waves," Fundam. Prikl. Mat. 12 (2006), no. 6, 213–229 (Russian); translation in J. Math. Sci. (N. Y.) 151 (2008), no. 1, 2781–2792.
- [26] Shelkovich, V. M., "The Riemann problem admitting δ -, δ' -shocks, and vacuum states (the vanishing viscosity approach)," J. Differential Equations 231 (2006), no. 2, 459–500.
- [27] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., "Pseudo-differential operators in the p -adic Lizorkin space. p -Adic mathematical physics, AIP Conf. Proc., 826, Amer. Inst. Phys., Melville, NY, 2006, pp. 195–205.
- [28] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., "Harmonic analysis in the p -adic Lizorkin spaces: fractional operators, pseudo-differential equations, p -adic wavelets," Tauberian theorems. J. Fourier Anal. Appl. 12 (2006), no. 4, 393–425.
- [29] Panov, E. Yu.; Shelkovich, V. M., " δ -shock waves as a new type of solutions to systems of conservation laws," J. Differential Equations 228 (2006), no. 1, 49–86.
- [30] Khrennikov, A. Yu.; Shelkovich, V. M., "Tauberian theorems for p -adic distributions," Integral Transforms Spec. Funct. 17 (2006), no. 2–3, 141–147.
- [31] Khrennikov, A. Yu.; Shelkovich, V. M., "Distributional asymptotics and p -adic Tauberian and Shannon–Kotelnikov theorems," Asymptot. Anal. 46 (2006), no. 2, 163–187.
- [32] Albeverio, S.; Khrennikov, A. Y.; Shelkovich, V. M., "Associated homogeneous p -adic distributions," J. Math. Anal. Appl. 313 (2006), no. 1, 64–83.
- [33] Danilov, V. G.; Shelkovich, V. M., "Delta-shock wave type solution of hyperbolic systems of conservation laws," Quart. Appl. Math. 63 (2005), no. 3, 401–427.
- [34] Shelkovich, V. M., "New versions of the Colombeau algebras," Math. Nachr. 278 (2005), no. 11, 1318–1340.
- [35] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., "Nonlinear singular problems of p -adic analysis: associative algebras of p -adic distributions," Izv. Ross. Akad. Nauk Ser. Mat. 69 (2005), no. 2, 3–44 (Russian); translation in Izv. Math. 69 (2005), no. 2, 221–263.
- [36] Danilov, V. G.; Shelkovich, V. M., "Dynamics of propagation and interaction of δ -shock waves in conservation law systems," J. Differential Equations 211 (2005), no. 2, 333–381.
- [37] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., " p -adic Colombeau–Egorov type theory of generalized functions," Math. Nachr. 278 (2005), no. 1–2, 3–16.
- [38] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., "Associative algebras of p -adic distributions," Tr. Mat. Inst. Steklova 245 (2004), Izbr. Vopr. p -Adich. Mat. Fiz. i Anal., 29–40 (Russian); translation in Proc. Steklov Inst. Math. 2004, no. 2 (245), 22–33.
- [39] Danilov, V. G.; Shelkovich, V. M., "Propagation and interaction of δ -shock waves of hyperbolic systems of conservation laws," Dokl. Akad. Nauk 394 (2004), no. 1, 10–14 (Russian).
- [40] Albeverio, S.; Khrennikov, A. Yu.; Shelkovich, V. M., "Associated homogeneous p -adic generalized functions," Dokl. Akad. Nauk 393 (2003), no. 3, 300–303 (Russian).

- [41] Danilov, V. G.; Shelkovich, V. M., Propagation and interaction of delta-shock waves of a hyperbolic system of conservation laws. *Hyperbolic problems: theory, numerics, applications*, Springer, Berlin, 2003, pp. 483–492.
- [42] Shelkovich, V. M., “Delta-shock waves of a class of hyperbolic systems of conservation laws. Patterns and waves,” (Saint Petersburg, 2002), AkademPrint, St. Petersburg, 2003, pp. 155–168.
- [43] Danilov, V. G.; Omel’yanov, G. A.; Shelkovich, V. M., “Weak asymptotics method and interaction of nonlinear waves. Asymptotic methods for wave and quantum problems,” *Amer. Math. Soc. Transl. Ser. 2*, 208, Amer. Math. Soc., Providence, RI, 2003, pp. 33–163.
- [44] Khrennikov, A. Yu.; Shelkovich, V. M.; Smolyanov, O. G., “Locally convex spaces of vector-valued distributions with multiplicative structures,” *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* 5 (2002), no. 4, 483–502.
- [45] Smolyanov, O. G.; Khrennikov, A. Yu.; Shelkovich, V. M., “Multiplicative structures in a linear space of vector-valued distributions,” *Dokl. Akad. Nauk* 383 (2002), no. 1, 28–31, (Russian).
- [46] Danilov, V. G.; Shelkovich, V. M., “Propagation and interaction of nonlinear waves to quasilinear equations. Hyperbolic problems: theory, numerics, applications,” Vol. I, II (Magdeburg, 2000), 267–276, *Internat. Ser. Numer. Math.*, 140, 141, Birkh?user, Basel, 2001.
- [47] Danilov, V. G.; Shelkovich, V. M., “Propagation and interaction of shock waves of quasilinear equation,” *Nonlinear Stud.* 8 (2001), no. 1, 135–169.
- [48] Danilov, V. G.; Maslov, V. P.; Shelkovich, V. M., “Algebras of the singularities of singular solutions of first-order quasilinear strictly hyperbolic systems,” *Teoret. Mat. Fiz.* 114 (1998), no. 1, 3–55 (Russian); translation in *Theoret. and Math. Phys.* 114 (1998), no. 1, 1–42.
- [49] Danilov, V. G.; Shelkovich, V. M., “Generalized solutions of nonlinear differential equations and the Maslov algebras of distributions. Generalized functions – linear and nonlinear problems (Novi Sad, 1996). *Integral Transform. Spec. Funct.* 6 (1998), no. 1–4, 171–180.
- [50] Shelkovich, V. M., “The Colombeau theory of generalized functions using harmonic regularizations,” *Mat. Zametki* 63 (1998), no. 2, 313–316 (Russian); translation in *Math. Notes* 63 (1998), no. 1–2, 275–278.
- [51] Shelkovich, V. M., “An associative-commutative algebra of distributions that includes multipliers, and generalized solutions of nonlinear equations,” *Mat. Zametki* 57 (1995), no. 5, 765–783, 800 (Russian); translation in *Math. Notes* 57 (1995), no. 5–6, 536–549.
- [52] Shelkovich, V. M., “An algebra of distributions and generalized solutions of nonlinear equations,” *Dokl. Akad. Nauk* 342 (1995), no. 5, 600–602 (Russian).
- [53] Shelkovich, V. M.; Yuzhakov, A. P., “The structure of a class of asymptotic distributions of V. K. Ivanov,” *Izv. Vyssh. Uchebn. Zaved. Mat.* 1991, no. 4, 70–73 (Russian); translation in *Soviet Math. (Iz. VUZ)* 35 (1991), no. 4, 70–72.
- [54] Shelkovich, V. M., “An associative algebra of distributions and multipliers,” *Dokl. Akad. Nauk SSSR* 314 (1990), no. 1, 159–164 (Russian); translation in *Soviet Math. Dokl.* 42 (1991), no. 2, 409–414.
- [55] Shelkovich, V. M., “An algebra of distributions connected with a light cone,” *Izv. Vyssh. Uchebn. Zaved. Mat.* 1983, no. 8, 84–87 (Russian).
- [56] Shelkovich, V. M., “An algebra of distributions with a singular carrier that is a point,” *Dokl. Akad. Nauk SSSR* 267 (1982), no. 1, 53–57 (Russian).

- [57] Shelkovich, V. M., "Multiplications of distributions generated by Wightman functions, and an algebra of hyperdistributions," *Izv. Vyssh. Uchebn. Zaved. Mat.* 1980, no. 2, 80–84 (Russian).
- [58] Kakichev, V. A.; Shelkovich, V. M., "Riemann's problem for a p -cone in Vladimirov algebras," *Mat. Zametki* 27 (1980), no. 6, 899–911, 989 (Russian).
- [59] Kakichev, V. A.; Shelkovich, V. M., "Solution of boundary problems for analytic functions of several variables in the algebras of Vladimirov," *Mat. Zametki* 22 (1977), no. 1, 51–60 (Russian).