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Scientia Agricola
http://dx.doi.org/10.1590/1678-992X-2016-0035

Growth and yield models for eucalyptus stands obtained by differential equations

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Edited by: Rafael Rubilar Pons

Received January 24, 2016
Accepted September 11, 2016

ABSTRACT: The main purpose of this study was to assess nonlinear models generated by integrating the basal area growth rate to estimate the growth and yield of forest stands. The database was collected from permanent sample units, in Paraopeba county, in the state of Minas Gerais, Brazil. The stands were represented by *Eucalyptus camaldulensis* × *Eucalyptus urophylla* hybrid trees, with 3 × 3 meters of spacing. The data were divided into two groups: fitting and validating databases. Two nonlinear models (Strategy A and Strategy B) were developed using differential equations to estimate the basal area growth and yield of the sample units. The logistic model was fitted to estimate the volumetric yield as a function of age, site index and basal area. The efficiency of the systems generated by logistic model and models obtained by differential equations (Strategy A and Strategy B) was also compared to the efficiency of the system estimated by the Clutter model (Strategy C). The projection models used to estimate basal area obtained by differential equations were compatible with forest growth and yield, and the logistic model with covariates was compatible with volumetric growth and yield. Strategy A and Strategy B generated different thinning and harvesting options for different site indices, which is biologically consistent.
Keywords: nonlinear models, compatible models, biological consistency

Introduction

An accurate estimation of forest growth and yield is fundamental to the achievement of forestry planning objectives. The stand growth and yield volume, both present and future, are the most important items of information if forestry planning is to be successful.

Moreover, an important positive in growth and yield studies is the opportunity to implement prognosis models. Growth models represent the relationship between the quantity of yield and growth and the various factors that explain or allow for the estimation of this growth (Davis et al., 2000) and several studies on forest growth and yield have been completed in the past including the following: Bonet et al. (2012), Burkhart (1971), Clutter (1963), Pienaar (1979), Pienaar and Turnbull (1973), Schumacher (1939) and Sullivan and Clutter (1972).

The sampling process for estimating the present yield of forest stands and their dynamics has led to the continued improvement of techniques used to construct growth and yield models. Growth models can be represented by differential equations or by systems with two or more differential equations (Wraith and Or, 1988). Clutter (1963) and Schumacher (1939) have used differential equations in their models, but most of them have used linear relationships only. Furthermore, Garcia (1980) developed a stochastic differential equation model for the height growth of even-aged stands in which the deterministic part is equivalent to the Bertalanffy-Richards model.

By using nonlinear models, important assumptions are incorporated in the problem of obtaining a theoretical relationship between the variables of interest, instead of an empirical description. Another advantage of nonlinear models lies in parameter interpretation and parsimony. In many situations, nonlinear models require fewer parameters than linear models, which can simplify the interpretation of the model results. Nonlinear yield models, with sigmoidal trend lines, represent the growth of individuals, or populations by presenting the same trend line of the yield over time. Examples using models to project growth by nonlinear equations are Koya and Goshu (2013) and Zeide (1993, 2004). In a unified sigmoid growth equation, Garcia (2005) states that a general model can simplify the study of properties such as the presence and nature of asymptotes and inflection points, and facilitate the development of more widely applicable software.

In this context, the main objective of the present study was to model basal area differential equations in order to generate nonlinear models that can estimate the growth and yield of forest stands.

Materials and Methods

Study area and database

The database used in this study was obtained from an area located in Paraopeba county (19°16′28″ S, 44°24′15″ W and altitude of 761m), in the state of Minas Gerais, Brazil. The average temperature in this region is 20.9 °C, the annual mean precipitation 1328 mm.

Unthinned stands of the hybrid *Eucalyptus camaldulensis* × *Eucalyptus urophylla,* cultivated in a 3 × 3m spacing, were used in this study. A database, collected from permanent plots each measuring 400 m², was used to estimate basal area growth, yield and volumetric yield. The number of measurements over time to permanent plots ranged from two to four. The database was divided into two random groups (Table 1): database to fit the models (N = 88 plots) and data to validate the models (N = 40 plots).

Table 1 – Descriptive statistics of variables related to stands of the *Eucalyptus camaldulensis* × *Eucalyptus urophylla.*

Variables	Fitting				Validating			
	Min.	Max.	Mean	CV (%)	Min.	Max.	Mean	CV (%)
S	22.5	32.5	27.5	11.26	22.5	32.5	27.5	10.63
A	1.3	7.7	3.8	37.87	1.3	7.4	3.3	38.18
N	750	1325	1078	9.60	625	1550	1089	14.08
B	2.87	27.21	17.32	21.15	2.87	28.31	13.89	37.01
V	52.32	364.82	160.24	35.47	52.32	340.78	144.46	42.05

Wherein: *S* = site index (m); *A* = age (years); *N* = number of trees per hectare (trees ha^{-1}); *B* = basal area (m^2 ha^{-1}); *V* = stand volume (m^3 ha^{-1}).

Development of the growth and yield models

The approach used in this study is quite different from that taken by Clutter (1963). In this study the dependent variable was not transformed and the growth function used had a nonlinear parameter combination. Two modeling strategies were developed (Strategies A and B). When implementing Strategy A, the first step was to choose a model that represented variations of the current annual increment (CAI) over time. The model chosen was $Y = \beta_0 A.e^{-\beta_1 X}$ (1) (Ratkowsky, 1990). Considering the methodology proposed by Clutter (1963) for linear approach and the expression (1), wherein Y is equal to the CAI in basal area of the stand and X is equal to the age of the stand, it is possible to compose the following expression for CAI:

$$CAI = \frac{dB}{dA} = \beta_0 A e^{-\beta_1 A} \tag{2}$$

wherein B = basal area of i^{th} stand (m^2 ha^{-1}), A = age of the stand (years), β_i = parameters of the model and e = base of the natural logarithm.

Expression (2) is a separable differential equation and was integrated to obtain a basal area yield function:

$$\int dB = \int \beta_0 A e^{-\beta_1 A} dA$$

$$B = \beta_0 \frac{\left(-e^{-\beta_1 A}\beta_1 A - e^{-\beta_1 A}\right)}{\beta_1^2} \tag{3}$$

The projection model for the basal area of the stand was derived by integrating equation (2) from basal areas B_1 to B_2 and from ages A_1 to A_2:

$$\int_{B_1}^{B_2} dB = \int_{A_1}^{A_2} \beta_0 A e^{-\beta_1 A} dA$$

$$B_2 - B_1 = \beta_0 \left[\frac{-e^{-\beta_1 A_2}\left(\beta_1 A_2 + 1\right) + e^{-\beta_1 A_1}\left(\beta_1 A_1 + 1\right)}{\beta_1^2}\right]$$

$$B_2 = B_1 + \beta_0 \left[\frac{-e^{-\beta_1 A_2}\left(\beta_1 A_2 + 1\right) + e^{-\beta_1 A_1}\left(\beta_1 A_1 + 1\right)}{\beta_1^2}\right] \tag{4}$$

wherein B_1 = basal area for age A_1, B_2 = basal area for age A_2, A_1 = present age, and A_2 = future age.

The basal area also depends on the stand site quality. Next, in expression (4) the effect of the site index was incorporated (S) as a covariate (Guimarães et al., 2009):

$$B_2 = B_1 = \left(\beta_{00} + \beta_{01} S\right)\left\{\frac{-e^{-(\beta_{10}+\beta_{11})A_2}\left[\left(\beta_{10} + \beta_{11} S\right) A_2 + 1\right] + e^{-(\beta_{10}+\beta_{11} S)A_1}\left[\left(\beta_{10} + \beta_{11} S\right) A_1 + 1\right]}{\left(\beta_{10} + \beta_{11} S\right)^2}\right\} \tag{5}$$

The next step was to choose a model to estimate the stand volume. The logistic model was used to estimate volumetric yield:

$$V_2 = \frac{\beta_0}{1 + e^{\left[(\beta_1 - A_2)/\beta_2\right]}} + \varepsilon \tag{6}$$

wherein V_2 = the stand volume for age A_2, and ε = stochastic error.

The logistic model, like other nonlinear models, can be fitted using the initial parameter values generated by interpreting them. The parameter β_0 represents the upper horizontal asymptote (UHA); specifically, the maximum response value (V_2) as time tends to $+\infty$. The parameter β_1 represents the inflexion point of the curve; in other words, the age (A_2) where the yield (V_2) reaches half of β_0. The parameter β_2 represents the difference between the age when the yield reaches approximately 73 % of β_0 and the age corresponding to the inflexion point. This interpretation is very useful, as the greatest limitation in using nonlinear models is the correct choice of initial parameters for the iteration process. Based on the fact that the variation of the total stand volume is explained by more than their age, the parameters of the logistic model were decomposed inserting the site index (S) and the basal area of the stand (B) as covariates (expression 7).

$$V_2 = \frac{\beta_{00} + \beta_{01} S + \beta_{01} B_2}{1 + e^{\left\{\left[(\beta_{10}+\beta_{11}S+\beta_{12}B_2)-A_2\right]/(\beta_{20}+\beta_{21}S+\beta_{22}B_2)\right\}}} + \varepsilon \tag{7}$$

To test the compatibility of the logistic model with covariates, the first derivative of the growth function was solved as a function of age.

To develop Strategy B, the expression (1) was replaced by the following CAI expression (Ratkowsky, 1990):

$$CAI = \frac{dB}{dA} = \left(\beta_0 A - \beta_1\right) e^{-\beta_0 A} \tag{8}$$

The replacement of the expression (1) by expression (8) is the only difference between Strategies A and B, i.e., the other steps are the same. Considering that the Clutter model is widely used in forest studies, Strategies A and B were compared to his model. To simplify the result interpretations, the Clutter model was called Strategy C. In summary, we had the following systems of equations for Strategies A, B and C:

Strategy A:

$$B_2 = B_1 = (\beta_{00} + \beta_{01}S)\left\{\frac{-e^{-(\beta_{10}+\beta_{11})A_2}\left[(\beta_{10} + \beta_{11}S)A_2 + 1\right] + e^{-(\beta_{10}+\beta_{11}S)A_1}\left[(\beta_{10} + \beta_{11}S)A_1 + 1\right]}{(\beta_{10} + \beta_{11}S)^2}\right\} \quad (5)$$

$$V_2 = \frac{\beta_{00} + \beta_{01}S + \beta_{01}B_2}{1 + e^{\{[(\beta_{10}+\beta_{11}S+\beta_{12}B_2)-A_2]/(\beta_{20}+\beta_{21}S+\beta_{22}B_2)\}}} + \varepsilon \quad (7)$$

Strategy B:

$$B_2 = B_1 + \left\{\frac{e^{-(\beta_{00}+\beta_{01}S)A_2}\left[-(\beta_{00} + \beta_{01}S)A_2 - 1 + (\beta_{10} + \beta_{11}S)\right] + e^{-(\beta_{00}+\beta_{01}S)A_1}\left[(\beta_{00} + \beta_{01}S)A_1 + 1 - (\beta_{10} + \beta_{11}S)\right]}{(\beta_{00} + \beta_{01}S)}\right\} \quad (9)$$

$$V_2 = \frac{\beta_{00} + \beta_{01}S + \beta_{01}B_2}{1 + e^{\{[(\beta_{10}+\beta_{11}S+\beta_{12}B_2)-A_2]/(\beta_{20}+\beta_{21}S+\beta_{22}B_2)\}}} + \varepsilon \quad (7)$$

Strategy C:

$$LnV_2 = \beta_0 + \beta_1\left(\frac{1}{A_2}\right) + \beta_2 S + \beta_3 LnB_2 + \varepsilon \quad (10)$$

$$LnB_2 = LnB_1\left(\frac{A_1}{A_2}\right) + \alpha_0\left(1 - \frac{A_1}{A_2}\right) + \alpha_1\left(1 - \frac{A_1}{A_2}\right)S + \varepsilon \quad (11)$$

wherein Ln = natural logarithm, and α_i = parameters of the model.

The stand's basal area projection and volume models were fitted by R statistical software (Version 2.10.1). The projection models of Strategies A and B (expressions 5, 7 and 9) were fitted using the *nlme* package. The Clutter model (expressions 10 and 11) was fitted using the least squares method in two stages (*systemfit* package).

Evaluation of the models

To compare the models evaluated in terms of accuracy, the following statistics were applied:

a) $\bar{R}^2$ as proposed by Kvalseth (1985)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2} \qquad \bar{R}^2 = 1 - \left(1 - R^2\right)\frac{(n-1)}{n-p}$$

wherein $\bar{R}^2$ = adjusted coefficient of determination; R^2 = coefficient of determination, n = number of observations, p = number of parameters. $\hat{Y}_i$ = the estimated values of stand basal area or volume by the model, Y_i = observed values of stand basal area or volume, $\bar{Y}$ = average of the stand basal area or volume.

b) Root mean square error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i = \hat{Y}_i\right)^2}{n-p}} \qquad RMSE(\%) = \frac{RMSE}{\bar{Y}}\ 100$$

wherein $RMSE$ = root mean square error.

c) Bias

$$B = \frac{\sum_{i=1}^{n} Y_i - \sum_{i=1}^{n}\hat{Y}_i}{n} \quad B(\%) = 100\frac{B}{\bar{Y}}$$

wherein B = bias.

Furthermore, error graphical analyses (%) were performed to check the quality of the model estimations. The values of the error (%) used in the construction of the graphs were expressed by:

$$Error_i(\%) = 100\frac{Y_i - \hat{Y}_i}{Y_i}\ .$$

Results and Discussion

Analysis of the growth and yield models

Table 2 presents the statistics corresponding to the fit of the basal area projection considering Strategies A, B and C.

The parameter α_1 of Clutter's system of equations was removed from the model because it was not significant (*p-value* > 0.05). In terms of the root mean square error [RMSE (%)], Strategy B had a better performance, followed by Strategies A and C. The results were similar using the statistical $\bar{R}^2$ and Bias (%), with Strategy B performing better than either Strategy A or C.

The results in Table 2 show that Strategies A, B and C presented similar performance in projecting the basal area with a slight advantage for Strategy B. On the

Table 2 – Statistics for the basal area projection models for the stands for the fit data.

Strategy A ($\bar{R}^2$ = 0.9255; RMSE % = 5.69 %; Bias (%) = 0.55 %)				
Parameter	Estimate	Standard error	*t*	*p*-value
β_{00}	25.64821	5.38068	4.77	< 0.001
β_{01}	-0.54550	0.18440	-2.96	0.004
β_{10}	1.13779	0.16258	7.00	< 0.001
β_{11}	-0.01382	0.00590	-3.02	0.003
Strategy B ($\bar{R}^2$ = 0.9349; RMSE % = 5.32; Bias (%) = 0.24 %)				
Parameter	Estimate	Standard error	*t*	*p*-value
β_{00}	0.85473	0.14296	5.98	< 0.001
β_{01}	-0.01830	0.00520	-3.52	< 0.001
β_{10}	-28.08757	5.19361	-5.41	< 0.001
β_{11}	0.60196	0.17786	3.38	< 0.001
Strategy C ($\bar{R}^2$ = 0.9229; RMSE %= 5.85; Bias (%) = -0.29 %)				
Parameter	Estimate	Standard error	*t*	*p*-value
α_0	3.564632	0.03804	93.69	< 0.001

Wherein: $\bar{R}^2$ = adjusted determination coefficient; RMSE = root mean square error.

other hand, if the basal area databases, from forest inventory, are available and it is not necessary to project this variable, it is possible to compare the volumes projected by Strategies A and B with the volume projected by the Clutter model. As Strategies A and B used the same expression to project the volume, i.e., the Logistics model, a comparison was then made between this model and Clutter's volumetric model (Table 3).

However, when comparing the logistics model (expression 7) with the Clutter model (expression 10), a number of inconsistent results were found; for example, lower cutting ages in less productive sites. Thus, expression 7 was rearranged by testing different combinations of S and B_2 as covariates in order to find consistent results. The combination of S and B_2 that best met consistent results is presented in Table 3.

All of the parameters were significant (*p-value* < 0.001). The models evaluated showed accurate volumetric growth and yield estimates (RMSE < 4 % and $\bar{R}^2$ > 0.98) and the Clutter model performed slightly better than the logistics model. This indicates that both models can be used for estimating stand volume. However, the logistics model presented smaller Bias (%) and often this is the model of choice.

Compatibility of the logistics model with covariates in estimating the volumetric yield

The compatibility of the logistic model was tested by taking the first derivative of the yield model as a function of age:

$$\frac{\partial V_i}{\partial A_i} = \frac{\hat{\beta}_{00} + \hat{\beta}_{01} S}{1 + e^{\left\{\left[\left(\hat{\beta}_{10} + \hat{\beta}_{11} B_i\right) - A_i\right] / \hat{\beta}_2\right\}}} \quad (12)$$

The basal area was estimated for Strategy B (9), site index of 20 m and ages from 2 to 4 years old. The basal area estimated for 2 years old was equal to 2.50 m^2 ha^{-1}. After integrating the growth equation, the yield estimated was 25.35 m^3 ha^{-1}. For three years old, the basal area was 7.65 m^2 ha^{-1} and the yield was 43.82 m^3 ha^{-1}. For four years old, the basal area was 11.03 m^2 ha^{-1} and the yield was 74.66 m^3 ha^{-1}. The same yields were obtained using equation (7), demonstrating the compatibility of the proposed estimation process.

Analysis of validation data of growth and yield models for calculating basal area and volume

Table 4 shows the RMSE (%) and the Bias (%) of the growth and yield projection used to estimate stand basal area (Strategies A, B and C) and volume. The comparison of the volumetric models was made in the same way to produce the results in Table 3.

In general, the models showed to be accurate when estimating basal area (RMSE < 8 % and Bias < 1.5 %) and volume (RMSE < 9 % and Bias < 5 %) in all sites. The results obtained with the validation data were similar to those obtained with the data used for fitting the models.

The residual distributions estimated with the basal area projection models are shown in Figure 1. Most errors were concentrated within the range of ± 10 % (Figure 1). Despite this, the models had a certain tendency to underestimate the stand basal area located above 25 m^2 ha^{-1}. The range of stand basal area observed, approximately from 8.1 to 28.3 m^2 ha^{-1}, was the same as estimated by both the proposed and the Strategy C models.

The residual distribution for the logistic model with covariates and the Clutter model for volumetric growth and yield estimates is shown in Figure 2. The majority of errors were ± 10 %. The low error from Figure 2 is in agreement with the low value of RMSE as reported in Table 4.

Table 5 shows the RMSE (%) and Bias (%) for the strategies' stand volume projections.

The results (Table 5) demonstrated that the strategies were accurate and similar to estimate stand volume. When comparing the RMSE values (%), Strategy B presented the best volume projection result followed by Strategy A and Strategy C in all sites. But, when comparing the value of Bias (%), the best result was obtained when using Strategy C.

Table 3 – Statistics of the logistics model with addition of covariates and statistics of the Clutter model to estimate volumetric growth and yield for the fit data.

Logistic with Covariates ($\bar{R}^2$ =0.9881; RMSE % = 3.80; Bias (%) = -0.03)				
Parameter	Estimate	Standard error	*t*	*p*-value
β_{00}	218.65999	23.50651	9.30	< 0.001
β_{01} S	10.83302	0.73223	14.79	< 0.001
β_{10}	27.06476	1.76727	15.31	< 0.001
β_{11} B_2	-0.91066	0.06159	-14.79	< 0.001
β_2	8.30707	0.52370	15.86	< 0.001
Clutter ($\bar{R}^2$ = 0.9890; RMSE % = 3.68; Bias (%) = 0.22)				
Parameter	Estimate	Standard error	*t*	*p*-value
β_0	1.40134	0.07886	17.77	< 0.001
β_1	-1.53391	0.07412	-20.69	< 0.001
β_2	0.02136	0.00111	19.18	< 0.001
β_3	1.20767	0.02577	46.86	< 0.001

Wherein: $\bar{R}^2$ = adjusted determination coefficient; RMSE = root mean square error; S = site index (S) and B_2 = basal area for age A_2.

Table 4 – Root mean square error [RMSE (%)] and relative bias [Bias (%)] for growth and yield projection models used to estimate the basal area and volume of *Eucalyptus camaldulensis* × *Eucalyptus urophylla* stands for the validation data.

Strategy	Site						General	
	22.5		27.5		32.5			
	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias
	%							
Basal area								
Strategy A	5.63	0.59	6.50	0.76	7.76	-0.42	6.52	0.59
Strategy B	5.65	1.35	6.20	0.45	7.79	-0.12	6.32	0.51
Strategy C	4.52	-0.27	6.60	0.73	7.83	-1.7	6.45	0.25
Volume								
Logistic	3.24	-2.59	3.45	0.92	8.29	-1.71	4.25	0.05
Clutter	5.2	-4.82	3.25	1.31	7.08	-3.12	4.13	-0.02

Wherein: RMSE = root mean square error.

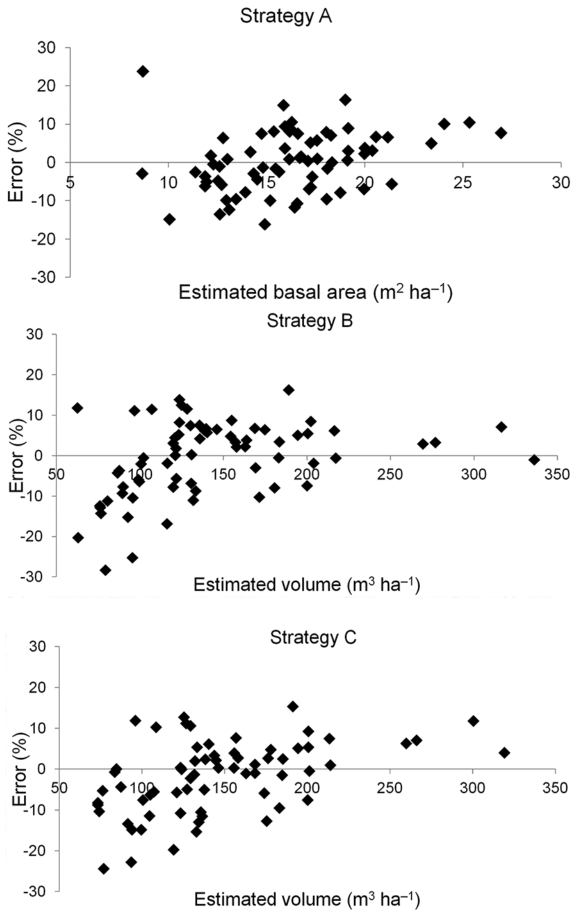

Figure 1 – Residual distribution (%) in function of the basal area estimated for Strategies A, B and C, respectively.

Table 5 – Root mean square error [RMSE (%)] and relative bias [Bias (%)] for the growth and yield equation strategies analyzed in volume of *Eucalyptus camaldulensis* × *Eucalyptus urophylla* stands for the validation data.

Strategy	Site						General	
	22.5		27.5		32.5			
	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias
	%							
Strategy A	7.82	-0.90	8.22	2.28	5.37	-1.16	7.98	1.37
Strategy B	7.75	-0.19	8.01	1.95	5.81	-0.64	7.83	1.30
Strategy C	7.51	-4.84	9.05	2.48	5.82	-3.12	8.65	0.65

Where in: RMSE = root mean square error.

Figure 3 shows the residual distribution as a function of the estimated volume for the projection growth and yield systems.

The residual distribution presented in Figure 3 is concentrated in the range of ± 20 %. The three strategies for volume projection have a similar residual distribution and the systems show a tendency to underestimate volume values above 250 $m^3\,ha^{-1}$.

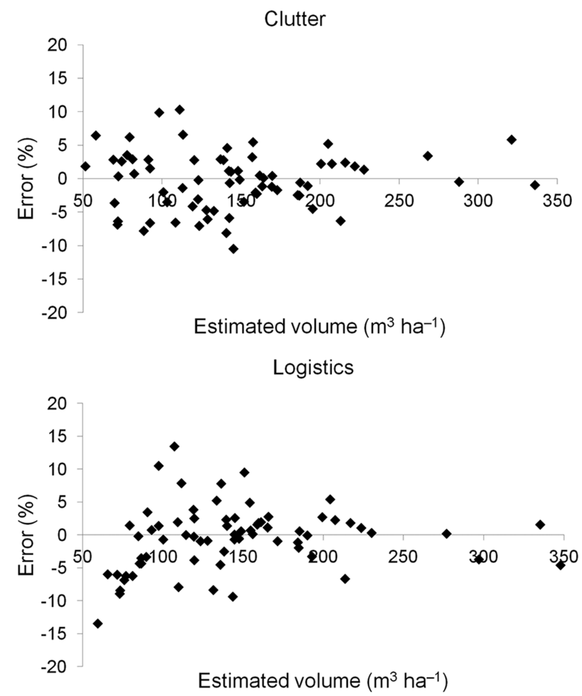

Figure 2 – Residual distribution (%) as a function of the volume estimated for the logistics model with covariates and the Clutter Model, respectively.

The real volume distribution of the stands is approximately between 52.3 $m^3\,ha^{-1}$ and 340.8 $m^3\,ha^{-1}$ (Table 1). Strategy B was the most accurate for estimating this range (between 62.56 $m^3\,ha^{-1}$ and 336.01 $m^3\,ha^{-1}$), followed by Strategy A (between 59.93 $m^3\,ha^{-1}$ and 330.04 $m^3\,ha^{-1}$) and Strategy C (between 47.57 $m^3\,ha^{-1}$ and 319.12 $m^3\,ha^{-1}$), respectively.

Application of the proposed strategies for volumetric growth and yield estimation

The estimations of the current annual increments and the mean annual increments in different site indexes are shown in Figures 4, 5, and 6.

Strategy A showed harvesting or thinning alternatives (where the MAI and CAI crossed each other) with approximately 4.6 years for the site index of 22.5 m and 4.2 years for the site index of 27.5 m (Figure 4). However, for the site index of 32.5 m, the curves of CAI and MAI did not intercept. Strategy B generated alternatives of harvesting or thinning approximately at 5.5 years for the site index of 2.5 m, 5.4 years for the site index of 27.5 m, and 5.0 years for the site index of 32.5 m. Strategy C, representing the Clutter models (Figure 6), generated alternatives at 5.1 years for the site index of 22.5 m, 4.9 years for the site index of 27.5 m, and 4.3 years for the site index of 32.5 m. Strategies A, B and C generated biologically consistent harvesting alternatives (i.e., higher rotations for less productive sites; Schumacher, 1939; Sevillano-Marco, 2009). This characteristic is very important to growth and yield

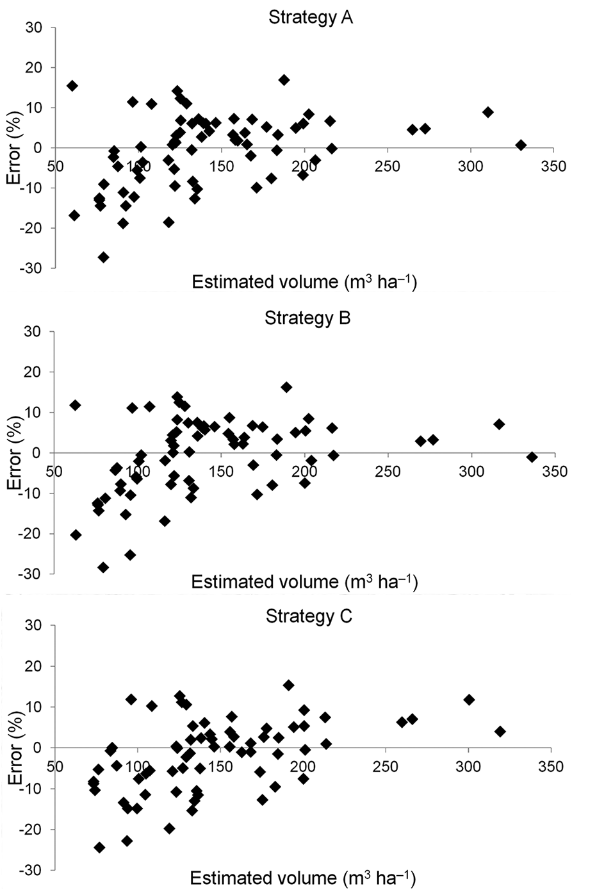

Figure 3 – Residual distribution (%) as a function of the volume estimated for the Strategies A, B and C, respectively.

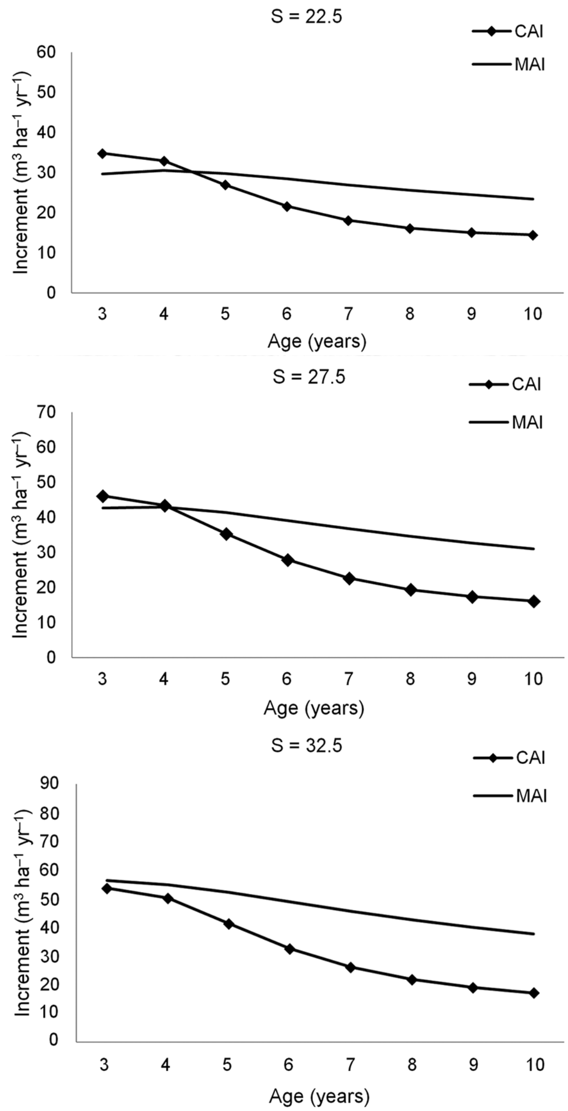

Figure 4 – Relationship between current annual increment (CAI) and mean annual increment (MAI) for Strategy A.

models because it constitutes a method for evaluating these models (Vanclay, 1994; Vanclay and Skovsgaard, 1997).

Conclusions

Models for projecting stand growth and yield, based on differential equations, can generate precise estimations. The basal area projection models obtained by using differential equations were compatible with forest growth and yield. The logistics model with covariates was compatible with volume growth and yield. Strategies A and B generated different harvesting/thinning alternatives for different site indexes, presenting biological consistency. These equation systems are alternatives for projecting the growth and yield of forest plantations.

Acknowledgments

We thank the Coordination for the Improvement of Higher Level Personnel (CAPES) institute for the scholarship and the Minas Gerais State Foundation for Research Support (FAPEMIG) as a research project sponsor.

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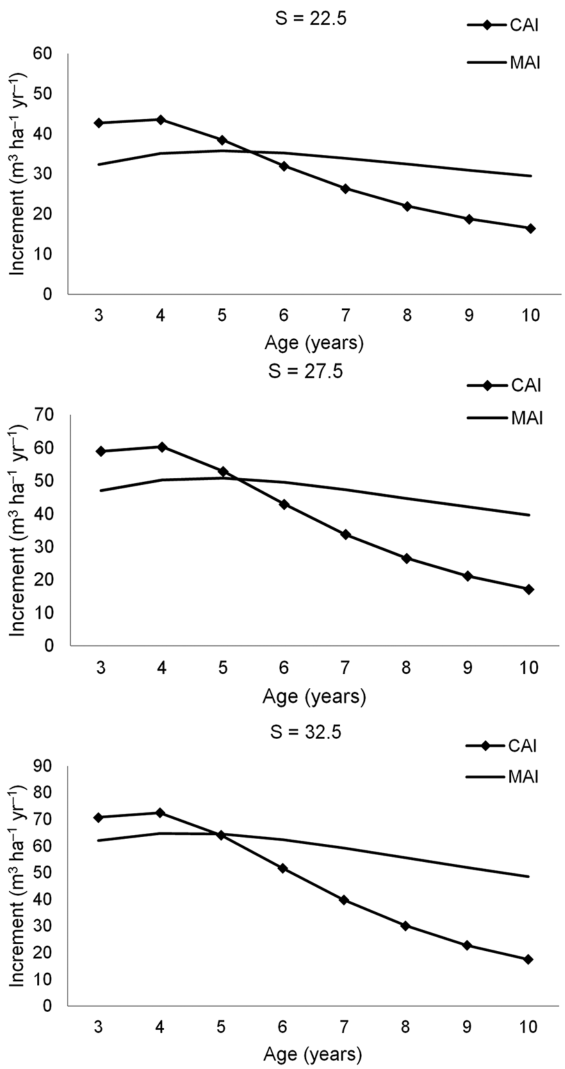

Figure 5 – Relationship between current annual increment (CAI) and mean annual increment (MAI) for Strategy B.

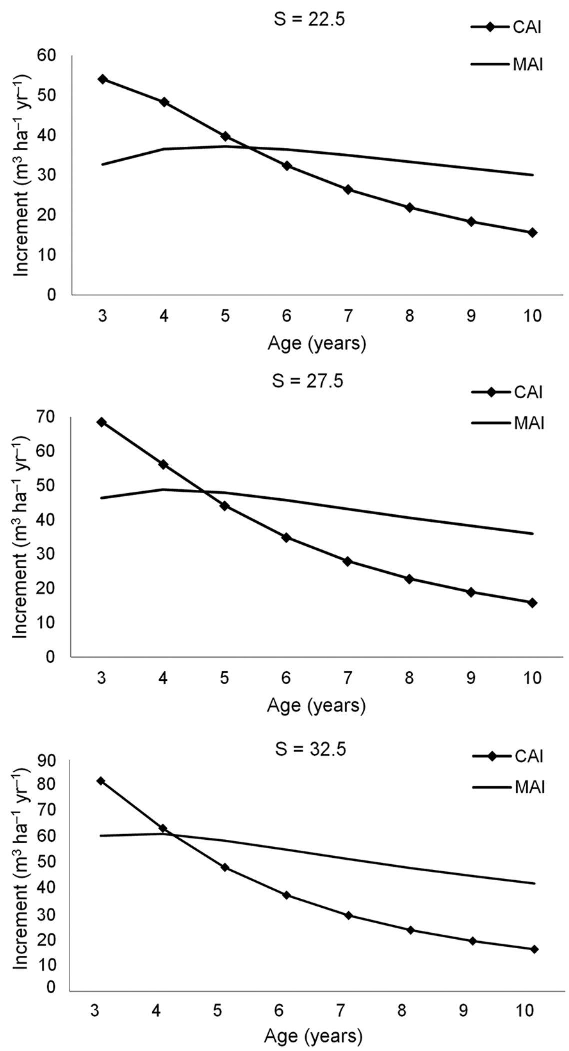

Figure 6 – Relationship between current annual increment (CAI) and mean annual increment (MAI) for Strategy C.

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