オセーン方程式の解

中　川　清　和

The solutions of the Oseen equation

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Abstract

We consider the Oseen's linearization of the stationary Navier-Stokes equation. We represent its solution in the integral formula and investigate its behavior with vanishing vicosity. Our result is as follows. If the density of the integral formula converges weakly in some Hilbert space, then the solution of the Oseen equation also converges in some sense. And we see that the difference of the limit functions at each point may occur, which eliminates the Euler-D'Alember paradox.

1. Introduction. Oseen[5)] derived the following linearized system from the stationary Navier-Stokes system.

$$\left.\begin{array}{ll} \mu\Delta \boldsymbol{u}_\mu+\rho U\partial\boldsymbol{u}_\mu/\partial x_1-\nabla q_\mu=0 & \text{in } E_3\backslash S, \\ \nabla\cdot\boldsymbol{u}_\mu=0 & \text{in } E_3\backslash S, \\ \boldsymbol{u}_\mu|_S=(U,\ 0,\ 0), \quad \lim\limits_{|x|\to\infty}\boldsymbol{u}_\mu=0. & \end{array}\right\} \tag{1}$$

Here $S=\{x=(x_1,\ x_2,\ x_3)\in E_3\ ;\ x_1=0,\ x_2^2+x_3^2\leqq 1\}$ and ρ is density of fluid and μ is the viscosity coefficient. The disk S moves with a constant velocity $(U,\ 0,\ 0)$. The velocity of fluid is given by $(-U+u_{\mu,1},\ u_{\mu,2},\ u_{\mu,3})$ and the pressure is given by $q_\mu-\rho(u_{\mu,1}^2+u_{\mu,2}^2+u_{\mu,3}^2)/2$ where $\boldsymbol{u}_\mu=(u_{\mu,1},\ u_{\mu,2},\ u_{\mu,3})$.

He investigated the behavior of the solutions of (1) as the viscosity coefficient μ tends to zero and gave the interesting suggestion which eliminated the Euler-D'Alember paradox. To this end, he treated the integral representation of the solution $(\boldsymbol{u}_\mu,\ q_\mu)$. Put $x'=(x_2,\ x_3)$ and $x=(x_1,\ x')$. Let $\boldsymbol{\phi}_\mu(x')=(\phi_{\mu,1}(x'),\ \phi_{\mu,2}(x'),\ \phi_{\mu,3}(x'))$ be the solution of the following system:

$$\int_S \sigma r_0^{-1}e^{-\sigma r_0}\phi_{\mu,1}(y')dy'-D_2\int_S(1-e^{-\sigma r_0})r_0^{-1}\phi_{\mu,2}(y')dy'$$
$$-D_3\int_S(1-e^{-\sigma r_0})r_0^{-1}\phi_{\mu,3}(y')dy'=U \quad \text{in } S,$$
$$-D_2\int_S(1-e^{-\sigma r_0})r_0^{-1}\phi_{\mu,1}(y')dy'+2\sigma\int_S r_0^{-1}e^{-\sigma r_0}\phi_{\mu,2}(y')dy'$$
$$-D_2\int_S(1-e^{-\sigma r_0})(x_2-y_2)r_0^{-2}\phi_{\mu,2}(y')dy'$$

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