Reconsideration of Timoshenko Beam Theoy on Phase Velocity for Transverse Elastic Waves

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Abstract

This study has been done according to the Timoshenko Beam Theory to clarify that natural frequencies obtained from the eigen functions and frequency equations of the lower order are positioned on the first mode curve of the phase velocity for transverse elastic waves, and that as to natural frequencies of the upper order, they are on both of the first and the second mode curves of the phase velocity for transverse elastic waves. For the second mode curve of the phase velocity for transverse elastic waves, the difference between the curve by Timoshenko Beam Theory and one by Pochhammer-Chree theory of three-dimensional treatment becomes larger with the decrease of wave length. On the other hand, for the first mode curve of the phase velocity for transverse elastic waves, the difference is extremely small in all ranges. This fact implies that the Timoshenko Beam Theory can be utilized as the theory which gives us one important part of the rigorously treated solutions in the region where computed natural frequencies are positioned only on the first mode curve of the phase velocity for transverse elastic waves.

Key words: Vibration, Shear Deformation, Phase Velocity

1. Introduction

Taking shear deformation into consideration, Timoshenko proposed the most precise theory on the transverse vibrations of beam. [1] In the year of 1922 [2], he proved himself a high preciseness of the theory based on the phase velocity for transverse elastic waves, which was driven by the plane strain theory of two-dimensional elastic theory. [3-7] Huang found that the eigen functions and frequency equations of a uniform beam with simple end conditions consist of two cases, one is the case of the lower order, and another, of the upper order. [8]

This paper has demonstrated that wave numbers calculated from the upper order natural frequencies are wave numbers of the first mode for the phase velocity of transverse elastic waves and at the same time, they are wave numbers of the second mode for the phase velocity of transverse elastic waves. For the second mode curve of the phase velocity for transverse elastic waves, the difference between the curve by Timoshenko and one by Pochhammer [9] and Chree [10] of three- dimensional treatment [11-15] becomes larger with the decrease of wave length. On the other hand, for the first mode curve, the difference is extremely small in all ranges. Therefore, if Timoshenko Beam Theory is applied to the case of the lower order natural frequencies, we can obtain the most precise results only in the case of small height of

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