PLAYFUL MATH – AN INTRODUCTION TO MATHEMATICAL GAMES

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"Children learn as they play. Most importantly, in play children learn how to learn."
(O. Fred Donaldson)

"We don't stop playing because we grow old; we grow old because we stop playing."
(George Bernard Shaw)

"Play is the highest form of research." (Albert Einstein)

INTRODUCTION

Playing games is vital not only for children, but for everyone. It is a social activity in which the participants (the players) interact with each other and have to make decisions. The ultimate goal of each player is to win. By many educators, play is considered to be the "work of childhood". It is important, however, to highlight a crucial difference between work (of adulthood) and play. Usually, if somebody is working, then this person is working for a reward (e. g. working for the salary, or to accomplish a certain goal, etc.), and in many cases, the intermediate process of working does not provide (much) enjoyment or satisfaction. In contrast to this, the process of playing is rewarding in itself: although players want to win in the end, this is not the essence of playing. All players can simultaneously enjoy the game, regardless whether they win or not. A truly good game is not played because the winner gets a prize in the end, but because it is fun to play it.

It is difficult to give a precise definition for the concept of a **mathematical game**. Instead, we list some features, which apply for most games (in particular for those we introduce in the next section).

- Usually there are **two players** (or teams) playing against each other.
- **Complete information.** This roughly means that in every turn, each of the players can make a perfectly logical decision based on the history of the game. The game has no gambling part; the outcome does not depend on luck, but purely on the strategy of the players.
- The **rules are simple** to understand, and usually there are only a few of them.
- The ultimate goal is **not winning, but understanding** the structure of the game.

- If a game always comes to an end in finitely many turns, one of the players always has a **winning strategy**. All the games we present are of this type.

Advantages of Playing Mathematical Games

- Kids are challenged to think in a playful way.
- There are several games at their level, mathematical knowledge is not necessary. They „only” need to think logically.
- Most of those games are motivated by themselves, because it is fun to play them. However, the instructors can always introduce some additional reward (e. g. bar of chocolate, candies, etc.) for the winner.
- The setup is usually very simple: most of them can be played on a piece of paper, or a chessboard, or you need some coins, etc.
- They fit into the program of the summer camp very easily (1-2 hours of length) wherever there is a gap, and can be played in an outdoor/indoor environment.

Brief Description of the Activity

1. In the beginning, instructors introduce the game to the kids (storytelling is important), maybe they play an example round to make sure that everyone understands the rules.
2. Kids are formed into groups of two (or more, preferably an even number). Each group gets a separate table, or piece of paper on which they can play the game.
3. Kids are given some time (10-15 minutes at least, depending on the difficulty of the game) to play within groups: members of the same group against each other.
4. After some time, the instructors would walk around the tables, and ask the kids, whether they are brave enough to play against her/him. (The instructor should know the structure of the game very well, of course.) If they are, then they should play a game against the instructor. Here at this point, the instructor needs to be very careful: if (s)he plays according to the winning strategy, (s)he might tell the kids too much. However, if the kids make a bad move, the instructor could play a good countermove in the game, to somehow suggest the kids the right direction.
5. The ultimate goal for the groups of kids is to find out the winning strategy (preferably by themselves, with some minor hints) and to beat the instructors.
6. Those who can beat an instructor, they would get a big reward (e. g. two bars of chocolate); the groups which were not successful, they would get a small reward (e. g. one bar of chocolate).

MOST IMPORTANTLY: If a kid says something which is wrong, the instructor should **never tell** this to the kid directly, but should **<u>ask a question</u> instead**: „Why do you think it is true? Can you justify your answer? Can you convince me?”

A BOUQUET OF MATHEMATICAL GAMES

Below some games are presented. Except the zeroth one, in all games the following two questions can be asked: 1. Who has a winning strategy? 2. What is the winning strategy?

0. WARM-UP EXERCISE

This game is played by everyone together. On Figure 1 one can see an ordinary dice (i. e. the numbers on opposite sides add up to 7) in a specified starting position, and a prescribed trajectory of the dice, as it is being rolled on a piece of paper. In each moment, one face of the dice touches the paper, and the players have to find out, which one is it.

All players stand up in a circle around a table, and they all look at the figure which is printed on a large piece of paper. Someone starts with the first number, and then her/his neighbor continues, and so on. On Figure 1, the first few numbers are already filled in.

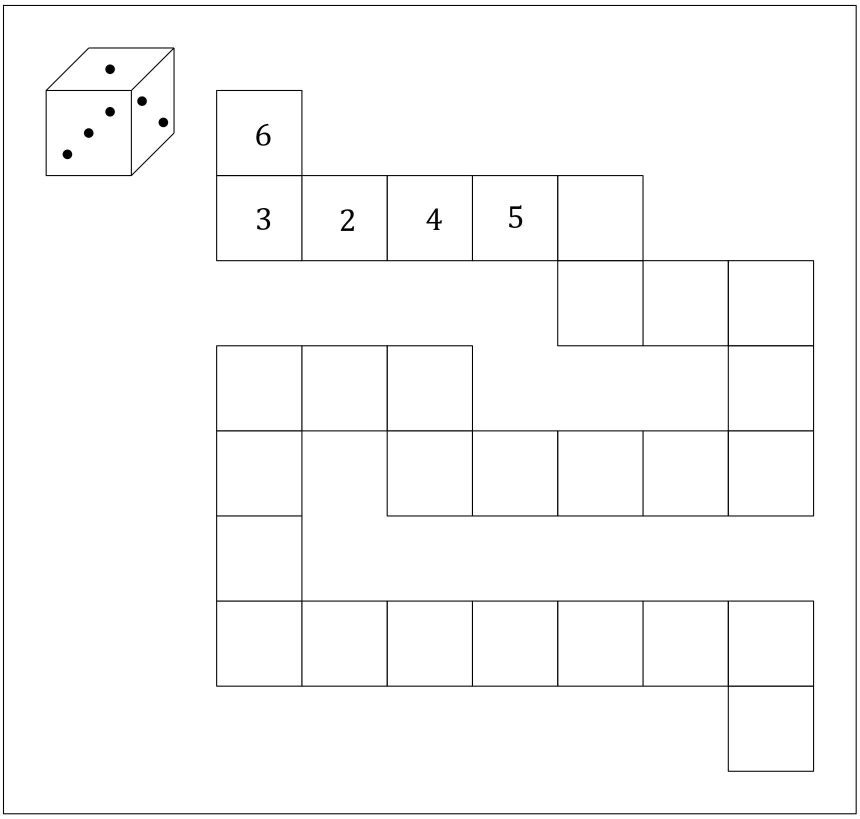


Figure 1

This game is challenging, because the players have to remember and update the orientation of the dice in their minds after each step. One can also introduce a time limit to make the game even more challenging: if a player cannot tell the upcoming number in a few seconds (say three), than the player is out. In this version, the game is played until only one player remains, and this last person is the winner.

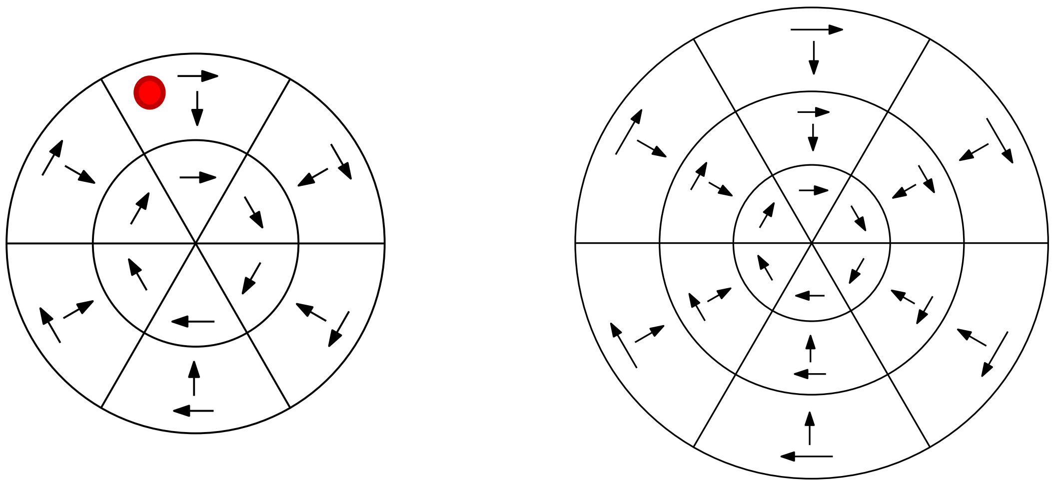
1. Collect the Coins!

This game is played by two players. On the table there is a pile of n coins (n is a positive integer to be specified later). The two players alternately take away either 1 or 2 coins from the pile. The winner is the one who can take away the last coin(s). Who has a winning strategy if

(a) $n = 6$, (b) $n = 8$, (c) $n = 10$, (d) $n = 2014$.

2. Coloring the Magic Windmill

In Wonderland flour is produced from grain in Magic Windmills. In order to make them work, their wheels have to be colored by two colors. Two players alternately paint the fields of a wheel (Figure 2 shows two wheels in (a) and (b), with 12 and 18 fields, respectively) according to the following rules: 1. If a field is painted, then the next must be one on which the arrows of the preceding field point. 2. The first field to be painted must lie in the outermost belt (e. g. the field marked with a ●). The winner is the player who paints the last field. Depending on the size of the Magic Windmill who has a winning strategy?



(a) (b)

Figure 2

Modified version: The game can be modified easily: instead of painting wheels that have 6-fold symmetry (as above), one can play on wheels with k-fold symmetry (where $k = 3, 4, 5, \ldots$).

3. The Lone Rook

A lone rook is standing on the top left field of a chessboard (Figure 3). Two players alternately move the rook either downwards or rightwards (the number of steps made by the rook in one turn can be arbitrary). The player who reaches the bottom right corner with the rook, wins.

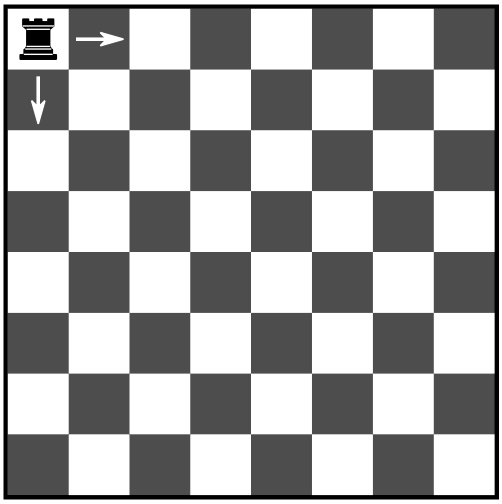


Figure 3

Modified versions: 1) If a player reaches the bottom right corner with the rook, loses.
2) The game can be played on any rectangular (non-square) „chessboard".

4. The Number 1 is Written on the Blackboard.

Two players take turns. In a turn, a player either adds 1 to the number on the blackboard, or multiplies it by 2. If a player gets over 14 in her/his turn, then (s)he loses.

5. Building a Wall

Two players alternately put × marks into the fields of a 3-by-3 board (see Figure 4). They both use the same mark! The player who can complete a straight line (either horizontal [×××], or vertical), wins.

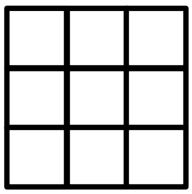


Figure 4

6. Dividing up a Bar of Chocolate

Two players are given a bar of chocolate of size n-by-m. They alternately pick and break a piece into two parts along a straight line. (Initially, there is only one piece: the bar itself). The player who breaks the last piece (i. e. after his turn, there are $n \cdot m$ pieces which are not allowed to be broken further), can eat everything ☺