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TOWARD A LEGAL DEONTIC LOGIC

*Howard Pospesil**

I. DEONTIC MIXTURES

Consider this argument (let's call it "Yugo"):

"Yugo"

All automobiles are required to be licensed.
All licensed vehicles are required to be insured.
All automobiles are vehicles.
Your Yugo is an automobile.
So, your Yugo is required to be insured.

In this paper I endeavor to answer the following questions:

Is "Yugo" valid (under some interpretation), that is, does its conclusion follow from its premises with logical necessity?
If "Yugo" is valid, is there a system of logic in which this fact can be demonstrated? If such a system is not currently available, can we develop one?
If "Yugo" is invalid, what feature makes it so?

We can represent the first premise of "Yugo" in standard predicate logic¹ if we lump the element of *requirement* together with the *licensing* concept to form the compound predicate *is required to be licensed*:

$(x)(Ax \rightarrow Lx)^2$

(Ax = x is an automobile, Lx = x is required to be licensed)

However, the predicate *is required to be licensed* does not occur in the second premise of "Yugo"; conversely, in this treatment of the argu-

* Professor of Philosophy, University of Miami. I have been assisted in the preparation of this paper by a sabbatical leave provided by the University of Miami. I have benefitted from discussions with Professors Edward Erwin, Risto Hilpinen, and Harvey Siegel. Professor Robert E. Rodes has made many good suggestions for improving the paper.

1 "Standard predicate logic" extends standard propositional logic to include the machinery of individual constants, variables, and quantifiers plus property and relational predicates. See *infra* note 5 for a definition of "standard propositional logic."

2 This formula may be read: "For any (individual) x, if it is A, then it is L."

ment, the predicate "is licensed" occurs in the second premise, but not in the first. As a result, no connection between these two premises is detected, and so "Yugo" is judged to be invalid in standard predicate logic.

If we are to find a logic that assesses "Yugo" as valid, we evidently must turn to some form of *deontic logic*,³ that is, to a logic that employs a logical operator with the meaning "it is required that."⁴ Let's see how "Yugo" fares when treated in an elementary form of deontic logic I shall call "standard deontic logic" (SDL). By SDL I understand a system of standard propositional logic⁵ augmented by the three (fully interdefineable⁶) deontic operators:

- O (It is required that)
- P (It is permitted that)
- F (It is forbidden that),

and embodying these deontic principles (and no others):

- (P1) Whatever is entailed by what is required is also required.⁷

3 Deontic logic is the logic of the expressions "required" (or "obligatory"), "permitted," and "forbidden." See G.H. von Wright, *Deontic Logic*, LX MIND 1, 1-15 (1951), for the first viable system of deontic logic. See Dagfinn Føllesdal & Risto Hilpinen, *Deontic Logic: An Introduction*, in DEONTIC LOGIC: INTRODUCTORY AND SYSTEMATIC READINGS 1, 1-35 (2d ed. 1981) for a survey of the field.

4 In some versions of deontic logic, including von Wright's, the deontic operators attach to names of acts (or act types) rather than to propositions. In such systems, the requirement operator will have the meaning "it is required to do," rather than "it is required that." In propositional deontic logic, it is *states of affairs* (not acts) that are required.

5 By "standard propositional logic" I mean any system of propositional logic that gives the customary two-valued truth-table definitions to the propositional connectives "and" (&), "or" (v), "if" (\rightarrow), "if and only if" (\leftrightarrow), and "not" (\neg).

6 $OA =_{df} \neg P \neg A =_{df} F \neg A$
 $PA =_{df} \neg O \neg A =_{df} \neg FA$
 $FA =_{df} O \neg A =_{df} \neg PA$

7 There is an ambiguity in P1 that is shown in the following (distinct) symbolic formulations of the principle:

If A entails B , then OA entails OB .

If the conjunction $A \& \dots \& M$ entails N , then the conjunction $OA \& \dots \& OM$ entails ON .

(For the second formulation we should allow for the limiting case of a "conjunction" that has only one conjunct.) We can illustrate the difference between these two formulations of P1 with the help of these two arguments:

(A1) $O[C \& (C \rightarrow D)] \vdash OD$

(A2) $OC \& O(C \rightarrow D) \vdash OD$

The premise of A1 has the general form OA , while the premise of A2 has the general form $OA \& OB$. Accordingly, the first formulation of P1 applies to A1, but not to A2.

(P2) Whatever is required is permitted.⁸

(P3) Whatever is logically necessary is required.⁹

“Yugo” may be represented in standard deontic logic as:

OB, OC, D, E ⊢ OF

(B = All automobiles are licensed, etc.)

In this symbolization the first premise of “Yugo” is understood to mean: “It is required that all automobiles be licensed”; the second premise and the conclusion are treated in a similar fashion. Of course, this symbolized argument is assessed as “invalid”; there are no discernible connections among its elements. Obviously, the logical structure of “Yugo” requires a more discriminating tool than SDL.

Suppose we expand SDL by incorporating the machinery of predicate logic. That is, we add to SDL individual variables (x, y), individual constants (a, b), property predicates (Fx, Gx), relational predicates ($Rxy, Sxyz$), and universal and existential quantifiers ($(x), (\exists x)$). We remain true to the spirit of SDL by requiring that deontic operators not fall within the scope of quantifiers. For lack of a better name, I will call such a system “standard deontic logic *plus* [quantifiers]” (SDL+).

To illustrate how sentences may be symbolized in SDL+ let’s apply that system to the following argument (which I take to be valid):

It is required that all automobiles be licensed.

It is required that all licensed automobiles be insured.

So, it is required that all automobiles be insured.

In symbols:

$O(x)(Ax \rightarrow Lx)$

$O(x)[(Lx \ \& \ Ax) \rightarrow Ix]$

$\vdash O(x)(Ax \rightarrow Ix)$

($Lx = x$ is licensed, $Ix = x$ is insured)

The second formulation of P1 applies to A2 and (with the help of the parenthetical comment above about the limiting case) to A1 as well. Obviously the second formulation of the principle has a greater scope of application than the first. Principle P1 should be understood in the second sense. If it is understood in the first sense, then we must add the agglomeration principle (discussed in Part II below) to P1 through P3 to capture SDL.

8 In Part II, I argue that P2 should be rejected by a legal deontic logic.

9 P3 is a logical consequence of P1 and the obviously true claim that something is required (thanks to the logical principle that every statement entails every logical truth).

SDL+ judges this argument to be valid because (a) SDL+ incorporates principle P1 (listed above) and (b) standard predicate logic determines the following symbolized argument to be valid:

$$\begin{aligned} & (x)(Ax \rightarrow Lx) \\ & (x)[(Lx \ \& \ Ax) \rightarrow Ix] \\ & \vdash (x)(Ax \rightarrow Ix) \end{aligned}$$

How does “Yugo” itself fare when treated in SDL+? The argument is readily symbolized in that system:

$$\begin{aligned} & O(x)(Ax \rightarrow Lx) \\ & O(x)[(Lx \ \& \ Vx) \rightarrow Ix] \\ & (x)(Ax \rightarrow Vx) \\ & Ac^{10} \\ & \vdash OIc \end{aligned}$$

$$(Vx = x \text{ is a vehicle, } c = \text{your Yugo})$$

“Yugo” is judged by SDL+ to be invalid for the simple reason that SDL+ ignores the third and fourth premises; it ignores those premises because they are non-deontic. Neither SDL nor SDL+ is equipped to deal with premise sets containing non-deontic statements.¹¹ From the vantage point of SDL+, the third and fourth premises of “Yugo” vanish; since the conclusion does not follow from the two deontic premises alone, SDL+ judges the argument to be invalid.

Let’s focus on this issue of *deontically mixed arguments* (my label for arguments such as “Yugo” that contain both deontic and non-deontic premises). Consider a stripped-down version of “Yugo” and its symbolization in SDL+:

“Short Yugo”

It is required that all automobiles be insured.

Your Yugo is an automobile.

So, it is required that your Yugo be insured.

$$O(x)(Ax \rightarrow Ix), Ac \vdash OIc$$

“Short Yugo” seems to exhibit a common form of legal argumentation: the application of a law to an individual case. And the argument does appear to be valid; it seems impossible that the premises be true but the conclusion false. However, because the second premise of “Short Yugo” is non-deontic and the conclusion does not follow

10 This formula may be read “individual c has property A,” or just “c is A.”

11 This is an over-simplification. SDL and SDL+ can tolerate non-deontic premises if they play a role (via non-deontic principles) in reaching deontic formulas. For example, SDL can treat the following symbolized argument adequately even though its second premise is non-deontic: $P \rightarrow OQ, P \vdash O(Q \vee R)$.

straight away from the first premise, SDL+ judges the argument to be invalid.

Can we enrich SDL+ so that the resulting system will assess "Short Yugo" as valid while avoiding obviously paradoxical results? Perhaps there is some deontic principle that can be added to the three already embraced by SDL+. This principle might appear promising:

(P4) From " $O(A \rightarrow B)$ " derive " $A \rightarrow OB$."

A system embracing principle P4 will indeed judge "Short Yugo" valid, as this proof shows:

- | | | |
|-----|---------------------------|---|
| (1) | $O(x)(Ax \rightarrow Ix)$ | Premise |
| (2) | Ac | Premise |
| (3) | $O(Ac \rightarrow Ic)$ | From line 1 by P1 ¹² |
| (4) | $Ac \rightarrow OIc$ | From line 3 by P4 |
| (5) | OIc | From lines 4 and 2 by <i>modus ponens</i> |

Unfortunately, such a system will judge to be valid many other arguments that are obviously invalid. Worse than that (if worse is possible), in a system that includes P4, if any requirement is violated, anything whatsoever becomes obligatory; that is, " $OA \ \& \ -A$ " entails " OB " in such a system.¹³ Obviously we must reject principle P4.

Compare "Short Yugo" with two other arguments that appear to have the same form:

"Uninsured"

It is required that all uninsured objects not be automobiles.
Your Yugo is not insured.
So, it is required that your Yugo not be an automobile.

¹² For the sake of brevity, I omit establishing that " $(x)(Ax \rightarrow Ix)$ " entails " $Ac \rightarrow Ic$ ".

¹³ In the proof below, the formula on line 7 is derived free of assumptions; this shows that it is logically true and a candidate for the move on line 8.

- | | | |
|------|---------------------------------------|--|
| (1) | $OA \ \& \ -A$ | Premise |
| (2) | $-A$ | Provisional assumption |
| (3) | A | Provisional assumption |
| (4) | $A \vee B$ | From line 3 by disjunctive addition |
| (5) | B | From lines 4 and 2 by disjunctive argument |
| (6) | $A \rightarrow B$ | From lines 3 and 5 by conditionalization |
| (7) | $-A \rightarrow (A \rightarrow B)$ | From lines 2 and 6 by conditionalization |
| (8) | $O[-A \rightarrow (A \rightarrow B)]$ | From line 7 by P3 |
| (9) | $-A \rightarrow O(A \rightarrow B)$ | From line 8 by P4 |
| (10) | $-A$ | From line 1 by simplification |
| (11) | $O(A \rightarrow B)$ | From lines 9 and 10 by <i>modus ponens</i> |
| (12) | OA | From line 1 by simplification |
| (13) | OB | From lines 11 and 12 by P1 |

$O(x)(-Ix \rightarrow -Ax), -Ic \vdash O-Ac$

"Fingerprints"

It is required that all nursery school operators have fingerprints on file.

Jill operates a nursery school.

So, it is required that Jill have fingerprints on file.

$O(x)(Nx \rightarrow Fx), Nj \vdash OFj$

($Nx = x$ is a nursery school operator, $Fx = x$ has fingerprints on file, $j = \text{Jill}$)

"Uninsured" is invalid. The conclusion will be false even if the premises are true. The law can no more require that your Yugo not be an automobile than it can require that some Lutherans not be Protestants.¹⁴ "Fingerprints," on the other hand, looks valid, but there is good reason to suspect that it is not. Here is the argument: If "Fingerprints" is valid, the following argument is valid as well, since the two arguments have essentially the same form.

It is required that all persons who do not have fingerprints on file not be nursery school operators.

Jill does not have fingerprints on file.

So, it is required that Jill not be a nursery school operator.

But if this argument and "Fingerprints" are both valid, then the result of melding them into a larger third argument will also be valid:¹⁵

It is required that all nursery school operators have fingerprints on file.

Jill operates a nursery school.

It is required that all persons who do not have fingerprints on file not be nursery school operators.

Jill does not have fingerprints on file.

So, it is required that Jill have fingerprints on file, and it is required that Jill not be a nursery school operator.

$O(x)(Nx \rightarrow Fx), Nj, O(x)(-Fx \rightarrow -Nx), -Fj \vdash OFj \ \& \ O-Nj$

(Universe of discourse:¹⁶ people)

14 Distinguish "The law requires that some Lutherans not be Protestants" from "The law requires of some Lutherans that they not be Protestants." The latter sentence, but not the former, could be true.

15 It is a principle of logic that if P entails Q and R entails S , then P and R together entail the conjunction $Q \ \& \ S$.

16 The universe of discourse is the class of individuals over which the variables range. When no universe is specified, it is unrestricted.

The first and third premises of this larger argument are logically equivalent according to SDL+,¹⁷ and that result appears to be correct. If an argument has two logically equivalent premises (*i.e.*, premises having the same content), one of them may be dropped without affecting the validity (or invalidity) of the argument. Also, conjoining two premises of an argument never alters the argument's validity. So, we may delete the third premise of the merged argument, and conjoin two of its remaining premises to reach a fourth argument:

"Scofflaw"

It is required that all nursery school operators have fingerprints on file.

Jill operates a nursery school, but does not have fingerprints on file.

So, it is required that Jill have fingerprints on file, and it is required that Jill not be a nursery school operator.

Clearly "Scofflaw" is invalid. What follows from its premises is only the weaker statement: It is required that Jill *either* have fingerprints on file *or* not operate a nursery school. (And according to SDL+ that statement follows from the first premise alone!) We have shown that if "Fingerprints" is valid, so is "Scofflaw"; but "Scofflaw" is invalid. Therefore, "Fingerprints" is an invalid argument (in spite of appearances), and SDL+ rightly rejects it.

So, "Short Yugo" is (or appears to be) valid, whereas "Uninsured" and "Fingerprints" are invalid. What accounts for this difference? I think the crucial difference is that the non-deontic premise of "Short Yugo" is in some sense a necessary truth, while the non-deontic premises of the other two arguments are not.¹⁸ Because the second premise of "Short Yugo" enjoys a kind of necessity, it presents an unalterable fact, while the second premises of the other two arguments, being contingent, concern what is alterable. Jill can stop operating a nursery school; your Yugo cannot stop being an automobile. Any event (like being crushed into a cube of steel) that caused your Yugo to cease being an automobile would also cause it to cease being a Yugo. Jill, however, can cease to operate a nursery school and still remain Jill. Jill can bring herself into compliance with the law either by not being a nursery school operator or by having fingerprints on file. Your Yugo can comply with the law only by being insured. I sub-

17 This is so because SDL+ incorporates principle P1, and standard predicate logic judges " $(x)(Nx \rightarrow Fx)$ " and " $(x)(\neg Fx \rightarrow \neg Nx)$ " to be logically equivalent, *i.e.*, to entail one another.

18 I am indebted to Professor Risto Hilpinen for the suggestion that modal logic may help solve the problem of deontically mixed arguments.

mit that that is the reason "Short Yugo" is valid and "Fingerprints" and "Uninsured" are not.

The second premise of "Short Yugo" is not demonstrable using the techniques of logic, but it enjoys some measure of necessity none the less. Let's describe it as "conceptually necessary" and understand conceptually necessary truths to embrace logical truths, analytical truths (definitional truths), and statements that assign individuals to appropriate fundamental categories.¹⁹ The latter kind of statement can be termed "categorical"; the second premise of "Short Yugo" is a categorical statement. Now we can broaden principle P3 of SDL+ ("Whatever is logically necessary is required") into P3' as follows:

(P3') Whatever is conceptually necessary is required.

We can strengthen SDL+ by adding to it a system of modal logic,²⁰ and we can dub the result "standard deontic logic *plus* [quantifiers] *plus* [modalities]" (SDL++). Let's take the conservative course of (a) incorporating the weakest of the common systems of modal logic, T,²¹ and (b) insisting that modal operators not fall within the scope of quantifiers. T incorporates these modal principles:

(P5) Whatever is logically true is necessary.

(P6) Whatever is necessarily true is true.

(P7) Whatever is entailed by a necessary truth is a necessary truth.

And for our purposes we will understand the concept of necessity in these principles to encompass conceptual necessities.

Using the box symbol to represent necessity we may symbolize "Short Yugo" in SDL++ as follows:

$$O(x)(Ax \rightarrow Ix), \Box Ac^{22} \vdash OIc$$

SDL++ permits a simple proof of validity for "Short Yugo":

(1) $O(x)(Ax \rightarrow Ix)$ Premise

(2) $\Box Ac$ Premise

19 Where F is a category term, F is a fundamental category for individual a if and only if belonging to F is constitutive of a 's identity. *Being human* is a fundamental category for Lisa, while *being Lutheran* is not.

20 Modal logic is the logic of the expressions "necessary" and "possible." See C. I. LEWIS & C. H. LANGFORD, *SYMBOLIC LOGIC* (1932) for the first viable system of modal logic.

21 See Robert Feys, *Les Logiques Nouvelles des Modalités*, XL *REVUE NÉOSCHOLASTIQUE DE PHILOSOPHIE* 517-53 (1937), and XLI *REVUE NÉOSCHOLASTIQUE DE PHILOSOPHIE* 217-52 (1938) (proposing System T); see also G.H. VON WRIGHT, *AN ESSAY IN MODAL LOGIC* (1951) (proposing an equivalent system, called "M").

22 This formula may be read "It is necessary that c is A " or "Necessarily c is A ."

- (3) OAc From line 2 by $P3'$
 (4) OIc From lines 1 and 3 by $P1^{23}$

There will be no comparable proof for "Uninsured" or "Fingerprints" because their second premises are not correctly modalized.

We can also construct in $SDL++$ a proof for "Yugo":

- (1) $O(x)(Ax \rightarrow Lx)$ Premise
 (2) $O(x)[(Lx \ \& \ Ax) \rightarrow Ix]$ Premise
 (3) $\Box(x)(Ax \rightarrow Vx)$ Premise
 (4) $\Box Ac$ Premise
 (5) $O(x)(Ax \rightarrow Vx)$ From line 3 by $P3'$
 (6) OAc From line 4 by $P3'$
 (7) OIc From lines 1, 2, 5, and 6 by
 $P1$

Because premises three and four are conceptually necessary we are warranted in symbolizing them with the box.

One troublesome consequence of $SDL++$ is that it makes all conceptual necessities obligatory. Thus, for example, it is (legally) required that all automobiles be vehicles (line 5 in the proof above) and that your Yugo be an automobile (line 6 above). This result is odd, but in my judgment tolerable. A deontic logic that embraces conceptual necessities must adopt one of these three positions:

- (1) All conceptual necessities are required.
 (2) No conceptual necessities are required.
 (3) Some conceptual necessities are required and some are not.

Options two and three are less tolerable than option one. Note that according to option two all conceptual impossibilities are permitted. Another argument for option one is that it accords the same treatment to logical and other necessities. The following consideration helps lessen discomfort with option one: the obligation conferred on conceptual necessities is empty in the sense that it cannot be violated.

II. INCONSISTENT OBLIGATIONS

Does $SDL++^{24}$ as set out above provide a satisfactory basis for a legal deontic logic? No, and one major reason is that in $SDL++$ $F1$ is held to be a logical contradiction.

²³ Here and elsewhere I omit the trivial step of conjoining lines before applying $P1$.

²⁴ $SDL++$ is the system embracing deontic principles $P1$, $P2$, and $P3'$, and modal principles $P5$, $P6$, and $P7$.

(F1) OS & O-S²⁵

We can establish that F1 is a contradiction in SDL++ by deriving from it an evident self-contradiction.²⁶

- | | |
|----------------|-----------------------------------|
| (1) OS & O-S | Premise |
| (2) OS | From line 1 by simplification |
| (3) O-S | From line 1 by simplification |
| (4) PS | From line 2 by P2 |
| (5) -O-S | From line 4 by definition of "P" |
| (6) O-S & -O-S | From lines 3 and 5 by conjunction |

The problem is that S1 (which is symbolized by F1) *could* be true, and therefore is not contradictory.

(S1) Jones is required to sell the diamond to Smith, and she is required not to sell it to Smith.

(S = Jones sells the diamond to Smith)

The fact of the matter is that it is possible to have inconsistent legal obligations, that is, to have two obligations that are inconsistent with each other.²⁷ (Distinguish that claim from the similar-sounding claim that it is possible to have a legal obligation to do the contradictory. I will deny the second claim below.) In what situation would S1 be true? Where Jones has entered into two contracts, one requiring her to sell the diamond to Smith and one requiring her to sell it to someone else instead.

In a satisfactory system of legal deontic logic F1 must be a contingent formula. That means we must give up one of the rules employed in the above proof. Simplification and conjunction are valid patterns if any inference patterns are, so the problematic rule must be either P2 or the definition of "P." That P2 is the culprit is evident from the fact that with its help (and without relying on the definition of "P") we can prove that "OS & O-S" entails the obviously false F2:

(F2) PS & P-S

F2 alleges falsely that Jones is legally free to sell the stone to Smith and also legally free to not do so.

- | | |
|--------------|-------------------------------|
| (1) OS & O-S | Premise |
| (2) OS | From line 1 by simplification |
| (3) O-S | From line 1 by simplification |

25 Note that F1 is a contradiction if and only if "PS v P-S" is a logical truth.

26 It is a principle of logic that only contradictions entail contradictions.

27 Professor Robert Rodes showed this fact to me. See *Helzberg's Diamond Shops, Inc. v. Valley West Des Moines Shopping Center, Inc.*, 564 F.2d 816 (8th Cir. 1977), for an example of inconsistent obligations.

- | | |
|--------------|-----------------------------------|
| (4) PS | From line 2 by P2 |
| (5) P-S | From line 3 by P2 |
| (6) PS & P-S | From lines 4 and 5 by conjunction |

So, we have no choice but to abandon P2.²⁸

The recognition that inconsistent obligations are possible (that F1 is not a contradiction) is likely to be the point at which *moral* deontic logic and *legal* deontic logic part company.²⁹ Ethicists typically admit that there can be *apparent* moral dilemmas, but deny that these are insoluble.³⁰ People can have inconsistent *prima facie* moral obligations, but (it is commonly held) only one of those obligations can be the individual's *actual* moral obligation.³¹ However, this approach is not going to succeed in legal deontic logic. A judge may decide that Jones' obligation to sell the jewel to Smith takes precedence over her obligation to sell it to Brown, but that will not show that she had no actual legal obligation to sell the gem to Brown. If that were so, she could not be justly required to pay damages for breaking the agreement to sell to Brown, but of course she can be.

It is easy to understand why crafters of a legal deontic logic would want to regard S1 as contradictory. Inconsistency is as undesirable in a set of laws as it is in a set of descriptive statements. When legislators or judges discover that one law contradicts another law (in the same system of laws), one of the laws must be set aside. What we need to remember here is that while *laws* must be consistent among themselves, it will always be possible for *individuals* to incur inconsistent

28 See BRIAN CHELLAS, *MODAL LOGIC: AN INTRODUCTION* 202 (1980), for an example of a deontic system that lacks P2. The system also rejects P3 and weakens P1 to the first version displayed in note 7. That system is not intended to apply specifically to legal deontic reasoning.

See also John Horty, *Nonmonotonic Foundations for Deontic Logic*, in *DEFEASIBLE DEONTIC LOGIC* 17, 20–23 (Donald Nute ed., 1997), for a discussion of Chellas's and van Fraassen's proposed solutions to the problem of inconsistent obligations.

29 Ronald Moore held that legal deontic logic differs from moral deontic logic in two formal respects: the contingency of S1, and the definition of the permission operator. Ronald Moore, *The Deontic Status of Legal Norms*, LXXXIII *ETHICS* 151, 151–58 (1972). However, he seems to have believed that inconsistent legal obligations arise only when the laws that give rise to them are inconsistent. I argue against that idea below.

30 Kant writes that "a conflict of duties and obligations is inconceivable." IMMANUEL KANT, *THE METAPHYSICS OF MORALS* 24 (John Ladd trans., Bobbs-Merrill 1965). *But see*, Bas C. van Fraassen, *Values and the Heart's Command*, LXX *J. PHIL.* 1, 5–19 (1973), and Ruth Barcan Marcus, *Moral Dilemmas and Consistency*, LXXVII *J. PHIL.* 3, 121–36 (1980). Van Fraassen argues explicitly that formulas of the form of S1 are contingent and that a moral deontic logic must reject principle P2.

31 This distinction was drawn by W.D. Ross. *See* W.D. ROSS, *THE RIGHT AND THE GOOD* 19–20 (1930).

obligations from a set of consistent laws. This symbolized argument illustrates the point:

- (PR1) $O(x)(Dx \rightarrow Ex)$
 (PR2) $O(x)(Fx \rightarrow -Ex)$
 (PR3) $ODg \ \& \ OFg$
 $\vdash OEg \ \& \ O-Eg$

The laws expressed by premises one and two are consistent; the inconsistency in obligations arises only with the addition of premise three (which, while deontic, is not a law). A system of legal deontic logic should be able to represent not only statements expressing laws, but also statements about the legal obligations incurred by individuals.

Deleting P2 is not the only change that is required by the recognition of the consistency of F1, as this proof shows.

- (1) $OS \ \& \ O-S$ Premise
 (2) OA From line 1 by P1³²

If "A" abbreviates "Murders occur" then "OA" is plainly false. For an appropriate assignment of meaning to "S," F1 (line 1) will be true. Hence F1 does not entail "OA." P1 is the culprit here, but P1 is at the heart of deontic logic. If we abandon it entirely, we abandon deontic logic. Fortunately, we can modify it rather than eliminating it.

- (P1') Whatever is entailed by what is consistently required is also required.

In symbols:

If the conjunction $A \ \& \ . . . \ \& \ M$ is consistent³³ and entails N , then the conjunction $OA \ \& \ . . . \ \& \ OM$ entails ON .³⁴

Having determined that in a legal deontic system F1 should count as a contingent formula, we have to consider the status of F3. Should it count as contradictory or (like F1) contingent?

- (F3) $O(S \ \& \ -S)$

³² Line 2 follows from line 1 in part because of the logical principle that contradictory statements entail any statement.

- (1) $S \ \& \ -S$ Premise
 (2) S From line 1 by simplification
 (3) $S \vee A$ From line 2 by disjunctive addition
 (4) $-S$ From line 1 by simplification
 (5) A From lines 3 and 4 by disjunctive argument

³³ A statement is consistent if and only if it is not a logical contradiction.

³⁴ We should allow for the limiting case of a "conjunction" that has only one conjunct.

(S3) Jones is required both to sell the diamond to Smith and not to sell it to Smith.

I believe that F3 (S3) should be regarded as a contradiction. Several considerations support that view. Note that F4 (below) is a theorem in any system that embraces P3, and is logically equivalent to F5 in any system that includes the deontic definitions stated above.³⁵

(F4) $O-(S \ \& \ -S)$

(F5) $F(S \ \& \ -S)$

Can the law both require (F3) and forbid (F5) a logically impossible state of affairs ($S \ \& \ -S$)? That question goes hand in hand with another: Can the law both require and forbid a logically necessary state of affairs? My intuition is that the law cannot accomplish these feats, that it can no more require Jones to both-sell-and-not-sell-the-diamond than it can require that some Lutherans not be Protestants, that the sum of two and two be five, or that your Yugo not be an automobile. The law can only require the bringing about of state of affairs A if that state is logically possible. If A is logically impossible then we cannot even understand what it is that the law is (supposedly) requiring. This is a minimal condition of rationality for statements of legal requirement, that what is required to occur be logically conceivable. The supposed state of affairs in which Jones both sells and does not sell the diamond to Smith does not meet this minimum condition.

Of course, the supreme lawgiver in nation N can utter the words "It is henceforth required by law that adultery will both be and not be a capital offense," but the mere fact that this sentence has been uttered by the lawgiver does not show that in N adultery both is and is not a capital offense. There are rational constraints on the content of legal requirements; logical conceivability is a minimal constraint. If that constraint is dropped, one has given up any hope that a *logic* of legal obligation can be developed.

Here is another consideration pointing in the same direction. Where A, B, and C are statements, it is generally held that if B is a component of A, and C is logically equivalent to B, then the substitution of C for B in A cannot change the truth value of A.³⁶ But then if F3 is held to be true (for some assignment of meaning to "S"), so is every statement asserting the requiredness of some contradictory state of affairs (since all contradictions are logically equivalent). That can't be tolerated.

35 See *supra* note 6.

36 This principle does not hold when B occurs in an intensional context; presumably F3 does not provide such a context.

Those who hold F3 and S3 to be contingent face a dilemma: They must maintain concerning statements of legal obligation whose constituents are contradictory either that none of them is contradictory or that some are and others are not. Both positions are puzzling. If statements of obligation having contradictory constituents are never contradictory, then presumably no statements of obligation are. Statements generally come in three logical varieties (logically true, contingent, contradictory); how are they to explain why this does not apply to statements of obligation? On the other hand, if some statements of obligation whose constituents are contradictory are themselves contradictory and others are not, how is this difference to be explained?

All of these considerations support the decision to regard F3 as a contradiction. We can accomplish that by incorporating into our system this principle:

(P8) Whatever is required is conceptually possible.

Now, having decided that F3 is a contradiction and F1 not, we are forced to give up the plausible *agglomeration principle* that holds in SDL (and SDL++):³⁷

$$(OA \ \& \ OB) \rightarrow O(A \ \& \ B)^{38}$$

This principle will hold in our system whenever *A* and *B* are consistent. What about the *converse agglomeration principle* (which holds in SDL):

$$O(A \ \& \ B) \rightarrow (OA \ \& \ OB)?$$

It will hold in our system (even when *A* and *B* are inconsistent).³⁹

Now that we have modified P1 and replaced P2 with P8 it is time to give up the label "SDL++" because we have moved too far from standard deontic logic. Let's call our modified system "legal deontic logic" (LDL).⁴⁰

37 Peter K. Shotch & Raymond E. Jennings, *Non-Kripkean Deontic Logic*, in *NEW STUDIES IN DEONTIC LOGIC* 149-62 (Risto Hilpinen ed., 1981) (arguing for the rejection of the agglomeration principle).

38 The analogue of the agglomeration principle is:

$$P(A \vee B) \rightarrow (PA \vee PB)$$

This is a theorem of SDL, but it holds in our system only when *A* and *B* are consistent. The converse is a theorem in both systems.

39 When *A* and *B* are consistent, the principle will hold thanks to P1'; and when they are inconsistent, it will hold thanks to the principle that a logical contradiction entails any statement.

40 As so far developed, LDL embraces deontic principles P1', P3', and P8, and modal principles P5, P6, and P7.

III. *DE RE* REQUIREMENTS

LDL is a useful tool for evaluating a wide range of arguments involving the concept of legal requirement, but it is not adequate for treating all such arguments. Consider these two examples:

"Handguns I"

It is required that all police officers carry handguns.
 It is required that there be police officers.
 So, it is required that there be people who carry handguns.

$O(x)(Px \rightarrow Hx), O(\exists x)Px^{41} \vdash O(\exists x)Hx$

(Universe of discourse: people; $Px = x$ is a police officer,
 $Hx = x$ carries a handgun)

"Handguns II"

It is required that all police officers carry handguns.
 It is required that there be police officers.
 So, there are people who are required to carry handguns.

"Handguns I" is obviously valid; this can be demonstrated in LDL (or even in SDL+). "Handguns II," on the other hand, is invalid. Suppose that there are no police officers in Community C (in violation of the legal requirement expressed in the second premise). While it is required that there be people in C who carry guns (the conclusion of "Handguns I"), there may be no specific individual to whom that requirement applies. The conclusion of "Handguns II" could be false while the premises are true. In its present form LDL is not equipped to assess the argument.

Note that in the conclusion of "Handguns II" the deontic term ("required") falls within the scope of a quantifier expression ("there are"). This will be more evident in the following symbolization of the conclusion:

$(\exists x)OHx^{42}$

Note that this formula does not belong to LDL as it is presently constituted or to any of the systems of deontic logic that we have examined because they permit the attachment of the O-operator only to *closed* (complete) formulas, not to *open* formulas such as "Hx." Let's call a sentence that contains both deontic and quantifier expressions *de re* if (at least) some of the deontic terms fall within the scope of quantifier

41 The existential quantifier may be read "there exists an (individual) x such that."

42 This formula may be read "There is an (individual) x such that it is required that x is H" or "There is an x that is required to be H."

terms, and *de dicto* if none of the deontic terms fall within the scope of quantifier terms.⁴³ A related distinction will apply to formulas. Until this section we have concentrated on arguments whose component deontic sentences can be interpreted *de dicto*. The conclusion of "Handgun II" requires *de re* treatment.

Let us modify LDL first by changing the formation rules so that *de re* formulas are accepted and second by adopting whatever principles seem necessary to cover this expansion. Note that when we attach the O-operator to an open formula such as "Hx" we seem to be changing its meaning (from "It is required that") to something like "is required to."

Consider these four formulas (two *de re* and two *de dicto*) and examples of statements they may be used to symbolize:

- | | |
|-----------------------|--|
| (F6) $O(x)Hx$ | (S6) It is required that all people carry handguns. |
| (F7) $(x)OHx$ | (S7) Each person is required to carry a handgun. |
| (F8) $O(\exists x)Hx$ | (S8) It is required that some people carry handguns. |
| (F9) $(\exists x)OHx$ | (S9) Some person is required to carry a handgun. |
- (Universe of discourse: people)

I believe that the four formulas (and the four statements they symbolize) stand in these logical relations to one another:

F6 entails every other formula. It seems evident that F6 entails F7. How could F6 be true while F7 was false? F6 entails F8 courtesy of principle P1'. Since (as I will argue below) F7 entails F9, by the transitivity of entailment F6 also entails F9.

F7 entails only F9. Why does F7 not entail F6? In Community C there are exactly two rules pertaining to the carrying of handguns by the police:

- (L1) Every police officer who is not on sick leave is required to carry a hand gun.
- (L2) No police officer on sick leave is permitted to carry a handgun.

(For this example we adopt "police officers" as our universe of discourse, and accordingly substitute "police officer(s)" for "person" and "people" in S6 through S8.) As it happens, none of the police officers of C are on sick leave. Therefore, S7 is true and S6 false; hence S7

43 This definition will have to be further complicated if we apply it to sentences that also contain modal terms.

(F7) does not entail S6 (F6). Why does F7 not entail F8? It is compatible with the rules in community C that every police officer be on sick leave; so S8 is false; hence S7 (F7) does not entail S8 (F8). F7's entailment of F9 is a consequence of the principle that what is true of all is true of some. This principle is built into the inference rules governing the quantifiers (in standard predicate logic).⁴⁴

F8 entails no other formula in the group. It is obvious that F8 entails neither F6 nor F7. To show that F8 does not entail F9 we switch examples. Community C has this law:

(L3) There must be a Director of Recycling.

However, no specific individual is required to hold the post. Hence S8A (F8) is true and S9A (F9) is false.

(S8A) It is required that some person be a Director of Recycling.

(S9A) It is required of some person that he or she be a Director of Recycling.

Therefore, F8 does not entail F9.

F9 entails no other formula in the group. It is obvious that F9 entails neither F6 nor F7. We showed above that F7 entails F9 but not F8; that proves that F9 does not entail F8.

We have found four cases of entailment among formulas F6 through F9. One of these (F7's entailment of F9) is covered by the principles of predicate logic, and another (F6's entailment of F8) results from deontic principle P1' plus the principles of predicate logic. We can include the remaining two cases (F6's entailment of both F7 and F9) by adding principle P9 to our system:

(P9) A universal *de dicto* requirement entails the corresponding *de re* requirement.

In symbols:

From " $O(x)Fx$ " derive " $(x)OFx$."

So far the extension of LDL into the realm of *de re* obligations seems rather straightforward. Matters get more complicated when we turn our attention to singular statements⁴⁵ like S10 and S11.

(S10) It is required that A1 carry a handgun.

44 Some, aware that in predicate logic " $(x)(Ax \rightarrow Bx)$ " does not entail " $(\exists x)(Ax \& Bx)$ " because of the notorious problem of existential import, may suppose also that " $(x)Ax$ " does not entail " $(\exists x)Ax$ " (or, more to the point, that " $(x)OHx$ " does not entail " $(\exists x)OHx$ "), but they would be mistaken.

45 A singular statement lacks quantifier terms (like "all" and "some") and contains a name (or an expression that functions like a name) of an individual. In stan-

(S11) Al is required to carry a handgun.

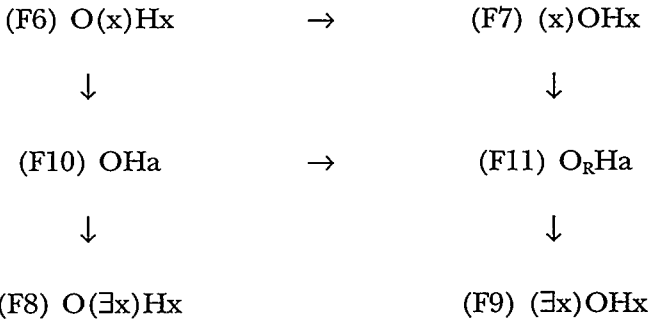
The *de re* - *de dicto* distinction as framed above does not apply to these statements because they lack quantifier terms; nevertheless S10 uses the characteristic *de dicto* expression "It is required that" and S11 the *de re* expression "is required to." Let's extend the distinction to embrace singular statements. We can mark the difference in our symbolizations of singular statements by attaching a subscript *R* (for *de re*) to the O-operator when symbolizing *de re* singular statements.

(F10) OHa

(F11) O_RHa

(When the O-operator in the symbolization of a singular statement lacks a subscript *R* we will read it as *de dicto*. Of course there is no need for this device in formulas that contain both deontic operators and quantifiers.) In a formal proof the universal (existential) instantiation of F7 (F9) will be F11 (rather than F10). F7 (F9) will be derived by universal (existential) generalization from F11 (and not from F10).

I believe that the following chart correctly depicts the entailment relations obtaining among formulas F6 through F11.



Of course the arrows signify entailment. The entailment relations that obtain because of the transitivity of entailment are shown only indirectly. (Thus F6's entailment of F8 is shown by means of two arrows.)

With one exception, the entailments depicted in this chart are already covered by the principles we have adopted so far (P1', P9, and the principles of predicate logic). The exception is F10's entailment of F11. To encompass that we adopt this principle:

(P10) A singular *de dicto* requirement entails the corresponding *de re* requirement.

dard predicate logic, it is presupposed that the individual named belongs to the universe of discourse.

In symbols:

From "OFa" derive "O_RFa."

Note that F11 does not entail F10. F10 means approximately "It is a consequence of the laws that a is H." F11 may be true as a consequence of laws taken together with certain facts. For instance, F11 may be true because it follows from F12 (a law) when it is coupled with F13 (a factual claim).

(F12) (x)(Gx → OHx)

(F13) Ga

If one held that F11 entails F10 one would be committed (by the transitivity of entailment) to the false view that F7 entails F8.

Up to this point principle P1' has been applied only to *de dicto* obligations. Should we permit it to apply also to *de re* singular statements? Apparently not; consider this argument:

It is required that all police officers carry handguns.

Al is required to be a police officer.

So, Al is required to carry a handgun.

$O(x)(Px \rightarrow Hx), O_RPa \vdash O_RHa$

This argument seems to be invalid. Suppose that Al is not a police officer (in spite of the fact that he is required to be one). It may well be that in his current civilian status he is not required to carry a handgun. So, the conclusion does not follow from the premises. But if we allowed P1' to apply to *de re* singular statements we could construct a "proof" for the argument.

(1) $O(x)(Px \rightarrow Hx)$ Premise

(2) O_RPa Premise

(3) $O(Pa \rightarrow Ha)$ From line 1 by P1'

(4) O_RHa From lines 2 and 3 by P1'

With the assistance of LDL fortified with *de re* obligation we are now in a position to make an interesting discovery about "Yugo." We saw in Part I that if the first two premises are interpreted *de dicto*, the argument is valid. That result holds whether the conclusion is understood *de dicto* or *de re*. What if one or both of those premises (and the conclusion) are interpreted *de re*? Let's consider first the case where both of these premises are interpreted *de re*:

Each automobile is required to be licensed.

$(x)(Ax \rightarrow OLx)$

Each licensed vehicle is required to be insured.

$(x)[(Lx \& Vx) \rightarrow OIx]$

All automobiles are vehicles.

$$\Box(x)(Ax \rightarrow Vx)$$

Your Yugo is an automobile.

$$\Box Ac$$

So, your Yugo is required to be insured.

$$\vdash O_R I c$$

Under this interpretation "Yugo" is invalid; the conclusion can be false while all the premises are true. Consider this scenario: The law covering insurance for vehicles includes just one exception and that is for vehicles operated only on a farm. Your Yugo falls under this exception, hence the conclusion is false. How, in that case, can premise two be true? Your Yugo (although required to be licensed) is, in fact, not licensed. As it happens, no *licensed* vehicle falls under the farm exemption; so, premise two is true. The scenario (which has the conclusion false) is compatible with the truth of all four premises.

Note that the scenario applies even if the first premise is interpreted *de dicto*. If the second premise is interpreted *de re* it makes a claim about things that *are* (rather than *are required to be*) licensed. The first premise (whether interpreted *de re* or *de dicto*) makes no claim about things that *are* licensed; hence there is no link between the first two premises (when the second premise is interpreted *de re*).

Finally, how shall we evaluate the argument when the first premise is understood *de re* and the second *de dicto*?

Each automobile is required to be licensed.

$$(x)(Ax \rightarrow OLx)$$

It is required that all licensed vehicles be insured.

$$O(x)[(Lx \ \& \ Vx) \rightarrow Ix]$$

Under this interpretation the argument is also invalid, as the following scenario shows. Your Yugo is unlicensed and driven only on a farm. There is a law that says that unlicensed farm vehicles are not required to be insured; hence, the conclusion of the argument is false. Because it is unlicensed, your Yugo does not come within the scope of the law expressed in premise two. The scenario is consistent with the truth of all four premises; hence, the argument is invalid. Does the scenario involve postulating inconsistent laws? No. The first premise might be true because of a law that every automobile for which a "license waiver" has not been obtained is required to be licensed, coupled with the fact that no such waivers have been obtained. Note that you can satisfy the law in the matter of the licensing of your Yugo by obtaining the waiver. You are not required to obtain insurance.

So we have reached this interesting and possibly surprising conclusion about "Yugo": The argument is valid if and only if its first two premises are interpreted *de dicto*. One benefit of LDL is that it can aid

our understanding of the structure of arguments about legal requirements expressed in English.

IV. METAPHYSICAL NECESSITY

Let's return briefly to the problem of deontically mixed arguments. We have learned how to accommodate arguments whose non-deontic premises are conceptually necessary. There are other deontically mixed arguments whose conclusions exhibit a weaker kind of necessity. Consider this argument (imagine it to be advanced in November of 1997) and its symbolization in LDL:

"De Dicto I.R.S."⁴⁶

It is required that everyone who earned more than \$13,400 in 1997 file an income tax return by April 15, 1998.

Sarah has already earned more than \$13,400 in 1997.

So, Sarah is required to file an income tax return by April 15, 1998.

$O(x)(Ex \rightarrow Fx), Es \vdash O_RFs$

(Universe of discourse: people; $Ex = x$ earned more than \$13,400 in 1997, $Fx = x$ files an income tax return by April 15, 1998, $s = Sarah$)

This argument seems to be valid; there appears to be no way for the premises to be true and the conclusion false. But LDL as so far developed will not judge it valid because the second premise is not conceptually necessary. That premise does exhibit an element of necessity, namely the impossibility of altering the past.⁴⁷ If Sarah has already earned more than \$13,400 in 1997 nothing that she or anyone else can do will change that fact. For want of a better name, let's say that the second premise of "De Dicto I.R.S." possesses *metaphysical* necessity.

Can we modify LDL so that it judges "De Dicto I.R.S." valid? One approach would be to further expand the meaning of the box symbol and strengthen principle P3' to include metaphysical necessity:

(P11) Whatever is conceptually or metaphysically necessary is required.

But this would have a consequence much too paradoxical to swallow: Every past event (because metaphysically necessary) becomes legally obligatory. Certainly Sarah was not required by law to earn more than

⁴⁶ I label the argument "*de dicto*" because of its first premise (in spite of the fact that the conclusion is interpreted *de re*).

⁴⁷ I take no position here on the question of the determination of future events; the point is the noncontroversial one that past events are fixed.

\$13,400 in 1997. We could note (as we did in connection with conceptual necessities) that these obligations concerning the past are empty in the sense that they cannot be violated,⁴⁸ but even so the consequence is too counterintuitive to be accepted. We have to find a different way to accommodate "*De Dicto* I.R.S."

Another possibility is to employ an operator (M) with the meaning "it is metaphysically necessary that" and then adopt this principle (reminiscent of P4):

(P12) From " $O(A \rightarrow B)$ " derive " $MA \rightarrow OB$."

This would enable us to symbolize and construct a proof of validity for "*De Dicto* I.R.S."

$O(x)(Ex \rightarrow Fx)$, $MEs \vdash O_RFs$

- | | | |
|-----|---------------------------|---|
| (1) | $O(x)(Ex \rightarrow Fx)$ | Premise |
| (2) | MEs | Premise |
| (3) | $O(Es \rightarrow Fs)$ | From line 1 by P1' |
| (4) | $MEs \rightarrow OFs$ | From line 3 by P12 |
| (5) | OFs | From lines 4 and 2 by <i>modus ponens</i> |
| (6) | O_RFs | From line 5 by P10 |

Unfortunately for this proposed solution, P12 succumbs to an analogue of the paradoxical result that plagued P4. With P12 on board, if any requirement is violated by a metaphysical necessity, anything whatsoever becomes obligatory; that is, " $OA \ \& \ M-A$ " entails " OB " in such a system. Obviously we must reject principle P12.

Perhaps we should weaken P12 like this:

(P12') From " $O(A \rightarrow B)$ " derive " $MA \rightarrow O_RB$."

P12' would be applicable only where the premise is *de dicto* and the consequent of the conclusion represents a *de re* singular statement. This principle would suffice for demonstrating the validity of "*De Dicto* I.R.S.," and it escapes the lethal problem noted above that affects P12. Whether P12' is a sound principle remains to be determined.

In the meantime, we must consider the possibility that "*De Dicto* I.R.S.," while seemingly valid, is actually invalid, even though the following similar argument is demonstrably valid (even in non-deontic logic).

"De Re I.R.S."

Everyone who earned more than \$13,400 in 1997 is required to file an income tax return by April 15, 1998.

48 That is, whenever " OA " is true because A is metaphysically necessary, A will be true. (But note that " $O-A$ " may also be true.)

Sarah has already earned more than \$13,400 in 1997.
So, Sarah is required to file an income tax return by April 15,
1998.

$(x)(Ex \rightarrow OFx), Es \vdash O_RFs$

V. CONCLUSION

To summarize, in this paper I have set out a system of deontic logic (LDL) tailored to legal reasoning, built upon standard predicate logic augmented by modal system T, and embracing these five deontic principles:⁴⁹

- (D1) Whatever is entailed by what is consistently required is also required.
- (D2) Whatever is conceptually necessary is required.
- (D3) Whatever is required is conceptually possible.
- (D4) A universal *de dicto* requirement entails the corresponding *de re* requirement.
- (D5) A singular *de dicto* requirement entails the corresponding *de re* requirement.

Aside from resolving the problem of metaphysical necessities in deontically mixed arguments (discussed briefly in Part IV), what tasks will be involved in the further development of LDL? At least three kinds of work may be attempted.

(1) *Formalization*. The approach taken to system development in this paper has been entirely informal. We need to lay down formation and inference rules for LDL and adopt a proof format. And a semantics must be provided so that systematic demonstrations of argument invalidity can be provided. A metatheoretical investigation of the system should be undertaken.

(2) *Revisions*. We should address the paradoxes of material implication, as they are particularly fierce in deontic logic.⁵⁰ In order to solve this problem it may be necessary to reject standard propositional

49 I employ a new system for numbering deontic principles. The new numbers correlate with the old as follows: D1=P1', D2=P3', D3=P8, D4=P9, and D5=P10.

50 These paradoxes involve the following patterns (in propositional logic):

$$P \vdash Q \rightarrow P$$

$$\neg P \vdash P \rightarrow Q$$

Note that this argument is valid in LDL (and even in SDL+):

It is forbidden that rapes occur.

So, it is required that whoever rapes kill his victims.

$$F(\exists x)(\exists y)Rxy \vdash O(x)(y)(Rxy \rightarrow Kxy)$$

logic in favor of some version of conditional logic.⁵¹ (A difficulty inherent in that approach is the rejection of certain plausible inference patterns such as chain argument.) We should consider also the impact of other well-known deontic paradoxes.⁵²

(3) *Extensions.* We should also consider the desirability of extending the system in certain directions, for instance, to accommodate *de re* modalities, conditional obligation, or temporal concepts. We have not considered the issue of iterated or nested operators, for example, whether either “ $OOA \rightarrow OA$,” “ $OA \rightarrow OOA$,” or “ $O(OA \rightarrow A)$ ” should be counted as a theorem in the system.

As its title indicates, the present paper represents just a beginning.

51 For the two best known systems of conditional logic, see David Lewis, *Counterfactuals and Comparative Possibility*, in *IFS* 57 (William L. Harper et al. eds., 1981); and Robert C. Stalnaker, *A Theory of Conditionals*, in *IFS*, *supra*, at 41.

52 Both kinds of deontic paradoxes are canvassed in Donald Nute & Xiaochang Yu, *Introduction*, in *DEFEASIBLE DEONTIC LOGIC* 4, 4–13 (Donald Nute ed., 1997).